Advanced Computational Mechanics I

Computer Homework 3

Due: Thursday, Nov 8, 2018

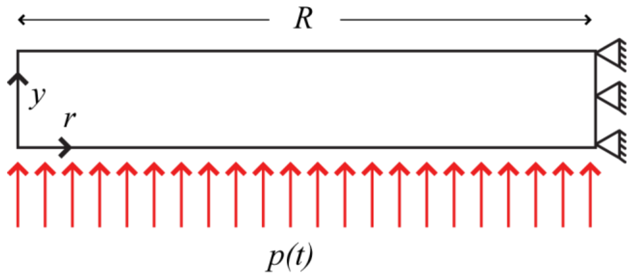

Figure 1 Problem schematic.

Problem statement: A circular rubber plate, shown in Fig. 1, is clamped at the edges and subjected to a blast load described by Friedlander's equation, given as

$$p(t) = Ae^{-t/\tau}(1 - t/\tau),$$

where A = 2 MPa is the initial pressure amplitude and τ = 0.2 ms is the duration of the load. The radius of the plate is R = 50 mm, and the thickness t = 8 mm. The rubber is described as a Neohookean material, with initial shear modulus μ_0 = 5 MPa and initial bulk modulus K_0 = 500 MPa. The reference density of the rubber is 0.001 g/mm^3.

For this assignment, it is recommended to use the following set of consistent units {g, mm, ms, N}, see http://www.dynasupport.com/howtos/general/consistent-units for more information.

Implementation details:

- The external pressure load is a follower force, i.e., you must use the current element coordinates to determine the outward normal direction of the surface to compute the prescribed traction.
- You may use either updated or total Lagrangian formulation for computing the internal forces.
- You code must incorporate the hoop strain, e.g.

$$F_{33} = \frac{r}{R} = \frac{x_I^e N_I^e}{X_J^e N_J^e} \qquad \text{and} \qquad \delta L_{33} = \frac{\delta v_r}{r}$$

- You may want to incorporate bulk viscous damping to improve stability. For large deformations, use the trace of the velocity gradient (**L**) as the volumetric strain rate.

Your report should contain the following:

Make sure that all plots are clearly labeled (axes & legends) with the appropriate units indicated. Plots should be clear (use the export setup feature in MATLAB to size plots and fonts consistently). **Append your code as an appendix to your report.**

1) Brief explanation of the numerical methods that are used to solve the problem

Describe how each of the finite element matrices are calculated and what formulation you are using for each (mass matrix, internal force, and external force).

2) Energy conservation

Determine how accurately the conservation of energy is satisfied during the computation, consider (kinetic energy, internal strain energy, external work, and viscous damping energy). Plot the evolution of from t = 0 to t = 5 ms.

Pick at least one of these

3) Mesh convergence study (easy difficulty)

Show how the maximum deflection of the top of the plate (over all time) varies as a function of element size.

4) One-point integration w/ hour glass control (moderate difficulty)

Implement hour glass control so that a one-point integration method can be used (this should speed the simulation up by 4x). Calculate and plot the hourglass energy as a function of time in conjunction with Part 2.

5) Implement a damage law (high difficulty)

Each integration point will now have a state variable D, such that the strain energy function is modified to be $\hat{\Psi}(\mathbf{C}) = (1-D)\Psi(\mathbf{C})$. The rate of damage accumulation is then given by $\dot{D} = e^{K(\lambda_1 - \lambda_c)}$, where λ_1 is the maximum eigenvalue of **C,** λ_c = 1.25 is the critical stretch, and K = 10 is a scaling factor. Vary the maximum pressure to determine blast magnitude that causes the plate to fail. Check whether the results are mesh dependent, i.e., what happens to the solution as the element size is decreased? For this part to be computationally feasible, you may need to implement one-point integration w/ hour glass control.