Assignment -7

CH6

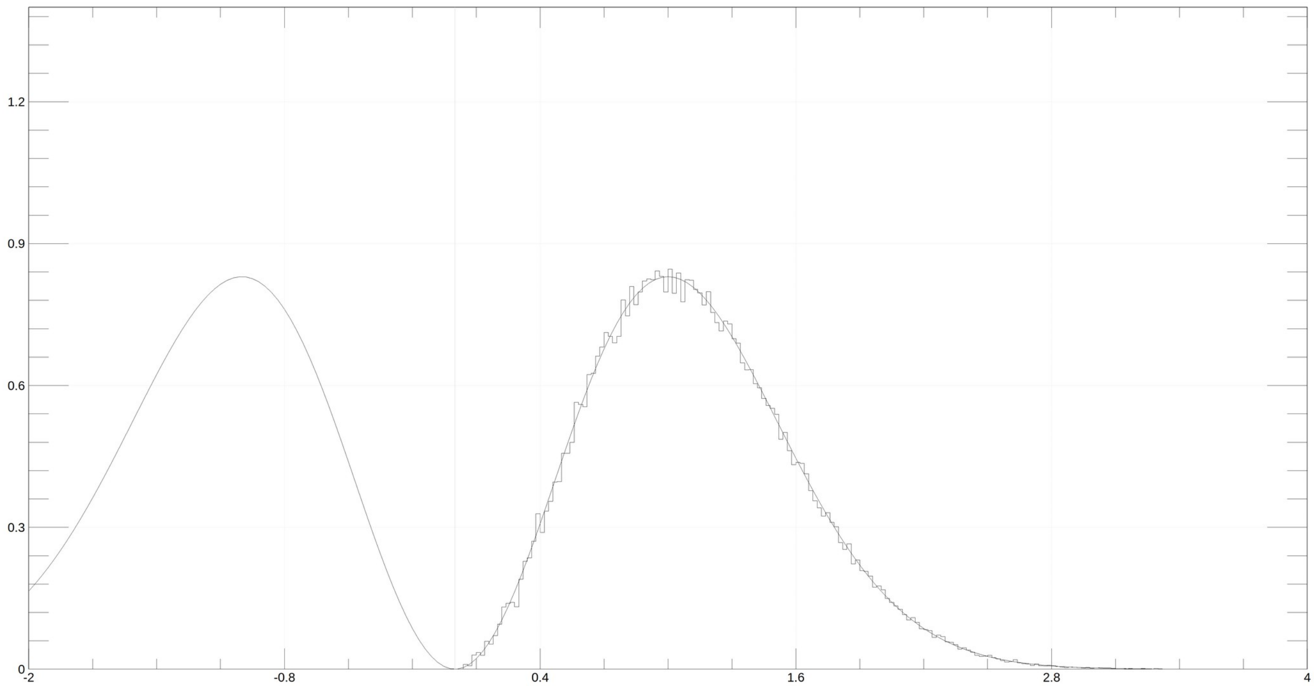
Codes have been submitted for problem 1-3.

CH7

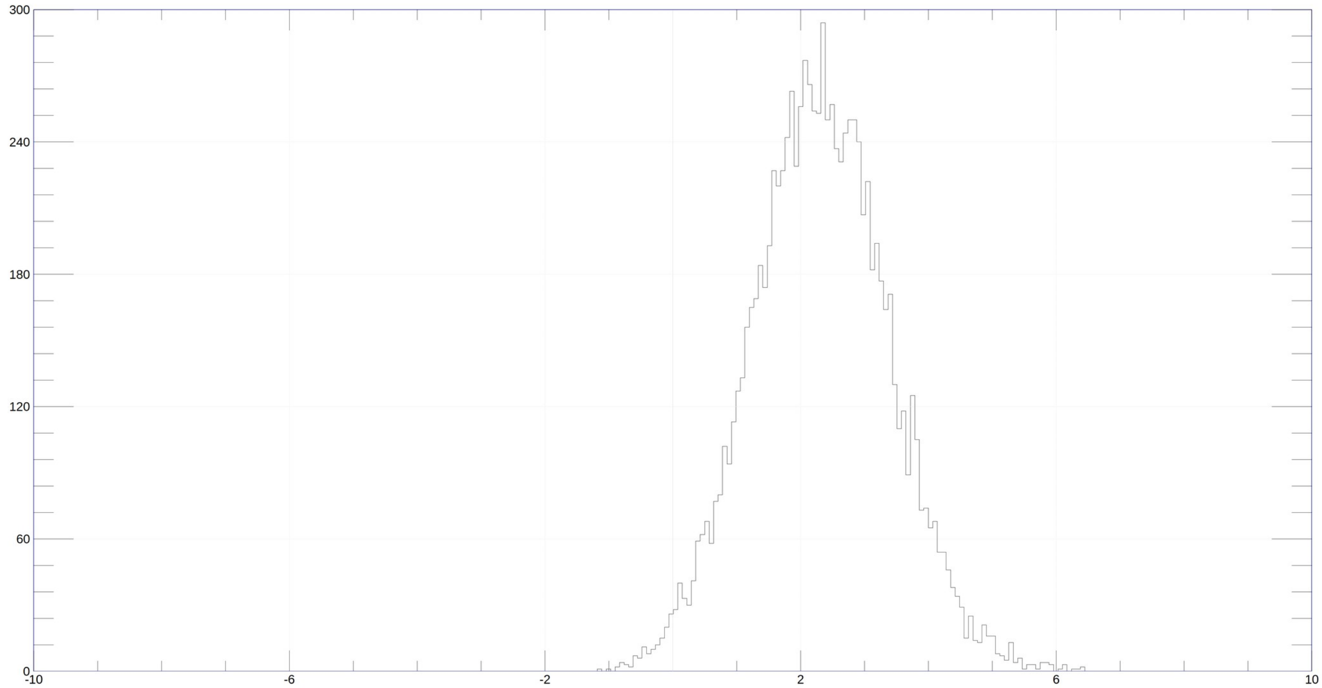
2. The Maxwell distribution is converted to a gamma distribution term with:

a) alpha=2 and beta=1.
b) Non-dimensionalizing the Maxwell PDF by putting (2*tau/m)=1
c) Putting V^2=X where V is the velocity and X is from the gamma distribution

The results obtained on comparison can be seen below:

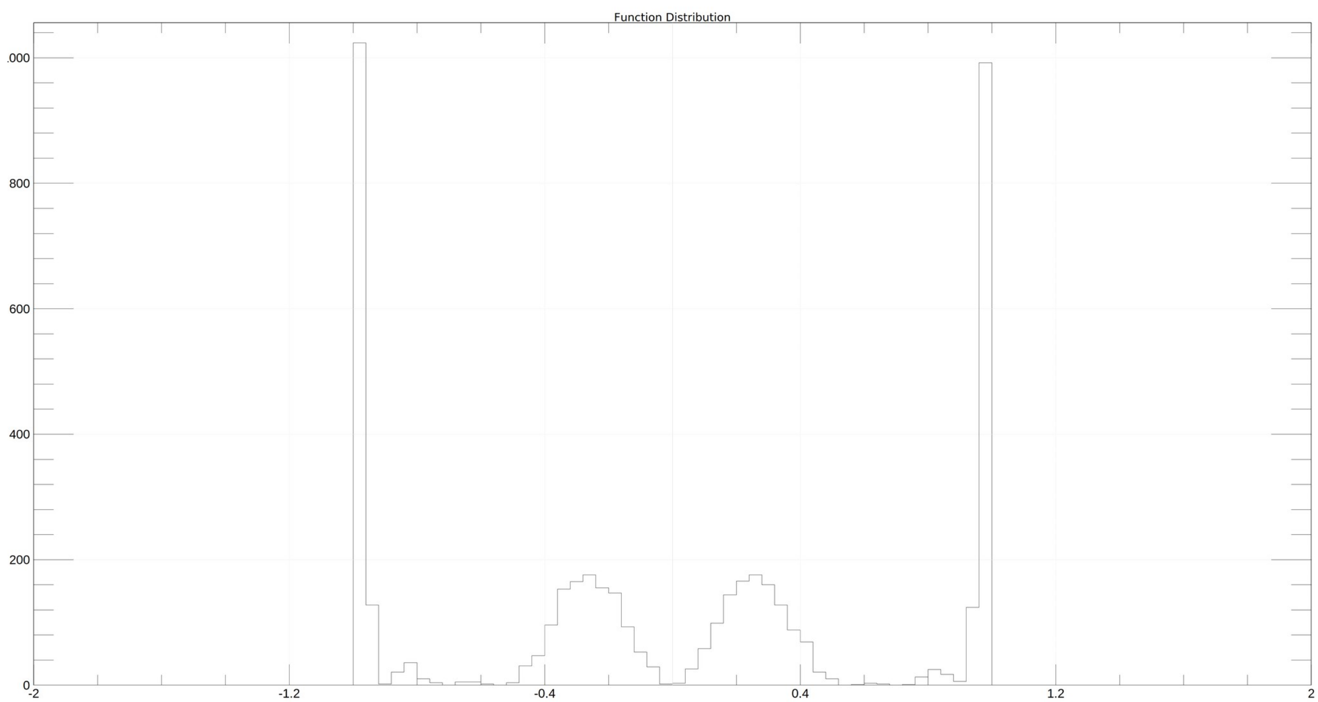


5. (a)Analytical Solution is given at the end.
   (b) The histogram (sum of two variates) approach gives the plot as:



(c) Code was written for plotting the rejection method as well as the analytical solution, but the plotter could not plot them for some reason. The code exists in the code file.

6. The distribution histogram using rejection method is obtained as:



Could not plot the actual function for comparison because of Legendre Polynomials (as per compiler)

Convolution of Gaussian with an exponential

Gaussian function $G(x)$ is given by:-

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}}$$

Exponential function $F(x)$ is given by:-

$$F(x) = e^{-\lambda x}$$

Convolution $I(y) = F(x) * G(x)$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty e^{-\lambda x} e^{\frac{-(y-x)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty e^{\frac{-(x^2+2\sigma^2\lambda x-2xy+y^2)}{2\sigma^2}} dx$$

Let $z = \frac{1}{\sqrt{2}\sigma}(x-(y-\sigma^2\lambda))$

$$\therefore dx = \sqrt{2}\sigma dz$$

At $x=\infty$, $z=\infty$

At $x=0$, $z = -\frac{(y-\sigma^2\lambda)}{\sqrt{2}\sigma}$

$$z^2 = \frac{(x^2+y^2+\sigma^4\lambda^2 - 2xy + 2\sigma^2\lambda x - 2y\sigma^2\lambda)}{2\sigma^2}$$

$$\Rightarrow \frac{(x^2+2\sigma^2\lambda x - 2xy + y^2)}{2\sigma^2}$$

$$= z^2 + \lambda\left(y - \frac{\sigma^2\lambda}{2}\right)$$

$$\therefore I(y) = \frac{1}{\sqrt{\pi}} e^{-\lambda\left(y-\frac{\sigma^2\lambda}{2}\right)} \int_{-\frac{(y-\sigma^2\lambda)}{\sqrt{2}\sigma}}^{\infty} e^{-z^2}\,dz$$

we know, $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-z^2}\,dz$

$$\therefore I(y) = e^{-\lambda\left(y-\frac{\sigma^2\lambda}{2}\right)} \left[1 + \operatorname{erf}\left(\frac{(y-\sigma^2\lambda)}{\sqrt{2}\sigma}\right)\right]$$

Putting $\sigma = 2$, & $\lambda = 3$,

we get $I(y) = \frac{3}{\sqrt{8\pi}} e^{\left(\frac{y^2}{8} + \frac{\cancel{2}y}{2} - 6\right)}$

$$\left[\left(1 + \operatorname{erf}\left(0.3535\,y - 4.2421\right)\right)\right]$$