Secant Method.

→ Secant method is non-bracketing iterative method for solving system of non-linear equation which uses two initial guess values.

→ Here, the guess values doesnot require that they must bracket the root.

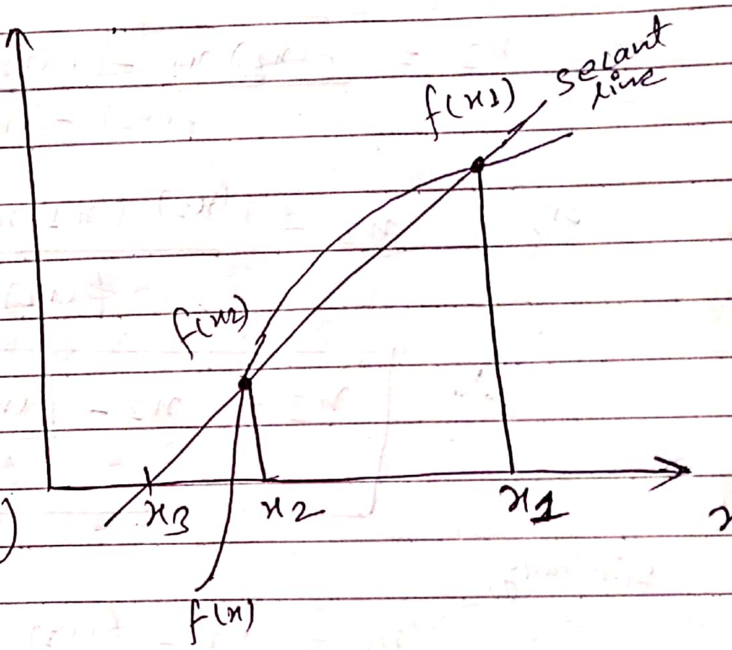

fig: Graphical depiction of secant method

Let $f(x)$ be a real and continious function and x_1 & x_2 are starting or initial guess values.

⇒ The slope of the secant line passing through $(x_1, f(x_1))$, $(x_2, f(x_2))$ and $(x_3, 0)$ is given by,

$$\frac{f(x_1)-0}{x_1-x_3} = \frac{f(x_2)-0}{x_2-x_3}$$

or, $$\frac{f(x_1)}{x_1-x_3} = \frac{f(x_2)}{x_2-x_3}$$

or, $$x_2 f(x_1) - x_3 f(x_1) = x_1 f(x_2) - x_3 f(x_2)$$

or, $$f(x_2) x_3 - f(x_1) x_3 = f(x_2) x_1 - x_2 f(x_1)$$

or, $$x_3 \left(f(x_2) - f(x_1) \right) = f(x_2) x_1 - x_2 f(x_1)$$

or, $$x_3 = \frac{f(x_2)\, x_1 - f(x_1) x_2}{f(x_2) - f(x_1)}$$

→ By adding and subtracting $f(x_2)\, x_2$ to the numerator & rearrange the terms, we get

$$x_3 = \frac{f(x_2)x_1 - f(x_1)x_2 + f(x_2)x_2 - f(x_2)x_2}{f(x_2) - f(x_1)}$$

or, $$x_3 = \frac{f(x_2)x_1 - f(x_2)x_2 - f(x_1)x_2 + f(x_2)x_2}{f(x_2) - f(x_1)}$$

or $$x_3 = \frac{f(x_2)(x_1 - x_2) - x_2(f(x_1) - f(x_2))}{f(x_2) - f(x_1)}$$

$$\therefore \boxed{x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}}$$

This is formula for Secant method.

Similarly,

$$x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)}$$

In general,

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}}$$

This is general formula for secant method.

(*) Given a function $f(x) = x^3 - 2x - 5$. compute the root of the function correct upto 3 decimal places.

Soln

Given, $f(x) = x^3 - 2x - 5$

Let $x_1 = 2$ & $x_2 = 3$ are two initial guess values.

$f(x_1) = f(2) = 2^3 - 2*2 - 5 = 8 - 9 = -1$

$f(x_2) = f(3) = 3^3 - 2*3 - 5 = 27 - 6 - 5 = 16$

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1st iteration,

We know, $x_3 = x_2 - \dfrac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$

or, $x_3 = 3 - \dfrac{16(3-2)}{16+1} = 2.0588$

$\therefore f(x_3) = f(2.0588) = -0.3911$

2nd iteration

$x_1 = x_2 = 3$

$x_2 = x_3 = 2.0588$

We know, $x_3 = x_2 - \dfrac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = 2.0588 - \dfrac{(-0.3911)(2.0588-3)}{(-0.3911) - 16}$

$= 2.0813$

$\therefore f(x_3) = f(2.0813) = -0.1468$

3rd iteration

$x_1 = x_2 = 2.0588$

$x_2 = x_3 = 2.0813$

We know,

$x_3 = x_2 - \dfrac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = 2.0813 - \dfrac{(-0.1468)(2.0813 - 2.0588)}{(-0.1468) - (-0.3911)}$

$= 2.0948$

$\therefore f(x_3) = f(2.0948) = 0.0030$

4th iteration

$x_1 = x_2 = 2.0813$

$x_2 = x_3 = 2.0948$

We know,

$x_3 = x_2 - \dfrac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = 2.0948 - \dfrac{0.0030(2.0948 - 2.0813)}{(0.0030 + 0.1468)}$

$= 2.0945$

Here, the value of x_3 in this iteration & previous iteration is same.

So, root $= 2.0945$ Ans