Algorithm for fixed point iteration method.

1. Input function $f(x)$ and re-arrange $f(x)=0$ as $x=g(x)$.
2. provide an initial guess value, x_0 and error tolerance, E.
3. Compute

$$x_1 = g(x_0)$$

4. Test for accuracy of x_1

if $\left|\frac{x_1 - x_0}{x_1}\right| > E$ then

set $x_0 = x_1$

and goto step 3.

otherwise

Set root $= x_1$ and print result.

Convergence of fixed point iteration method.

→ The convergence of fixed point iteration method depends upon the arrangement of $g(x)$.

→ The various patterns of behaviour of the iteration process of the fixed point are shown in below figure.

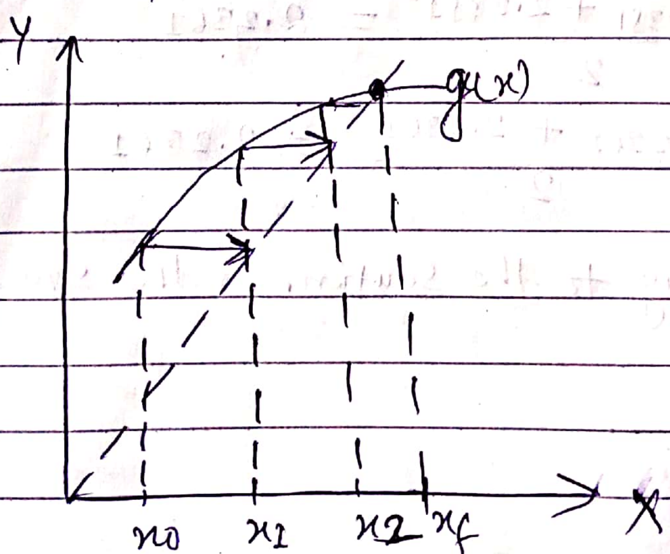

fig: Monotone convergence

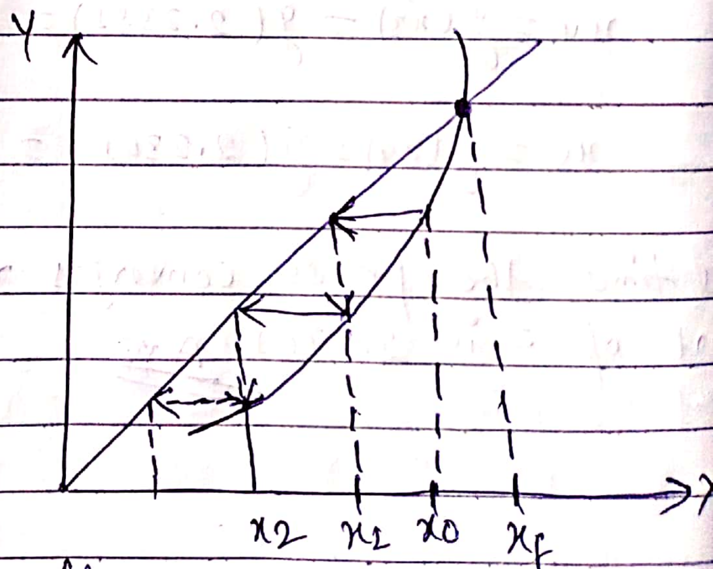

fig: Monotone divergence

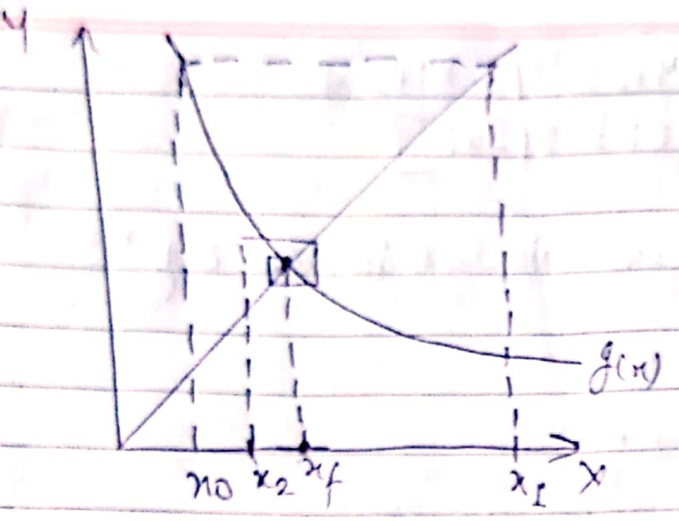

fig: Spiral Convergence

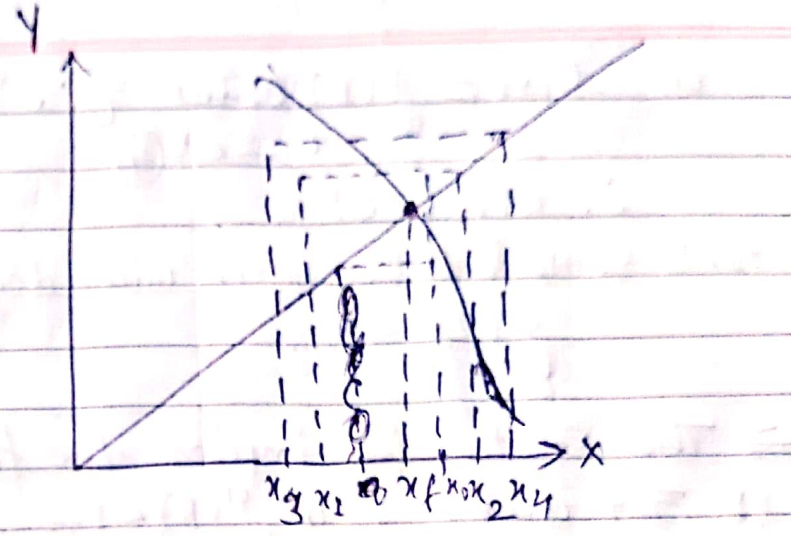

fig: Spiral divergence

→ From the figure, we can noticed that, if the slope of $g(x)$ is positive, then the convergence is monotonic with "staircase" behaviour, and if the slope of $g(x)$ is negative, then the convergence is oscillatory in behaviour.

→ The iterative formula is,

$$x_{i+1} = g(x_i) \quad \text{——— eq}^n \ (1)$$

Let x_f be a root of function $f(x)$ then

$$x_f = g(x_f) \quad \text{——— eq}^n \ (2)$$

Subtracting eq^n (1) from eq^n (2), we get,

$$x_f - x_{i+1} = g(x_f) - g(x_i) \quad \text{——— eq}^n \ (3)$$

According to mean value theorem, there is atleast one point $x = R$, in the interval $[x_f, x_i]$ such that

$$g'(R) = \frac{g(x_f) - g(x_i)}{x_f - x_i}$$

$$\text{or, } g(x_f) - g(x_i) = (x_f - x_i)\, g'(R)$$

Now, equation (3) becomes,

$$x_f - x_{i+1} = (x_f - x_i)\, g'(R) \quad [\because g(x_f) - g(x_i) = (x_f - x_i)\, g'(R)]$$

or, $e_{i+1} = g'(R)\, e_i$ $\leftarrow$ eqn (4) $\left[\because e_{i+1} = x_f - x_{i+1} \text{ \& } e_i = x_f - x_i\right]$

$\therefore e_{i+1} \propto e_i$

This shows that the error will decrease with each iteration only if

$$g'(R) < 1$$

$\rightarrow$ The equation (4) implies the following

(1) Error decreases if $g'(R) < 1$.

(2) Error grows if $g'(R) > 1$.

(3) If $g'(R)$ is positive then the convergence is monotonic.

(4) If $g'(R)$ is negative then the convergence is oscillatory.

(5) The error at this iteration is roughly proportional to (or less than) the error in the previous steps. So, fixed point iteration method is linearly convergent.