Newton Raphson Method (NRM)

Consider a graph $y = f(x)$ as shown in figure. Let us assume that x_1 is an approximate root of $f(x) = 0$. Draw a tangent at the curve $f(x)$ at $x = x_1$. The point of intersection of this tangent with x-axis gives the second approximation of the root.

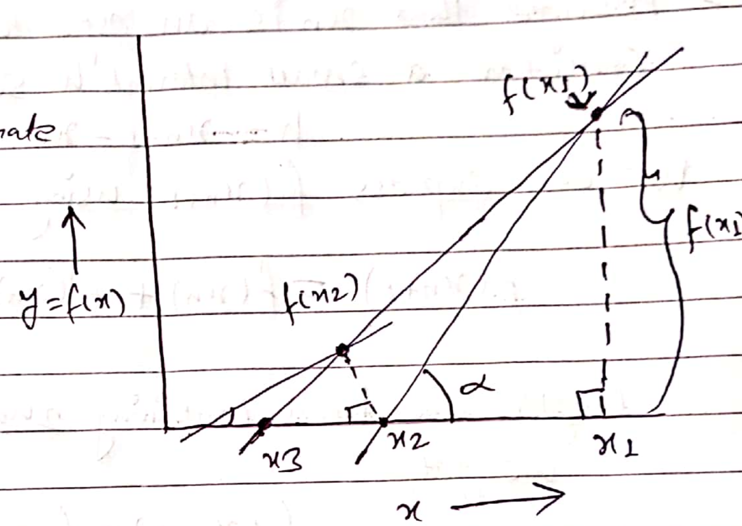

fig: Newton Raphson method

The slope of the tangent is given by,

$$\tan\alpha = P/b = \frac{f(x_1)}{x_1 - x_2} = f'(x_1)$$

or, $(x_1 - x_2) . f'(x_1) = f(x_1)$

or, $$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

The next approximation would be

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

So, In general,

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

This is called Newton Raphson formula for computing root.

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Deriving Newton-Raphson formula using Taylor Series Expansion

→ Assume that x_n is an estimate of a root of the function $f(x)$. Consider a small interval 'h' such that

$$h = x_{n+1} - x_n$$

We can express $f(x_{n+1})$ using taylor series expansion as follows:

$$f(x_{n+1}) = f(x_n) + f'(x_n)\,h + f''(x_n)\,\frac{h^2}{2!} + \cdots\cdots$$

Neglect the terms containing 2nd and higher order derivatives, we get,

$$f(x_{n+1}) = f(x_n) + f'(x_n)\,h.$$

If x_{n+1} is a root of $f(x)$ then

$$f(x_{n+1}) = 0 = f(x_n) + f'(x_n)\,h$$

or, $$f(x_n) + f'(x_n)\,h = 0$$

or, $$f'(x_n)\cdot h = -f(x_n)$$

or, $$h = -\frac{f(x_n)}{f'(x_n)}$$

or, $$x_{n+1} - x_n = -\frac{f(x_n)}{f'(x_n)}$$

or, $$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

This is Newton Raphson formula.

Example of NRM

(*) Find the root of the equation $f(x) = x^2 - 3x + 2$ in the vicinity of $x = 0$ using Newton Raphson method.

Soln

Given, $f(x) = x^2 - 3x + 2$

$f'(x) = 2x - 3$

Iteration 1

Let, Initial guess value $(x_1) = 0$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0 - \frac{2}{-3} = 2/3 = 0.6667$$

Iteration 2

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.6667 - \frac{0.4444}{-1.6667} = 0.9333$$

Iteration 3

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.9333 - \frac{0.071}{-1.334} = 0.9959$$

Iteration 4

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.9959 - \frac{0.0041}{-1.0082} = 0.9999$$

Iteration 5

$$x_6 = x_5 - \frac{f(x_5)}{f'(x_5)} = 0.9999 - \frac{0.0001}{-1.0002} = 1.000$$

So, the root of the equation is 1. Answer

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