UNIT 5

Conic Section

Learning Outcomes:

At the end of this chapter, students will be enabled to

- define conic section, circle, parabola, ellipse, hyperbola and related terms and give example,
- obtain the equations of above conic sections and sketch the graphs of the conic sections using MATLAB /Mathematica,
- classify the conic sections by eccentricity,
- solve the related problems,
- find the polar equations of lines, circles, ellipse, parabola and hyperbola.

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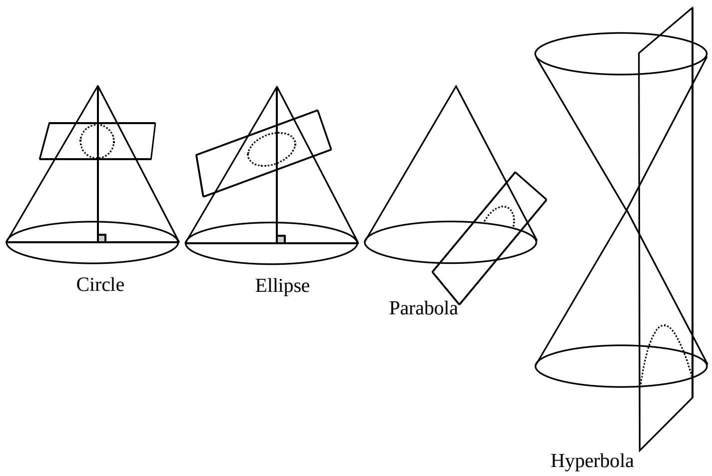
Conic Section

3.1.1 A conic section (or simply conic) is a curved obtained by the intersection of the surface of the cone with a plane. The three types of conic sections are the parabola, hyperbola and the ellipse. The circle is type of ellipse, and is sometimes considered to be a fourth type of conic section.

Conic sections can be generated by the intersection with a plane with a cone. A cone has two identically shaped parts called nappes. One nappe is what most people mean by "con". Conic sections are generated by the intersection of a plane with a cone. If the plane is parallel to the axis of revolution, then the conic section is a hyperbola. If the plane is parallel to the generating line, the conic section is a parabola. If the plane is per perpendicular to the axis of revolution, the conic section is a circle. If the plane intersects one nappe at an angle to the axis (other than 90º), then the conic section is an ellipse. The following are the curves (conic section) obtained when a cone is intersection by a plane in different positions.



i) A circle is formed when the plane is parallel to the base of the cone, its intersection with the cone is therefore a set of points equidistant from a common point.

ii) If a plane intersects a cone at a given angle with the axis greater than the semi-variable angle then the section is the ellipse.

iii) A parabola is formed when the plane is parallel to the surface of the cone, resulting in a U-shaped curve that lies on the plane.

iv) If a plane intersects the double right cone such that the angle between the axis and the plane be less than the semi-vertical angle, then the section is a hyperbola.

Usually a conic section is defined in the following ways:

"A conic section is the locus of a point, which moves in a plane such data its distance from a fixed point is in a constant ratio to its distance from a fixed straight line.

The fixed point is called the focus, the fixed line is called the direction and the constant ratio is called the eccentricity of the conic section. The eccentricity is denoted by e. Or,

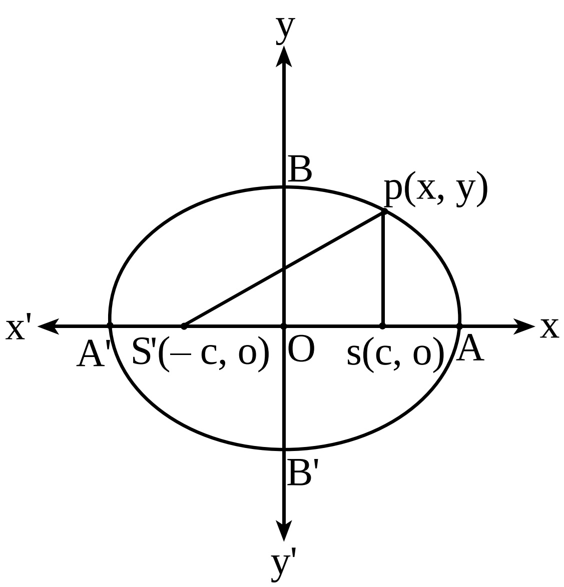
The conic section (or a conic) is called an ellipse, a parabola or a hyperbola according as the eccentricity, $e < 1$, $e = 1$ or $e > 1$.

According to the syllabus prescribed by CDC, only ellipse and hyperbola are entertained in grade XII.

<u>3.1.2 Ellipse:</u>

An ellipse is the locus of a point which moves in a plane such that its distance from a fixed point (focus) of the plane bears a constant ratio, which is less than 1 to its distance from a fixed line called direction lying in the plane.

3.1.3 Equation of ellipse in standard form:



Solution: Let 0(0, 0) be the centre of an ellipse where s(c, 0) and s' (– c, 0) be the foci of an ellipse. The major axis of an ellipse is taken along x-axis and minor axis is taken along y-axis such that AA' = 2a (say) and BB' = 2 b (say), then SS' = 2C. If P(n, y0 be any point on the ellipse, then

PS + PS' = 2a(say)

Or, $\sqrt{(x-c)^2 + (y-0)^2} + \sqrt{(x+c)^2 + (y-0)^2} = 2a$

Or, $\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$

Or, $\sqrt{(x-c)^2 + y^2} = 20 - \sqrt{(x+c)^2 + y^2}$

Squaring both sides; we get

$$(x-c)^2 + y^2 = 4a^2 - 2.2a\sqrt{(x+c)^2 + y^2} + (x+c)^2 + y^2$$

Or, $x^2 - 2x + c^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + x^2 + 2cx + c^2 + y^2$

Or, $-4cx = 4a^2 - 4a\sqrt{(x+c)^2 + y^2}$

Or, $4a\sqrt{(x+c)^2 + y^2} = 4a^2 + 4cx$

Or, $a\sqrt{(x+c)^2 + y^2} = a^2 + cx$

Squaring both sides; we get

$$a^2 \{(x+c)^2 + y^2\} = (a^2 + (x)^2$$

or, $a^2 (x^2 + 2 (x + c^2 + y^2) = a^4 + 2a^2 cx + c^2 x^2$

or, $a^2x^2 + 2a^2cx + a^2c^2 + a^2y^2 = a^4 + 2a^2 cx + c^2x^2$

or, $a^2x^2 + a^2c^2 + a^2y^2 = a^4 + c^2x^2$

or, $a^2x^2 - c^2 x^2 + a^2 y^2 = a^4 - a^2c^2$

or, $x^2 (a^2 - c^2) + a^2y^2 = a^2 (a^2 - c^2)$

or, $\dfrac{x^2 (a^2 - c^2)}{a^2 (a^2 c^2)} + \dfrac{a^2 + y^2}{a^2 (a^2 - c^2)} = 1$

or, $\dfrac{x^2}{a^2} + \dfrac{y^2}{(a^2 - c^2)} = 1$

In a Δ PSP' ; $2a > 2c \Rightarrow a > c \Rightarrow (a^2 - c^2) > 0$

Therefore,

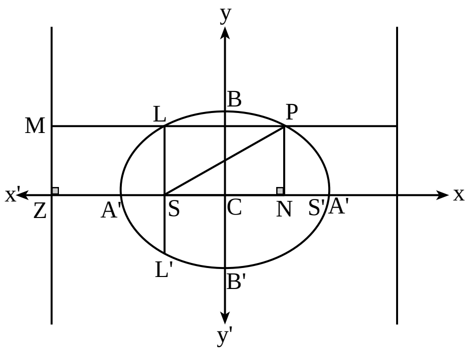
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \text{ where } b^2 = a^2 - c^2$$

Which is the equation of an ellipse centered at the origin with its major axis on the x-axis.

Similarly, the equation of an ellipse centered at the origin with its major axis on the y-axis is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ Or, simply we can take $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ but here, b > a.

Alternative methods:

Let s be the focus and zm be the equation to the direction. Draw sz perpendicular to zm. Divide sz internally at A and externally at A' in the ratio e:1 So, that



SA = e.AZ ………… (i)

And SA' = e.A'Z ………… (ii)

Since, A and A' are collinear where c is the mid-point of AA' and let AA' = 2a

Now, From (i) and (ii);

SA + SA' = eAZ + RA'Z

Or, AA' = e (AZ + A'Z)

Or, AA' = e (CZ – CA + CA' + CZ)

Or, AA' = e(2CZ

Or, 2a = 2e. CZ

Or, CZ = $\frac{a}{e}$

Again, subtracting eq^n (i) from eq^n (ii); we get

SA' – SA = eA'Z – eAZ.

Or, (CS + CA') – (CA – CS) = e (A'Z – AZ)

Or, CS + CA' – CA + CS = e(AA')

Or, 2CS = e. 2a

Or, CS = ae

Let us take C as an origin and CA' and CB are taken along x-axis and y-axis respectively. Then S is at (– ae, 0) and the equation to the direction is $x = \frac{a}{e}$.

Let p(x, y)be any point on the ellipse, then SP= e. PM where PM ⊥ ZM.

SP = e PM

Or, SP = e. ZN [∵ PM = ZN]

Or, $\sqrt{(x + ae)^2 + y^2} = e\left(x + \frac{a}{e}\right)$

Squaring both sides; we get

$$(x + ae)^2 + y_2 = e^2\left(x + \frac{a}{e}\right)^2$$

Or, $x^2 + 2xae + a^2 e^2 + y^2 = e^2 e\left(x^2 + 2x.\frac{a}{e} + \frac{a^2}{e^e}\right)$

Or, $x^2 – e^2x^2 + a^2 e^2y^2 – a^2 = 0$

Or, $x^2 (1 – e^2) + a^2 (e^2 – 1) + y_2= 0$

Or, $x^2 (1 – e^2) + y^2 = – a^2(e^2 – 1)$.

Or, $x^2 (1 – e^2) + y^2 = a^2 (1 – e^2)$

Or, $\frac{x^2 (1 – e^2)}{a^2(1 – e^2)} + \frac{y^2}{a^2 (1 – e^2)} = 1$

Or, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ……….. (1)

Where $b^2 = a^2 (1 – e^2)$

Equation (i) is an equation to an ellipse.

Note: In case of $b^2 = a^2 (1 – e^2)$

Or, $\frac{b^2}{a^2} = 1 – e^2$

Or, $e^2 = 1 – \frac{b^2}{a^2}$

Or, $e = \sqrt{1 – \frac{b^2}{a^2}}$

3.1.4 Sketching of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

i) The curve is symmetrical to both axis as it has even power of x and y respectively because an changing (x, y) to (x, – y) the equation of the curve does not change. Again, the curve is symmetrical to y-axis because on changing (x, y) to (–x, y) the equation of the curve does not change.

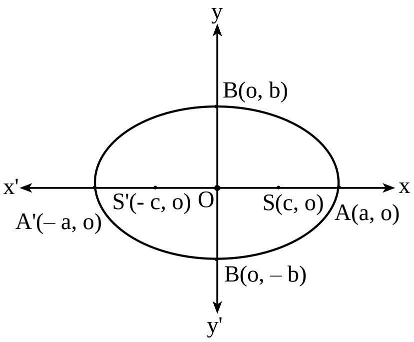
ii) The curve is symmetrical with respect to origin because on changing (x, y) to (–x, –y), the equation does not change.

iii) The curve intersect the y-axis at (0, ± b) and then x-axis at (±a, 0).

Note:

i) The distance between A(a, 0) and A'(– a, 0) is called the major axis of the ellipse which is also called longer axis.

ii) The distance between B(0, b) is called minor axis of the ellipse which is also called shorter axis.



Some Terminologies of ellipse:

i) Major axis: The longest diameter of an ellipse.

ii) Minor axis: The shortest diameter of an ellipse.

iii) Centre: The point of the intersection between major axis and minor axis of an ellipse.

iv) Foci: Foci of an ellipse are two fixed points on its major axis such that sum of the distances of any point on the ellipse from these two points is constant.

i.e. PS + PS' = Constant.

v) Directrices of an ellipse: Two parallel lines on the outside of an ellipse perpendicular to the major axis.

vi) Vertices of an ellipse: The vertex is the point of intersection of the line perpendicular to the direction which passes through the focus cuts the ellipse.

vii) Latus return of an ellipse: The chord of an ellipse through its one focus and perpendicular to the major axis is called the later rectum of the ellipse.

viii) Eccentricity: The distance from the fixed point in a plane bears a constant ratio less than the distance from a fixed line in a plane. If is non-negative real number that uniquely characterizer its shape.

iii) The points of intersection of the ellipse and the axes are the vertices and co-vertices of the ellipse i.e. $(\pm a, 0)$ is vertices and $(0, \pm b)$ are co-vertices.

iv) The point of the intersection between major axes are called a centre of the ellipse.

(A) For an ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $a > b$

i) Length of major axis = 2a

ii) Length of minor axis = 2b

iii) Centre of ellipse: c(0, 0)

iv) Foci of ellipse: S(ae, 0), S'(– ae, 0): $(\pm ae, 0)$ or, $(\pm c, 0)$where $c^2 = a^2 - b^2$

v) Vertices of ellipse: A(a, 0), A' (– a, 0) ": $(\pm a, 0)$

vi) Equation of directrix: $x = \pm\frac{a}{e}$.

vii) Length of latus rectum (L.R) = $\frac{2b^2}{a}$

viii) Equation of latus rectum $x = \pm$ ae.

ix) Eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}}$

x) Distance between foci = 2ae.

xi) Distance between directrix = $\frac{2a}{e}$

B) For an ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $a < b$

i) Length of major axis = 2b

ii) Length of minor axis = 2a

iii) Centre = C(0, 0)

iv) Foci = $(0, \pm be)$

v) Vertices = $(0, \pm b)$

vi) Equation of directrix: $y = \pm\frac{b}{e}$

vii) Length of latus rectum = $\frac{2a^2}{b}$

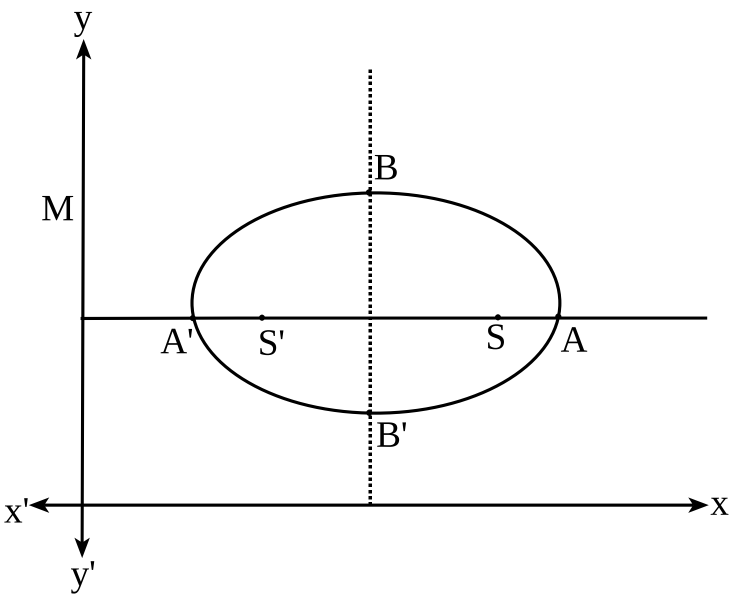
viii) Equation of latus rectum is $y = \pm be$

ix) Eccentricity (e) = $\sqrt{1 - \frac{a^2}{b^2}}$

x) Distance between foci = 2be

xi) Distance between directrices = $\frac{2b}{e}$

3.1.5 Equation of an ellipse if a centre is not at the origin.



The equation of the ellipse with centre (h, k) where the axis are parallel to the axes of co-ordinates is of the form.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \; a > b > 0.$$

Represents an ellipse of the same size and shape as the one represented by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ but is centre is shifted to a point (h, k). Then the major and minor axes are parallel to x-axis and y-axis. As we have proved that $x^2 + y^2 = r^2$ and $(x-b)^2 + (y-k)^2 = r^2$ represent circler of the same size with the centers (0, 0) and (h, k) respectively. Similarly, the equation of parabola $y^2 = 4ax$ and $(y-k)^2 = 4a(x-h)$ represents the vertex (0, 0) in first parabola and vertex (h, k) in second parabola. By the above method. The equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ changer to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

if we let $x - h = x$ and $y - k = y$

When $b > a > 0$ then equation of an ellipse

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ changes to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if we replace $x - h = x$ and $y - k = y$.

The important formulae of an ellipse are listed in following table:

Ellipse	Center	Vertex	Focus	Major axis	Minor axis	Eccentricity	Length of Lalas rectum	Equation of Direction
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $a>b>0$	(0, 0)	(± a, 0)	(± ae, 0)	2a	2b	$e^2 = 1-\frac{b^2}{a^2}$	$\frac{2b^2}{a}$	$x = \pm\frac{a}{e}$
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $a<b<0$	(0, 0)	(0, ± be)	(0, ± be)	2b	2a	$e^2 = 1-\frac{b^2}{a^2}$	$\frac{2a^2}{b}$	$y = \pm\frac{b}{e}$
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $a>b>0$	(h, k)	(h ± a, k)	(h ± ae, k)	2a	2b	$e^2 = 1-\frac{b^2}{a^2}$	$\frac{2b^2}{a}$	$x = h\pm\frac{a}{e}$
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $b>a>0$	(h, k)	(h, k ± b)	(h, k ± be)	2b	2a	$e^2 = 1-\frac{b^2}{a^2}$	$\frac{2a^2}{b}$	$y = k\pm\frac{k}{e}$

Worked out examples

Example-1 Find the eccentricity co-ordinates of the vertices and foci and also the lengths of the major axis and minor axis of the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$

Solution: Here,

The given equation of the ellipse is $\frac{x^2}{9}+\frac{y^2}{16}=1$ ……… (i)

Comparing equation (i) with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

We get

$a^2 = 9 \Rightarrow a = 3$

$b^2 = 16 \Rightarrow b = 4$

Since, b > a. So, major axis is along y-axis

Eccentricity (e) = $\sqrt{1-\frac{a^2}{b^2}}$

$= \sqrt{1-\frac{9}{16}} \qquad = \sqrt{\frac{16-9}{16}} = \sqrt{\frac{7}{4}}$

Co-ordinate of vertices = (0, ± b).

= (0, ± 4)

Foci of the ellipse = (0, ± be) $= \left(0, \pm 4 \times \frac{\sqrt{7}}{4}\right) = 0, \pm\sqrt{7}$

Length of the major axis = 2a = 2 × 3 = 6

Length of the minor axis = 2b = 2 × 4 = 8

Example - 2: Find the eccentricity and the foci of the ellipse $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$

Solution: Here,

The given equation of an ellipse is, $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$

Or, $\frac{\{x-(-2)\}^2}{4^2} + \frac{(y-5)^2}{9} = 1$

Comparing this equation with $\frac{(x-h)^2}{a^2} + 9\,\frac{(y-k)^2}{b^2} = 1$

We get,

h = – 2, k = 5, a = 4 and b = 3

Since a > b the major axis is along x-axis.

We have,

Eccentricity (e) $= \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{(3)^2}{4^2}} = \sqrt{1-\frac{9}{16}} = \sqrt{\frac{7}{16}} = \sqrt{\frac{7}{4}}$

Foci = (h ± ae, k) $= \left(-2 \pm 4.\frac{\sqrt{7}}{4}, 5\right)$

$= \quad (-2 \pm\sqrt{7}, 5)$

Examples - 3: Find the foci of the ellipse; $\frac{(x-1)^2}{25} + \frac{y^2}{16} = 1$

Solution: Here,

Given equation of ellipse is $\frac{(x-1)^2}{25} + \frac{y^2}{16} = 1$

Or, $\frac{(x-1)^2}{25} + \frac{(y-0)^2}{4^2} = 1$

Comparing this equation with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$;

We get, h = 1, k = 0, a = 5 and b = 4

Since, b > a; the major axis is along x -axis.

We have,

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25 - 16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Foci = (h ± ae, k)

$$= \left(1 \pm 5 \times \frac{3}{5}, 0\right) = (1 \pm 3, 0) = (4, 0) \text{ and } (-2, 0)$$

Example-4:

Find the equation of the ellipse in the standard form whose focus is at (– 2, 0) and vertex at (5, 0)

Solution: Given,

Vertex (a, 0) = (5, 0)

∴ $a = 5$

Focus = (–ae, 0) = (– 2, 0)

Or, $ae = 2$

Or, $5e = 2$

Or, $e = \frac{2}{5}$

We have,

$b^2 = a^2 (1 - e^2)$

or, $b^2 = 5^2 \left\{1 - \left(\frac{2}{5}\right)^2\right\}$

Or, $b^2 = 25 \left(1 - \frac{4}{25}\right)$

or, $b^2 = 25 \frac{(25 - 4)}{25}$

or, $b^2 = 21$

or, $b = \sqrt{21}$

The required equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Or, $\frac{x^2}{5^2} + \frac{y^2}{21} = 1$

Or, $\frac{x^2}{25} + \frac{y^2}{21} = 1$ is the required equation of an ellipse.

Example 5: Find the eccentricity and the coordinates of the foci of the curve

$$9x^2 + 5y^2 – 30y = 0.$$

Solution:

The given equation ellipse is

$9x^2 + 5y^2 – 30y = 0$

Or, $9x^2 + 5 (y^2 – 2.y. 3 + 3^2 – 3^2) = 0$

Or, $9x^2 + 5 \{(y – 3)^2 – 9\} = 0$

Or, $9x^2 + 5 (y – 3)^2 – 45 = 0.$

Or, $9x^2 + 5 (y – 3)^2 = 45.$

Or, $\frac{9x^2}{45} + \frac{5(y-3)^2}{45} = 1.$

Or, $\frac{x^2}{5} + \frac{(y-3)^2}{3^2} = 1$

Or, $\frac{(x^2\ 0)^2}{(\sqrt{5^2})} + \frac{(y-3)^2}{3^2} = 1$

Comparing this equation with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

We get, $a = \sqrt{5}$, $b = 3$, $h = 0$, $k = 3$

Since $b > a$, so major axis is along y -axis we have,

Eccentricity (e) $= \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{5}{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

Foci $= (h , k \pm be) = \left(0, 3 \pm 3. \frac{2}{3}\right) = (0, 3 \pm 2) = (0, 5)$ and $(0, 1)$

Example -6: Find the equation of the ellipse in the standard form with a vertex at (0, 8) and passing through $\left(3, \frac{32}{5}\right)$.

Solution:

Let the equation of ellipse in standard form be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ……………… (i)

Given, vertex = (0, 8)

i.e. (0, b) = (0, 8)

or, b = 8

Subtracting the value of b is eq^n (i); we get. $\frac{x^2}{a^2} + \frac{y^2}{8^2} = 1$ ……. (ii)

Since, equation (ii) passes through a point $\left(3, \frac{32}{5}\right)$

$$\frac{3^2}{a^2} + \left(\frac{32}{5}\right)^2 \frac{1}{64} = 1$$

Or, $\frac{9}{a^2} ++ \frac{1024}{25} \times \frac{1}{64} = 1$

Or, $\frac{9}{a^2} + \frac{16}{25} = 1$

Or, $\frac{9}{a^2} = 1 - \frac{16}{25}$

Or, $\frac{9}{a^2} = \frac{9}{25}$

Or, $a^2 = 25$

Now, the equation to an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Or, $\frac{x^2}{25} + \frac{y^2}{64} = 1$

Or, $64x^2 + 25y^2 = 1600$

Example-7: Find the equation of the ellipse whose major axis is twice its minor axis and passes through the point (0, 1).

Solution: Here,

Let the equation of an ellipse with a > b is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ ……….. (i)}$$

By the question,

Major axis = 2 (minor axis)

Or, $2a = 2.(2b)$

Or, $a = 2b$ …………… (ii)

Also, if equation (i) passes through the point (0, 1), then

$$\frac{0}{a^2} + \frac{1^2}{b^2} = 1$$

Or, $0 + \frac{1}{b^2} = 1$

Or, $\frac{1}{b^2} = 1$

Or, $b^2 = 1$

Then,

$a = 2b$

Or, $a^2 = 4b^2 = 4 \times 1 = 4$

Putting the values of a^2 and b^2 in eq^n (i)

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

Or, $\frac{x^2 + 4y^2}{4} = 1.$

Or, $x^2 + 4y^2 = 4$ which is the required equation of an ellipse.

Example-8: Show that: $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ represents the equation of an ellipse. Find its center, vertex and focus.

Solution: Here,

The given equation to an ellipse is;

$$9x^3 + 4y^2 - 18x - 16y - 11 = 0$$

Or, $9x^2 - 18x + 4y^2 - 16y = 1$

Or, $9(x^2 - 2x) + 4(y^2 - 4y) = 11$

Or, $9(x^2 - 2.x.1 + 1^2) + 4(y^2 - 2.y.2 + 2^2) = 11 + 9 + 16$

Or, $9(x - 1)^2 + 4(y - 2)^2 = 36$

Or, $9(x - 1)^2 + 4(y - 2)^2 = 36$

Or, $\frac{9(x-1)^2}{36} + \frac{4(y-2)^2}{36} = 1$

Or, $\frac{(x-1)^2}{4} + \frac{(y-2)^2}{3^2} = 1.$

Comparing this equation with $\frac{(x-h)^2}{x} + \frac{(y-k)^2}{b^2} = 1$;

We get,

$a = 2, b = 3, h = 1$ and $k = 2$

Since, b > a, So, the major axis is parallel to y – axis.

Centre = (h, k) = (1, 2)_

Vertex = (h, k ± b) = (1, 2 ± 3) = (1, 5) and (1, – 1).

Eccentricity (e) = $\sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

Focus = (b, k ± be) = $\left(1, 2 \pm 3.\frac{\sqrt{5}}{3}\right)$ = (1, 2 ± $\sqrt{5}$)

Example-9: Find the equation of the ellipse in the standard position with a focus at (0, – 5) and eccentricity $\frac{1}{3}$.

Solution: Given,

Focus = (0, – be) = (0, – 5)

Or, – be = – 5

Or, be = 5

Eccentricity (e) = $\frac{1}{3}$

Or, $b.\frac{1}{3} = 5$

Or, b = 15 or $b^2 = 225$

Again, we have

$$e^2 = 1 - \frac{a^2}{b^2}$$

Or, $\left(\frac{1}{3}\right)^2 = 1 - \frac{a^2}{15^2}$

Or, $\frac{1}{9} = 1 - \frac{a^2}{225}$

Or, $\frac{a^2}{225} = 1 - \frac{1}{9}$

Or, $a^2 = \frac{8}{9} \times 225$

Or, $a^2 = 8 \times 25$

∴ $a^2 = 200$

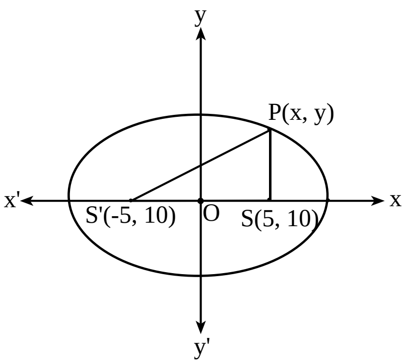
∴ The equation to an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Or, $\dfrac{x^2}{200} + \dfrac{y^2}{225} = 1$

Example 10: A point moves in a plane in such a way that the sum of its distances from two fixed points (5, 0) and (– 5, 0) is always 12. Find the locus of a point.

Solution:

Let P(x, y) be any point on the locus of an ellipse and let two points be S(5, 0) and S'(– 5, 0).



Using distance formula,

PS = $\sqrt{(x-5)^2 + y^2}$

= $\sqrt{(x-5)^2 + y^2}$

PS' $\sqrt{(x+5)^2 + (y-0)^2}$

= $\sqrt{(x+5)^2 + y^2}$

By the question

$PS' - PS = 12$

Or, $PS' = PS + 12$

Or, $PS' = PS + 12$

Or, $\sqrt{(x+5)^2 + (y-0)^2} = \sqrt{(x-5)^2 + y^2} + 12.$

Or, $\sqrt{(c+5)^2 + y^2} = \sqrt{(x-5)^2 + y^2} + 12.$

Squaring both sides; we get

$(x+5)^2 + y^2 = (x-5)^2 + y^2 + 2.12.\sqrt{(x-5)^2 + y^2} + 144.$

$x^2 + 2.\,X.5 + 5^2 = x^2 - 2.x.\,5 + 5^2 + 24\sqrt{(x-5)^2 + y^2} + 144$

$20x - 144 = 24\sqrt{(x-5)^2 + y^2}$

$5x - 36 = 6\sqrt{(x-5)^2 + y^2}$

Squaring both sides;

$$25x^2 – 360x + 1296 = 36 \{(x – 5)^2 + y^2\}$$

$$25 x^2 – 360 x + 1296 = 36 (x^2 – 10x + 25 + y^2)$$

$$25x^2 – 360x + 1296 = 36x^2 – 360x + 900 +36y^2$$

Or, $– 11x^2 – 36y^2 = – 396$

Or, $11x^2 + 36y^2 = 396$

Or, $\frac{11x^2}{396} + \frac{36y^2}{396} = 1$

Or, $\frac{x^2}{\frac{326}{11}} + \frac{y^2}{\frac{396}{36}} = 1$

Or, $\frac{x^2}{36} + \frac{y^2}{11} = 1$ is the required equation to an ellipse.

Example -11: Find the equation of the ellipse passing through the points (– 3, 1) and (– 2, 2). Find also its eccentricity.

Solution: Here,

Let the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ……….. (i)

Since, equation (i) pass through the points (– 3, 1) and (– 2, 2). We get

$$\frac{(– 3)^2}{a^2} + \frac{(– 1)^2}{b^2} = 1$$

Or, $\frac{9}{a^2} + \frac{1}{b^2} = 1$ ….. (ii)

And,

$$\frac{(– 2)^2}{a^2} + \frac{(2)^2}{b^2} = 1$$

Or, $\frac{4}{a^2} + \frac{4}{b^2} = 1$

Or, $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$ ………. (ii)

Subtracting eq^n (ii) from eq^n (i) we get

$$\frac{9}{a^2} + \frac{1}{b^2} = 1$$

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$$

$$\underline{(-)\ (-)\quad (-)}$$

$$\frac{8}{a_2} = \frac{3}{4}$$

$$a^2 = \frac{32}{3}$$

Putting the value of a^2in eq^n (ii). Then,

$$\frac{1}{\frac{32}{3}} + \frac{1}{b^2} = \frac{1}{4}$$

Or, $\frac{1}{b^2} = \frac{1}{4} - \frac{3}{32}$

Or, $\frac{1}{b^2} = \frac{32 - 12}{4 \times 32}$

Or, $\frac{1}{b^2} = \frac{20}{4 \times 32}$

Or, $\frac{1}{b^2} = \frac{5}{32}$

Or, $b^2 = \frac{32}{5}$

Since, $a^2 > b^2$

So, $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{\frac{32}{5}}{\frac{32}{3}}$

$$e^2 = 1 - \frac{32}{5} \times \frac{3}{32} = 1 - \frac{3}{5} = \frac{2}{5}$$

Eccentricity (e) = $\sqrt{\frac{2}{5}}$ units.

Exercise- 3.1

1. Find the length of major axes, minor axis, eccentricity, co-ordinates of foci and vertices.

   a) $25x^2 + 4y^2 = 100$ b) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

c) $\frac{x^2}{1^2}+\frac{y^2}{9} = 1$ d) $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$

2. Find the center, vertices the eccentricity, foci and equations of directries of the following ellipses:

a) $\frac{(x+6)^2}{4} + \frac{y^2}{36} = 1$ b) $9x^2 + 5y^2 - 30y = 0$

c) $\frac{(x+2)^2}{2} + y^2 = 5$ d) $\frac{x^2}{5} + \frac{(y+2)^2}{3} = 1$

e) $9x^2 + 5y^2 - 30y = 0$ f) $x^2 + 4y^2 - 4x + 24y + 24 = 0$

g) $9x^2 + 4y^2 + 40y + 18x + 73 = 0$

3. Find the equation of the ellipse in standard position satisfying the given conditions:

a) Vertices at $(\pm 5, 0)$ and foci at $(\pm 4, 0)$

b) Foci at $(\pm 2, 0)$ and eccentricity $= \frac{1}{2}$

c) Foci at $\left(\frac{10}{3}, 0\right)$ and eccentricity $\frac{1}{3}$

d) A focus at (0, 3) and length of minor axis 8.

e) A vertex at (0, 8) and passing through $\left(3, \frac{32}{5}\right)$

4. Find the equation of the ellipse in the standard position whose latus rectum is equal to half its major axis and which passes through the point $(\sqrt{6}, 1)$

5. Find the equation of the ellipse whose latus rectum is 3 and eccentricity is $\frac{1}{\sqrt{2}}$.

6. Find the equation of the ellipse whose distance between two foci is 8 and the semi-latus return is 6.

7. Deduce the equation of ellipse in the standard position if a focus is at (0, – 5) and eccentricity is $\frac{1}{3}$.

Answers:

1. (a) 6; 4; $\frac{\sqrt{21}}{5}$; $(0, \pm\sqrt{21})$; (0, 0) b) 6; 10; $\frac{4}{5}$ ' $(0, \pm 4)$ ' (0, 0)

c) 6; $4\sqrt{3}$, $\frac{1}{2}$, $(\pm\sqrt{3}, 0)$; (0, 0) d) 8; 6; $\frac{\sqrt{7}}{4}$; $(-2 \pm\sqrt{7}, 5)$; (– 2, 5)

2. (a) center = (– 6, 0); (0, ± 6); $\frac{2\sqrt{2}}{3}$, (– 6, ± 4$\sqrt{2}$); y = ±$\frac{9}{\sqrt{2}}$

b) center = (0, 3); (0, 3), (0, 6) e = $\frac{2}{3}$, (0, 1), (0, 5), y = 3 ±$\frac{3}{\frac{2}{3}}$ = $\frac{15}{2}$, $\frac{-3}{2}$

c) (– 2, 0)' (– 2 ±$\sqrt{10}$, 0), $\frac{1}{\sqrt{2}}$, (– 2 ±$\sqrt{5}$, 0), – 2 ± 2 $\sqrt{5}$

d) (0, – 2); (±$\sqrt{5}$, – 2), $\sqrt{\frac{2}{5}}$, (±, $\sqrt{2}$, – 2), x = ± $\frac{5}{\sqrt{2}}$.

e) (0, 3), (0, 0) & (0, 6); $\frac{2}{3}$, (0, 1) and (0, 5), x = 3 ± $\frac{9}{2}$

f) (–1, – 5); (– 1, – 2); (– 1, – 8); $\frac{\sqrt{5}}{3}$; (– 1, – 5 ±$\sqrt{5}$)

3. a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ b) $3x^2 + 4y^2 = 48$ (c) $5x^2 + 9y^2 = 125$

d) $\frac{x^2}{16} + \frac{y^2}{25} = 1$ e) $\frac{x^2}{25} + \frac{y^2}{64} = 1$

4. $x^2 + 2y^2 = 8$

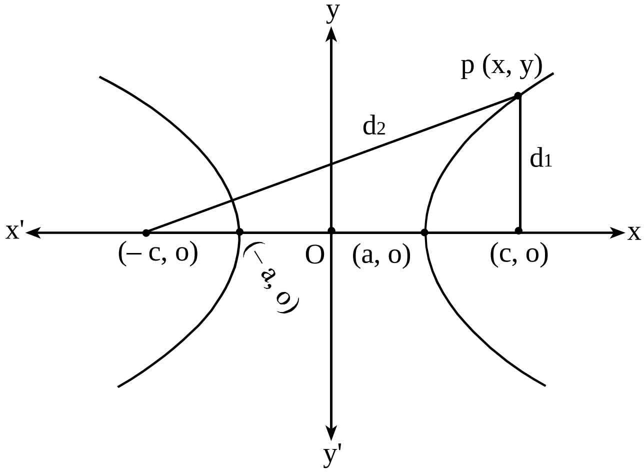
5. $x^2 + 2y^2 = 9$

6. $\frac{x^2}{64} + \frac{y^2}{48} = 1$

7. $\frac{x^2}{200} + \frac{y^2}{225} = 1$

3.1.6 The Hyperbola:

A hyperbola is the locus of a point which moves so that its distance from a fixed point (the focus) bears a constant ratio, greater than one, to its distance from a fixed straight line (the directrix)



Let (– c, 0) and (c, 0) be the foci of a hyperbola centered at the origin. The hyperbola is the set of all points (x, y) such that the difference of the distances from (x, y) to the foci is constant. If (a, 0) is a vector of the hyperbola, the distance from (–c, 0) to

(a, 0) is a – (– c) = (a + c). The distance from (c, 0) to (a, 0) is (c, – a). The difference of the distances from the foci to the vertex is

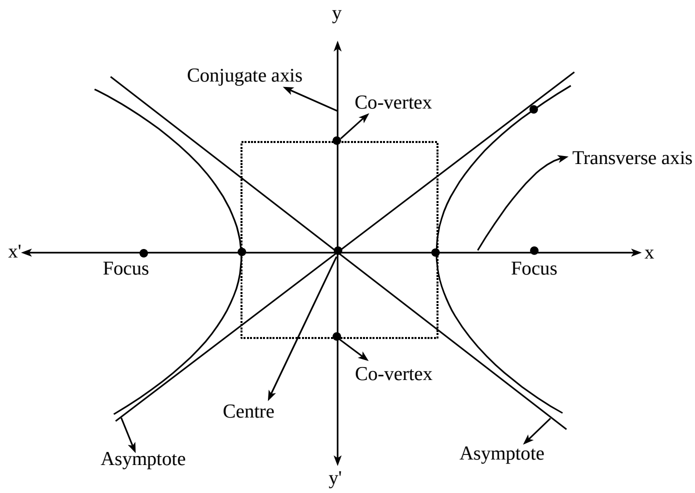
(a + c) – (c – a) = a + c – c + a = 2a

If (x, y) is a point on the hyperbola, we can define;

d_1 = the distance from (– c, 0) to (x, y).

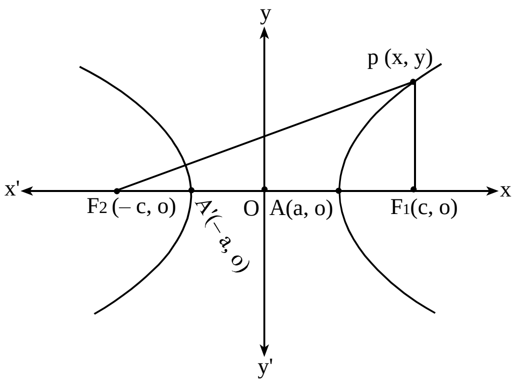
d_2 = the distance from (c, 0) to (x, y)

By definition of a hyperbola, $|d_2 - d_1|$ is constant for any point (x, y) on the hyperbola. It follows that: $|d_1 - d_2| = 2a$ for any point on the hyperbola.



Every hyperbola has two axes of symmetry. The transverse axis is a line segment that passes through the centre of the hyperbola and has vertices as its end points. The foci lie on the line that contains the transverse axis. The conjugate axis is perpendicular to the transverse axis and has the co-vertices as its end points. The centre of a hyperbola is the mid-point of both the transverse and conjugate axes. Every parabola also has two asymptotes that pass through its centre.

3.1.7 The equation of a hyperbola centered at the origin.



Let O be the centre of a hyperbola whose F(c, 0) and F'(– c, 0) be its foci. Let P(x, y) be any point on the hyperbola and join PS and PS' respectively. Let A(a,0) and A' (– a, 0) be two vertices of a hyperbola such that:

$AA' < F_1F_2$

Or, $2a < 2C$

Or, $a < C$.

By the definition if hyperbola

$PF_2 - PF_1 = 2a$ (say)

Or, $\sqrt{(x+c)^2+(y-0)^2}-\sqrt{(x-c)^2+(y-0)^2} = 2a$

Or, $\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2} = 2a$

Or, $\sqrt{(x+c)^2+y^2} = 2a+\sqrt{(x-c)^2+y^2}$

Squaring both sides we get,

$(\sqrt{(x+c)^2+y^2}\ )= (2a+\sqrt{(x-c)^2+y^2}\ )^2$

Or, $(x+c)^2 + y_2= 4a^2 + 2.2a\sqrt{(x-c)^2+y^2} + (x-c)^2 + y^2$

Or, $x^2 + 2cx + c^2 = 4a^2 + 4a\sqrt{(x-c)^2+y^2} + x^2 - 2cx + c^2$

Or, $4cx = 4a^2 + 4a\sqrt{(x-c)^2+y^2}$

or, $cx - a^2 = a\sqrt{(x-c)^2+y^2}$

Squaring both-sides; we get

Or, $\dfrac{c^2x^2 - 2cx\, a^2 + a^4}{a^2} = x^2 - 2xc + c^2 + y^2$

Or, $c^2x^2 - 2a^2cx + a^4 = a^2x^2 - 2ca^2x + a^2c^2 + a^2y^2$

Or, $c^2x^2 - a^2x^2 - a^2\, y^2 = a^2c^2 - a^4$

Or, $x^2\,(c^2 - a^2) - a^2\, y^2 = a^2\,(c^2 - a^2)$

Or, $\dfrac{x^2\,(c^2-a^2)}{a^2\,(c^2-a^2)} - \dfrac{a^2\, y^2}{a^2\,(c^2-a^2)} = 1$

Or, $\dfrac{x^2}{a^2} - \dfrac{y^2}{c^2-a^2} = 1$

Or, $\dfrac{x^2}{a^2} - \dfrac{y^2}{c^2-a^2} = 1[c^2 - a^2 = b^2]$

Or, $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ which is the required equation to a hyperbola.

Note:

i) Centre = 0 (0, 0)

ii) Vertices = (±, a, 0)

iii) Foci = (±, c, 0) where, $c = \sqrt{a^2+b^2}$

iv) Length of transverse axis = 2a

v) Length of conjugate axis = 2b

vi) $e = \dfrac{c}{a} = \dfrac{\sqrt{a^2+b^2}}{a}$

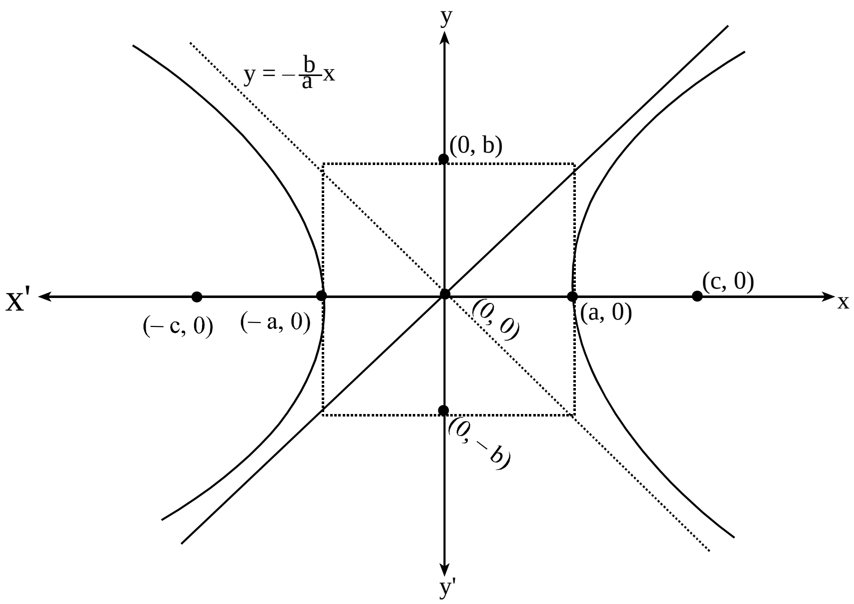
vii) Length of latus rectum = $\frac{2b^2}{a}$

viii) Hyperbola in which a = b is called an equilateral hyperbola.

ix) The equations of the asymptotes are $y = \pm \frac{a}{b}.x$

The vertices, co-vertices and foci are related by the equation $c^2 = a^2 + b^2$. when we are given the equation of a hyperbola, we can use this relationship to identity its vertices and foci.

The standard form of a hyperbola with centre (0, 0) and transversal axis on the x-axis $y = \frac{b}{a}x$

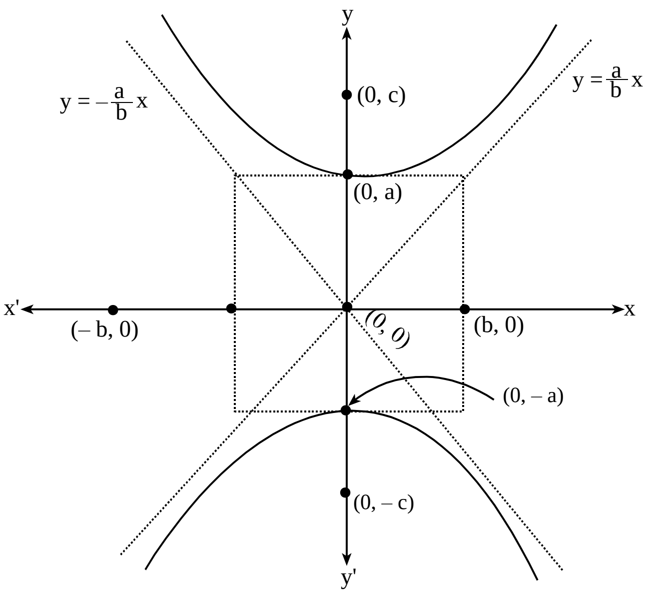


Its equation has the form;

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The standard form of a hyperbola with centre (0, 0) and transversal axis lies on the y-axis on the y-axis

Its equation has the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$



3.1.8 The equation to a hyperbola Not Centered at the Origin.

Solution: The equation to a hyperbola with the center at (0, 0) derived as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

If a hyperbola is translated h units horizontally and k unit vertically, the center of a hyperbola will be (h, k). The translation result is (x – h) and (y – k) replaced instead of x and y respecting. So, the equation to a hyperbola to a hyperbola in the standard form with centre (h, k) where the transversal axis parallel to the x-axis is

$$\frac{(x-h)^2}{a^2} + \frac{(x-k)^2}{b^2} = 1$$

Where,

i) The length of the transversal axis is 2a.

ii) The co-ordinates of the vertices are $(h \pm a, k)$.

iii) The length of the conjugate axis is 2b.

iv) The co-ordinate of the foci are $(h \pm c, k)$

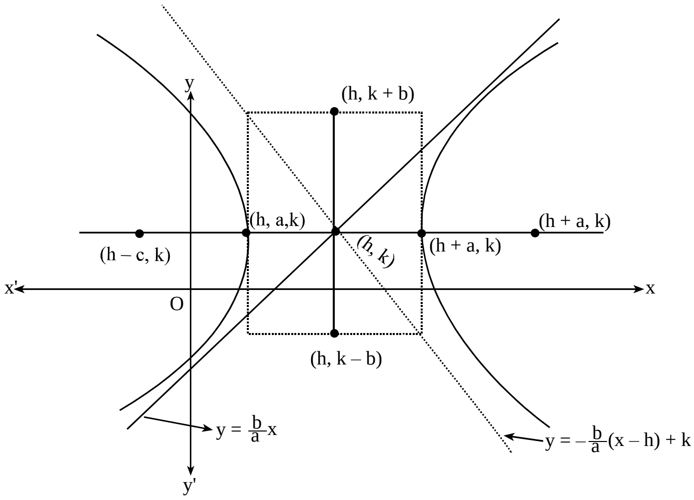
v) The distance of foci is 2c, where $c^2 = a^2 + b^2$

vi) The co-ordinate of vertices are $(h, k \pm b)$.

vii) The distance between two foci is 2c, where $c^2 = a^2 + b^2$.

viii) The equation of the asymptotes are $y = \pm\frac{b}{a}\ (x - h) + k$.

Along with above information, what horizontal hyperbola is shown below:



Similarly, the standard form of the equation of a hyperbola with centre (h, k) and transverse axis parallel to the y - axis is $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$

Where,

i) The length of the transverse axis is 2b.

ii) The length of the conjugate axis is 2a.

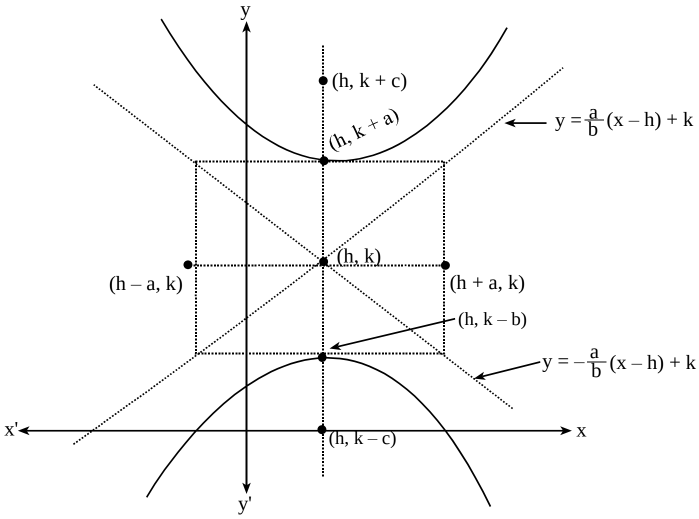
iii) The co-ordinates of vertices are $(h, k \pm b)$.

iv) The co-ordinates of co-vertices are $(h \pm a, k)$

v) The co-ordinates of the foci are 2c where $c^2 = a^2 + b^2$

vi) The co-ordinates of the foci are $(h, k \pm c)$

The equation of the asymptotes are $y = \pm\frac{a}{b}\ (x - h) + k$



Vertical hyperbola with the centre (h, k).

<u>Worked out Examples:</u>

Example: Find the eccentricity and foci of the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$

Solution:

The given equation to a hyperbola is $\frac{x^2}{25} - \frac{y^2}{16} = 1$ ………….. (i)

Comparing this equation (i) with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, We get;

$a^2 = 25 \Rightarrow a = 5$

$b^2 = 16 \Rightarrow b = 4$

We have, Eccentricity (e) = $\sqrt{1 + \frac{b^2}{a^2}}$

$= \sqrt{1 + \frac{16}{25}} = \sqrt{\frac{41}{25}} = \sqrt{\frac{41}{5}}$

$\therefore$ Eccentricity (e) = $\frac{\sqrt{41}}{5}$

$\therefore$ Foci = $(\pm ae, 0) = \left(\pm 5 \times \frac{\sqrt{41}}{5}, 0\right) = (\pm\sqrt{41}, 0)$

Example -2: Find the eccentricity and foci of the hyperbola $\frac{x^2}{36} - \frac{y^2}{64} = 1$

Solution:

The given equation to a hyperbola is $\frac{x^2}{36} - \frac{y^2}{64} = 1$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$a^2 = 36 \Rightarrow a = 6$

$b^2 = 64 \Rightarrow b = 8$

We have,

Eccentricity (e) = $\sqrt{1 + \frac{b^2}{a^2}}$

= $\sqrt{1 + \frac{64}{36}} \sqrt{\frac{100}{36}} = \frac{10}{6} = \frac{5}{3}$

$\therefore$ Foci = $(\pm ae, 0)$ $\left(\pm 6 \times \frac{5}{3}, 0\right)$

= $(\pm 10, 0)$

Example -3: Show that the equations $9x^2 - 16y + 18x + 32y - 151 = 0$ represents a hyperbola. Also find its eccentricity.

Solution:

The given equation to a hyperbola is

$9x^2 - 16y^2 + 18x + 32y - 151 = 0$

Or, $9x^2 - 18x - 16y^2 + 32y = 151$

Or, $9(x^2 - 2x) - 16(y^2 - 2y) = 151$

Or, $9(x^2 - 2.x + 1 + 1^2) - 16(y^2 - 2.y.1 + 1^2) = 9 - 16 + 151.$

Or, $9(x-1)^2 - 16(y-1)^2 = 144$

Or $\frac{9(x-1)^2}{144} - \frac{16(y-1)^2}{144} = 1$

Or, $\frac{(x-1)^2}{12} - \frac{(y-1)^2}{9} = 1$

Or, $\frac{(x-1)^2}{(2\sqrt{3})^2} - \frac{(y-1)^2}{32} = 1$

Which is in the form of $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

So that it represents the equation to a hyperbola.

Then,

$a^2 = 12 \Rightarrow a = 2\sqrt{3}$

$b^2 = 9 \Rightarrow b = 3$

Eccentricity (e) = $\sqrt{1 + \frac{b^2}{a^2}}$

$= \sqrt{1 + \frac{9}{12}}$

$= \sqrt{\frac{21}{12}}$

$= \sqrt{\frac{7}{4}}$

$= \frac{\sqrt{7}}{2}$

Example-4: Determine the equation of the hyperbola with a focus at (– 5, 0) and a vertex at (3, 0)

Solution: Given,

Transversal axis (2a) = 4 $\Rightarrow$ a = 2

Conjugate axis (2b) = 5

Or, 2b = 5

Or, $b = \frac{5}{2}$

The equation of hyperbola in the standard position is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Or, $\frac{x^2}{2^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$

Or, $\frac{x^2}{4} - \frac{y^2}{\frac{25}{4}} = 1$

$$\frac{x^2}{4} - \frac{4y^2}{25} = 1$$

Or, $\frac{25x^2 - 16y^2}{100} = 1$

Or, $25x^2 - 16y^2 = 100$

Example-5: Find the equation of hyperbola with a focus at (7, 0) and a vertex at (5, 0).

Solution: Given,

Vertex = (5, 0)

Or, (a, 0) = (5, 0)

Or, a = 5

Also, focus = (7, 0)

Or, (ae, 0) = (7, 0)

Or, ae = 7

Or, 5.e = 7

Or, $e = \frac{7}{5}$

We have,

$$e^2 = 1 + \frac{b^2}{a^2}$$

Or, $\left(\frac{7}{5}\right)^2 = 1 + \frac{b^2}{5^2}$

Or, $\frac{49}{25} = \frac{25 + b^2}{25}$

Or, $49 = 25 + b^2$

Or, $49 - 25 = b^2$

Or, $b^2 = 24$

The equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Or, $\frac{x^2}{16} - \frac{y^2}{32} = 1$

Example-6: Find the equation of the hyperbola with vertex at (0, 8) and passing through the point $(4, 8\sqrt{2})$

Solution: Given,

Vertex = (0, 8) = (0, b)

$\therefore \quad b = 8$

The equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Or, $\frac{x^2}{a^2} - \frac{y^2}{64} = -1$ ……………….. (i)

Since, equation (i) passes through a point $(4, 8\sqrt{2})$

$$\frac{4^2}{a^2} - \frac{(8\sqrt{2})^2}{64} = -1$$

Or, $\frac{16}{a^2} - \frac{64 \times 2}{64} = -1$

Or, $\frac{16}{a^2} = 2 - 1$

Or, $\frac{16}{a^2} = 16$

The equation (i) reduces to;

$\frac{x^2}{16} - \frac{y^2}{64} = -1$ Which is the required equation of a hyperbola

Exercise

1. Find the coordinates of Centre, vertices, eccentricities and foci of the hyperbola.

   (a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ (b) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (c) $\frac{(x+1)^2}{144} - \frac{(y-1)^2}{25} = 1$

   (d) $9x^2 - 16y^2 + 36x + 32y - 124 = 0$

2. Find the equation of a hyperbola in standard position such that its transverse and conjugate axes are respectively 4 and 5.

3 (a) Find the equation of hyperbola with focus at (7,0) and a vertex at (5,0).

(b) Determine the equation of the hyperbola with a focus at (–5,0) and a vertex at (3,0).

(c) Find the equation of a hyperbola with a focus at (–7,0) and eccentricity $\frac{7}{4}$.

4 (a) Obtain the equation of hyperbola with a focus at (6,0) and a vertex at (4,0).

(b) Find the equation of the hyperbola with focus at (–5,0) and vertex (2,0)

5. The coordinates of the vertices of a hyperbola are (9,2) and (1,2) and the distance

6. Find the equation of the hyperbola whose coordinates of the foci of a hyperbola are (± 6,0) and its latus rectum is of 10 units.

Answers:

1 (a) (0,0); (± 4,0), $\frac{5}{4}$, (± 5,0) (b) (0,0); (± 3,0), $\frac{5}{3}$, (± 5,0)

(c) (–1,1); (–13,1) and (11,1) , $\frac{13}{12}$, (± 4,1); (12,1)

(d) (2,1); (– 6, 1) and (2,1) , $\frac{5}{4}$, (–7,1); (3,1)

2. $\frac{x^2}{4} - \frac{y^2}{25} = 1$

3. (a) $\frac{x^2}{25} - \frac{y^2}{10} = 1$ (b) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (c) $\frac{x^2}{16} - \frac{y^2}{33} = 1$

4. (a) $\frac{x^2}{16} - \frac{y^2}{20} = 1$ (b) $21x^2 - 4y^2 = 84$

5. $9x^2 - 16y^2 - 90x + 64y + 17 = 0$ and $\frac{9}{2}$ units

6. $\frac{x^2}{16} - \frac{y^2}{20} = 1$.

Project works

1. Construct an ellipse using a rectangle.
2. Fix a point on the middle of the ceiling of your classroom. Find the distance between that point and four corners of the floor.

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