UNIT

Algebra

6

Permutation and Combination

Learning Outcomes:

At the end of this chapter, students will be enable to…………

- Solve the problems related to permutation and combinations.

Permutation and Combination

1.1 Introduction:

we very often come across problems in which we have to compute the number of ways a set of objects can be arranged under some conditions. In this unit, we discuss about the fundamental principle of counting along with permutation and combination. In some cases, we consider to an order but in some cases, we do not consider its order. To tackle such problems, we need to study different counting methods. The study of counting method comes under the field of combinatorial algebra. There are two useful basic principles of counting which is called fundamental principles of counting which are based under the additional and multiplication operations. First one is used in counting the number of ways in which either of two works can be done where one of two works is possible to do at a time. And the second is used in counting the number of ways in which a work followed by a second work can be done.

Example: 1

Let us consider an example, we need to travel from Kathmandu to Fiji Island. We must change flights, first at Singapore and then at Sydney, Australia.There are 6 different flights available from Kathmandu to Singapore 5 from Singapore to Sydney and 3 from Sydney to Fiji.

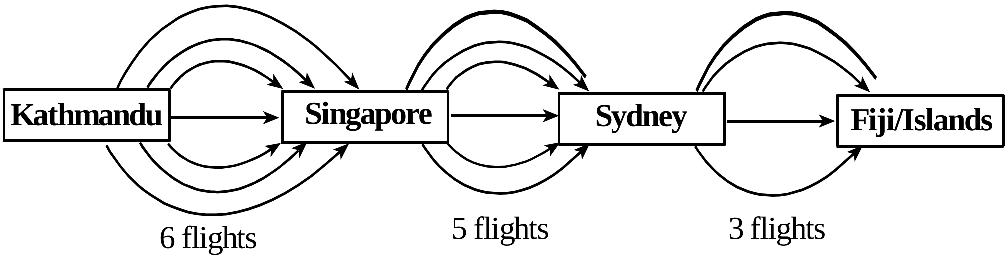


Figure - I

Rough:

A = {1, 2, 3, 4, 5, 6}

B = {a, b, c, d, e}

Probable Cases:

{ (1, a), (2,a), (3,a), (4,a), (5, a), (6, a)

(1, b), ...(6,b)

....

....

(1,e), (2,e), ...(6,e) }

i. How many ways exist of making a flight plan from Kathmandu to Fiji ?

If we are in Singapore, there are 5 flights to Sydney. For each of these 5 flights we have 3 different flights from Sydney to Fiji. Thus, we have total $5 \times 3 = 15$ ways of travelling from Singapore to Fiji.

ii. How many ways do we have to reach Singapore from Kathmandu?

In the first place there are 6 flights; for each of these 6 flights, we have 15 further ways of reaching Fiji from Singapore. Thus, the total number of ways P of travelling from Kathmandu to Fiji is $6 \times 15 = 90$ ways.

In other ways,

$$\begin{pmatrix} \text{No. of flights from} \\ \text{KTM to Singapore} \end{pmatrix} \times \begin{pmatrix} \text{No. of flights from} \\ \text{Singapore to Sydeny} \end{pmatrix} \times \begin{pmatrix} \text{No. of flights from} \\ \text{Sydeny to Fiji} \end{pmatrix}$$

Basic Principle of counting under multiple basic:

If an event is formed of stages, where one stage can be accomplished in m ways, another stage can be accomplished in n ways and next stage can be accomplished is p ways and so on, then the total number of ways to accomplish the event is $m \times n \times p$.

Example:1

If there are two jobs such that first job can be done in 'm' ways and second job can be done in 'n' ways than two jobs in succession can be done in mn ways.

Example:2

In a certain election, there are three candidates for president. Five for secretary and only two for the treasure. Find in how many ways the election may turn out.

Solution:

There are 3 choices for president.

There are 5 choices for secretary.

There are 2 choices for treasure.

Since, they are all independent of one another the total number of ways in which the election may turn out = $3 \times 5 \times 2 = 30$ ways.

Example:3

How many numbers of plates of vehicle consisting of 4 different digits be made out of integers 4, 5, 6, 7, 8, 9?

Solution:

To form a numbers of plates of 4 digits, we have to arrange the given digits 4, 5, 6, 7, 8, 9 taking four at a time.

Indeed, there are **six** choices for thousands place value. After filling thousand places, five integer are left. Hundreds place can be filled by **five** ways. By similar manner, ten place can be filled by **four** ways and unit place can be filled by **three** ways. By basic principle of counting.

Number of four digits can be filled = $6 \times 5 \times 4 \times 3 = 360$ ways.

Example:4

How many numbers of plates of vehicle consisting of at least 4 different digits be made out of integers 4, 5, 6, 7, 8, 9?

Solution:

At least 4 different digits means we have to take the numbers of minimum four digits i.e. numbers of 4 different digits + Numbers of 5 different digits + Numbers of 6 different digits = $6 \times 5 \times 4 \times 3 + 6 \times 5 \times 4 \times 3 \times 2 + 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 1800$

Example:5

How many numbers of plates of vehicle consisting of at most 4 different digits be made out of integers 4, 5, 6, 7, 8, 9?

Solution:

At most 4 different digits means we have to take the numbers of maximum four digits i.e. numbers of 4 different digits + Numbers of 3 different digits + Numbers of 2 different digits + Numbers of 1 digit

$= 6 \times 5 \times 4 \times 3 + 6 \times 5 \times 4 + 6 \times 5 + 6 = 360 + 120 + 30 + 6 = 516$

Example:6

How many number of four different digits each greater than 5,000 can be formed from the digit 2, 4, 5, 7, 8, 0?

Solution: Here,

Number of four digits greater than 5,000 must begin with either 5 or 7 or 8.

$\therefore$ Thousands place can be filled up by 3 different ways.

When thousands place is filled up.

Hundred places can be filled up by 5 different ways.

Tenth place can be filled up by 4 different way.

Unit place can be filled up by 3 different ways.

By the basic principle of counting.

Required number greater than 5,000 is $3 \times 5 \times 4 \times 3 = 180$ ways.

Example:7

How many numbers of three different digits less than 500 can be formed from the integers 1, 2, 3, 4, 5, 6?

Solution: Here,

The given integers are 1, 2, 3, 4, 5, 6. Since the number should be made from three different digits less than 500.

The hundred digits should be either 1 or 2 or 3 or 4. So, there are 4 choices. When the hundreds place has been filled up, there will be 5 numbers left, So, there are 5 choices for ten's place. In the same way, there are 4 choicer for unit's place.

By basic principle of counting, total number of ways = $4 \times 5 \times 4 = 80$ ways.

Example:8

How many numbers are there between 100 and 1,000 such that every digit is either 2 or 9?

Solution:

We know that the numbers between 100 and 1,000 are 3 digit numbers. Since, each digit is either 2 or 9, so there are 2 choices for unit's(one's), 2 choices for ten's for ten's and 2 choices for hundred's place.

By basic principle of counting,

Total number of arrangements = $2 \times 2 \times 2 = 8$

Exercise - 1.1(A)

1. A cinema hall has three entrance doors and seven exits doors. In how many different ways can a person enter and leave the cinema hall.
2. In a college, there are 6 doors, in how many ways can a person enter the college and leave by different door.
3. In how many ways can a woman send three of her children to five different colleges of a certain town?
4. If there are seven running track between two village P and Q. In how many ways can a woman go from a village to the other and return by a different tracker.
5. How many number between 4,000 and 5,000 can be formed with the digits 2, 3, 4, 5, 6, 7 ? How many of their number are divisible by 5 ?
6. A student can choose one of 4 monitors, one of 2 keyboards, one of 4 computers and one of 3 printers. Find the number of possible systems that a student can choose from.
7. A boy can select one of 6 different mathematic books, one of 3 different chemistry books and one of 4 different science books. In how many different ways can a student select a book of mathematics a book of chemistry and a book of science ?
8. There are 3 different roads from city A to city B and 2 different roads from city B to city C. In how many ways can a man go from city A to city C through by city B ?
9. In a college's ID cards have 5 digit numbers.
   a) How many ID cards can be formed if repetition of the digit is allowed?
   b) How many ID cards can be formed if repetition of the digit is not allowed?
10. Using the digits 1, 2, 3 and 5 how many 4 digit number can be formed if:
    a) The first digit must be 1 and repletion of the digits is allowed?
    b) The first digit must be 1 and repetition of the digits is not allowed?
    c) The number must be divisible by 2 and repetition is allowed?

d) Number must be divisible by 2 and repetition is not allowed.

Answers

1.	18 ways					2.	30 ways
3.	60 ways					4.	35 ways
5.	60, 12					6.	96
7.	72					8.	6
9.	a)	1,00,000				b)	30,240
10.	a)	64;	b)	6;	c) 64;	d)	6

1.2 Factorial Notation:

We are all familiar with multiplication. The factorial notation is a symbol that we use to represent a multiplication operation. But it is more than just a symbol. The factorial notation is the exclamation mark, and you will see it directly following a number. For example, you will see it as 5! or 3!. You read there as 'five factorial' and ' three factorial'. **Factorial means the product of all the positive integers equal to and less than our chosen number**. So,

$1! = 1$

$2! = 2 \times 1 = 2\ (2–1)$

$3! = 3 \times 2 \times 1 = 3\ (3–1)\ (3–2)$

$4! = 4 \times 3 \times 2 \times 1 = 4\ (4–1)\ (4–2)\ (4–3)$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 5\ (5 – 1) \times (5 – 2) \times (5 – 3) \times (5 – 4)$

$n! \ = n \times (n – 1) \times (n – 2) \times (n – 3)\ \ldots..3.2.1$

Definition: The continued product of the first n- positive integers is denoted by n! or $\lfloor n$

Remark: $0! = 1$

We know that;

$n! = n.(n – 1)!$

Putting $n = 1$. Then,

$1! = 1.\ (1 – 1)!$

Or, $1 \times 1 = 0!$

$\therefore$ $0! = 1.$

Example : Find the value of: $\frac{9! - 6!}{4!}$

Solution: Here,

$$\frac{9! - 6!}{4!}$$

$$= \frac{9 \times 8 \times 7 \times 6! - 6!}{4!}$$

$$= \frac{6!\,(9 \times 8 \times 7 - 1)}{4!}$$

$$= \frac{6 \times 5 \times 4! \times 503}{4!}$$

$$= 15{,}090$$

A permutation is a mathematical technique that determines the number of possible **arrangements** in a set when the order of the arrangements matters. Common mathematical problems involve choosing only several items from a set of items with a certain **order**. The number of permutations that can be formed taken r at a time out of n given objects is expressed in the following way

$$p(n, r) \text{ or } {}^{n}p_{r} = \frac{n!}{(n - r)!}$$

Where;

n = the total number of elements in a set.

r = the number of selected elements arranged in a set.

put r = n, $p(n, n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$

There are various types of permutations:

At first let's include them as following **formulae**:

1. Permutation of n different objects for r selection of objects is given by:

$$P(n, r) = \frac{n!}{(n-r)!}$$

If r = n then $p(n, n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$

So, total permutation arranging n objects anywhere = **n!**

2. Permutation of n objects **not all different** (taken all at a time) : $\frac{n!}{p!q!r!}$

   Where p is an objects of an one kind.
   q is an object of second kind.
   r is an object of third kind.

Eg:- STATISTICS n = 10

S is repeated for 3 times so let, p = 3, T is repeated for 3 times so let, q =3

I is repeate for 2 times so let, r =2

Now, Permutation = 10!/3! 3! 2! = ...

3. Permutation of n different objects taken r at a time with repetition is $\mathbf{n^r}$.

ie. n.n.n.n.n.n... r times = n^r

4. **Circular permutation:** The number of permutations of a set of n objects arranged in a circle is (n – 1)!.

   Special case: Circular permutations of n object to form a necklace or bracelet is $\frac{(n-1)!}{2}$. In such cases the clockwise and anticlockwise arrangements are identical.

Theorem 1: **(Permutation without repetition)**

The total number of permutations of a set of n objects taken r at a time is given by $p(n, r) = n (n-1) (n-2) (n-3) \ldots (n-r+1) \; (n \geq r)$.

Proof: The number of permutations of a set of n objects taken r at a time is equivalent to the number of ways in which r position can be filled up by n objects. Now, there are n = (n – 0) choices to fill up the first place. When the first place has been filled up, there will be left (n – 1) choices to fill up second position. i.e. there are (n – 1) choices to fill up the second position. Similarly, there are (n – 2) choices to fill up the third position and soon. Ultimately to fill up the r^{th} position there are

$n - (r - 1) = (n - r + 1)$ choices. Then, by basic principle of counting,

total number of ways = n (n – 1) (n – 2) ….. (n – r + 1)

$\therefore \quad p(n, r) = n (n-1) (n-2) (n-3) \ldots\ldots\ldots\ldots (n-r+1)$

$$= \frac{n(n-1)(n-2)(n-3)\ldots\ldots\ldots(n-r+1).(n-r)\ldots\ldots\ldots.3.2}{(n-r)\ldots\ldots\ldots3.2}$$

$$= \frac{n!}{(n-r)!}$$

Remarks:

i) $^{n}p_{o} = \frac{n!}{n!} = \frac{n!}{n!} = 1$

ii) $^{n}p_{n} = \frac{n!}{(n-n)!} = \frac{n!}{o!} = \frac{n!}{1} = n!$

iii) $^{n}p_{1} = \frac{n!}{(n-1)\,!} = \frac{n.\,(n-1)\,!}{(n-1)!} = n$

iv) $^{(n-1)}p_{r} + r^{(n-1)}p_{r-1} = {}^{n}p_{r}.$

i.e. $^{(n-1)}p_{r} + r.\ ^{(n-1)}p_{r-1}$

$$= \frac{(n-1)!}{(n-1-r)!} + r.\frac{(n-1)\,!}{(n-1-r+1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r.\frac{(n-1)!}{(n-r)\,!}$$

$$= \frac{(n-1)\,!}{(n-r-1)!} + r.\frac{(n-1)!}{(n-r)\,(n-r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)\,!}\left(1 + \frac{r}{n-r}\right)$$

$$= \frac{(n-1)!\ \ (n-r+r)}{(n-r-1)!\ \ (n-r)}$$

$$= \frac{n(n-1)!}{(n-r)\,(n-r-1)\,!}$$

$$= \frac{n!}{(n-r)!}$$

$= {}^{n}p_{r} =$ proved.

Example 1: In how many different ways can 6 passenger sit in a bus having 20 vacant seats ?

Solution:

Number of vacant seats (n) = 20

Number of passengers (r) = 6.

∴ 6 passenger is a 20 vacant seats can be arranged as $^{20}p_{6}$

$$= \frac{20!}{(20-6)!}$$

$$= \frac{20!}{14!} = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14!}{14!}$$

$$= 2,79,07,200$$

Example 2: How many 6 different digit telephone number can be formed from the integers 0,1,2, ..., 9 by using the integers at once, which begins with 98?

Solution:

Since, first two places are occupied by 9 and 8, remaining 4 places can be filled up by 0, 1, 2, 3, 4, 5, 6, 7.

4 digits in 8 places can be arranged as ${}^8p_4$ i.e. p(8, 4)

$$= \frac{8!}{(8-4)!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} = 1680 \text{ ways.}$$

Theorem2: (Permutations of object not all different)

If there are n objects of which p objects are of one kind, q objects are of second kind, r objects are of third kind and all the rest (if any) are different, then the number of permutations of their objects taken all at a time is $\frac{n!}{p!q!r!}$

Proof:-Let us suppose x be the required number of permutation that p be an object of first kind which can be replaced by p new objects. It is different from each other and different from the rest. Since, this p different objects can be arranged in p! ways, each of x permutations will produce p! different permutations without changing the position of any other object. Hence, there will be $x \times p!$ permutation in all. In the same manner, if q be the second object of second kind which is replaced by q new objects different from one another and from the remaining objects $x \times p!$ permutations the total permutations is $x \times p! \times q!$.

Similarly, if the r of the objects be replaced by r different objects different from each other and different from the rest, the number of permutations will become

$$x \times p! \times q! \times q! \times r!$$

But the number of permutations of n different things taken all at a time is n!

Thus,

$$x \times p! \times q! \times r! = n!$$

Or, $$x = \frac{n!}{p!q!r!}$$

$\therefore$ The total number of permutation = $\dfrac{n!}{p!q!r!}$

The general formula for this situation is $\dfrac{n!}{r_1!r_2!r_3! \ldots. r_k!}$

Where n - elements is a set and r_1 are alike, r_2 are alike, r_3 are alike and so on through r_k.

Example 1:

Find the number of permutation of the letters of the word 'STATISTICS' taken all at a time.

Solution:

Number of letters of the word 'STATISTICS' is (n) = 10.

p = No. of letters of S = 3

q = No. of letters of T = 3

r = No. of letters of I = 2

$\therefore$ Required number of arrangement = $\dfrac{n!}{p!q!r!}$

$= \dfrac{10!}{3!3!2!} = \dfrac{10 \times 9 \times 8 \times 7 \times 6 \times 4 \times 3!}{\times 3! \times 3 \times 2 \times 1 \times 2 \times 1 \times 1 \times 1}$

= 5,04,000.

Permutations of objects with repetition:

Theorem: The number of permutations of n different objects taken r at a time when repetition are allowed is n^r.

Proof: Let us suppose that we have to fill up r position with n different objects. The first position can be filled up by any of the n objects. Since, the repetition is allowed. So, there are n choices to fill up the second position also it can be filled up by n objects. The object which occupies the first position can also be placed in the second position. The first two positions can be filled up in $n \times n = n^2$ ways. Similarly, the third, fourth, fifth positions can be filled up in $n \times n \times n \times n \times n = n^5$ ways.

Proceeding in the same way, the r^{th} places can be filled up by n objects in $n \times n \times n \times n \times \ldots\ldots\ldots$ r times = $\mathbf{n^r}$ ways.

Example 1 : In how many ways can 4 letters be posted in six letter boxes ?

Solution: Here,

Since there are six letter boxes.

The first letter can be posted in 6 ways.

The second letter can be posted in 6 ways.

The third letter can be posted in 6 ways.

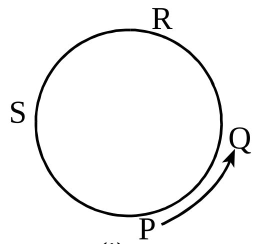
The four letter can be posted in 6 ways.

So, n =6 and r =4

$\therefore$ The total number of ways = $(6 \times 6 \times 6 \times 6)$ ways = 6^4 ways = 61296 ways.

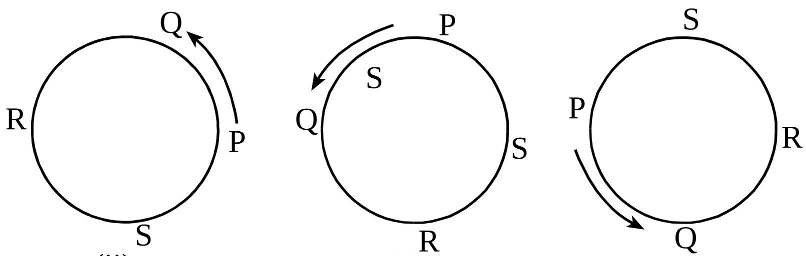
<u>Circular Permutations:</u>

The arrangements we have considered so far are linear. There are also arrangements in closed loops, called circular arrangements. Consider four persons P,Q, R and S who are to be arranged along a circle. It's one circular arrangement is as shown in adjoining figure.



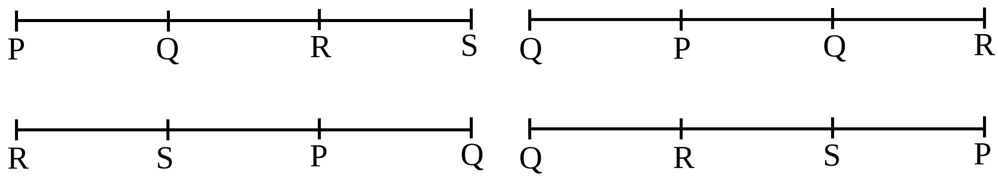
(i)

Shifting P, Q, R, S one position in anticlock wise direction. Then we get the following arrangements as shown:



(ii) (ii) (iii)

Arrangements as shown in figure (i), (ii), (iii) and (iv) are not different as relative position of none of the four persons P, Q, R, S is changed. But in case of linear arrangements the four arrangements are.



Thus, it is clear that corresponding to a single circular arrangement of four different things there will be 4 different linear arrangements. Let the number of different things be n and the number of their circular permutation be *x*.

Now, for one circular permutation, number of linear arrangements is n.

For x circular arrangements number of linear arrangements = nx ……….(i)

But number of linear arrangements of n different things = n! ……..(2)

From (1) and (2), we get

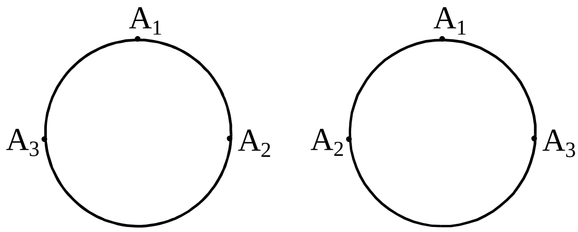
$nx = n!$

$$x = \frac{n!}{n} = \frac{n(n-1)!}{n} = (n-1)!$$

Distinction between clockwise and anti-clockwise arrangements.

Here,

Consider the following circular arrangements.



(i)   (ii)

In figure (I) the order is clockwise whereas in figure (II), the other is anti-clockwise. There are two different arrangements when distinction is made between the clockwise and the anti-clockwise arrangements of n different objects around a circle, then the number of arrangements = (n – 1)!

If there is no distinction between clockwise and anticlockwise arrangements, then the number of ways will be $\frac{(n-1)!}{2}$ as in the case of forming a neckline or bracelet out of n differently coloured beads.

Example 1: In how many ways can 8 people and a host be seated in a circular table of a party ?

Solution:

Total number of people = n = 8 + 1 = 9

Number of circular arrangements = (n -1)! = (9 – 1) ! = 8 !

Example 2: Find the number of ways in which 6 different beads can be arranged in a necklace.

Solution: No of beads = n = 6

Six beads can be arranged on a circle in (6 – 1)! = 5! ways.

In this case clockwise and anti-clock wise arrangements are the same.

$\therefore$ The number of arrangements = $\frac{1}{2} \times 5! = \frac{1}{2} \times 120 = 60$ ways.

Worked out examples:

1. Find the value of n if 3p(n, 4) = p(n, 3)

Solution: Given,

$3.p\,(n, 4) = p(n, 3)$

Or, $3.\dfrac{n!}{(n-4)!} = \dfrac{n!}{(n-3)!}$

Or, $\dfrac{3}{(n-4)\,(n-3)!} = \dfrac{1}{(n-3)!}$

Or, $3 = n - 4$

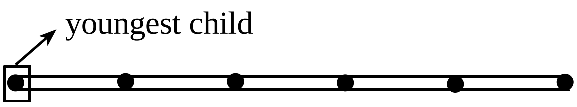
Or, $3 + 4 = n$

Or, $n = 7$

$\therefore$ $n = 7$

2. Six children are to be seated on a bench. How many arrangements are possible if the youngest child sits at the left end of the bench:

Solution: Here,



Let us fix the youngest child at the left end of the bench. Then, remaining five children can be arranged in p(5, 5) ways.

$= 5!$

$= 5 \times 4 \times 3 \times 2 \times 1 = 120$

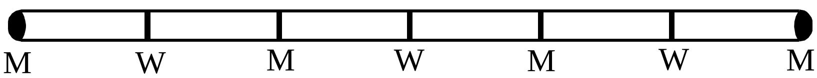
$\therefore$ Required number of arrangements = 120 ways.

3. Find the number of ways in which 4 men and 3 women can be seated in a row having seven seats so that the men and the women must alternate.

Solution: Here

Number of men = 4

Number of woman = 3



If men and woman sit alternatively, Then men should be in odd places, (i.e. 1^{st}, 3^{rd}, 5^{th}, 7^{th}) and women should be in even place (i.e. 2^{nd}, 4^{th} and 6^{th})

Now, 4 men is 4 seats can be arranged in p(4, 4) ways

$= \dfrac{4!}{(4-4)!} = \dfrac{4!}{0!}$

$= 4 \times 3 \times 2 \times 1$

= 24 ways

3women is 3 seats can be arranged in p(3, 3) ways.

$$= \frac{3!}{(3-3)!} = \frac{3 \times 2 \times 1}{0!} = 6 \text{ ways.}$$

Total number of arrangements = (24 × 6) ways = 144 ways.

4. How many words can be formed from the letters of the word 'ENGLISH' ? How many of these do not begin with E? How many of these begin with E and do not end with H ?

Solution:

Total number letters in the word 'ENGLISH = 7 | E | N | G | L | I | S | H |

So, the letters can be arranged in p(7, 7) ways = 7! = 7.6.5.4.3.2.1 = 5040 = Total Ararngement

Next, for begin with E, let us fix E at first place, then the remaining 6 letters can be arranged in p(6, 6) ways i.e. 6! = 720 ways

Hence, the number of arrangements that do not begin with E = (5040 – 720) ways = 4320 ways.

Note: Total = 7 . 6. 5. 4.3.2.1 = 5040

Do not begin with E = 6. 6.5.4.3.2.1 = 4320

Again.

Let us fix **E** at first place and **H** at last. (i. begin with E and End with H)

Then remaining 5 letter can be arranged in 5! ways.

= 5 × 4 × 3 × 2 × 1 = 120 ways.

Hence, the number of arrangements that begin with E and do not end with H = (720 – 120) = 600 ways

Note: Total = 7.6.5.4.3.2.1

Begin with E and do not end with H = 1. 5. 5.4.3.2.1 = 600

5. In how many ways can the letters of the word "ARRANGE" be arranged so that no two R's come together ?

Solution:

The given word is 'ARRANGE'

Total number of letters (n) = 7

Number of A's (p) = 2

Number of R's (q) = 2

And the remaining letters are single.

$\therefore$ Total number of arrangements = $\dfrac{n!}{p!q!}$

$= \dfrac{7!}{2!2!}$

$= \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2\,! \times 2 \times 1}$

$= 7 \times 6 \times 30$

$= 1260$

To find the number of arrangements in which two R's come together, consider two R's as a single letter i.e. '

A	RR	A	N	G	E

The number of letter will be 6.

$\therefore$ Total number of arrangements in which two R's come together is $\dfrac{6!}{2!}$

$= \dfrac{6 \times 5 \times 4 \times 3 \times 2}{2!} = 120 \times 3 = 360.$

Required number of arrangements in which no two R's come together

= 1260 – 360 = 900. (do not come together = Total – Come together)

6. How many numbers of four digits can be formed from the digits 4, 5, 6, 7, 8 ? How many of these number are divisible by 5 ? How many of these number are not divisible by 5 ?

Solution:

The given digits number are 4,5,6,7,8. The number of 4 different digits number that can be formed from the given digits i.e. 4, 5, 6, 7,8 is

$P(5, 4) = \dfrac{5!}{(5-4)!} = \dfrac{5 \times 4 \times 3 \times 2 \times 1}{1!}$

$= 120$

For the digit divisible by 5, we must fix 5 at the unit place So, remaining 3 digits from 4 digits can be arranged as p(4, 3)

$= \dfrac{4!}{(4-3)!} = \dfrac{4 \times 3 \times 2 \times 1}{1!} = 24$ ways.

Number of 4 digit numbers which are divisible by 5 = 24

Hence, the number of 4 different digits number which are not divisible by 5 are (120 – 24) = 96.

7. Find the total number of words which can be formed using letter of the word 'FAILURE' so that consonants always occupy odd place.

Solution:

F	A	I	L	U	R	E

No. of letter of consonants = F, L, R.

No. of letter of vowels = A, I, U, E

Number of arrangements of consonants at odd places i.e. 1^{st}, 3^{rd}, 5^{th} and 7^{th} can be arranged as p(4, 3) ways i.e.

$$\frac{4!}{(4-3)!} = \frac{4 \times 3 \times 2 \times 1}{1!} = 24$$

Then remaining 4 places may be filled with remaining letters in 4! ways so,

Required number of arrangements = 24 × 4! = 576 ways.

8. 18 guest have to sit, half on each side of a long table. Four particular guests desire to sit on one particular side and three other on other side. Find the number of ways of seating arrangement.

Solution:

Number of guest of a particular side = 9.

Number of particular guest = 4

Total number of ways of sitting 4 particular guest on a particular side]

$$= {}^{9}p_{4} = \frac{9!}{(9-4)!} = \frac{9!}{5!}$$

Similarly,

Total number of ways of sitting another particular guests on the other side

$${}^{9}p_{3} = \frac{9!}{(9-3)!} = \frac{9!}{6!}$$

After these two arrangement the remaining 11 guests can sit on remaining 11 seats in 11! ways

∴ Total number of sitting arrangements = $\frac{9! \times 9! \times 11!}{5! \times 6!}$

9. In how many ways can 5 persons be seated round a circular table when two particular person sit together ?

Let two particular persons be seated together. The persons in two places can be arranged in 2! ways = 2 × 1 = 2 ways.

Then remaining 3 persons may sit in remaining three places in 3! ways = 3 × 2 × 1 = 6 ways.

∴ Total number of ways = 2 × 6 = 12 ways.

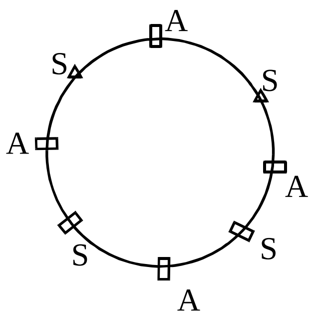
Note: - Since two of them sit together, n=4

Circular permutaiton = (n-1)! = (4-1)! = 3!

But those two can be arranged(interchanged) in 2! ways

Hence, total permutaions = 2! . 3! = 12

10. In how many ways can 4 Art students and 4 Science students be arranged alternately at a round table ?



Solution: Here,

Let A denotes Art student S denotes Science student.

∴ 4 Art students at a round table can be arranged in (4 – 1)!
= 3! ways
= 3 × 2 × 1 = 6 ways.

Since Art and science students must be arranged alternatively. So, 4 science students in 4 seats can be arranged in p(4, 4) ways i.e. 4! ways = 4 × 3 × 2 × 1 = 24 ways.

Total number of arrangements = 6 × 24 = 144 ways,

11. In how many ways can 6 different beads be strung on a necklace ?

Solution:

No. of beads (n) = 6.

Since clockwise and anticlockwise arrangement are same in the necklace

∴ The total number of arrangements $= \frac{(n-1)!}{2} = \frac{(6-1)!}{2} = \frac{5!}{2} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2} =$ 60.

12. (a) In how many ways can 4 letter be posted in six letter boxes ?

Solution:

Number of letter boxes = 6

Number of letters = 4

The first letters can be posted in 6 ways.

The second letters can be posted in 6 days.

The third letter can be posted in 6 ways.

The four letter can be posted in 6 ways.

∴ The total number of ways = 6 × 6 × 6 × 6 ways,
$= 6^4$ = 1296 ways.

b) How many even numbers of 4 digits can be formed when repetition of digits is allowed ?

Solution:

There are 5 choices for the digit in the unit's place.

There are 10 choices for the digit in the digits in the ten's place

There are 9 choices for the digit in the hundred's place.

There are 8 choices for the digit in the thousand's place.

∴ No. of 4-digits even numbers = 5 × 10 × 9 × 8 = 3600 ways

c) In how many ways can 5 prizes be distributed among 6 students so that each students may receive any numbers of prizes ?

Here,

Number of students (n) = 6

Number of prizes (r) = 5

∴ The number of ways in which the prizes can be distributed = $n^r = 6^5$

13. In how many ways can the letters of word "TOPIC" be arranged so that
    a) All the vowels are always together ?
    b) The vowels may occupy only odd positions ?
    c) The relative positions of vowels and consonants are not changed ?

Solution:

a) If two vowels come together, than the number of object is 4. So, they can be arranged in p(4, 4) ways

$$= \frac{4!}{(4-4)!} = \frac{4 \times 3 \times 2 \times 1}{0!} = 24 \text{ ways.}$$

Also, two vowels can be arranged in 2 ways i.e. 2! ways

= 2 × 1 ways = 2 ways.

∴ Total number of arrangement = 24 × 2 = 48 ways.

b) There are only 2 vowels and three odd places.

∴ 2 vowels in 3 places can be arranged in p(3, 2) ways

$$P(3, 2) = \frac{3!}{(3-2)!} = \frac{3 \times 2 \times 1}{1!} = 6 \text{ ways}$$

And, the remaining three letters can be arranged in p(3, 3) ways = 3! = 6 ways.

∴ Required number of arrangement 6 × 6 = 36 ways.

c) Two rowels can be arranged in 2! ways and 3 consonants can be arranged in 3! ways.

Therefore, total number of arrangement = 2! × 3! = 2 × 1 × 3 × 2 × 1 = 12 ways.

14. In how many ways can the letters of the word Logic be arranged so that

(i) vowels may occupy odd positions?

(ii) no vowel are toether?

Solution:

There are 5 letters in LOGIC, among them 2 are vowels and 3 are consonants.

(i) If vowels occupy odd positions i.e. 1^{st} , 2^{nd} and 3^{rd} positions then they can be arrannded in P(3,2) ways = 6 ways

And the 3 consonants can be arranged in p(3, 3) ways = 3! ways = 6 ways

So, total arrangement = 6 × 6 = 36 ways

(ii) Total arrangements (permutation) = 5 ! = 120

If two vowels are together they are regarded as a single letters then there will be only 4 letters namely, L, (O, I), G, C and can be arranged in 4! ways.

But, the vowels are different, so, placing together, they can be arranged in 2! ways

Now, arrangement that vowels come together = 2! × 4! ways = 2×24 = 48 ways

Hence, arangement that no vowel are together

= Total arrangements – arrangements that vowels come together = 120 – 48 = 72.

Exercise 1.1 (B)

1. In how many ways can eight people be seated in a row of eight seats so that two particular person are always together ?
2. Six children are to be seated on a bench. How many arrangements are possible if the youngest child sits at the left end of the bench.
3. In a certain election of parliament, there are three candidates for president, five for secretary and only to for the treasurer. Find in how many ways the election may turn out.
4. How many different numbers of five digits can be formed with the digits 0, 1, 2, 3, 4
5. How many numbers between 3000 and 4000 can be formed with the digits 2,3,4,5,6,7 ?
6. How many **four digits odd** number can be formed using the digits 0,1,2,3,4,5 no digit being repeated ?

   [Hint: Total odd of four digit – odd of four digit begining with 0

   =3. 5. 4 . 3 - 1. 3. 4. 3 = 180 – 36 = 144]
7. How many numbers of three different digits less than 500 can be formed from the integer 1,2,3,4,5,6?

8. In how many ways can seven boys and three girls be seated in a row containing thirteen seats.
    i) If they may sit anywhere.
    ii) If the boys and girls must alternate.
    iii) If all three girls are together ?
9. There are 5 boys and 4 girls. It is asked to sit in a row so that girls occupy the even places. How many ways such arrangements are possible?
10. In how many ways 5 boys and 4 girls can be seated in a row so that no two girls are together.
11. a. In how many ways can 9 boys sit in a row of 9 seats so that three particular boys are always together?
    b. In a library, there are six different books, and need to arrange in a cubboard. Find the number of ways in which two particular books are (a) always together (b) not together.
12. a. In how many ways can ten people be seated in a round table if two people insists on sitting next to each other ?
    b. In how many ways can 5 Biology group students and 5 physical group students be arranged alternately at a round table ?
    c. In how many ways can eight different coloured beads be made into a bracelet?
    d. In how many ways can a garland of 10 flower be made?
13. How many arrangements can be made from the following words:
    i) INTERVAL Ans: 8!
    ii) MATHEMATICS Ans: $\frac{11!}{2!22!}$
    iii) MISSISSIPPI Ans: $\frac{11!}{4!4!2!}$
    iv) EXAMINATION Ans: $\frac{11!}{2!2!2!}$
14. In how many ways can a man post 5 letters in four different boxes. Ans: 1024
15. There are 3 prizes to be distributed among 5 students so that each students may receive any number of prizes ? Ans: 125
16. In how many way can the letter of word "COMPUTER" be arranged so that
    i. All the vowels are always together.
    ii. The vowels may occupy only odd positions. Ans: i) 3! ii) 2880 ways.

17. In how many ways can the letter of the word "SUNDAY" be arranged ? How many of these arrangements do not begin with S ? How many begin with S and do not end with **a** ? Ans: 120 ways.

18. In how many ways can the letters of the word, "CALCULUS" be arranged so that two L's do not come together ? Ans: 3780

19. In how many ways can the letters of the word "MONDAY" be arranged ? How many of these arrangements do not begin with M ? How many begins with M and does not end with Y ? Ans: 96

20. In how many ways can the letters of the word "TERMINAL" be arranged so that:
    a. All vowels are always together ? Ans: 4320
    b. The vowels may occupy only odd positions? Ans: 2880
    c. The relative positions of vowels and consonants Ans: 720

Answer:

(1)	10080	(2)	120 ways.	(3)	30	(4)	96	(5)	60	(6)	144
(7)	80	(8)	(i) 3628800	(ii)	30240	(iii)	241920	(9)	2880	(10)	43200

(11) (i) 30240 (ii) a) 240 ways b) 480 ways

(12) (i) 80640 (ii) 2880 (iii) 2520 (iv) 181440

(13) i) 8! ii) $\frac{11!}{2!22!}$ iii) $\frac{11!}{4!4!2!}$ iv) $\frac{11!}{2!2!2!}$

(14) 1024 (15) 125 (16) i) 3! ii) 2880 ways.

(17) 120 ways. (18) 3780 (19) 96

(20) a. 4320 b. 2880 c. 720

1.1.4 Combination

A combination is a mathematic technique that determiner the number of possible arrangement in a collection of items where the order of selection does not matter. Indeed, in a combination, you can select the items in any order. But, in permutations, the order of the items is essential for example, the arrangement *x*y and y*x* are equal in combination which is considered as one arrangement.

The selections (groups) of a number of things taken some or all of them at a time are called combinations. In combination, the order is not considered. The total number of combinations of *n* distinct things taken r ($1 \leq r \leq n$) at a time is denoted by ^nc_r or by c(n, r)

Theorem: The total number of combinations of n objects taken r at a time, c(n, r) is given by the expression $c(n, r) = \frac{n!}{(n-r)!r!}$

Solution:

Let x be the required number of combinations. Each of these combinations has r different objects. So, these r objects among themselves can be arranged in r! different ways. So, for each combination, there are r! permutation. Hence, for the x combinations, there are x. r! different permutations. Since, these are all possible permutations of x objects taken r at a time.

We have,

$$x.r! = p(n, r)$$

or, $$x\, r! = \frac{n!}{(n-r)!}$$

or, $$x = \frac{n!}{(n-r)!\, r!}$$

$$\therefore \quad {}^nc_r = \frac{n!}{(n-r)!\, r!} \quad \ldots\ldots\ldots\ (i)$$

Property 1; when r = 0,

$${}^nc_o = \frac{n!}{(n-r)!\, o!} = \frac{n!}{n! \times 1} = 1$$

Property 2: Putting r = 1 in eq^n (i)

$${}^nc_1 = \frac{n!}{(n-1)!\, 1!} = \frac{n(n-1)!}{(n-1)!\, 1!} = n$$

Property 3 Putting r = 2 in eq^n (i)

$${}^nc_2 = \frac{n!}{(n-2)!\, 2!} = \frac{n(n-1)(n-2)!}{(n-2)! \times 2 \times 1} = \frac{n(n-1)}{2}$$

Property 4: Putting r = n in eq^n (i)

$$^nc_n = \frac{n!}{(n-n)!\,n!} = \frac{n!}{o!.n!} = \frac{1}{1} = 1$$

Property 5: Complementary combinations:

$$^nc_{(n-r)} = \frac{n!}{(n-(n-r))!\,(n-r!)!}$$

$$= \frac{n!}{(n-n+r)!\,(n-r)!}$$

$$= \frac{n!}{r!\,(n-r)!}$$

$$= \frac{n!}{(n-r!)\ \ r!}$$

$$= {}^nc_r$$

$$\therefore \quad {}^nc_{(n-r)} = {}^nc_r$$

Property 6: $^nc_r + {}^nc_{(r-1)} = (n+1)_{c_r}$

Solution : Here,

Proof:

$$^nc_r + {}^nc_{r-1}$$

$$= \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-(r-1))!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\ \ r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n!}{(n-r!)\ \ r!} + \frac{n!}{(n-r+1)\,(n-r)!(r-1)!}$$

$$= \frac{n!}{(n-r)!\,r(r-1)!} + \frac{n!}{(n-r+1)\,(n-r)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{1}{r} + \frac{1}{(n-r+1)}\right]$$

$$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{n-r+1+r}{r\,(n-r+1)}\right]$$

$$= \frac{n!}{(n-r)!(r-1)!} \cdot \frac{(n+1)}{r\,(n-r+1)}$$

$$= \frac{(n+1).n!}{r(r-1)!(n-r+1)\,(n-r)\,!}$$

$= \dfrac{(n+1)!}{r!(n-r+1)!}$

$= \dfrac{(n+1)!}{(n+1-r)!\,r!}$

$= {}^{(n+1)}c_r$

$\therefore\quad {}^nc_r + {}^nc_{(r-1)} = {}^{(n+1)}c_r$

Property 7 : If ${}^nc_r = {}^nc_{r'}$ then $r + r' = n$ or $r = r'$

Solution:

${}^nc_r = {}^nc_{r'}$

or, $c(n, r) = c(n, r')$

or, $c(n, r) = c(n, n - r')$

or, $c(n, r) = c(n, n - r')$ [$\because$ Property 5)

or, $r = n - r'$

or, $r + r' = n$

Also,

$C(n, r) = c(n, r')$

Or, $r = r'$

$\therefore$ If ${}^nc_r = {}^nc_{r'}$ then either $r = r'$ or $r + r' = n$

<u>Worked out examples:</u>

1. Find the value of 'r' and c(r, 2); ${}^9c_{2r} = {}^9c_{(3r-1)}$

Solution:

${}^9c_{2r} = {}^9c_{(3r-1)}$

Then, $2r = 3r - 1$ [$\because {}^nc_r = {}^nc_{r'} \Rightarrow r = r'$]

Or, $1 = 3r - 2r$

Or, $r = 1$

And, ${}^9c_{2r} = {}^9c_{(3r-1)}$

Or, $2r + 3r - 1 = 9$ [$\because {}^nc_r = {}^nc_{r'} \Rightarrow r + r' = n$]

Or, $5r = 9 + 1$

Or, $5r = 10$

Or, $5r = 10$

Or, $r = \frac{10}{5} = 2$

$\therefore$ $r = 2.$

Also,

$$C(r, 4) = \frac{r!}{(r-1)!1!} = \frac{2!}{(2-1)!1!} = \frac{2 \times 1}{1 \times 1 \times 1 \times 1} = 2.$$

2. In an examination, an examiner has to secure A+ grade in each of the five subjects. In how many ways can the examinee fail to secure A+ grade ?

Solution:

Total number of subjects(n) = 5

An examiner has to secure A+ grade in each of the five subject i.e. in either 1 or 2 or 3 or 4 or 5 subjects.

$\therefore$ Total number of ways by which the candidates fails to secure A+ grade.

$= C(5, 1) + C(5, 2) + C(5, 3) + C(5, 4) + C(5, 5)$

$$= \frac{5!}{(5-1)!1!} + \frac{5!}{(5-2)!\,2!} + \frac{5!}{(5-3)!3!} + \frac{5!}{(5-4)!\,4!} + \frac{5!}{(5-5)!\,5!}$$

$$= \frac{5!}{4!1!} + \frac{5!}{3!2!} + \frac{5!}{2!3!} + \frac{5!}{2!3!} + \frac{5!}{1!4!} + \frac{5!}{0!5!}$$

$$= \frac{5.4!}{4! \times 1} + \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} + \frac{5 \times 4 \times 3!}{2! \times 3!} + \frac{5 \times 4!}{1 \times 1 \times 4!} + 1$$

$= 5 + 10 + 10 + 5 + 1$

$= 31$

3. In an examination a candidates has to pass in each of the four subjects. In how many ways can the candidate fail ?

Solution:

A candidate can fail in an examination if he fails either in 1 or 2 or 3 or 4 subjects.

Total number of ways by which he can fail = ${}^4c_1 + {}^4c_2 + {}^4c_3 + {}^4c_4$

$$= \frac{4!}{(4-1)!1!} + \frac{4!}{(4-2)!2!} + \frac{4!}{(4-3)!3!} + \frac{4!}{(4-4)!4!}$$

$$= \frac{4!}{3! \times 1} + \frac{4!}{2! \times 2!} + \frac{4!}{1! \times 3!} + \frac{4!}{0! \times 4!}$$

$$= \frac{4 \times 3!}{3!} + \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} + \frac{4 \times 3!}{3!} + \frac{1}{1}$$

$= 4 + 6 + 4 + 1$

$= 15$

4. If there are 10 persons in a party and each two of them shakes hands with each other, how many handshakes happen in the party?

Solution:

Number of persons (n) = 10

Number of persons shaking hands (r) = 2

Total number of hands shakes = C(n , r)

= C(10, 20

$$= \frac{10!}{(10-2)!\ 2!}$$

$$= \frac{10 \times 9 \times 8!}{8! \times 2 \times 1}$$

= 45.

5. From 10 persons, in how many ways can a selection of 4 be made when two particular persons are always included ?

Solution:

Total number of players = 10

Number of players to be selected = 4

When two particular players are excluded, then we have to select 4 players out of = 10 – 2 = 8.

Required number of selections = C(8, 4)

$$= \frac{8!}{(8-4)!4!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1}$$

= 70 ways.

6. Find the number of ways in which 5 courser out of 8 can be selected when 3 course are compulsory ?

Solution:

Total number of friends = 5

Required number of ways =

= C(5, 1) + C(5, 2) + C(5, 3) + C(5, 4) + C(5, 5)

$$= \frac{5!}{(5-1)!\ 1!} + \frac{5!}{(5-2)!\ 2!} + \frac{5!}{(54-3)!\ 3!} + \frac{5!}{(5-4)!4!} + \frac{5!}{(5-5)!5!}$$

$$= \frac{5!}{4!1!} + \frac{5!}{3!2!} + \frac{5!}{2!3!} + \frac{5!}{1!4!} + \frac{5!}{0!5!}$$

= 5 + 5 × 2 + 5 × 2 + 5 + 1

= 5 + 10 + 10 + 6 = 31

7. From 6 gentleman and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at least two gentlemen ?

Solution:

The selection of the members in the committee can be made as follow:

Gentlemen (6)	Ladies (4)	Selection
2	3	$^6c_2 \times {}^4c_3$
3	2	$^6c_3 \times {}^4c_2$
4	1	$^6c_4 \times {}^4c_1$
5	0	$^6c_5 \times {}^4c_0$

$\therefore$ Required number of combinations = $^6c_2 \times {}^4c_3 + {}^6c_3 \times {}^4c_2 + {}^6c_4 \times {}^4c_1 + {}^6c_5 \times {}^4c_0$

$$= \frac{6!}{(6-2)!2!} \times \frac{4!}{(4-3)!3!} + \frac{6!}{(6-3)!3!} \times \frac{4!}{(4-2)!2!} + \frac{6!}{(6-4)!4!} \times \frac{4!}{(4-1)!1!} + \frac{6!}{(6-5)!5!} \times \frac{4!}{(4-0)!0!}$$

$$= \frac{6!}{4!2!} \times \frac{6!}{1!3!} + \frac{6!}{3!3!} \times \frac{4!}{2!2!} + \frac{6!}{2!4!} \times \frac{4!}{3!1!} + \frac{6!}{1!5!} \times \frac{4!}{4!0!}$$

$$= \frac{6 \times 5 \times 4!}{4!2 \times 1} \times \frac{4 \times 3!}{1 \times 3!} + \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2 \times 1} \times \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} + \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!} \times \frac{4 \times 3!}{3! \times 1} + \frac{6 \times 5!}{1 \times 1 \times 5!} \times \frac{4!}{4!1}$$

$= 15 \times 4 + 120 + 15 \times 4 + 6 \times 1$

$= 60 + 120 + 60 + 6$

$= 246.$

8. In how many ways a committee of five can be formed out of 4 men and 3 women so that it includes at least one women ?

Solution:

The selection of member of five person can be made as:

Men (4)	women (3)	Selection
4	1	$^4c_4 \times {}^3c_1$
3	2	$^4c_3 \times {}^3c_2$
2	3	$^4c_2 \times {}^3c_3$

$\therefore$ The total number of committee that can be formed as:

$^4c_4 \times {}^3c_1 + {}^4c_3 \times {}^3c_2 + {}^4c_2 \times {}^3c_3$

$$= \frac{4!}{(4-4)!4!} \times \frac{3!}{(3-1)!\,1!} + \frac{4!}{(4-3)!\,3!} \times \frac{3!}{(3-2)!\,2!} + \frac{4!}{(4-2)!\,2!} \times \frac{3}{(3-3)!3!}$$

$$= \frac{1}{0!} \times \frac{3 \times 2!}{2! \times 1} + \frac{4 \times 3!}{1!3!} \times \frac{3 \times 2!}{0! \times 2!} + \frac{4 \times 3 \times 2!}{2! \times 2 \times 1} \times \frac{3!}{0! \times 3!}$$

$= \quad 1 \times 3 + 4 \times 3 + 6 \times 1 = 3 + 12 + 6 = 21.$

9. From a group of 12 boys and 9 Girls, how many committee consisting of 4 boys and 3 Girls are possible ?

Solution:

4boys can be selected from 12 boys in C(12, 4) ways.

3 Girls can be selected from 9 girls in C(9, 3) ways.

∴ Total number of committees = C(12, 4) × C(9, 30

$$= \frac{12!}{(12-4)!\,4!} \times \frac{9!}{(9-3)!3!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8!}{8! \times 4 \times 3 \times 2 \times 1} \times \frac{9 \times 8 \times 7 \times 6!}{6! \times 3 \times 2 \times 1}$$

$$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} \times \frac{9 \times 8 \times 7}{3 \times 2 \times 1}$$

$= \quad 3 \times 5 \times 11 \times 3 \times 3 \times 4 \times 7$

$=$ 41580 ways.

10. From 4 mathematician, 6 statistician and 5 economists, how many committees of 6 members can be formed so as to include 2 member from each category ?

Solution:

2mathematicians can be selected from 4 mathematician in C(4, 2) ways.

2 statisticians can be selected from 6 statistician in C(6, 2) ways.

2 economists can be selected from 5 economist in C(5, 2) ways,

∴ Number of committees = C(4, 2) × C(6, 2) × C(5, 2)

$$= \frac{4!}{(4-2)!2!} \times \frac{6!}{(6-2)!\,2!} \times \frac{5!}{(5-2)!\,2!}$$

$$= \frac{4!}{2!\,2!} \times \frac{6!}{4!\,2!} \times \frac{5!}{3!\,2!}$$

$$= \frac{4 \times 3 \times 2!}{2! \times 2 \times 1} \times \frac{6 \times 5 \times 4!}{4! \times 2 \times 1} \times \frac{5 \times 4 \times 3!}{3! \times 2 \times 1}$$

$= \quad 6 \times 15 \times 100$

$=$ 900 ways.

11. A person has got 12 acquaintance of whom 8 are relative. In how many ways can he invite 7 guests so that 5 of them may be relatives?

Solution:

Number of relatives = 8

Number of other = 12 – 8 = 4

5 relatives can be selected from 8 in C(8, 5) ways.

2other can be selected from 4 in C(4, 2) ways.

Total number of selection = C(8, 5) × C(4, 2)

$$= \frac{8!}{(8-5)!\,5!} \times \frac{4!}{(4-2)!\,2!}$$

$$= \frac{8 \times 7 \times 6 \times 5!}{3! \times 5!} \times \frac{4 \times 3 \times 2!}{2! \times 2!}$$

$$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1}$$

$= \quad 4 \times 7 \times 2 \times 2 \times 3$

$= \quad 16 \times 21$

= 336 ways.

12. A man has 5 friends. In how many ways can he invite one or more of them to a dinner.

Solution:

Since a man can invite one, two three, four or five from 5 friends.

∴ Total number of selection is ${}^4c_1 + {}^5c_2 + {}^5c_3 + {}^5c_4 + {}^5c_5$

$$= \frac{5!}{(5-1)!\,1!} + \frac{5!}{(5-2)!\,2!} + \frac{5!}{(5-3)!\,3!} + \frac{5!}{(5-4)!\,4!} + \frac{5!}{(5-5)!5!}$$

$$= \frac{5 \times 4!}{4! \times 1} + \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} + \frac{5 \times 4 \times 3!}{2! \times 3!} + \frac{5 \times 4!}{1! \times 4!} + \frac{5!}{0! \times 5!}$$

$= \quad 5 + 10 + 10 + 5 + 1$

$= \quad 31$

13. From 10 players in how many ways can a selection of 4 be made, when one particular player is always included when two particular players are excluded ?

Solution:

Total number of players = 10.

Number of players to be selected = 4.

When one particular player is always included then we have to select 4 – 1 = 3 player out of 10 – 1 = 9.

Total number of selection = C(9, 3)

$= \dfrac{9!}{(9-3)!\,3!}$

$= \dfrac{9 \times 8 \times 7 \times 6!}{6! \times 3 \times 2 \times 1}$

$= 3 \times 4 \times 7$

$= 84$

Again, when two particular players are excluded.

We have to select 4 persons out of (10 – 2) = 8.

Required number of selections = C(8, 4) ways

$= \dfrac{8!}{(8-4)!\,4!}$

$= \dfrac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1} = 70$ ways.

Exercise1.1(c)

1. (a) If ${}^{n}c_{9} = {}^{n}c_{7}$, find the value ${}^{n}c_{3}$.

   (b) If ${}^{15}c_{r} = {}^{15}c_{(r+2)}$, find the value of r.

2 (a) In an examination paper containing 10 questions a candidate has to answer 7 questions only, in how many ways can be choose the questions ?

   (b) Find the number of ways in which 5 courses out of 8 can be selected when 3 courses are compulsory.

   (c) A person has got 12 acquaintances of whom 8are relatives. In how many ways can be invite 7 guests so that 5 of them may be relatives?

   (d) A committee is to be chosen from 12 men and 8 women and is to consist of 3 men and 2 women. How many such committee can be formed?

   (e) How many different sums of money can be made from 4 coins of different denominators?

3. How many committees can be formed from a set of 8 men and 6 women if each committee contains 5 men and 4 women?

4. In how many ways a committee of five can be formed out of 4 men and 3women so that it includes at least one women?

5. In how many ways a committee of three person can be formed out of 4 men and 3 women so that it includes at least one woman?

6. An examination paper consisting of 10 questions, is divided into two groups A and B. Group A contains 6 questions. In how many ways can an examinee attempt 7 questions selecting at least two questions from each group?

7. From 3 men and 7 women a committee of 5 is to be formed. In how many ways can this be done so as to include at least one man?
8. In how many ways a committee of three can be formed out of 5 men and 2 women so that it always consists at least one women?
9. A committee of 5 is to be formed out of 6 gents and 4 ladies. In how many ways can this be done when at least two ladies are to be included?
10. A committee of five is to be constituted from six boys and five girls. In how many ways can this be done so as to include at least one boy and one girl?
11. A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he/she is not permitted to attempt more than 4 from any group. In how many different ways can be make up his choice.
12. From 6 gentleman and 4 ladies a committee of 5 is to be formed. In how many ways can there be done so as to include at least one lady?
13. If C(n, r – 1) = 36, C(n, r) = 84 and C(n, r + 1) = 126, find the value of r and n.
14. Prove that C(n, r) + C(n, r – 1) = C(n + 1, r), where C(n, r) is the combination of n things taken r at a time.
15. A box contains 7 red, 6 white and 4 blue balls. In many ways can a selection of 3 balls be made, if all the balls are are red?
16. An examination paper consisting of 10 questions, is divided into two groups P and Q. Group P contains 6 questions. In how many ways can an examinee attempt 7 questions?
    (a) Selecting 4 from group P and 3 from group Q ?
    (b) Selecting at least two questions from each group?
17. In how many ways can 6 member forming a committee out of 12 be selected so that:
    (a) Two particular members must be included.
    (b) Two particular members must be excluded.
18. 153 matches were played in a cricket tournament. If every team played a match with one another, then find the total number of teams in a tournament. [Hint: ${}^{n}c_{2} = 153$]

Answers

1. (a) 455 (b) $\frac{13}{2}$

2. 2(a) 120 (b) 10 (c) 336 (d) 6160 (e) 15

3. 840 4. 21 5. 19 6. 116 7. 231 8. 25 9. 186 10. 455

11. 200 12. 246 13. r = 3 , n= 9. 15. 35 16. (a) 60 (b) 116

17. (a) 210 (b) 210 18. 18.

Objective questions.

Tick the best answers:

1. The number of ways in which 4 boys and 3 girls can be arranged in a row containing seven seats is:
   (a) 6!   (b) 7!   (c) 4! 3!   (d) 3.4!
2. The number of permutations of n distinct objects taken r at a time is given by
   (b) n!   (b) $\frac{n!}{(n-1)!}$ (c) $\frac{n!}{(n-1)!r!}$ (d) n^r
3. In how many ways can 8 people and a host be arranged in a circular table of a party?
   (a) 8! ways   (b) 7! ways   (c) 9! ways   (d) (9!– 8!) ways.
4. The number of ways in which 3 prizes can be distributed among 4 boys if no boy gets all the prizes is;
   (a) 64   (b) 60   (c) 4   (d) 3!4!
5. If P(n,r) = C(n,r–1), then which one of the following is true?
   (a) ${}^{12}c_7$ (b) ${}^{11}c_6$ (c) ${}^{12}c_5$ (d) ${}^{12}c_6$

(b) If a polygon has same number of sides as its diagonals, then the number of sides is:
   (a) 5   (b) 6   (c) 7   (d) 8

(c) Out of 6 books, in how many ways can a set of one or more books be chosen?
   (a) 2^6 (b) $2^6–2$ (c) $1+2^6$ (d) $2^6 –1$

(d) There are 5 subjects in an examination .in how many ways students may fail ?
   (a) 32   (b) 30   (c) 31   (d) 62

(e) The number of ways in which a student can select one or more questions out of 12 each having an alternative is:
   (a) 3^{12} (b) $3^{12}–1$ (c) 12^3 (d) 21^2

(b) The number of possible outcomes in a throw of n ordinary dice in which at least one of the dices shows an odd number is
   (a) 6^n (b) $6^n–1$ (c) $3^n–1$ (d) $6^n –3^n$

Answers

1	2	3	4	5	6	7	8	9	10
b	b	a	b	b	a	d	c	b	d

Project works and activities

1. You have recently taken license plate making. Each license plate contains 4 letters and 3 numbers. Your friend asked you to make them in following:
   A license plate containing no vowels
   Total possibilities…………………….
   A license plate containing no repeating letters or numbers
   Total possibilities…………………….

❑❑❑