3

Sequence and Series Conti..

1.6.1 Sum of the Natural Numbers:

All the counting numbers which is started from 1 i.e. 1, 2, 3, 4 ………… are said to be natural numbers. Indeed, in this section we derive the formulae for the sum of first n-natural numbers, squares of first n-natural numbers and cubes of first n-natural numbers

(i) Sum of first n-natural numbers

The first n-natural numbers are 1, 2, 3, ………. n and let S_n be its sum. i.e.

$S_n = 1 + 2 + 3 + 4 + \dots + n$

Here, first term (a) = 1, common difference (d) =1, no. of terms = n

We have,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n - 1).\ 1] = \frac{n}{2}[2 + n - 1]$$

$$= \frac{n}{2}[n + 1] = \frac{n(n + 1)}{2}$$

$$\therefore S_n = \frac{n(n + 1)}{2}$$

ii) Sum of first n-even natural numbers.

The first n-even natural numbers are 2, 4, 6, 8, ………. , n terms and le S_n be its sum. Then,

$S_n = 2 + 4 + 6 + 8 + \dots$ to n terms

Or, $S_n = \frac{n}{2}[2 \times 2 + (n - 1).2]$

$$= \frac{n}{2}[4 + 2n - 2] = \frac{n}{2}[2n + 2] = \frac{2n(n + 1)}{2} = n(n + 1)$$

Next Method:-

$\mathbf{S_n} = 2 + 4 + 6 + ...$

$= 2\,(1 + 2 + 3 + ...)$

$= 2.\, n(n+1)/2$

$= n(n+1)$

iii) Sum of first n - odd natural numbers.

The first n-odd natural numbers are;

1, 3, 5, ………….and let S_n be its sum.

$S_n = 1 + 3 + 5 + \ldots\ldots\ldots\ldots$ to n terms

$= \frac{n}{2}[2 \times 1 + (n-1) \times 2] = \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2} \times 2n = n^2$

(iv) Sum of squares of first n- natural numbers

Sum of first n-natural number are 1, 2, 3, 4, …………. n where $1^2, 2^2, 3^2, \ldots\ldots\ldots, n^2$ are the squares of first n-natural numbers and let S_n be its sum. Then,

$S_n = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^2$

Now, we will use the following **identity** to find the value of S_n;

We know that,

$x^3 - (x-1)^3 = 3x^2 - 3x + 1$

Substituting, x = 1, 2, 3, 4 …………… n; we get,

$1^3 - (1-1)^3 = 3.1^2 - 3.1 + 1$

$2^3 - (2-1)^3 = 3.2^2 - 3.2 + 1$

$3^3 - (3-1)^3 = 3.3^2 - 3.3 + 1$

…………………………….

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$n^3 - (n-1)^3 = 3.n^2 - 3.n + 1$

Adding above equations we get

$n^3 - (1-1)^3 = 3\,(1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^2) - 3\,(1 + 2 + 3 + \ldots\ldots\ldots + n) + n$

or, $n^3 - o^3 = 3S_n - 3.\,\frac{n(n+1)}{2} + n$

or, $n^3 + \frac{3n(n+1)}{2} - n = 3S_n$

or, $3S_n = \frac{2n^3 + 3n\,(n+1) - 2n}{2}$

or, $S_n = \dfrac{2n^3 + 3n^2 + 3n - 2n}{2 \times 3}$

or, $S_n = \dfrac{2n^3 + 3n^2 + n}{2 \times 3}$

or, $S_n = \dfrac{n(2n^2 + 3n + 1)}{6}$

or, $S_n = \dfrac{n\{2n^2 + 2n + n + 1\}}{6}$

or, $S_n = \dfrac{n\{2n(n+1) + 1(n + 1)\}}{6}$

or, $S_n = \dfrac{n(n + 1)(2n + 1)}{6}$

$\therefore S_n = \dfrac{n(n + 1)(2n + 1)}{6}$ is the required formula for the sum of square of first n-natural numbers.

Example:
Find the sum of the squares of first 50-natural numbers.
Solution:
Number of terms (n) = 50
We know that the sum of the squares of first n-natural numbers (S_n) = ?

Then, $S_n = \dfrac{n(n + 1)(2n + 1)}{6}$

Or, $S_{50} = \dfrac{50(50 + 1)(2 \times 50 + 1)}{6}$

$= \dfrac{50 \times 51 \times 101}{6}$

$= 25 \times 27 \times 101$

$= 42925$

(v) Sum of cubes of the first n-natural numbers:
Solution:
Sum of first n-natural number are 1, 2, 3, 4, ……….n and 1^3, 2^3,3^3,………….., n^3 are the cubes of first n-natural numbers and let $S_n = 1^3 + 2^3 + 3^3 + \dots + n^3$
We Know that,
$x^4 - (x - 1)^4 = (x^2)^2 - \{(x - 1)^2\}^2$
$= (x^2)^2 - \{(x - 1)^2\}^2$
$= (x^2 + (x - 1)^2)(x^2 - (x - 1)^2)$
$= (x^2 + x^2 - 2x + 1)(x^2 - x^2 + 2x - 1)$

$= (2x^2 - 2x + 1)\ (2x - 1)$
$= 4x^3 - 4x^2 + 2x - 2x^2 + 2x - 1$
$= 4x^3 - 6x^2 + 4x - 1$
$\therefore x^4 - (x - 1)^4 = 4x^3 + 4x - 1$
Putting x = 1, 2, 3, 4, ……………… n is succession, we have.
$1^4 - (1 - 1)^4 = 4.1^3 - 6.1^2 + 4.1 - 1$
$2^4 - (2 - 1)^4 = 4.2^3 - 6.2^2 + 4.2 - 1.$
$3^4 - (3 - 1)^4 = 4.3^3 - 6.3^2 + 4.3 - 1.$
………………………………….
………………………………….
………………………………….
$n^4 - (n - 1)^4 = 4.n^3 - 6.n^2 + 4.n - 1$
Adding above equations; we get
$n^4 - (n - n)^4 = 4(1^3 + 2^3 + 3^3 + \ldots + n^3) - 6(1^2 + 2^2 + 3^2 + \ldots. + n^2) + 4(1 + 2 + \ldots + n) - n$
or, $n^4 - 0^4 = 4S_n - \frac{6n(n + 1)\ (2n + 1)}{6} + \frac{4n(n + 1)}{2} - n$
or, $n^4 = 4S_n - n(n + 1)\ (2n + 1) + 2n\ (n + 1) - n$
or, $n^4 + n(n + 1)\ (2n + 1) - 2n(n + 1) + n = 4S_n$
or, $n\{n^3 + (n + 1).\ (2n + 1) - 2(n + 1) + 1\} = 4S_n$
or, $n(n^3 + 2n^2 + n + 2n + 1 - 2n - 2 + 1) = 4S_n$
or, $n(n^3 + 2n^2 + n) = 4S_n.$
or, $n^2\ (n^2 + 2n + 1) = 4S_n.$
or, $\frac{n^2\ (n + 1)^2}{4} = S_n$
or, $S_n = \left[\frac{n\ (n + 1)}{2}\right]^2$

Corollary 1

If $1^3, 2^3, 3^3$,…………… n^3 be the cube of first n-natural numbers and S_n be its sum. Then

$$S_n = \left[\frac{n\ (n + 1)}{2}\right]^2 = \frac{n^2(n + 1)^2}{4}$$

Proof:-

$S_n = 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots. + n^3$
$S_1 = 1^3 = 1^2$
$S_2 = r^3 + 2^3 = 1 + 8 = 9 = (1 + 2)^2$
$S_3 = 1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36 = (1 + 2 + 3)^2$

$S_4 = 1^3 + 2^3 + 3^3 + 4^3 = 1 + 8 + 27 + 64 = 100 = (1 + 2 + 3 + 4)^2$

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$S_n = 1^3 + 2^3 + 3^3 \ldots\ldots\ldots + n^3 = (1 + 2 + 3 + \ldots\ldots\ldots\ldots + n)^2$

$= \left[\frac{n\,(n+1)}{2}\right]^2$

$= \frac{n^2(n+1)^2}{4}$

Remarks:

1.$\Sigma n = 1 + 2 + 3 + \ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}.$

2. $\Sigma n^2 = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^2 = \frac{n(n+1)\,(2n+1)}{6}$

3. $\Sigma n^3 = 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + n^3 = \left[\frac{n\,(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}.$

4. $\Sigma 1 = n$, $\Sigma 2 = 2n$, $\Sigma 3 = 3n$ and soon.

1.6.2 Sum of the series using the method of successive differences:

Sum of n terms in a sequence can be evaluated only if we know the type of sequence it is. Usually, we consider arithmetic progression at first if not we go to the geometric progression while calculating the sum of n number of terms. If the first difference or first ratio of the given series may not be in A.S. or G.S. But the difference or ratio of the successive terms of the given series will be in A.S. or G.S we can find the n^{th} terms and corresponding sum of n terms as well.

Example:

Find the n^{th} term and the sum of the first n terms of the series: 1 + 3 + 6 + 10 +……….

Solution: Here,

The given series is 1, 3, 6, 10, ………..and s be its sum.

$S = 1 + 3 + 6 + 10 + \ldots\ldots\ldots\ldots + t_n$

$S = 1 + 3 + 6 + \ldots\ldots\ldots\ldots\ldots\ldots t_{n-1} + t_n$

Subtracting above series; we get

$$\begin{aligned} S &= 1 + 3 + 6 + 10 + \ldots\ldots\ldots + t_n \\ -S &= \pm 0 \pm 1 \pm 3 \pm 6 \pm \ldots\ldots\ldots \pm t_{n-1} \pm t_n \\ \hline 0 &= 1 + 2 + 3 + 4 + \ldots\ldots\ldots\ldots + (t_n - t_{n-1}) - t_n \end{aligned}$$

Or, $t_n = 1 + 2 + 3 + 4 + \ldots\ldots\ldots\ldots +$ n terms

Or, $t_n = \frac{n(n+1)}{2}$

Then, $S_n = \Sigma t_n$

$= \Sigma \frac{n(n+1)}{2}$

$= \frac{1}{2} [\Sigma (n^2 + n)]$

$= \frac{1}{2} [\Sigma n^2 + \Sigma n]$

$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$

$= \frac{1}{2}\frac{n(n+1)}{2}\left[\frac{(2+1)}{3} + 1\right]$

$= \frac{n(n+1)}{4} \cdot \frac{(2n+1+3)}{3}$

$= \frac{n(n+1)(2n+4)}{12} = \frac{n(n+1)(n+2)}{6}$

1.6.3 Arithmetico -Geometric Series:

In mathematics an arithmetico-geometric sequence/progression is the result of the term by term multiplication of a geometric progression with the corresponding terms of an arithmetic progression. Then, the n^{th} term of an arithmetico-geometric sequence is the product of the n^{th} term of an arithmetic sequence and n^{th} term of a geometric one.

The terms of an A.S. are a, a + d, a + 2d, ….

The terms of a G.S. are 1, r, r^2,……….

A series of the type a.1 + (a + d). r + (a + 2d).r^2 + ……………

Whose each term is the product of the corresponding terms of an A.S. and G.S. is known as the arithmetic-geometric series.

Example:- $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots = 1.1 + 4.\frac{1}{5} + 7.\frac{1}{5^2} + 10.\frac{1}{5^3} + \ldots$

where, 1, 4, 7, 10,... are in A.P. and $1, \frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3}$ are in G.P.

Note:- See example -5 for the sum.

<u>**Worked out Examples:**</u>

Example 1

Find the sum to n terms of the series; 1.5 + 3.7 + 5.9 + ……..

Solution: Here, The given series are

1.5 + 3.7 + 5.9 + . …………

Let S_n be its sum then,

$S_n = 1.5 + 3.7 + 5.9 + \ldots\ldots +n$ terms

The r^{th} term of 1, 3, 5, 7, ………… is $1 + (r - 1) \times 2$

$= 1 + 2r - 2$

$= (2r - 1)$

Again, The r^{th}terms of 5, 7, 9,11, ………. is

$5 + (r - 1) \times 2$

$= 5 + 2r - 2$

$= 2r + 3$

The general term of the given series $(t_r) = (2r - 1)(2r + 3)$

$= 2r(2r + 3) - 1(2r + 3)$

$= 4r^2 + 6r - 2r - 3$

$= 4r^2 + 4r - 3$

Thus, the sum of n terms of the given series is $S_n = \sum_{r=1}^{n} t_r$

Or, $S_n = \sum_{r=1}^{n} (4r^2 + 4r - 3)$

$= \sum_{r=1}^{n} 4r^2 + \sum_{r=1}^{n} 4r - \sum_{r=1}^{n} 3$

$= 4 \sum_{r=1}^{n} r^2 + 4 \sum_{r=1}^{n} r - 3n$

$= 4(1^2 + 2^2 + 3^2 + \ldots\ldots + n^2) + 4(1 + 2 + 3 + \ldots + n) - 3n$

$= 4.\dfrac{n(n+1)(2n+1)}{6} + \dfrac{4n(n+1)}{2} - 3n$

$= \dfrac{2n(n+1)(2n+1)}{3} + \dfrac{4n(n+1)}{2} - \dfrac{3n}{1}$

$= \dfrac{4n(n+1)(2n+1) + 12n(n+1) - 18n}{6}$

$= \dfrac{4n(2n^2 + n + 2n + 1) + 12n^2 + 12n - 18n}{6}$

$= \dfrac{8n^3 + 12n^2 + 4n + 12n^2 + 12n - 18n}{6}$

$= \dfrac{8n^3 + 24n^2 + 16n - 18n}{6}$

$= \dfrac{4n^3 + 12n^2 - n}{3}$

$$= \frac{n(4n^2 + 12n - 1)}{3}$$

Example 2:

Find the sum to n terms of the series: $1^2.1 + 2^2.3 + 3^2.5 + \ldots\ldots\ldots$

Solution: Here, Give series is

$1^2.1 + 2^2.3 + 3^2.5 + \ldots\ldots\ldots\ldots\ldots$

Let $S_n = 1^2 .1 + 2^2.3 + 3^2.5 + \ldots$ to n terms

The n^{th} term of $1^2, 2^2, 3^3, \ldots$ is n^2

The n^{th} term of 1, 3, 5, … n terms is $1 + (n - 1) \times 2 = 1 + 2n - 2 = 2n - 1$

The n^{th} term of the give series $(t_n) = n^2. (2n - 1) = 2n^3 - n^2$

Sum of n^{th} terms $(S_n) = \Sigma t_n = \Sigma\, (2n^3 - n^2)$

Or, $S_n = 2\Sigma n^3 - \Sigma n^2$.

$$= 2 . \frac{n^2 (n + 1)^2}{4} - \frac{n(n + 1) (2n +1)}{6}$$

$$= \frac{n (n + 1)}{2}\left[\frac{n(n + 1)}{1} - \frac{(2n + 1)}{3}\right]$$

$$= \frac{n (n + 1)}{2}\left[\frac{n(n + 1) - (2n+1)}{3}\right]$$

$$= \frac{n(n + 1) (3n^2 + n - 1)}{6}$$

Example3

Find the sum of n terms of the series $6 + 66 + 666 + \ldots\ldots\ldots\ldots$

Solution: Here,

The given series is $6 + 66 + 666 + \ldots\ldots\ldots$ and S_n be its sum. Then,

$S_n = 6 + 66 + 666 + \ldots\ldots\ldots + n$ terms.

$= 6\,[1 + 11 + 111 + \ldots\ldots\ldots\ldots$ n terms]

$= \frac{6}{9} \times 9\,[1 + 11 + 111 + \ldots\ldots\ldots\ldots$ n terms]

$= \frac{2}{3}.\,[9 + 99 + 999 + \ldots\ldots\ldots\ldots$ n terms]

$= \frac{2}{3}\,[(10 - 1) + (100 - 1) + (1000 - 1) + \ldots\ldots\ldots\ldots$n terms]

$= \frac{2}{3}\,[(10 + 100 + 1000 + \ldots\ldots\ldots\ldots\ldots\ldots$+n terms$) - (1 + 1 + 1 + \ldots\ldots\ldots\ldots$n terms)]

$$= \frac{2}{3}\left[\frac{10(10^n - 1)}{(10 - 1)} - n\right]$$

$= \frac{2}{3} \times \frac{10}{9} (10^n - 1) - \frac{2n}{3}$

$= \frac{20}{27} (10^n - 1) - \frac{2n}{3}$

Example4

Find the sum of n terms of the series 0.6 + 0. 66 + 0.666 + ………..

Solution: Here,

The given series is 0.6 + 0.66 +0.666 + ……… and S_n be its sum. Then,

$S_n = 0.6 + 0.66 + 0.666 + \ldots\ldots + \text{n terms.}$

$= 0.6\ [0.1 + 0.11 + 0.111 + \ldots\ldots \text{ n terms}]$

$= \frac{6}{9} \times 9\ [0.1 + 0.11 + 0.111 + \ldots\ldots \text{ n terms}]$

$= \frac{2}{3}\ [0.9 + 0.99 + 0.999 + \ldots\ldots \text{ n terms}]$

$= \frac{2}{3}\ [(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \ldots\ldots \text{n terms}]$

$= \frac{2}{3}\ [(1 + 1 + 1 + \ldots\ldots + \text{n terms}) - (0.1 + 0.01 + 0.001 + \ldots\ldots \text{n terms})]$

$= \frac{2}{3} \left[n - 0.1 \left\{ \frac{1-(0.1)^n}{1-0.1} \right\} \right]$

$= \frac{2}{3} \left[n - \frac{1}{9} \{1 - (0.1)^n\} \right]$

Example 5:

Find the sum of n-terms of series: $1 + \frac{4}{5} + \frac{7}{5^2} + \ldots\ldots$

Solution: Given,

The given series is;

$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots\ldots$

Then,

n^{th} term of 1, 4, 7, 10, …… is $1 + (n - 1) \times 3 = 1 + 3n - 3 = (3n - 2)$

and n^{th} term of $5^0, 5^1, 5^2, 5^3, \ldots\ldots$ is 5^{n-1}.

$\therefore$ The general term, i.e. n^{th} term of the given series is $\frac{(3^{n-2})}{5^{n-1}}$.

Let S_n be its sum of n terms of the given series. Then

$S_n = 1 + \frac{4}{5} + \frac{7}{5^2} + \ldots\ldots + \frac{3n-5}{5^{n-2}} + \frac{3n-2}{5^{n-1}}$

$$\pm\frac{1}{5}Sn = \pm\frac{1}{5} \pm \frac{4}{5^2} \pm \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \pm \frac{3n-5}{5^{n-1}} \pm \frac{3n-2}{5^n}$$

$$\frac{4}{5}Sn = 1 + \frac{3}{5} + \frac{3}{5^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{3}{5^{n-1}} - \frac{3n-2}{5^n}$$

$$\frac{4}{5}Sn = 1 + 3\left(\frac{1}{5} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{5^{n-1}}\right) - \frac{3n-2}{5^n}$$

Or, $\frac{4}{5}S_n = 1 + 3.\dfrac{\frac{1}{5}\left(1 - \frac{1}{5^{n-1}}\right)}{1-\frac{1}{5}} - \frac{3n-2}{5^n}$

Or, $\frac{4}{5}S_n = 1 + 3 \times \frac{1}{5} \times \frac{5}{4}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{3n-2}{5^n}$

Or, $\frac{4}{5}S_n = 1 + \frac{3}{4}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{3n-2}{5^n}$

Or, $\frac{4}{5}S_n = 1 + \frac{3}{4} - \frac{15}{4.5^n} - \frac{3n-2}{5^n}$

Or, $\frac{4}{5}S_n = \frac{7}{4} - \frac{15 + 12n - 8}{4.5^n}$

Or, $\frac{4}{5}S_n = \frac{7}{4} - \frac{12n + 7}{4.5^n}$

Or, $\frac{4}{5}S_n = \frac{28 - 12n - 7}{4.5^n}$

Or, $S_n = \frac{21 - 12n}{4.5^n} \times \frac{5}{4}$

$S_n = \frac{5\,(21 - 12n)}{16.5^n}$

Example 6

Find the sum to n-terms of the series: $\frac{2}{5} - \frac{6}{5^2} + \frac{10}{5^3} - \frac{16}{5^4} + \ldots\ldots$

Solution: Given series is

$\frac{2}{5} - \frac{6}{5^2} + \frac{10}{5^3} - \frac{14}{5^4} + \ldots\ldots\ldots$

Let S_n be its sum. Then,

$S_n = \frac{2}{5} - \frac{6}{5^2} + \frac{10}{5^3} - \frac{14}{5^4} + \ldots\ldots\ldots\ldots + (-1)^n\frac{4n-6}{5^{n-1}} + (-1)^{n+1}\frac{4n-2}{5^n}$ ....(i)

Or, $\frac{1}{5}S_n = \frac{2}{5^2} - \frac{6}{5^3} + \frac{10}{5^4} \ldots\ldots\ldots\ldots\ldots + (-1)^n.\frac{4n-6}{5^n} + (-1)^{n+1}.\frac{4n-2}{5^{n+1}}$ ...(ii)

On adding (i) and (ii), we get

Or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{4}{5^2} + \frac{4}{5^3} - \frac{4}{5^4} + \text{........} + (-1)^n\frac{4}{5^n} + (-1)^{n+1}.\frac{4n-2}{5^n+1}.$

Or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{4}{5^2}\left[1 - \frac{1}{5} + \frac{1}{5^2} - \text{........ (n – 1) terms}\right] + (-1)^{n+1}.\frac{4n-2}{5^n+1}.$

Or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{4}{5^2} \times \frac{\left[1-\left(\frac{1}{5}\right)^{n-1}\right]}{\left(1-\frac{1}{5}\right)} + (-1)^{n+1}\frac{(4n-2)}{5^{n-1}}$

Or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{4}{5^2} \times \frac{5}{4}\left(1-\frac{1}{5^{n-1}}\right) + (-1)^{n+1}\frac{(4n-2)}{5^{n+1}}$

Or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{1}{5}\left(1-\frac{1}{5^{n-1}}\right) + (-1)^{n+1}\left(\frac{4n-2}{5^{n+1}}\right)$

$\frac{6}{5}S_n = \frac{2}{5} - \frac{2}{15}\left(1-\frac{(-1)^{n-1}}{5^{n-1}}\right) + (-1)^{n+1}.\frac{4n-2}{5^{n+1}}$

$\frac{6}{5}S_n = \frac{2}{5} - \frac{2}{15} + \frac{2}{3}\frac{(-1)^{n-1}}{5^n} + (-1)^{n+1}.\frac{4n-2}{5^{n+1}}$

$= \frac{4}{15} + \frac{2}{3}.\frac{(-1)^{n-1}}{5^n} + (-1)^{n+1}\frac{4n-2}{5^{n+1}}$

$S_n = \frac{5}{6}\left(\frac{4}{15} + \frac{2}{3}.\frac{(-1)^{n-1}}{5^n} + (-1)^{n+1}.\frac{4n-2}{6.5^{n+1}}\right)$

$= \frac{2}{9} + \frac{1}{9}.\frac{(-1)^{n-1}}{5^{n-1}} + (-1)^{n+1}\frac{(4n-2)}{6.5^n}$

$= \frac{2}{9} + \frac{(-1)^{n-1}}{9.5^{n-1}} + (-1)^{n+1}.\frac{(4n-2)}{6.5^n}$

Example 7

Sum to infinity:

$1- 5x + 9x^2 - 13x^3 + \dots \dots \dots$ to ∞, when, $|x| < 1$

Solution:-

Let $S_\infty = 1- 5x + 9x^2 - 13x^3 + \dots \dots \dots$ to ∞ .....(i). Then

$xS_\infty = \quad x - 5x^2 + 9x^3 + \dots \dots \dots$ to ∞ ....(ii)

Adding,

$(1 + x) S_\infty = 1- 4x + 4x^2 - 4x^3 + \dots \dots \dots$ to ∞

$\Rightarrow (1 + x) S_\infty = 1- 4x(1 - x + x^2 - \dots \dots \dots$ to $\infty)$

$\Rightarrow (1 + x) S_\infty = 1- 4x. \frac{1}{1-(-x)}$ [since, $S_\infty = \frac{\text{first term}}{1-\text{comon ratio}}$]

$\Rightarrow (1 + x) S_\infty = 1- \frac{4x}{1-(-x)}$

$\Rightarrow (1 + x)\, S_\infty = 1 - \frac{4x}{1+x}$

$\Rightarrow (1 + x)\, S_\infty = \frac{1+x-4x}{1+x}$

$\Rightarrow (1 + x)\, S_\infty = \frac{1-3x}{1+x}$

$\Rightarrow S_\infty = \frac{1-3x}{(1+x)^2}$

Exercise 1.6 (B)

1.Find the n^{th} term and the sum of the first n-terms of each of the following series:

(a) 1 + 4 + 9 + 16 + ……………..

(b) 1.3 + 2.5 + 3.7 + ………..

(c) 3 + 7 + 13 + 21 +31 + ………….

(d) 5 + 7 + 13 + 31 + 85 + ………..

2. Find the general term and also the sum of n-terms of the series.

1.n + 2. (n – 1) + 3.(n – 2) + ………….

3.Find the sum to n-terms of the series.

(a) $3.2 + 5.2^2 + 7.2^3 + \ldots$

(b) $1^2.1 + 2^2.3 + 3^2.5 + \ldots$

(c) $1.1^2 + 4.2^2 + 7.3^2 + \ldots$

(d) $1^2.2 + 2^2.3 + 3^2.4 + \ldots$

3. Find the sum of n^{th} terms of the series:

(a) $1 + \frac{4}{5} + \frac{7}{5^2} + \ldots$

(b) $1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots$

(c) $\frac{2}{5} - \frac{6}{5^2} + \frac{10}{5^3} - \frac{14}{5^4} + \ldots$

(d) $\frac{1}{3} + \frac{4}{3^2} + \frac{7}{3^3} + \frac{10}{3^4} + \ldots$

4. Find the sum of n-terms of the following series:

(a) 5 + 55 + 555 + 5555 + …………..

(b) 0.6 + 0.66 +0.666 + 0.6666 + ………

(c) $(x + a) + (x^2 + 2a) + (x^3 + 3a) + \ldots$

(d) $1 + 2x + 3x^2 + 4x^3 \ldots$

5. Find the sum to infinity:

(a) $\frac{1}{3} + \frac{4}{3^2} + \frac{7}{3^3} + \frac{10}{3^4} + \ldots$

(b) $1 + 3x + 5x^2 + 7x^3 + \ldots\ldots..\infty$ $(|x| < 1)$

(c) $1 - 5x + 9x^2 - 13x^3 + \ldots\ldots\ldots\ldots\ldots \infty$ $(|x| < 1)$

6. If $|x| < 1$ and $y = x + x^2 + x^3 + \ldots\ldots\ldots\ldots\ldots$ Prove that $x = \dfrac{y}{1 + y}$

Answers:

1(a) $n^2, \dfrac{n(n + 1)(2n + 1)}{6}$ (b) $2n^2 + 2, \dfrac{n(n + 1)(4n + 5)}{12}$

(c) $n^2 + n + 1, \dfrac{n}{3}(n^2 + 3n + 5)$ (d) $4 + 3^{n-1}, 4n + \dfrac{1}{2}(3^n - 1)$

2. $(n - r + 1), \dfrac{n(n + 1)(n + 2)}{6}$

3. (a) $2 + 2^{n+1} + (n - 1)2^{n+2}$ (b) $\dfrac{1}{6}n(n + 1)(3n^2 + n - 1)$

(c) $\dfrac{n(n + 1)(9n^2 + n - 4)}{12}$ (d) $(n^3 + n^2), \dfrac{n(n + 1)(3n^2 + n + 2)}{12}$

3. (a) $\dfrac{35}{15} - \dfrac{12n + 7}{16.5^{n-1}}$

(b) . $4 - \dfrac{n}{2^{n-1}} - \dfrac{1}{2^{n-2}}$

(c) $\dfrac{2}{9} + \dfrac{(-1)^{n-1}}{9.5^{n-1}} + \dfrac{(-1)^{n+1}(4n - 2)}{9.5^{n-1}}$

(d) $\dfrac{1}{4 \times 3^n}(5 \times 3^n - 6n - 5)$

4(a) $\dfrac{5}{9}\left[\dfrac{10(10^n - 1)}{9} - n\right]$

(b) $\dfrac{6}{9}\left[n - \dfrac{1}{9}\left(1 - \dfrac{1}{10^n}\right)\right]$

(c) $\dfrac{x(x^n - 1)}{(x - 1)} + \dfrac{n(n + 1)q}{2}$

(d) $\dfrac{1 - x^n}{(1 - x)^2} - \dfrac{nx^n}{(1 - x)}$

5. (i) $\dfrac{5}{4}$ (ii) $\dfrac{1 + x}{(1 - x)^2}$ (iii) $\dfrac{1 - 3x}{(1 + x)^2}$

Objective Questions

1. The common ratio of the G.P whose nth term $t_n = 2(5^n)$, $n \in N$ is
   (a) 2 (b) 3 (c) 25 (d) 5
2. The Geometric means between 1 and 64 are :
   (a) 3, 12 (b) 4, 9 (c) 2, 18 (d) 7 , 6
3. If (2a –b) ,b,(2c–b) are in G.P then
   (a) a, b, c are in G.P.
   (b) a, b, c are in A.P.
   (c) a, b, c are in H.P.
   (d) a, b, c are equal.
4. The next term of the sequence 2,6,12, 20 ……….is
   (a) 30 (b) 48 (c) 36 (d) 28
5. If $t_n = (2n+3)$, then $s_{10} =$
   (a) 120 (b) 130 (c) 140 (d) 110
6. The general term of the series 1 + 4 + 9 + 16 +25 +………..is :
   (a) $\frac{n(n+1)}{2}$ (b) $n^2 + n$ (c) n^2 (d) $\frac{n^2(n+2)}{4}$
7. If the sum of first n terms of a series be $5n^2 + 2n$, then its second term is
   (a) 7 (b) 17 (c) 24 (d) 42
8. The nth term of the series 12.1+22.3+ 32.5 +………is :
   (a) $n^2.n$ (b) $2n^3–1$ (c) n (d) $2n^3 – n^2$
9. If a,b,carei in A.P. then, $\frac{a-b}{b-c} =$
   (a) 1 (b) $\frac{a}{b}$ (c) $\frac{a}{c}$ (d) 0
10. If a,b,c,d,e are in G.P. then $\frac{e}{c} =$
    (a) $\frac{d}{c}$ (b) $\frac{b}{a}$ (c) $\frac{c}{b}$ (d) $\frac{d}{b}$

Answers

1	2	3	4	5	6	7	8	9	10
d	b	c	a	c	c	b	d	a	d

❑❑❑