N.M. BCA 4th sem / CSIT 3rd sem

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Meaning of solution of non-linear equation

⇒ A ~~linear~~ non-linear equation is of the form $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n$ where a's are constant ($a_0 \neq 0$) and n is a positive integer called a polynomial in x of degree 'n'.

⇒ The polynomial $f(x) = 0$ is called an algebric equation of degree 'n'.

⇒ The value α of x for which $f(x) = 0$ is caused root of the equation.

⇒ The process of finding the roots of an equation is known as solution of that equation.

Types of Equation

(1) Algebric equation

⇒ An equation of ~~type~~ form $p = 0$ is called algebric equation where p is a polynomial.

⇒ The algebric equation can be polynomial equation. like linear eqn (eg: $ax + b = c$), quadratic equation (eg: $ax^2 + bx + c = 0$), cubic eqn ($ax^3 + bx^2 + cx + d = 0$), trigonometric equation (eg. $\cos 2x = 1 + 4\sin x$). etc

(2) polynomial equation

⇒ A polynomial equation of degree 'n' is of the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0$ where a's are real numbers. (sometimes caused the coefficient of the polynomial)

eg: $f(x) = 4x^3 - 3x^2 + 2$ is a polynomial eqn of degree 3.
$f(x) = x^7 - 4x^5 + 1$ is a " " " " 7.

(3) Transcendental equation

⇒ These are non-algebric equation which includes trigonometric, exponential and logarithmic functions.

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Example: $2\sin x - x = 0$

$e^x \sin x - \frac{1}{2}x = 0$

$\log x - 1 = 0$.

Methods of Solution of Non-linear Equations.

→ There are different methods to find the solution of non-linear equation which are as follows.

1. Direct analytical method.
2. Graphical method.
3. Trial and Error method.
4. Iterative method.

Direct analytical method.

→ In direct analytical method, the roots can be found without any calculations. So it is used for simple equation like:

$x^2 - 3x + 2 = 0$

→ It is only used for certain simple cases.

→ It cannot be used for equation like $2\sin x - x = 0$ in which it is difficult to find soln without any calculations.

Graphical Method.

→ This method involves plotting the given function in a graph and finds the point where the function cuts the x-axis.

→ The point where function cuts x-axis is called root of the equation.

→ This method gives only approximate values.

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Trial and Error method.

→ This method involves a series of guesses for x, each time evaluating the function to see whether it is closed to zero.

→ The value of ~~function~~ x that causes the function value closed to 0 is one of the approximate root of the equation.

→ This method is time consuming and has low accuracy.

Example: Consider an equation

$3x+4=10$.

→ To use trial and error method, we have to substitute different values for variable x.

Equation	Substituting value	Values of L.H.S	Values of R.H.S.
$3x+4=10$	$x=1$	$3*1+4=7$	10
	$x=2$	$3*2+4=10$	10
	$x=3$	$3*3+4=13$	10
	$x=4$	$3*4+4=16$	10

Here, Substituting value $x=2$, we get L.H.S = R.H.S so, 2 is solution.

Iterative method.

→ Iterative method usually begins with an approximate value of root known as initial guess which is then refined iteration by iteration, until the desired level of accuracy is obtained.

→ It is also known as algorithmic approaches.

→ Iterative method can be grouped into 2 categories

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1. Bracketing method
   a. Bisection method.
   b. False position method.
2. Non-bracketing (open-end) method.