Fixed point iteration method.

→ Let, any function in the form of $f(x) = 0$ Can be manipulated such that x is on the left-hand side of the equation such that

$$x = g(x)$$

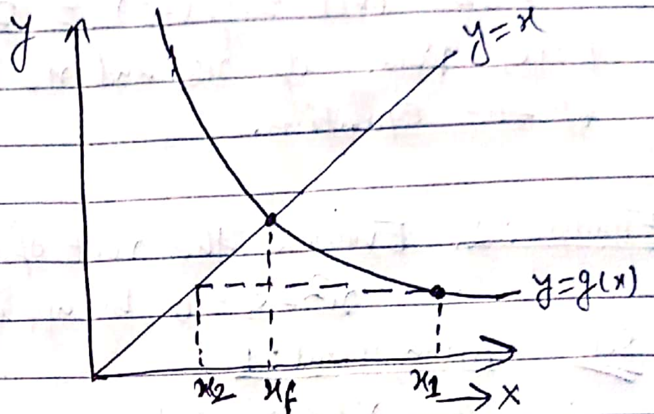

fig: Fixed point method

→ The root of the equation is given by the point of intersection of curve $y = g(x)$ and $y = x$.

→ This intersection point is called fixed point of $g(x)$.

→ The transformation equation can be obtained either by algebric manipulation or by simply adding x to both side of equation.

→ Let x_0 be the initial approximation then next approximation is given by

$$x_1 = g(x_0)$$

Similarly,

$$x_2 = g(x_1)$$

In general,

$$\boxed{x_{i+1} = g(x_i)} \text{ for } i = 0, 1, 2, \ldots$$

→ This is general formula for fixed-point iteration method.

Example: Locate the root of the equation $x^2 + x - 2 = 0$ using the fixed-point method.

Soln The given equation can be re-arranged as,

$$x = 2 - x^2 \qquad [\text{i.e } x = g(x)]$$

Let the initial guess value, $x_0 = 0$.

$$x_1 = g(x_0) = g(0) = 2 - (0)^2 = 2$$

$$x_2 = g(x_1) = g(2) = 2 - (2)^2 = 2 - 4 = -2$$

$x_3 = g(x_2) = g(-2) = 2-(-2)^2 = 2-4 = -2.$

Here, value of x_3 and x_2 is same. So, -2 is one of the root of the equation.

Example: 2. Evaluate the root of ~~the~~ 5 using the equation $x^2-5=0$ by applying the fixed-point iteration method.

Soln Arrangement: 1

Letus re-arrange the equation as.

$$x^2-5=0$$

$$\text{or, } x^2 = 5$$

$$\text{or, } x = 5/x \qquad [\because x = g(x)]$$

Assume, $x_0 = 1$ Then

$$x_1 = g(x_0) = g(1) = 5/1 = 5$$

$$x_2 = g(x_1) = g(5) = 5/5 = 1$$

$$x_3 = g(x_2) = g(1) = 5/1 = 5$$

$$x_4 = g(x_3) = g(5) = 5/5 = 1.$$

The process doesnot converge to the solution. This type of divergence is known as oscillatory divergence.

Arrangement: 2

Letus consider another form of $g(x)$.

$$x^2-5=0$$

Adding x to both sides, we get

$$x + x^2 - 5 = 0 + x$$

$$\text{or, } x = x^2 + x - 5 \qquad [x = g(x)]$$

Assume, $x_0 = 0$, Then

$$x_1 = g(0) = (0)^2 + 0 - 5 = -5$$

$$x_2 = g(x_1) = g(-5) = (-5)^2 + (-5) - 5 = 15$$

$$x_3 = g(x_2) = g(15) = (15)^2 + 15 - 5 = 235$$

$x_4 = g(x_3) = g(235) = (235)^2 + 235 - 5 = 55455$

Again it doesnot converge. Rather it diverges rapidly. This type of divergence is known as monotone divergence.

Arrangement: 3

Letus consider another form of $g(x)$.

$$x^2 - 5 = 0$$

or, $x^2 = 5$

or, $x = 5/x$

Adding x to both side of the equation, we get

$$x + x = \frac{5}{x} + x$$

or, $x = \frac{5/x + x}{2}$ $\quad [x = g(x)]$

Assume, $x_0 = 1$, Then

$$x_1 = g(1) = \frac{5/1 + 1}{2} = 3$$

$$x_2 = g(x_1) = g(3) = \frac{5/3 + 1}{2} = 2.3333$$

$$x_3 = g(x_2) = g(2.3333) = \frac{5/2.3333 + 2.3333}{2} = 2.2381$$

$$x_4 = g(x_3) = g(2.2381) = \frac{5/2.2381 + 2.2381}{2} = 2.2361$$

$$x_5 = g(x_4) = g(2.2361) = \frac{5/2.2361 + 2.2361}{2} = 2.2361$$

This time, the process converges rapidly to the solution. So, the square root of 5 is 2.2361 Ans