Unit → 5

Circle

A circle is defined as the locus of a point which moves in a plane such that its distance from a fixed point in that plane is always constant.

P(x, y)

Some Important Formulae of Circle

i. Standard equation of a circle with center (h, k) and radius r is

$(x - h)^2 + (y - k)^2 = r^2$

r P (x, y)

Eg: Find the equation of the circle with center (4, 5) and radius 7

Solution:

Centre $(h, k) = (4, 5)$

Radius $(r) = 7$

Equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 4)^2 + (y - 5)^2 = 7^2$

or, $x^2 - 8x + 16 + y^2 - 10y + 25 = 49$

$\therefore$ $x^2 + y^2 - 8x - 10y - 8 = 0.$

(h, k) = (0, 0)

ii. If center is origin i.e. (0, 0) and radius r then the equation is $x^2 + y^2 = r^2$

Eg: Find the equation of the circle with center (0, 0) and radius 5.

Solution:

Centre $(h, k) = (0, 0)$

Radius $(r) = 5$

Equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 0)^2 + (y - 0)^2 = 5^2$

$\therefore$ $x^2 + y^2 = 25.$

iii. If the circle touches x – axis then, k = r

So, equation is $(x-h)^2 + (y-k)^2 = k^2$

Eg:- Find the equation of the circle with center (2, 3) and touches x-axis.

Solution: = (h, k)

Since the circle touches the x-axis, so r = k = 3

Centre $(h, k) = (2, 3)$

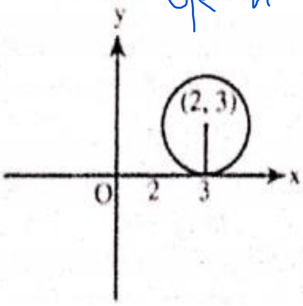
$\therefore k = 3$

Equation of circle is $(x-h)^2 + (y-k)^2 = r^2$

or, $(x-2)^2 + (y-3)^2 = 3^2$

or, $x^2 - 4x + 4 + y^2 - 6y + 9 = 9$

$\therefore x^2 + y^2 - 4x - 6y + 4 = 0.$



iv. If the circle touches y – axis then, h = r

So, equation is $(x-h)^2 + (y-k)^2 = h^2$

v. If the circle touches both the axis then, h = k = r

So, the equation is $(x-h)^2 + (y-h)^2 = h^2$

vi. General equation of the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

where, radius = $\sqrt{g^2 + f^2 - c}$ = r

and center = $(-g, -f)$ = (h, k)

$(x^2 + 2\cdot g\cdot x + g^2) + (y^2 + 2\cdot f y + f^2) = g^2 + f^2 - c$

$\Rightarrow \{x-(-g)\}^2 + \{y-(-f)\}^2 = (\sqrt{g^2+f^2-c})^2$

Eg: Find the equation of the circle passing through (0, 0), (4, 0) and (0, 3). Also, find the radius and center.

Solution:

Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

If equation (i) passes through the points (0, 0), (4, 0) and (0, 3), so

$0^2 + 0^2 + 2g\cdot 0 + 2f\cdot 0 + c = 0 \Rightarrow c = 0.$

Also, $4^2 + 0^2 + 2g\cdot 4 + 2f\cdot 0 + c = 0$

$\Rightarrow 16 + 8g + 0 = 0$ $[\because c = 0]$

$\Rightarrow g = -2.$

And, $0^2 + 3^2 + 2g\cdot 0 + 2f\cdot 3 + c = 0$

$\Rightarrow 9 + 6f + 0 = 0$ $[\because c = 0]$

$\Rightarrow f = -\frac{3}{2}.$

Putting the value of g, f and c in (i)

$x^2 + y^2 + 2\cdot(-2)\cdot x + 2\cdot\left(-\frac{3}{2}\right)\cdot y + 0 = 0$

$\therefore x^2 + y^2 - 4x - 3y = 0.$

(0, 3)

(0, 0) (4, 0)

Concyclic

Note :- Now, comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get,

$g = -2$, $f = -3/2$ and $c = 0$

where, radius $= \sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + \left(-\frac{3}{2}\right)^2 - 0} = 5/2$

and center $= (-g, -f) = (2, 3/2)$

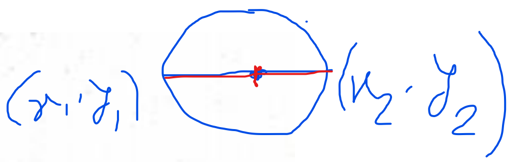
vii. If (x_1, y_1) and (x_2, y_2) are two ends of a diameter of a circle then the equation of the circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Eg: Find the equation of the circle which has (1, 3) and (4, 5) as the ends of a diameter.

Solution

Given, $(x_1, y_1) = (1, 3)$

$(x_2, y_2) = (4, 5)$



Equation of circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 1)(x - 4) + (y - 3)(y - 5) = 0$

or, $x^2 - 4x - x + 4 + y^2 - 3y - 5y + 15 = 0$

$\therefore$ $x^2 + y^2 - 5x - 8y + 19 = 0.$

viii. Any circle concentric to the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ is $x^2 + y^2 + 2gx + 2fy + k = 0$

6. Find the equation of circle concentric with $x^2 + y^2 + x + 2y + 3 = 0$ and through the point (1, 1).

Solution

Equation of any circle concentric with

$x^2 + y^2 + x + 2y + 3 = 0$ is

$x^2 + y^2 + x + 2y + k = 0$ ... (i)

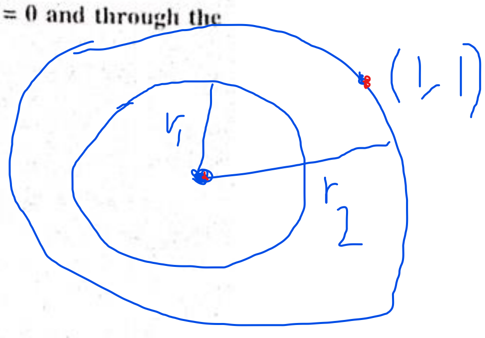
If equation (i) passes through the point (1, 1)

Then, $1^2 + 1^2 + 1 + 2 \times 1 + k = 0$

or, $k = -5$

Putting the value of k in (i), we get,

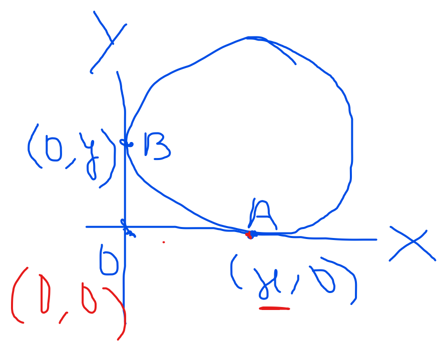
$x^2 + y^2 + x + 2y - 5 = 0.$



ix. Lengths of intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ ……. are

Intercept on x – axis (y = 0) $= 2\sqrt{g^2 - c}$ and

Intercept on y – axis (x = 0) $= 2\sqrt{f^2 - c}$ respectively.



Eg:-

4. Find the equation to the circle which passes through the origin and cuts off intercepts equal to 3 and 4 from the x-axis respectively. and y-axis

Solution

Here, (3, 0) and (0, 4) are end points of diameter of circle since $\angle XOY = 90°$.

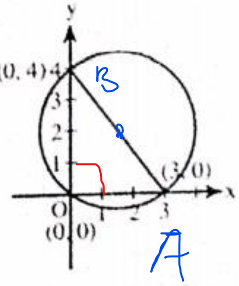
$\therefore \quad (x_1, y_1) = (3, 0)$ and $(x_2, y_2) = (0, 4)$

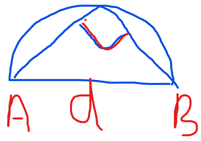
Equation of circle is $\quad (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $\quad (x - 3)(x - 0) + (y - 0)(y - 4) = 0$

or, $\quad x^2 - 3x + y^2 - 4y = 0$

$\therefore \quad x^2 + y^2 - 3x - 4y = 0.$





Note: Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get $g = -3/2$, $f = -2$ and $c = 0$. Then,

Intercept on x – axis = $2\sqrt{g^2 - c}$ = … … = 3

Intercept on y – axis = $2\sqrt{f^2 - c}$ = … … = 4

♦●♦●♦