Numerical Methods UNIT – IV Numerical Differentiation

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B. Tech I Year (II-Semester) May/ June 2014 **Subject: T 264- Numerical Methods**
ECE – B Section

UNIT – IV **Numerical Differentiation and Integration** **Faculty Name: N V Nagendram**

Numerical Differentiation and Integration – Differentiation using finite differences – Trapezoidal Rule – Simpson's 1/3 Rule – Simpson's 1/8 Rule.

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Planned Topics Lectures

4.1 Introduction

4.2 Numerical Differentiation

4.3 Formulae for Derivatives using finite differences
- (i) Derivatives using Newton's Forward difference formula
- (ii) Derivatives using Newton's Backward difference formula
- (iii) Derivatives using Newton's Central difference formula

4.4 Maxima and Minima of a Tabulated Function

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Lecture-1

Introduction:

Engineers and scientists are frequently faced with the problem of differentiation or Integration of some functions. If the functions have a closed form representation and are amenable for standard calculus methods, then differentiation and integration can be carried out.

However, in many situations, we may not know the exact functions. We will be knowing only, the values of the functions at a discrete set of points. In some instances, the functions are known but they are so complicated that analytic differentiation, integration is difficult. In both these situations, we seek the help of numerical techniques to obtain the estimates of derivatives or integrals. The method of obtaining the derivative of a function using a numerical technique is known as numerical differentiation.

The method of finding the value of an integral of the form $\int_a^b f(x)\,dx$ using numerical techniques is called "Numerical Integration".

In this section, we discuss various numerical differentiation and numerical integration methods. We have to understand that while analytical methods give exact answers, the numerical techniques provide only approximate answers.

Definition: Numerical differentiation:

Numerical differentiation is the process by which we can find the derivative or derivatives of a function at some values of the independent variable when we are given a set of values of that function.

Uses of Numerical differentiation:

The numerical differentiation techniques can be used in the following situations:

01. The function values corresponding to distinct values of the argument are known but the function is unknown.

    For example, we may knowing the values of f(x) at various values of x, say x_i, i = 1,2,3,…, n in a tabulated form.

02. The function to be differentiated is complicated, and so, it is difficult to differentiate by usual procedures.

Numerical differentiation is the process of calculating the value of the derivative of a function at some assigned value of x from the given set of data points (x_i, y_i = f(x_i)), i = 0,1,2,…, n which correspond to the values of an unknown function y = f(x). To find $\frac{dy}{dx}$, we first replace the exact relation y = f(x) by the best interpolating polynomial y = ϕ(x) as we know earlier and then differentiate the latter as many times as we desire. The choice of the interpolation formula to be used, will depend on the assigned value of x at which $\frac{dy}{dx}$ is desired.

If the points are equally spaced and $\frac{dy}{dx}$ is required near the beginning of the table, we use **Newton-Gregory's Forward Interpolation Formula**.

If we require the derivative at the end of the table, we employ **Newton-Gregory's Backward Interpolation Formula**.

If the value of the derivative is required near the middle of the table, we use one of the **Central Difference Interpolation Formula**.

If the values of x are not Equi-spaced, we use **Newton's Divided difference Interpolation Formula** or $\frac{dy}{dx}$ to get the derivative value.

Formulae for Derivatives:

Consider the function y = f (x) which is tabulated for the values x_i ($= x_0 + ih$), i = 0, 1, 2,...., n.

Derivatives using Newton's Forward Difference Formula:

Suppose that we are given a set of values (x_i, y_i), i = 0,1,2,...., n.

We want to find the derivative of y = f (x) passing through the (n + 1)points, at a point nearer to the starting value at x = x_0.

Newton's Forward Difference Interpolation Formula is

$$y = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots\ldots\ldots\ldots \quad \text{------------------ (1)}$$

Where $p = \dfrac{x - x_0}{h}$ ------------------ (2)

On differentiation (1) w.r.t., p we have

On differentiation (2) w.r.t. x we have, $\dfrac{dp}{dx} \approx \dfrac{1}{h}$

$$\frac{dy}{dx} = \frac{dy}{dp}\cdot\frac{dp}{dx} = \frac{1}{h}\left[\begin{array}{l}\Delta y_0 + \dfrac{2p-1}{2}\Delta^2 y_0 + \dfrac{3p^2 - 6p + 2}{6}\Delta^3 y_0 \\ + \dfrac{4p^3 - 18p^2 + 22p - 6}{24}\Delta^4 y_0 + \ldots\end{array}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Equation (3) gives the value of $\dfrac{dy}{dx}$ at any point x which may be anywhere in the interval.

At x = x_0 and p = 0, hence putting p = 0, equation (3) gives

$$\left(\frac{dy}{dx}\right)_{x\approx x_1} = \left(\frac{dy}{dp}\right)_{p\approx 1} = \frac{1}{\mathrm{h}}\left[\begin{matrix}\Delta \mathrm{y}0 + \frac{1}{2}\Delta^2 y_0 + \frac{1}{6}\Delta^3 y_0 \\ + \frac{4p^3 - 18p^2 + 22p - 6}{24}\Delta^4 y_0 + \ldots\end{matrix}\right] \quad \text{......................(3)}$$

Again on differentiation (3) we get

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{\mathrm{d}}{\mathrm{dp}}\left(\frac{dy}{dx}\right).\frac{dp}{dx} = \frac{\mathrm{d}}{\mathrm{dp}}\left(\frac{dy}{dx}\right).\frac{dp}{dx} = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(p-1)}{}\Delta^3 y_0 + \frac{6p^2 - 18p + 11}{12}\Delta^4 y_0 + \ldots\right]$$

From which we obtain

$$\left(\frac{d^2y}{dx^2}\right) x \approx x_0 = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + ..\right]$$ at x = x_0 and p = 0 ……. (5)

Similarly, $\left(\frac{d^3y}{dx^3}\right) x \approx x_0 = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + ...\right]$ ……. (6)

Derivatives using Newton's Backward Difference Formula:

Newton's Backward Difference Interpolation Formula is

$$y(x) = y_n + p\,\Delta y_n + \frac{p(p+1)}{2!}\Delta^2 y_n + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_n + \ldots\ldots\ldots\ldots \quad \text{------------------ (7)}$$

Where p = $\frac{x - x_n}{h}$ ------------------ (8)

On differentiation (7) w.r.t., p we have

$$\frac{dy}{dp} = \left[\Delta y_n + \frac{2p+1}{2}\Delta^2 y_n + \frac{3p^2+6p+2}{6}\Delta^3 y_n + \frac{4p^3+18p^2+22p+6}{24}\Delta^4 y_n + \ldots\right].$$

On differentiation (8) w.r.t. x we have, $\frac{dp}{dx} \approx \frac{1}{h}$ Now

$$\frac{dy}{dx} = \frac{dy}{dp}.\frac{dp}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2p+1}{2}\Delta^2 y_n + \frac{3p^2+6p+2}{6}\Delta^3 y_n + \frac{4p^3+18p^2+22p+6}{24}\Delta^4 y_n + ..\right] \quad \ldots\ldots\ldots\ldots\ldots(9)$$

Equation (9) gives the value of $\frac{dy}{dx}$ at any point x which may be anywhere in the interval.

At x = x_n and p = 0, hence putting p = 0, equation (9) gives

$$\left(\frac{dy}{dx}\right)x \approx x_{n_1} = \left(\frac{dy}{dx}\right)x_n = \frac{1}{h}\left[\Delta y n + \frac{1}{2}\Delta^2 y_n + \frac{1}{3}\Delta^3 y_n + \frac{1}{4}\Delta^4 y_n + \ldots\right] \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

Again on differentiation (09) we obtain

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx}.\frac{dp}{dx} = \frac{\mathrm{d}}{\mathrm{dp}}\left(\frac{dy}{dx}\right).\frac{dp}{dx} = \frac{\mathrm{d}}{\mathrm{dp}}\left(\frac{dy}{dx}\right).\frac{dp}{dx}$$

$$= \frac{1}{h^2}\left[\Delta^2 y_n + \frac{(p+1)}{}\Delta^3 y_n + \frac{6p^2+18p+11}{12}\Delta^4 y_n + .\right]$$

From which we obtain

$$\left(\frac{d^2y}{dx^2}\right) x \approx x_n = \frac{1}{h^2}\left[\Delta^2 y_n + \Delta^3 y_n + \frac{11}{12}\Delta^4 y_n + \frac{5}{6}\Delta^5 y_n + ..\right]$$ at $x = x_n$ and $p = 0$

Similarly, $\left(\frac{d^3y}{dx^3}\right) x \approx x_n = \frac{1}{h^3}\left[\Delta^3 y_n - \frac{3}{2}\Delta^4 y_0 + ...\right]$ .... (12)

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Lecture-4

Derivatives using Central Difference Formula:

The Stirling's Formula is given by

$$y = y_0 + \frac{p}{2}(\Delta y_0 + \Delta y_{-1}) + \frac{p^2}{2}\Delta^2 y_{-1} + \frac{p^3 - p}{12}(\Delta^3 y_{-1} + \Delta^3 y_{-2}) + \frac{p^4 - p^2}{24}\Delta^4 y_{-2} + \ldots\ldots. \quad (13)$$

Where p = $\frac{x - x_0}{h}$

On differentiation (1) w.r.t., x we have

$$\frac{dy}{dx} = \frac{dy}{dp}.\frac{dp}{dx} = \frac{1}{h}.\frac{dy}{dp} = \frac{1}{h}\left[\begin{array}{l} \frac{1}{2}(\Delta y_0 + \Delta y_{-1}) + \frac{p}{2}\Delta^2 y_{-1} + \frac{3p^2 - 1}{12}(\Delta^3 y_{-1} + \Delta^3 y_{-2}) \\ + \frac{(2p^3 - p)}{12}\Delta^4 y_{-2} \ + .\frac{5p^4 - 15p^2 - 4}{240}(\Delta^5 y_{-2} + \Delta^5 y_{-3}) \\ + \ \ldots\ldots.. \end{array}\right]$$

At x = x_0 and p = 0, hence putting p = 0, we get

$$\left(\frac{dy}{dx}\right) at \ \ x \approx x_0 = \frac{1}{h}\left[\frac{1}{2}(\Delta y_0 + \Delta y_{-1}) - \frac{1}{12}(\Delta^3 y_{-1} + \Delta^3 y_{-2}) + \frac{1}{60}(\Delta^5 y_{-2} + \Delta^5 y_{-3}) + \ldots.\right]$$

Similarly,

$$\left(\frac{d^2 y}{dx^2}\right) x \approx x_0 = \frac{1}{h^2}\left[\Delta^2 y_{-1} - 1/12\,\Delta^4 y_{-2} + \frac{1}{90}\Delta^6 y_{-3} - .\right]$$ at x= x_n and p = 0

Similarly, $\left(\frac{d^3 y}{dx^3}\right) x \approx x_0 = \frac{1}{h^3}\left[\frac{1}{2}(\Delta^3 y_{-1} + \Delta^3 y_{-2}) + \ldots\ldots\right]$

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Lecture-5

Maxima and Minima of a tabulated Function:

Given a set of data points (x_i, y_i) , i = 0,1,2,....., n, we can get the interpolating polynomial of degree n. Now we wish to estimate the value of x at which the curve is maximum or minimum.

We know that the maximum and minimum values of a function can be determined by equating the first derivative to zero and solving for the variable. The same procedure can be applied to find the maxima and minima of a tabulated function. Assume that the points are equally spaced with a step size of h.

Consider Newton's forward difference interpolation formula

$y \approx y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-3)}{3!}\Delta^3 y_0 + \ldots\ldots\ldots\ldots$ On differentiation it w.r.t. p, we get $\frac{dy}{dp} = \left[\Delta y_n + \frac{2p-1}{2}\Delta^2 y_n + \frac{3p^2-6p+2}{6}\Delta^3 y_n + ..\right]$ ....(1)

For maxima and minima $\frac{dy}{dp} \approx 0$. hence equating the RHS of (1) to zero and for simplicity only upto 3rd differences we obtain

$$\left[\Delta y_n + \frac{2p-1}{2}\Delta^2 y_n + \frac{3p^2-6p+2}{6}\Delta^3 y_n + ..\right] = 0$$

Re-arranging this as a quadratic in p, we get

$$\left(\frac{1}{2}\Delta^3 y_0\right)p^2 + \left(\Delta^2 y_0 - \Delta^3 y_0\right)p + \left(\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0\right). = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Substituting the values of $\Delta y_0, \Delta^2 y_0, \frac{1}{3}\Delta^3 y_0$ from the difference table, We solve the equation (2) for p. Then the corresponding values of x are given by x = x_0 + ph at which y is maximum or minimum.

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Tutorial-01

Problem #01: Given the following data find $f'(6)$

x:	0	2	3	4	7	9
y:	4	26	58	112	466	922

[**Ans.** $f'(6) = 135$]

Problem #02: Find the first and second derivatives of the function tabulated below at the point x = 1.5.

x:	1.5	2.0	2.5	3.0	3.5	4.0
y:	3.375	7.0	13.625	24.0	38.875	59.0

[**Ans.** $f'(6) = 4.75$; $f''(6) = 9$]

Problem #03: Find the first and second derivatives of the function tabulated below at the point x = 0.6.

x:	0.4	0.5	0.6	0.7	0.8
y:	1.5836	1.7974	2.0442	2.2375	2.6511

[**Ans.** $f'(6) = 2.64442$; $f''(6) = 3.6475$]

Problem #04: the following table of values of x and y is given.

X	0	1	2	3	4	5	6
Y	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

Find dy/dx when (i) x = 1 (ii) x = 3 (iii) x = 6. Also find $\frac{d^2y}{dx^2}$ when x = 3 ?

[**Ans.** $f'(1) = 0.3948$; $f''(3) = 0.3341$; $f'(6) = 0.2731$]

Problem #05: Given the following data find $f'(1)$

x:	1.0	1.5	2.0	2.5	3.0
f(x):	27	106.75	324	783.75	1621

[**Ans.** $f'(1) = 77$]

Problem #06: The population of Vijayawada town as obtained from census data is shown as the following table:

Year	1951	1961	1971	1981	1991
Population in thous	19.96	39.65	58.81	77.21	94.61

Estimate the growth of the population in 1981.[**Ans.** $f'(1981) = 1.8$ Thous.]

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UNIT – IV **Numerical Differentiation** **Faculty Name: N V Nagendram**
Solutions **& Integration** **Tutorial-01**

Problem #01: Given the following data find $f'(6)$

x:	0	2	3	4	7	9
y:	4	26	58	112	466	922

[**Ans.** $f'(6) = 135$]

Solution: Given data on observation arguments are not equally spaced, we will use Newton's divided formula.The divided difference table as below:

x	y= f(x)	Δf(x)	Δ²f(x)	Δ³f(x)	Δ⁴f(x)
0	4				
		$\frac{26-4}{2-0}=11$			
2	26		$\frac{32-11}{3-0}=7$		
		$\frac{58-26}{3-2}=32$		$\frac{11-7}{4-0}=1$	
3	58		$\frac{54-32}{4-2}=11$		0
		$\frac{112-58}{4-3}=54$		$\frac{16-11}{7-2}=1$	
4	112		$\frac{118-54}{4-0}=16$		0
		$\frac{466-112}{7-4}=118$		$\frac{22-16}{9-3}=1$	
7	466		$\frac{228-118}{9-4}=22$		
		$\frac{922-466}{9-7}=228$			
9	922				

By Newton's divided difference formula,

$$y = f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$= 4 + (x-0)\times 11 + (x-0)(x-2)\times 7 + +(x-0)(x-2)(x-3)\times 1 + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$= 4 + 11x + 7(x^2 - 2x) + x(x^2 - 5x + 6) \quad = (x^3 + 2x^2 + 3x + 4)$$ **is** the solution.

Problem #02: Find the first and second derivatives of the function tabulated below at the point x = 1.5.

x:	1.5	2.0	2.5	3.0	3.5	4.0
y:	3.375	7.0	13.625	24.0	38.875	59.0

[**Ans.** $f'(6) = 4.75$; $f''(6) = 9$]

Solution: the difference diagonal table as below:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375					
		3.625				
2.0	7.0		3			
		6.625		0.75		
2.5	13.625		3.75		0	
		10.375		0.75		0
3.0	24.0		4.5		0	
		14.875		0.75		
3.5	38.875		5.25			
		20.125				
4.0	59.0					

Here $x_0 = 1.5$, $y_0 = 3.375$, $h = 0.5$

By Newton's Difference Formula, we have

$$\left[\frac{dy}{dx}\right] at\ x \approx x_0 \ \approx \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - ....\right]$$

$$\therefore \left[\frac{dy}{dx}\right] at\ x \approx 1.5 \approx \frac{1}{0.5}\left[3.625 - \frac{1}{2}(3) + \frac{1}{3}(0.75) - \frac{1}{4}(0) + \frac{1}{5}(0)\right]$$

$$\left[\frac{dy}{dx}\right] at\ x \approx 1.5 \approx \frac{1}{0.5}\left[3.625 - 1.5 + 0.25\right]$$

$$\therefore \left[\frac{dy}{dx}\right] at\ x \approx 1.5 \approx \frac{2.375}{0.5} \approx 4.75$$

And

$$\left[\frac{d^2 y}{dx^2}\right] at\ x \approx x_0 \ \approx \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + ...\right]$$

$$\therefore \left[\frac{d^2 y}{dx^2}\right] at\ x \approx x_0 \ \approx \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + ...\right]$$

$$\therefore \left[\frac{d^2 y}{dx^2}\right] at\ x \approx 1.5 \approx \frac{1}{(0.5)^2}\left[3 - 0.75 + \frac{11}{12}(0) - \frac{5}{6}(0)\right]$$

$$\therefore \left[\frac{d^2 y}{dx^2}\right] at\ x \approx 1.5 \approx \frac{2.25}{(0.25)} \approx 9$$ Hence the solution.

Problem #03: Find the first and second derivatives of the function tabulated below at the point x = 0.6.

x:	0.4	0.5	0.6	0.7	0.8
y:	1.5836	1.7974	2.0442	2.2375	2.6511

[**Ans.** $f'(6) = 2.64442$; $f''(6) = 3.6475$]

Try urself……

Problem #04: the following table of values of x and y is given.

X	0	1	2	3	4	5	6
Y	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

Find dy/dx when (i) x = 1 (ii) x = 3 (iii) x = 6. Also find $\frac{d^2y}{dx^2}$ when x = 3 ?

[**Ans.** $f'(1) = 0.3948$; $f''(3) = 0.3341$; $f'(6) = 0.2731$]

Try urself……

Problem #05: Given the following data find $f'(1)$

x:	1.0	1.5	2.0	2.5	3.0
f(x):	27	106.75	324	783.75	1621

[**Ans.** $f'(1) = 77$]

Try urself……

Problem #06: The population of Vijayawada town as obtained from census data is shown as the following table:

Year	1951	1961	1971	1981	1991
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Estimate the growth of the population in 1981.[**Ans.** $f'(1981)$ =1.8 Thous.]

Try urself......

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Tutorial-02

Problem #01: The table given below reveals the velocity v of a body during the specified time t. Find the acceleration at t = 1.1:

t:	1.0	1.1	1.2	1.3	1.4
v:	43.1	47.7	52.1	56.4	60.8

[**Ans.** $f'(1.1)$ =44.9166]

Problem #02: Find the value of $f'(x)$ at x = 0.04 from the following table using Bessel's formula.

X	0.01	0.02	0.03	0.04	0.05	0.06
f(x)	0.1023	0.1047	0.1071	0.1096	0.1122	0.1148

[**Ans.** $f'(0.04)$ =0.25625]

Problem #03: Find the value of cos(1.74) from the following table.

X	1.7	1.74	1.78	1.82	1.86
Sin x	0.9857	0.9916	0.9781	0.9691	0.9584

[**Ans.** $f'(1.74)$=-0.0178]

Problem #04: A rod is rotating in a plane. The following table gives the angle θ in radians through which the rod has turned for various values of the time t sec.

T	0.0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and the angular acceleration of the rod, when t = 0.6 sec. [**Ans.** 3.8167, 6.75 rad/sec^{2}.]

Problem #05: The velocity v of a particle moving in a straight line covers a distance x in time t. They are related as follows: Find f′(15):

X	0	10	20	30	40
Y	45	60	65	54	42

[**Ans.** $f'(15)$=-0.05416]

Problem #06: Find the value of $f'(4)$ from the following table.

X	1	2	4	8	10
Sin x	0	1	5	21	27

[**Ans.** $f'(4)$=2.833]

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Tutorial-03

Problem #01: Find the first and second derivative of the function tabulated below at x = 0.6.

X	0.4	0.5	0.6	0.7	0.8
Y	1.5836	1.7974	2.0442	2.3275	2.6511

[**Ans.** $f'(0.6)=2.6444$; $f''(0.6) = 3.6475$]

Problem #02: Find dy/dx at x = 7.5 from the following table.

X	7.47	7.48	7.49	7.50	7.51	7.52	7.53
Y	0.193	0.195	0.198	0.201	0.203	0.206	0.208

[**Ans.** $f'(7.5)=0.225$]

Problem #03: Find the first and second derivative of the function tabulated below at x = 1.4.

X	1.0	1.2	1.4	1.6	1.8	2.0
Y	0.000	0.128	0.544	1.296	2.432	4.000

[**Ans.** $f'(1.4)=4.68$; $f''(1.4) = 9.6$]

Problem #04: Find dy/dx at x = 1.76 and x = 1.72 from the following table.

X	1.72	1.73	1.74	1.75	1.76
Y	0.17907	0.17728	0.17552	0.17377	0.17204

[**Ans.** $f'(1.72)= -0.1819$; $f''(1.72) = -0.1719$]

Problem #05: Find $f'(0)$ and $\int_0^9 f(x)\,dx$

X	0	2	3	4	7	9
f(x)	4	26	58	110	460	920

[**Ans.** $f'(0)=3$; $\int_0^9 f(x)\,dx = 2175.75$]

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B. Tech I Year (II-Semester) May/ June 2014 **T 264- Numerical Methods**
ECE - B Section

UNIT – IV **Numerical Differentiation & Integration** **Faculty Name: N V Nagendram**

Tutorial-04

Problem #01: From the following table Find the value of x for which y is maximum and find this value of y.

X	1.2	1.3	1.4	1.5	1.6
Y	0.9320	0.9636	0.9855	0.9975	0.9996

[**Ans.** p= 3.7577; max. y = 1.00]

Problem #02: From the following table Find the value of x correct to 4 decimals for which y is minimum and find this value of y.

X	0.60	0.65	0.70	0.75
Y	0.6221	0.6155	0.6138	0.6170

[**Ans.** p= 1.8469;min. y = 0.6137]

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B. Tech I Year (II-Semester) May/ June 2014 **T 264- Numerical Methods**
ECE - B Section

UNIT – IV **Numerical Differentiation** **Faculty Name: N V Nagendram**
Solutions **& Integration** **Tutorial-04**

Problem #01: From the following table Find the value of x for which y is maximum and find this value of y.

X	1.2	1.3	1.4	1.5	1.6
Y	0.9320	0.9636	0.9855	0.9975	0.9996

[**Ans.** p= 3.7577; max. y = 1.00]

Solution: The difference table is as below

X	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1.2	0.9320			
		0.0316		
1.3	0.9636		- 0.0097	
		0.0219		- 0.0002
1.4	0.9855		- 0.0099	
		0.0120		0
1.5	0.9975		- 0.0099	
		0.0021		
1.6	0.9996			

Here h = 0.1

Taking $x_0 = 1.2$, we have $y_0 = 0.9320$, $\Delta y_0 = 0.0316$, $\Delta^2 y = -0.0097$ and $\Delta^3 y = -0.0002$.

$\therefore$ Newton's Forward Difference Formula, terminated after second differences, gives as y 0.9320 + p(0.0316) + $\frac{p(p-1)}{2}(-0.0097)$ .... (1)

$$\therefore \frac{dy}{dp} \approx 0.0316 + \frac{2p-1}{2}(-0.0097)$$

For y to be maximum,

$$\frac{dy}{dp} \approx 0 \quad \Rightarrow \quad \frac{dy}{dp} \approx 0.0316 + \frac{2p-1}{2}(-0.0097) = 0 \quad \Rightarrow 2(0.0316) = (2p-1)(0.0097)$$

$$\Rightarrow 2p-1 = \frac{0.0632}{0.0097} \approx 6.51546 \Rightarrow \boxed{p = 3.7577}$$

Hence $x = x_0 + ph = 1.2 + (3.7577)(0.1) = 1.5758$.
So, y is maximum when $x = 1.5758 = 1.58$
Putting $p = 3.7577$ in (1), the maximum value of y

$$= 0.9320 + (3.7577)(0.0316) + \frac{(3.7577)(3.7577-1)}{2}(-0.0097)$$

$= 0.9320 + 0.11874 - 0.0502586 = 1.00048 = 1.00$

$\boxed{\text{Therefore } y=1.0}$

Hence the solution.

Problem #02: From the following table Find the value of x correct to 4 decimals for which y is minimum and find this value of y.

X	0.60	0.65	0.70	0.75
Y	0.6221	0.6155	0.6138	0.6170

Solution: The difference table is [**Ans.** p= 1.8469;min. y = 0.6137]

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$
0.60	0.6221			
		- 0.0066		
0.65	0.6155		0.0049	
		- 0.0017		0
0.70	0.6138		0.0049	
		0.0032		
0.75	0.6170			

Taking $x_0 = 0.60$, we have $y_0 = 0.6221$, $\Delta y_0 = -0.0066$, $\Delta^2 y_0 = 0.0049$ and $\Delta^3 y = 0$.

$\therefore$ Newton's Forward Difference Formula, terminated after second differences, gives as $y= 0.6221 + p(-0.0066) + \frac{p(p-1)}{2}(0.0049)$ .... (1)

$\therefore \frac{dy}{dp} \approx -0.0066 + \frac{2p-1}{2}(0.0049)$ For y to be minimum,

$\frac{dy}{dp} \approx 0 \quad \Rightarrow \quad \frac{dy}{dp} \approx -0.0066 + \frac{2p-1}{2}(0.0049) = 0 \quad \Rightarrow 2(0.0066) = (2p-1)(0.0049)$

$\Rightarrow 2p-1 = \frac{2(0.0066)}{0.0049} \quad \Rightarrow \boxed{p = 1.8469}$

Hence $x = x_0 + ph = 0.60 + (1.8469)(0.05) = 0.6923$.
So, y is minimum when $x = 0.6923 = 0.6923$
Putting $p = 1.8469$ in (1), the minimum value of y

$$= 0.6221 + (1.8469)(-0.0066) + \frac{(1.8469)(1.8469-1)}{2}(0.0049)$$

$= 0.6221 - 0.01219 + 0.0038 = 0.6137$. $\boxed{\therefore y=0.6137}$

Hence the solution.

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ECE - B Section

UNIT – IV **Numerical Differentiation** **Faculty Name: N V Nagendram**

Problems & Integration **Tutorial-05**

Problem #01 Compute the first and second derivatives for the following table of data at x = -3 and x = 0

X	-3	-2	-1	0	1	2	3
Y	-33	-12	-3	0	3	12	33

[**Ans.** $\left(\frac{dy}{dx}\right)x = x_0 = 29$; $\left(\frac{d^2y}{dx^2}\right)x = x_0 = -18$]

Solution: the difference table is as below:

X	y	Δy	Δ²y	Δ³y	Δ⁴y	Δ⁵y	Δ⁵y	Δ⁶y
-3	-33							
		21						
-2	-12		-12					
		9		6				
-1	-3		-6		0			
		3		6		0	0	
0	0		0		0			0
		3		6		0	0	
1	3		6		0			
		9		6				
2	12		12					
		21						
3	33							

Here $x_0 = -3$, $y_0 = -33$ and h = 1; By Newton's Forward Difference Formula, we have

$$\left(\frac{dy}{dx}\right)x = x_0 = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - \frac{1}{6}\Delta^6 y_0 + ...\right]$$

$$\therefore \left(\frac{dy}{dx}\right)x = x_0 = \frac{1}{1}\left[21 - \frac{1}{2}(-12) + \frac{1}{3}(6) - \frac{1}{4}(0) + \frac{1}{5}(0) - \frac{1}{6}(0)\right]$$

$$\left(\frac{dy}{dx}\right)x = x_0 = \frac{1}{1}[21 + 6 + 2 - 0 + 0 - 0] = 29 \quad \Rightarrow \left(\frac{dy}{dx}\right)x = x_0 = 29$$

$$\left(\frac{d^2y}{dx^2}\right)x = x_0 = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + ...\right]$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)x = x_0 = \frac{1}{1^2}\left[(-12) - 6 + \frac{11}{12}(0) - \frac{5}{6}(0)\right] \Rightarrow \left(\frac{d^2y}{dx^2}\right)x = x_0 = -18$$ is solution.

Problem #02 Compute the first and second derivatives for the following table of data at x = 1.1

X	1.0	1.2	1.4	1.6	1.8	2.0
Y	0.000	0.128	0.544	1.296	2.432	4.000

[**Ans.** $\left(\frac{dy}{dx}\right)x = x_0 = 0.64$; $\left(\frac{d^2y}{dx^2}\right)x = x_0 = 6.6$]

Solution: the difference table is as below:

X	y	Δy	Δ^2y	Δ^3y	Δ^4y	Δ^5y
1.0	0.000					
		0.128				
1.2	0.128		0.288			
		0.416		0.048		
1.4	0.544		0.336		0	
		0.752		0.048		0
1.6	1.296		0.384		0	
		1.136		0.048		
1.8	2.432		0.432			
		1.568				
2.0	4.000					

Here x_0 = 1, y_0 = 0 and h = 0.2

By Newton's Forward Difference Formula, we have

$$\left(\frac{dy}{dx}\right)x = x_0 = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - .....\right]$$

$$\therefore \left(\frac{dy}{dx}\right)x = x_0 = \frac{1}{0.2}\left[0.128 - \frac{1}{2}(0.288) + \frac{1}{3}(0.048) - \frac{1}{4}(0) + \frac{1}{5}(0)\right]$$

$$\left(\frac{dy}{dx}\right)x = x_0 = \frac{1}{0.2}[0.128 - 0.144 + 0.016 - 0 + 0] = 0 \Rightarrow \left(\frac{dy}{dx}\right)x = 1.1 = 0.128/2 \approx 0.64$$

$$\left(\frac{d^2y}{dx^2}\right)x = x_0 = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + ...\right]$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)x = x_0 = \frac{1}{0.2^2}\left[(0.288) - 0.048 + \frac{11}{12}(0) - \frac{5}{6}(0)\right]$$

$$\left(\frac{d^2y}{dx^2}\right)x = x_0 = \frac{1}{0.04}[0.240] \approx 6 \quad \Rightarrow \left(\frac{d^2y}{dx^2}\right)x = 1.1 = 6 \text{ X } 1.1 = 6.6$$

Hence the solution.

UNIT – IV **Numerical Integration** **Faculty Name: N V Nagendram**

Theory **Lecture-06**

Introduction:

We know that a definite integral of the form $\int_a^b f(x)\,dx$ represents the area under the curve y = f(x), enclosed between the limits x = a and x = b. This integration is possible only if f(x) is explicitly given and if it is integrable. The problem of numerical integration can be stated as follows:

Given set of (n + 1) data points (x_i, y_i), i = 0,1,2,.......,n of the function y = f(x), where f(x) is not known explicitly, it is required to evaluate $\int_{x_0}^{x_n} f(x)\,dx$.

The problem of numerical integration, like that of numerical differentiation is solved by replacing f(x) with an interpolating polynomial $P_n(x)$ and obtaining $\int_{x_0}^{x_n} P_n(x)\,dx$ which is approximately taken as the value of $\int_{x_0}^{x_n} f(x)\,dx$. Numerical Integration is also known as “Numerical Quadrature”.

Newton-Cote's Quadrature Formula (General Quadrature Formula):

This is the most popular and widely used numerical integration formula. It forms the basis for a number of numerical integration methods known as Newton-Cote's methods.

Derivation of Newton-Cotes formula:

Let the interval [a, b] be divided into n equal sub-intervals such that $a = x_0 < x_1 < x_2 < x_3 \ldots\ldots < x_n = b$. Then $x_n = x_0 + nh$.

Newton forward difference formula is

$$y(x) = y(x_0 + ph) = P_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots\ldots\ldots\ldots \quad (1)$$

Where $p = \dfrac{x - x_0}{h}$ .Now, instead of f(x) we will replace it by this interpolating polynomial.

$\therefore \int_{x_0}^{x_n} f(x)\,dx = \int_{x_0}^{x_n} P_n(x)\,dx$, where $P_n(x)$ is an interpolating polynomial of degree n

$$= \int_{x_0}^{x_0+nh} P_n(x)\,dx = \int_{x_0}^{x_0+nh} \left[y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots\ldots \right] dx$$

Since $x = x_0 + ph$, $dx = h.dp$ and hence the above integral becomes

$$\int_{x_0}^{x_n} f(x)\,dx = h\int_0^n \left[y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots\ldots \right] dp$$

$$= h\left[y_0(p) + \frac{p^2\Delta y_0}{2} + \frac{1}{2}\left(\frac{p^3}{3} - \frac{p^2}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{p^4}{4} - 3\frac{p^3}{3} + 2\frac{p^2}{2}\right)\Delta^3 y_0 + \ldots\ldots \right]$$

$$= h\left[ny_0 + \frac{n^2\Delta y_0}{2} + \frac{1}{2}\left(\frac{n^3}{3} - \frac{n^2}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^4}{4} - 3\frac{n^3}{3} + 2\frac{n^2}{2}\right)\Delta^3 y_0 + \ldots\ldots \right]$$

$$= nh\left[y_0 + \frac{n\Delta y_0}{2} + \frac{1}{2}\left(\frac{n^2}{3} - \frac{n}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^3}{4} - 3\frac{n^2}{3} + 2\frac{n}{2}\right)\Delta^3 y_0 + \ldots\ldots \right]$$

$$= nh\left[y_0 + \frac{n\Delta y_0}{2} + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \left(\frac{n^4}{5} - 3\frac{n^3}{2} + 11\frac{n^2}{3} - 3n\right)\frac{\Delta^4 y_0}{4!} + \ldots \right]$$

This is called **Newton-Cote's Quadrature for)mula.** ....(2)

Reference topic on Trapezoidal Rule:

Definition: The process of finding or evaluating a definite integral

$\boxed{I = \int_a^b f(x)\,dx}$ from a set of numerical values of the integrand f(x). If it is applied to the integration of a function of a single variable the process is known as "**Quadrature**".

The problem of numerical integration is solved by first approximating the integrand by a polynomial with the help of an interpolation formula and then integrating this expression between the desired limits.

Thus, to evaluate the definite integral $\int_a^b f(x)\,dx$ first expression the function f(x) by an interpolation formula say p(x) and then $\boxed{\int_a^b f(x)\,dx \sim \int_a^b p(x)\,dx}$.

The error E is such type of approximate given by,

$$\boxed{\int_a^b f(x)\,dx - \int_a^b p(x)\,dx = \int_a^b [f(x) - p(x)]\,dx}.$$

Definition: Trapezoidal Rule:

Here the function f(x) is approximated by a first- order polynomial $P_1(x)$ which passes through two points.

Putting n = 1 in the above general formula, all differences higher than the first will become zero (since other differences do not exist if n = 1) and we get

$$\int_{x_0}^{x_1} f(x)\,dx = \int_{x_0}^{x_0+h} f(x)\,dx = h\left[y_0 + \frac{1}{2}\Delta y_0\right] = h\left[y_0 + \frac{1}{2}(y_1 - y_0)\right] = \frac{h}{2}(y_0 + y_1)$$

and $\int_{x_1}^{x_2} f(x)\,dx = \int_{x_0+h}^{x_0+2h} f(x)\,dx = h\left[y_1 + \frac{1}{2}\Delta y_1\right] = h\left[y_1 + \frac{1}{2}(y_2 - y_1)\right] = \frac{h}{2}(y_1 + y_2)$

$$\int_{x_2}^{x_3} f(x)\,dx = \int_{x_0+2h}^{x_0+3h} f(x)\,dx = h\left[y_2 + \frac{1}{2}\Delta y_2\right] = h\left[y_2 + \frac{1}{2}(y_3 - y_2)\right] = \frac{h}{2}(y_2 + y_3)$$

....

Finally,

$$\int_{x_0+(n-1)h}^{x_0+nh} f(x)\,dx = \frac{h}{2}(y_{n-1} + y_n)$$

Hence,

$$\int_{x_0}^{x_n} f(x)\,dx = \int_{x_0}^{x_0+nh} f(x)\,dx = \int_{x_0}^{x_0+h} f(x)\,dx + \int_{x_0+h}^{x_0+2h} f(x)\,dx + \int_{x_0+2h}^{x_0+3h} f(x)\,dx + \ldots.. + \int_{x_0+(n-1)h}^{x_0+nh} f(x)\,dx$$

$$= \frac{h}{2}[y_0 + y_1] + \frac{h}{2}[y_1 + y_2] + \ldots\ldots\ldots + \frac{h}{2}(y_{n-1} + y_n)$$

$$= \frac{h}{2}\left[(y_0 + y_1) - 2(y_1 + y_2 + y_3 + y_4 + \ldots\ldots + y_{n-2} + y_{n-1}\right]$$

.... (3)

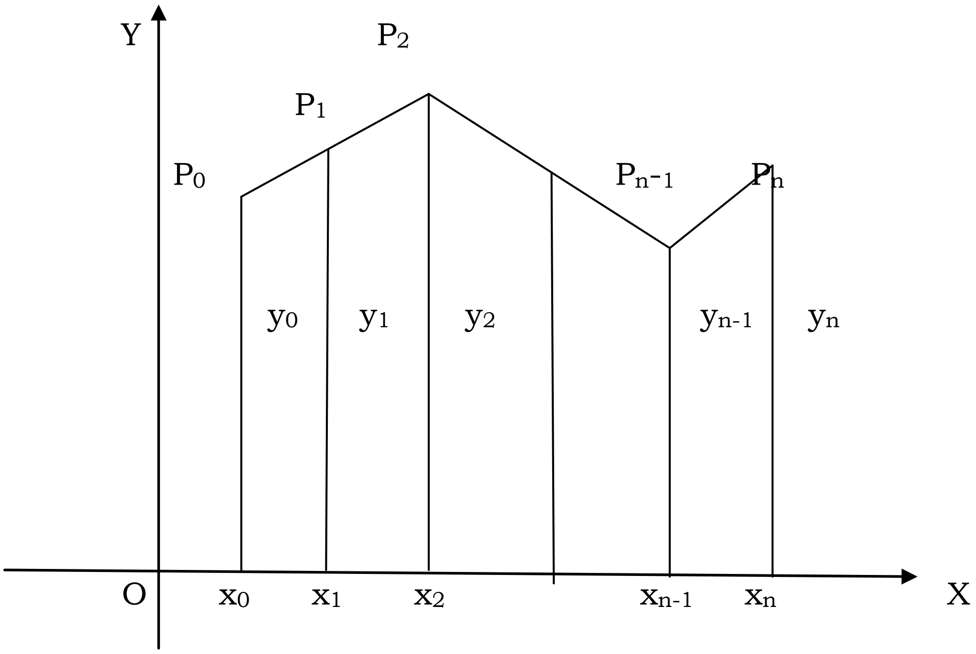
$$\int_{x_0}^{x_n} f(x)\,dx \sim$$

$$\frac{h}{2}\left[sum\ of\ the\ first\ and\ last\ ordnates) - 2(sum\ of\ the\ remaining\ ordinates)\right]$$

This is known as **Trapezoidal Rule**.

Geometrical Interpretation:

Consider the points $P_0(x_0, y_0)$, $P_1(x_1, y_1)$,$P_2(x_2, y_2)$, $P_3(x_3, y_3)$,....., $P_n(x_n, y_n)$. Suppose the curve y = f(x) passing through the above points be approximated by the union of the line segments joining (P_0, P_1), (P_1, P_2), (P_2, P_3),,.....,(P_{n-1}, P_n).



Geometrically, the curve y = f(x) is replaced by n straight line segments joining the points (x_0, y_0) and (x_1, y_1); (x_2, y_2) and (x_3, y_3); ,....., (x_{n-1}, y_{n-1}) and (x_n, y_n). The area bounded by the curve y = f(x), x- axis and the ordinates $x = x_0$ and $x = x_n$ is then approximately equal to the sum of the areas of the n trapezium as shown in the figure above.

The total area given by

$$\frac{h}{2}[y_0 + y_1] + \frac{h}{2}[y_1 + y_2] + \frac{h}{2}[y_2 + y_3] + \ldots\ldots\ldots + \frac{h}{2}(y_{n-1} + y_n)$$

$$= \frac{h}{2}[(y_0 + 2(y_1 + y_2 + y_3 + y_4 + \ldots\ldots + y_{n-1}) + y_n] = \int_{x_0}^{x_n} f(x)\,dx \text{ approximately.}$$

Note: Throughout this Trapezoidal rule method is very simple for calculation purposes of numerical integration; the error in this case is significant.

Note: The accuracy of the result can be improved by increasing the number of intervals or by decreasing the value of h.

Reference topic on Simpson's 1/3 Rule

This is another popular and important method. Here, the function f(x) is approximated by a second order polynomial $P_2(x)$ which passes through three successive points.

Putting n = 2 in Newton-Cotes Quadrature formula i.e., by replacing the curve y = f(x) by n/2 parabolas, we have

$$\int_{x_0}^{x_2} f(x)\,dx = 2\text{h}\left[\text{y}_0 + \frac{2}{2}\Delta y_0 + \frac{2(4-3)}{12}\Delta^2 y_0 \right] = 2\text{h}\left[\text{y}_0 + \Delta y_0 + \frac{1}{6}\Delta^2 y_0 \right]$$

$$= 2h\left[y_0 + (\text{y}_1 - y_0) + \frac{1}{6}(y_2 - 2y_1 + \text{y}_0) \right] \quad = 2h\left[\frac{1}{6}y_0 + \frac{2}{3}y_1 + \frac{1}{6}y_2 \right]$$

$$= \frac{2h}{6}\left[y_0 + 4y_1 + y_2 \right] = \frac{h}{3}\left[y_0 + 4y_1 + y_2 \right]$$

Similarly, $\int_{x_2}^{x_4} f(x)\,dx = \frac{h}{3}\left[y_2 + 4y_3 + y_4 \right]$

………………………………………………

$\int_{x_{n-2}}^{x_n} f(x)\,dx = \frac{h}{3}\left[y_{n-2} + 4y_{n-1} + y_n \right]$ Adding all these integrals, we obtain

$$\int_{x_0}^{x_2} f(x)\,dx = \int_{x_0}^{x_2} f(x)\,dx + \int_{x_2}^{x_4} f(x)\,dx + \ldots\ldots\ldots + \int_{x_{n-2}}^{x_n} f(x)\,dx$$

$$= \frac{h}{3}\left[y_0 + 4y_1 + y_2 \right] + \frac{h}{3}\left[y_2 + 4y_3 + y_4 \right] + \ldots\ldots\ldots + \frac{h}{3}\left[y_{n-2} + 4y_{n-1} + y_n \right]$$

$$= \frac{h}{3}\left[(y_0 + 4y_1 + y_2) + (y_2 + 4y_3 + y_4) + \ldots\ldots\ldots\ldots + (y_{n-2} + 4y_{n-1} + y_n)\right]$$

$$= \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_2 + y_3 + y_5 + y_{n-1}) + 2(y_2 + y_4 + y_6 + \ldots\ldots + y_{n-2})\right]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$= \frac{h}{3}\left[\begin{array}{l} sum\ of\ the\ first\ and\ last\ ordnates) + 4(sum\ of\ the\ odd\ ordinates) \\ +\ 2(sum\ of\ the\ remaining\ even\ ordinates \end{array}\right]$$

With the convention that y_0, y_2, y_4,….., y_{2n} are even ordinates and y_1, y_3, y_5,….., y_{2n-1} are odd ordinates.

This is known as **Simpson's 1/3 rule** or simply **Simpson's rule**.

Note: This rule requires the given interval must be divided into even number of equal sub-intervals of width h.

Reference topic on Simpson's 3/8 Rule:

Simpson's 1/3 rule was derived using three points that fit a Quadratic equation. We can extend this approach by incorporating four successive points so that the rule can be exact for a polynomial f(x) of degree 3. Putting n = 3 in Newton-Cote's Quadrature formula, all differences higher than the third will become zero and we obtain

$$\int_{x_0}^{x_3} f(x)\,dx = 3\mathrm{h}\left[\mathrm{y}_0 + \frac{3}{2}\Delta y_0 + \frac{3(6-3)}{12}\Delta^2 y_0 + \frac{3(3-2)^2}{24}\Delta^3 y_0 \right]$$

$$\int_{x_0}^{x_3} f(x)\,dx = 3\mathrm{h}\left[\mathrm{y}_0 + \frac{3}{2}\Delta y_0 + \frac{3}{4}\Delta^2 y_0 + \frac{1}{8}\Delta^3 y_0 \right]$$

$$\int_{x_0}^{x_3} f(x)\,dx = 3\mathrm{h}\left[\mathrm{y}_0 + \frac{3}{2}(y_1 - y_0) + \frac{3}{4}(y_2 - 2y_1 + y_0) + \frac{1}{8}(y_3 - 3y_2 + 3y_1 - y_0) \right]$$

$$\int_{x_0}^{x_3} f(x)\,dx = \frac{3}{8}\mathrm{h}\left[\mathrm{y}_0 + 3y_1 + 3y_2 + y_3\right]$$

Similarly,

$$\int_{x_3}^{x_6} f(x)\,dx = \frac{3}{8}\mathrm{h}\left[\mathrm{y}_3 + 3y_4 + 3y_5 + y_6\right] \quad \text{and so on.}$$

Adding all these integrals, from x_0 to x_n, where n is a multiple of 3, we get

$$\int_{x_0}^{x_n} f(x)\,dx = \int_{x_0}^{x_3} f(x)\,dx + \int_{x_3}^{x_6} f(x)\,dx + \dots\dots\dots\dots + \int_{x_{n-3}}^{x_n} f(x)\,dx$$

$$= \frac{3h}{8}\left[(y_0 + 3y_1 + 3y_2 + y_3) + (y_3 + 3y_4 + 3y_5 + y_6) + \dots\dots + (y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n)\right]$$

$$= \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \dots\dots\dots + y_{n-1}) + 2(y_3 + y_6 + y_9 + \dots\dots\dots + y_n)\right]$$

$$\dots\dots\dots\dots\dots\dots\dots \quad (5)$$

Equation (5) is called **Simpson's 3/8 rule** which is applicable only when n is multiple of 3. This rule is not so accurate as Simpson's 1/3 rule.

Note: while there is no restriction for the number of intervals in Trapezoidal rule, number of sub-intervals n in the case of Simpson's 1/3 rule must be even, for Simpson's 3/8 rule must be multiple of 3.

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B. Tech I Year (II-Semester) May/ June 2014 **T 264- Numerical Methods**
ECE - B Section

UNIT – IV Numerical Differentiation **Faculty Name: N V Nagendram**
Problems & Integration **Tutorial-06**

Problem # 01 Evaluate $\int_0^{\pi/2} e^{\sin x}\, dx$ taking h = π/6? [**Ans.**3.0815]

Problem # 02 Evaluate $\int_0^{1} x^3\, dx$ with five sub-intervals by Trapezoidal rule?

[**Ans.**0.26]

Problem # 03 When a train is moving ar 30 m/sec, steam is shut off and brakes are supplied. The spped of the train per second after t sec. Is given by using Simpson's rule, determine the distance moved by the train in 40 seconds?

Time	0	5	10	15	20	25	30	35	40
speed	30	24	19.5	16	13.6	11.7	10	8.5	7.0

[**Ans.573.4367**]

Problem #04 Evaluate $\int_0^{\pi} t \sin t\, dt$ using the trapezoidal rule? [**Ans.**3.07]

Problem #05 Compute $\int_0^{4} e^{x}\, dx$ by simpson's 1/3 rule with 10 sub-divisions?

[**Ans.5**3.6055]

Problem #05 Find the value of $\int_1^{2} \frac{dx}{x}$ by Simpson's rule. Hence obtain approximate value of $\log_e 2$? [**Ans.0**.6931]

Problem #06 Evaluate $\int_0^{4} e^{x}\, dx$ by using Trapezoidal and Simpson's rule. Also compare your result with the exact value of the integral?
[**Ans.**(i) 57.992 (ii) 53.864 (iii) 53.5981]

Problem #07 Evaluate $\int_0^{2} e^{-x^2}\, dx$ by using Simpson's rule taking h = 0.25?

[**Ans.** 0.63486]

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UNIT – IV Numerical Differentiation **Faculty Name: N V Nagendram**
Problems & Integration **Tutorial-07**

Problem #01 Dividing the range into 10 equal parts, find the approximate value of $\int_0^{\pi} \sin x \, dx$ by (i) Trapezoidal Rule (ii) Simpson's Rule.

[**Ans.** (i) 1.9843 (ii) 2.0009]

Problem #02 A rocket is launched from the ground. Its acceleration measured every 5 sec. Is tabulated below. Find the velocity and the position of the rocket at t = 40 seconds. Use Trapezoidal rule as well as Simpson's rule?

T	0	5	10	15	20	25	30	35	40
a(t)	40.0	42.25	48.50	51.25	54.35	59.48	61.5	64.3	68.7

[**Ans.** velocity 2194.9;position 87796;velocity 2197.5;position 87900]

Problem #03 Evaluate the following integral using Simpson's 1/3 rule for n = 4 $\int_1^2 \frac{e^x}{x} dx$? [Ans. 3.0592]

Problem #04 Evaluate $\int_0^1 \frac{1}{1+x} dx$ (i) by Trapezoidal Rule and Simpson's 1/3 rule (ii) Using Simpson's 3/8 rule? [Ans.0.69485;0.6931;0.6932]

Problem #05 Given that

X	4.0	4.2	4.4	4.6	4.8	5.0	5.2
Log x	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

Evaluate $\int_4^{5.2} \log x \, dx$ by Simpson's 3/8 rule? [Ans. 1.827847]

Problem #06 Evaluate $\int_0^1 \sqrt{1+x^4} \, dx$ using Simpson's 3/8 rule?[**Ans.**1.08942]

Problem #07 Evaluate $\int_0^6 \frac{1}{1+x} dx$ using Simpson's 1/3 and 3/8 rule and compare the result with actual value? [**Ans.**1.9586; 1.9659;1.94591]

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UNIT – IV Numerical Differentiation **Faculty Name: N V Nagendram**
Problems & Integration **Tutorial-08**

Problem #01 Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ using Simpson's 3/8 rule taking h = 1/6

Hence obtain an approximate value of π? [**Ans. 3.1416**]

Problem #02 Evaluate $\int_0^1 \sqrt{1+x^3}\,dx$ using (i)Simpson's 1/3 rule and (ii) Trapezoidal rule? [**Ans.**1.1114; 1.11226]

Problem #03 The table below shows the temperature f(t) as a function of time

t	1	2	3	4	5	6	7
f(t)	81	75	80	83	78	70	60

Use Simpson's 1/3 rule method to estimate $\int_0^7 f(t)\,dt$? [**Ans. 403.6667**]

Problem #04 Evaluate $\int_{0.6}^{2.0} y\,dx$ using Trapezoidal rule given data below

x	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
y	1.23	1.58	2.03	4.32	6.25	8.38	10.23	12.45

[**Ans.10.416]**

Problem #05 Calculate $\int_1^2 \frac{dx}{x}$ Using Simpson's rule and Trapezoidal rule take h = 0.25 in the given range? [**Ans.0.697]**

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UNIT – IV **Numerical Differentiation & Integration** **Faculty Name: N V Nagendram**

Objective Type Questions **OTQ-01**

01. If $y_0 = 0$, $y_1 = 10$, $y_2 = 18$, $y_3 = 25$ and $h = 2$ then the value of $\int_0^6 y\,dx$ by Simpson's 3/8 rule is .... [81.75]

02. The n th difference of a polynomial of degree n is .... [constant]

03. Trapezoidal rule states that ....

$$[\frac{h}{2}[(y_0 + y_1) - 2(y_1 + y_2 + y_3 + y_4 + ... + y_{n-2} + y_{n-1}]$$

04. f(x) is given by

x	0	1	2	3	4	5	6
f(x)	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$

Then the value of $\int_0^1 f(x)\,dx$ by Simpson's 3/8 rule is .... [1.966]

05. f(x) is given by,

x	0	0.5	1.0
f(x)	1	0.8	0.5

Then by Trapezoidal rule $\int_0^1 f(x)\,dx$ is..................... [0.775]

06. If f(0) = 1, $f(\frac{\pi}{6})$ = 1.6487, $f(\frac{\pi}{3})$ = 2.3632, $f(\frac{\pi}{2})$ = 2.7182, then the value of $\int_0^{\pi/2} f(x)\,dx$ by Simpson's 3/8 rule is .... [0.0911]

07. If f(0) = 1, f(1) = 0.5, f(2) = 0.2, f(3) = 0.1, f(4) = 0.0588, f(5) = 0.0385, f(6) = 0.027, then the value of $\int_0^6 f(x)\,dx$ by Simpson's 3/8 rule is [1.3571]

08. by Simpson's 3/8 rule, $\int_a^b f(x)\,dx =$

$$\frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + ... + y_{n-1}) + 2(y_3 + y_6 + y_9 + ... + y_n)]$$