Convergence of NRM

→ Let x_n be the estimated root of function $f(x)$.
→ x_n and x_{n+1} are two consecutive iterations.
→ The taylor series expansion is,

$$f(x_{n+1}) = f(x_n) + f'(x_n)\,h + f''(x_n)\,\frac{h^2}{2!} + \cdots$$

or, $$f(x_{n+1}) = f(x_n) + f'(x_n)(x_{n+1} - x_n) + f''(x_n)\,\frac{(x_{n+1}-x_n)^2}{2!} + \cdots$$

$\rightarrow$ eqⁿ ① [where $h = x_{n+1} - x_n$

Neglecting 3rd and higher order derivatives from eqⁿ ①, we get

$$f(x_{n+1}) = f(x_n) + f'(x_n)(x_{n+1} - x_n) + f''(x_n)\,\frac{(x_{n+1}-x_n)^2}{2!}$$

Let x_r be the real root and $x_{n+1} = x_r$,

$$f(x_{n+1}) = 0 = f(x_n) + f'(x_n)(x_r - x_n) + f''(x_n)\,\frac{(x_r - x_n)^2}{2!} \rightarrow \text{eq}^n\ ②$$

The iterative formula for NRM is,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore f(x_n) = (x_n - x_{n+1}) * f'(x_n)$$

Substituting value of $f(x_n)$ in eqⁿ ②, we get

$$(x_n - x_{n+1})\,f'(x_n) + f'(x_n)(x_r - x_n) + \frac{f''(x_n)}{2}(x_r - x_n)^2 = 0$$

or, $$f'(x_n)\,(\cancel{x_n} - x_{n+1} + x_r - \cancel{x_n}) + \frac{f''(x_n)}{2}(x_r - x_n)^2 = 0$$

or, $$f'(x_n)\,(x_r - x_{n+1}) + \frac{f''(x_n)}{2}(x_r - x_n)^2 = 0$$

or, $$f'(x_n) * e_{n+1} + \frac{f''(x_n)}{2}(e_n)^2 = 0$$

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or, $f'(x_n)\, e_{n+1} = \dfrac{-f''(x_n)}{2}\, e_n^2 = 0$

or, $e_{n+1} = \dfrac{-f''(x_n)\, e_n^2}{2 f'(x_n)}$

$\therefore \boxed{e_{n+1} \propto e_n^2}$

This shows that, error is proportional to the square of the error in previous iteration. Therefore, NRM is quadratic convergence.

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