Balance-assist bicycle weave mode identification

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1 Introduction

Research into the balance-assist bicycle is done with a model of a bicycle. The parameters of this model are for a different bicycle than the balance-assist bicycle. It is unknown how well the model represents the balance-assist bicycle. It is important for the balance-assist bicycle to be properly modelled, because the controller is designed based on the (thus far inaccurate) model. If the model is not representative of the bicycle, the designed controller may be invalid.

The goal of this experiment is to identify the actual weave mode of the balance-assist bicycle for different controller gains.

2 Methods

The weave mode will be identified by manually perturbing the bicycle at the seat post, and measuring the consequent roll rate. The weave mode can be found be fitting a decaying oscillation to this data. The following equation, defined by Kooijman et al. [1], is used to fit to the roll rate data:

$$c_1 + e^{dt}(c_2 \cos(\omega t) + c_3 \sin(\omega t)) \tag{1}$$

The imaginary part of the eigenvalue is given by the frequency ω. The real part is given by the damping $d.c_1$, c_2 and c_3 are constants. An example of the function fitted to roll rate data can be seen in figure 1. Eigenvalues were measured with the balance-assist system on with a gain of -6, -8 and -8 at speeds of 6, 8, 10, 12, 14, 16 and 18 km/h. For each velocity, the real and imaginary part of the eigenvalues were averaged over three perturbations.

The theoretical eigenvalues have been calculated for the *rigid bicycle* with an added balance-assist controller model. Table 1 shows the parameters of the bicycle model. The rigid bicycle parameters have been chosen because, of all the bicycles that have been measured, the rigid bicycle most closely represents the mass distribution of the balance-assist bicycle. The control law for the modelled controller is given by:

$$T_\delta = gain(5.0 - v)\dot{\phi} \quad \text{for} \quad 0.0 < v < 5.0 \tag{2}$$
$$T_\delta = 0 \quad \text{for} \quad v > 5.0 \tag{3}$$

3 Results

The mean, minimal and maximal r^2 value for the fitted function on the roll data can be seen in table 2. A plot of the measured eigenvalues for different gains of the balance-assist controller can be seen in figure 2. Furthermore, the measured eigenvalues for gain -6 can be seen in table 3, for gain -8 in table 4 and for gain -10 in table 5.

4 Discussion

Overall it can be seen that the theoretical eigenvalues of the rigid bicycle with modelled balance-assist controller do not line up with the measured eigenvalues of the balance-assist bicycle. The imaginary part of the measured eigenvalues are consistently lower than the theoretical eigenvalues.

It is interesting that the weave mode is similarly damped for all three gains. In contrast, the imaginary part of the eigenvalues increases as the gain gets larger. This would suggest that there is no reason to set the gain any larger than -6.

Table 1: Bicycle parameters used for the theoretical eigenvalues.

Parameter	**Value**
IBxx	2.63751918806
IBxz	0.65355108991
IByy	4.27679806219
IBzz	1.94277851254
IFxx	0.0524475128396
IFyy	0.0983720589324
IHxx	0.343873849682
IHxz	-0.0918999164196
IHyy	0.339925043172
IHzz	0.10310836332
IRxx	0.0700963804567
IRyy	0.12934189355
c	0.0599399125715
g	9.81
la	0.317489078499
mB	22.9
mF	1.55
mH	5.4
mR	4.9
rF	0.335572561302
rR	0.332528026978
v	1.0
w	1.06378677685
xB	0.334844476225
xH	0.818396445062
zB	-0.735628402212
zH	-0.98585673135

Table 2: Mean, maximal and minimal r^2 for the three gains.

Gain	**Mean**	**Minimal**	**Maximal**
-6	0.98	0.81	1.00
-8	0.99	0.98	1.00
-10	0.97	0.82	0.99

Table 3: Real and imaginary parts of the measured eigenvalues for a gain of -6.

Velocity (m/s)	**Real part**	**Imaginary part**
1.66	-3.10437	6.90899
2.22	-1.73230	7.47695
2.78	-1.07677	7.81142
3.33	-0.64501	7.79933
3.89	-0.57419	6.93977
4.44	-0.73719	7.33274
5.00	-0.82591	4.65962

Table 4: Real and imaginary parts of the measured eigenvalues for a gain of -8.

Velocity (m/s)	**Real part**	**Imaginary part**
1.66	-1.96625	8.97518
2.22	-1.67178	9.59152
2.78	-1.08789	9.82538
3.33	-0.82441	9.27701
3.89	-0.56166	8.26214
4.44	-0.70618	5.97152
5.00	-0.90672	5.32524

Table 5: Real and imaginary parts of the measured eigenvalues for a gain of -10.

Velocity (m/s)	**Real part**	**Imaginary part**
1.66	-3.73303	11.03922
2.22	-1.83437	10.90979
2.78	-1.08183	10.84567
3.33	-0.78383	10.49930
3.89	-0.59061	9.07418
4.44	-0.53405	6.87337
5.00	-0.02263	4.16736

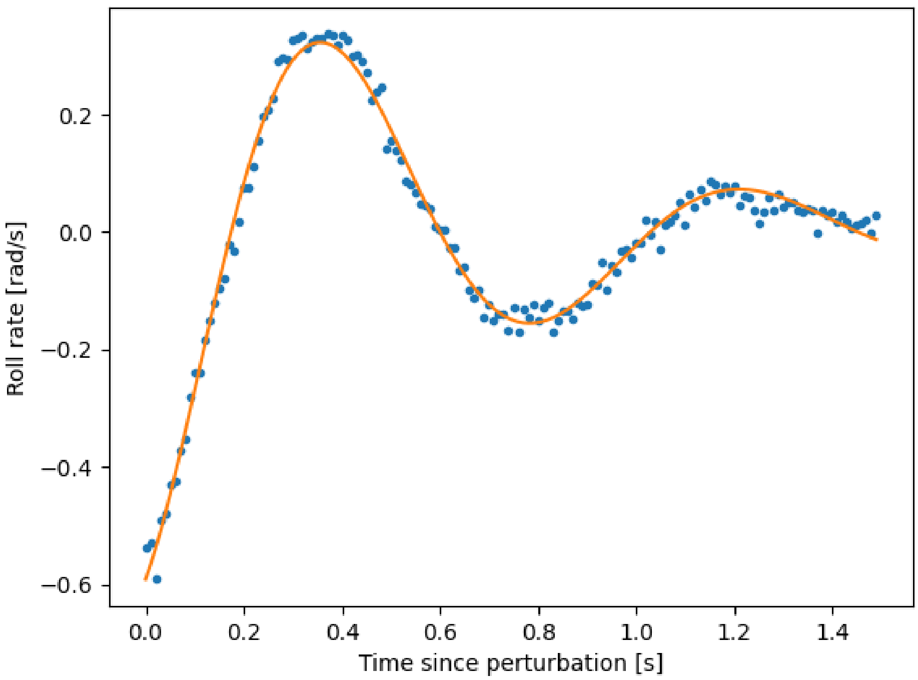

Figure 1: Example of a fit of a decaying oscillation to roll rate data.

Looking at the r^2 values in table 2, it can be seen that the function in equation 1 is fitted best to the roll data for a gain of -8. Both the -6 and -8 gains have a much lower minimal r^2. However, the average r^2 for these gains is still close to 1.

Future work can adjust the parameters of the bicycle model to better represent the measured eigenvalues. For example, the *symfit* package [2] could be used to fit the symbolic equations of motion of the bicycle model to the measured eigenvalues. Another approach would be to optimize the gain of the theoretical controller to better approximate the measured eigenvalues. The theoretical controller is not identical to the actual controller on the bicycle. By assuming the parameters of the rigid bicycle to be approximately correct, the difference in the theoretical and measured eigenvalues should be due to the difference in balance-assist controller.

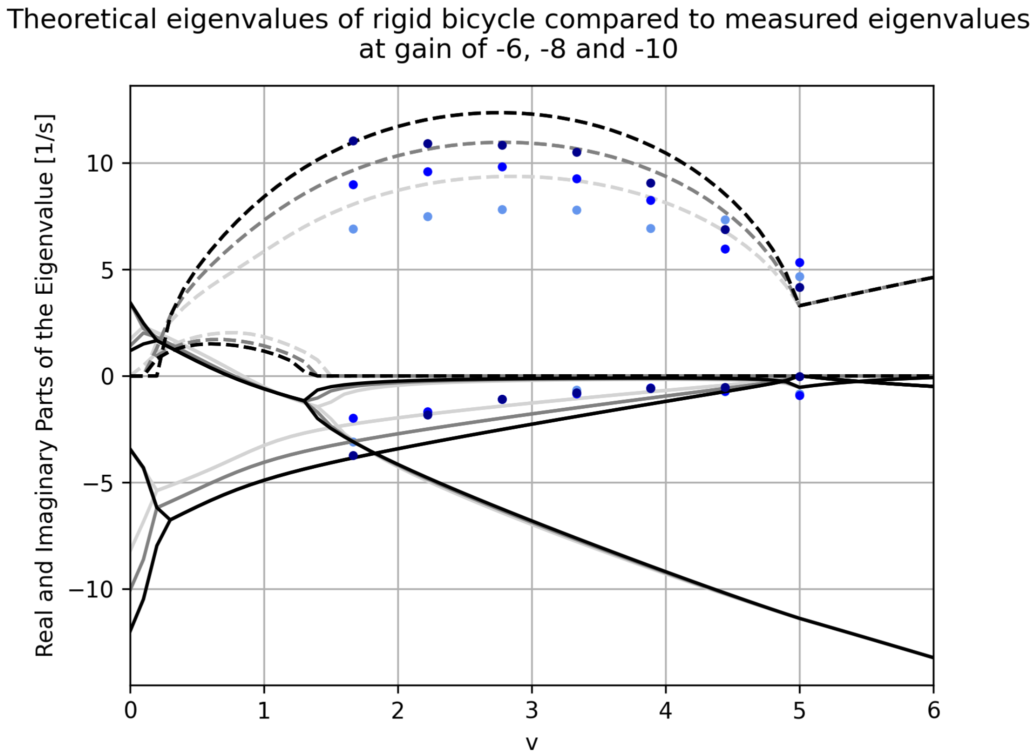

Figure 2: Theoretical eigenvalues of the rigid bicycle with added balance-assist and control compared to the measured eigenvalues of the balance-assist bicycle. The lightest color represents a gain of -6, the medium color a gain of -8 and the dark color a gain of -10.

References

[1] J.D.G. Kooijman, A.L. Schwab, and J.P. Meijaard. Experimental validation of a model of an uncontrolled bicycle. *Multibody Syst Dyn*, 19:115–132, 2008.

[2] Martin Roelfs and Peter C Kroon. Symfit.