

# Balance-assist bicycle weave mode identification

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## 1 Introduction

Research into the balance-assist bicycle is done with a model of a bicycle. The parameters of this model are for a different bicycle than the balance-assist bicycle. It is unknown how well the model represents the balance-assist bicycle. It is important for the balance-assist bicycle to be properly modelled, because the controller is designed based on the (thus far inaccurate) model. If the model is not representative of the balance-assist bicycle, the designed controller may be invalid.

The goal of this experiment is to identify the actual weave mode of the balance-assist bicycle for different controller gains.

## 2 Methods

The weave mode will be identified by manually perturbing the bicycle at the seat post, and measuring the consequent roll rate. The weave mode can be found by fitting a decaying oscillation to this data. The following equation, defined by Kooijman et al. [1], is used to fit to the roll rate data:

$$c_1 + e^{dt}(c_2 \cos(\omega t) + c_3 \sin(\omega t)) \tag{1}$$

The imaginary part of the eigenvalue is given by the frequency  $\omega$ . The real part is given by the damping  $d$ . An example of the function fitted to roll rate

data can be seen in figure 1. Eigenvalues were measured with the balance-assist system on with a gain of -6, -8 and -8 at speeds of 6, 8, 10, 12, 14, 16 and 18 km/h. For each velocity, the real and imaginary part of the eigenvalues were averaged over three perturbations.

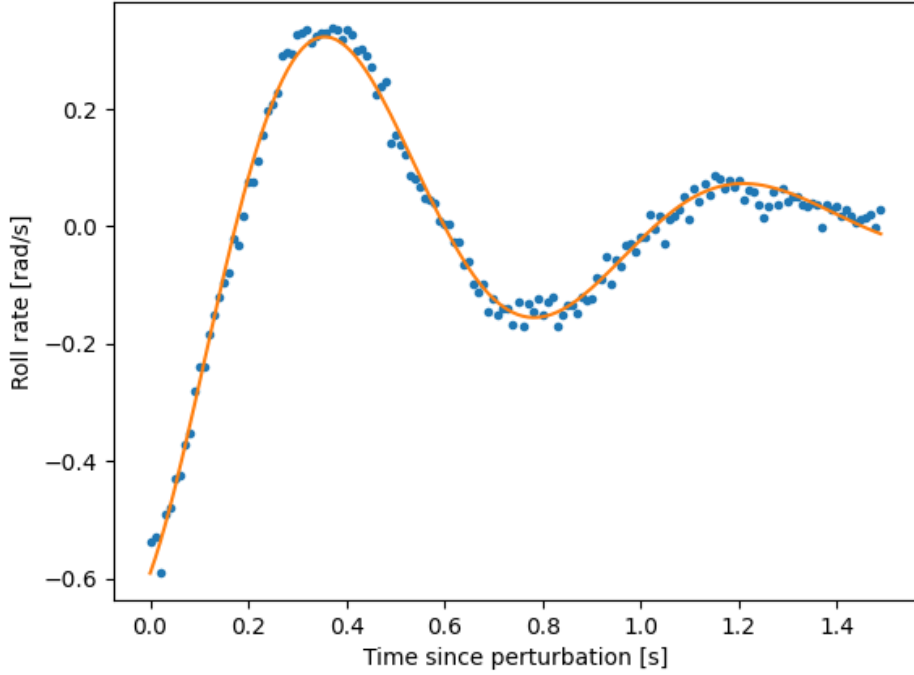


Figure 1: Example of a fit of a decaying oscillation to roll rate data.

### 3 Results

The mean, minimal and maximal  $r^2$  value for the fitted function on the roll data can be seen in table ?? A plot of the measured eigenvalues for different gains of the balance-assist controller can be seen in figure 2. Furthermore, the measured eigenvalues for gain -6 can be seen in table 2, for gain -8 in table 3 and for gain -10 in table 4.

Theoretical eigenvalues of rigid bicycle compared to measured eigenvalues at gain of -6, -8 and -10

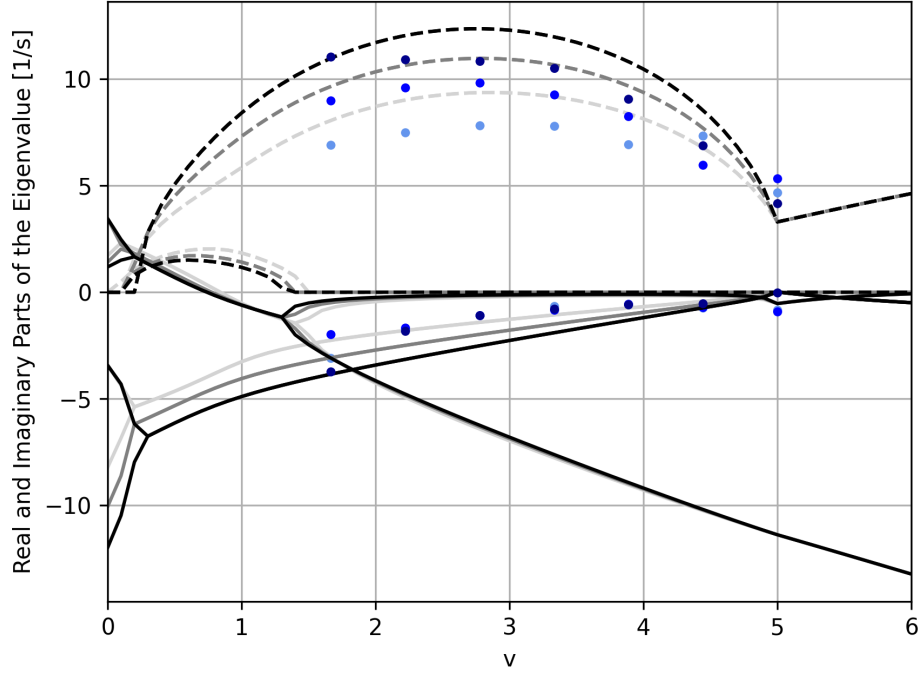


Figure 2: Theoretical eigenvalues of the rigid bicycle with added balance-assist and control compared to the measured eigenvalues of the balance-assist bicycle. The lightest color represents a gain of -6, the medium color a gain of -8 and the dark color a gain of -10.

Table 1: Mean, maximal and minimal  $r^2$  for the three gains.

Gain	Mean	Minimal	Maximal
-6	0.98	0.81	1.00
-8	0.99	0.98	1.00
-10	0.97	0.82	0.99

Table 2: Real and imaginary parts of the measured eigenvalues for a gain of -6.

Velocity (m/s)	Real part	Imaginary part
1.66	-3.10437	6.90899
2.22	-1.73230	7.47695
2.78	-1.07677	7.81142
3.33	-0.64501	7.79933
3.89	-0.57419	6.93977
4.44	-0.73719	7.33274
5.00	-0.82591	4.65962

Table 3: Real and imaginary parts of the measured eigenvalues for a gain of -8.

Velocity (m/s)	Real part	Imaginary part
1.66	-1.96625	8.97518
2.22	-1.67178	9.59152
2.78	-1.08789	9.82538
3.33	-0.82441	9.27701
3.89	-0.56166	8.26214
4.44	-0.70618	5.97152
5.00	-0.90672	5.32524

Table 4: Real and imaginary parts of the measured eigenvalues for a gain of -10.

Velocity (m/s)	Real part	Imaginary part
1.66	-3.73303	11.03922
2.22	-1.83437	10.90979
2.78	-1.08183	10.84567
3.33	-0.78383	10.49930
3.89	-0.59061	9.07418
4.44	-0.53405	6.87337
5.00	-0.02263	4.16736

## 4 Discussion

It is interesting that the weave mode is similarly damped for all three gains. In contrast, the imaginary part of the eigenvalues increases as the gain gets larger. This would suggest that there is no reason to set the gain any larger than -6.

Looking at the  $r^2$  values in table 1, it can be seen that the function in equation 1 is fitted best to the roll data for a gain of -8. Both the -6 and -8 gains have a much lower minimal  $r^2$ . However, the average  $r^2$  for these gains is still close to 1.

Future work can adjust the parameters of the bicycle model to better represent the measured eigenvalues. For example, the *symfit* package [2] could be used to fit the symbolic equations of motion of the bicycle model to the measured eigenvalues. Another approach would be to optimize the gain of the theoretical controller to better approximate the measured eigenvalues. The theoretical controller is not identical to the actual controller on the bicycle. By assuming the parameters of the rigid bicycle approximately correct, the difference in the theoretical and measured eigenvalues should be due to the difference in balance-assist controller.

## References

- [1] J.D.G. Kooijman, A.L. Schwab, and J.P. Meijaard. Experimental validation of a model of an uncontrolled bicycle. *Multibody Syst Dyn*, 19:115–132, 2008.
- [2] Martin Roelfs and Peter C Kroon. Symfit.