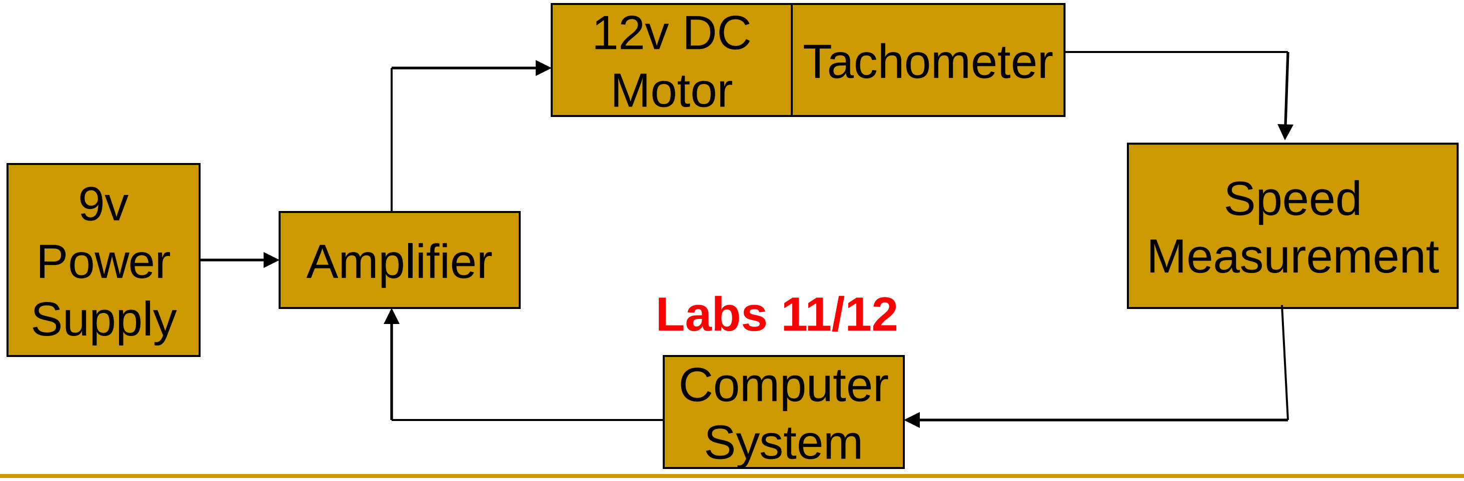
Lab 11. Speed Control of a D.C. motor

Motor Characterization

Motor Speed Control Project

1. Generate PWM waveform
2. Amplify the waveform to drive the motor
3. Measure motor speed
4. Measure motor parameters
5. Control speed with a PID controller

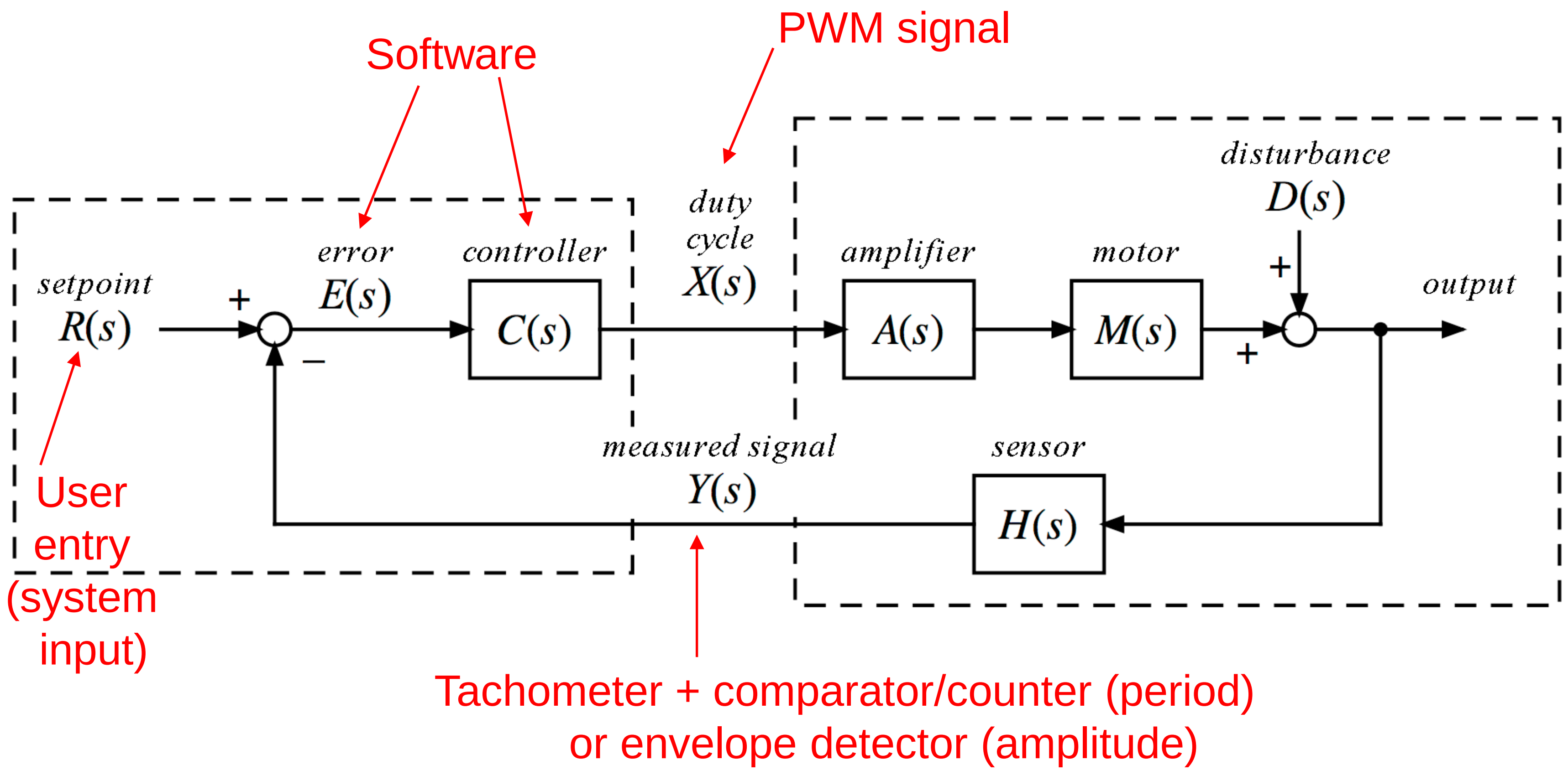


Goals of this lab

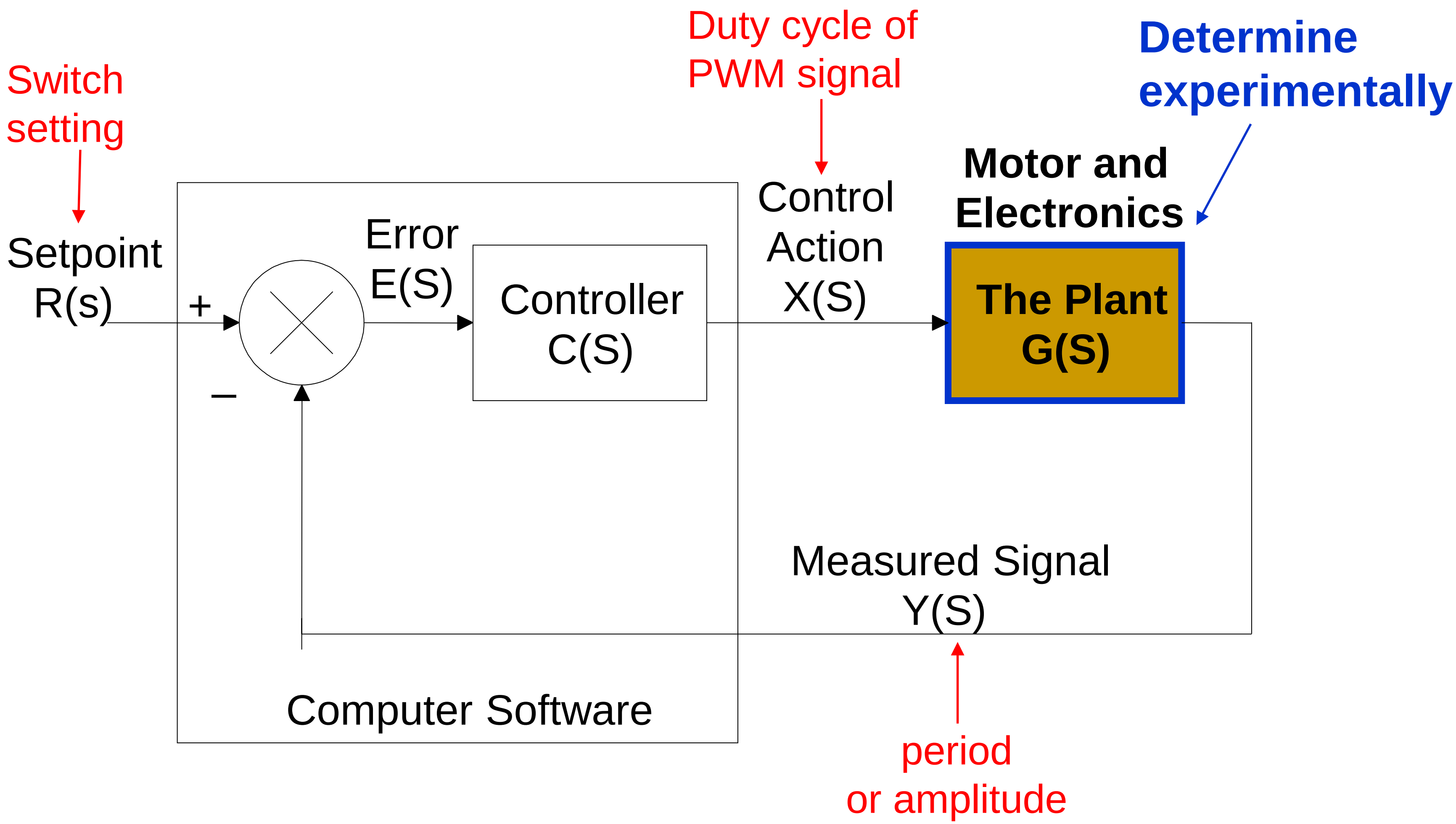
- Experimentally determine the control system model of the motor/hardware setup
  - Measure response to a step input
    (determine time constant, gain, etc.)
- This model will be used in the design of a speed controller

Motor control system modeled as a feedback system

(Frequency domain model)



Simplified system model



What goes into the plant $G(s)$?

- Electrical dynamics (motor winding has inductance and resistance)
- Mechanical dynamics (motor rotor has inertia and experiences friction)
- Sensor dynamics (filter has capacitance and resistance)

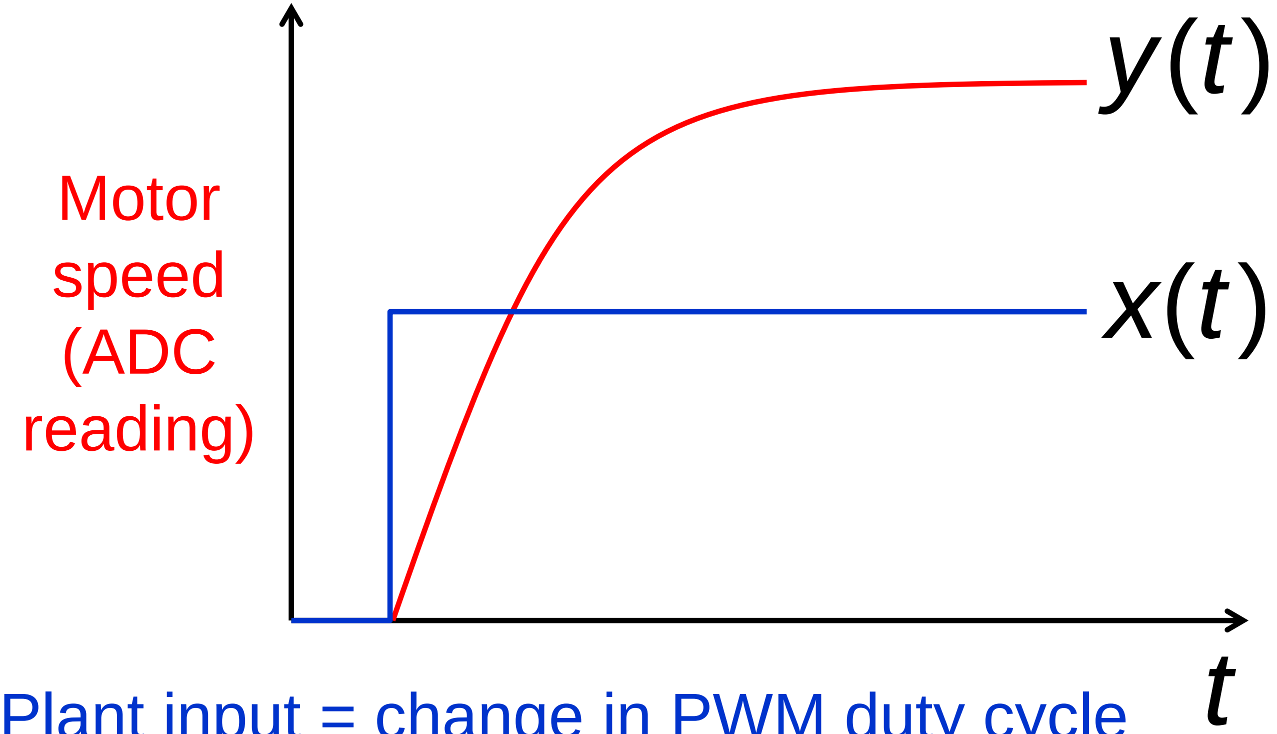
OVERALL: A 3rd order model

An Empirical Modeling Approach

- Experimentally determine “plant” model, G(s)

1. Apply a “step input” to the Plant
   - step change in the duty cycle of the PWM signal driving the motor
2. Measure the motor system “response” to this step input
   - measure speed change over time
3. Derive parameters of G(s) from the measured response

Response $y(t)$ of a 1st-order system to a step input $x(t)$



Plant input = change in PWM duty cycle (at t = 0)

First-order system model

System equation:

$$Kx(t) = \tau \frac{dy}{dt} + y(t)$$

x(t) = system input
y(t) = system output
K = gain
τ = time constant

Solution if step input applied at t=0 (step response):

$$\Delta y(t) = K \Delta x(t)(1 - e^{-t/\tau})$$

Δx = input change at time t=0

Laplace transform (plant transfer function):

$$G(s) = \frac{Y(s)}{X(s)} = \frac{K}{\tau s + 1}$$

Experimentally determining $G(s)$ for the first-order system

- After the transient period (*t* large), study output *y:*

$$\Delta y = K \Delta x$$

$$K = \frac{\Delta y}{\Delta x}$$

Experimentally measure change in y (after large *t*) to compute gain, *K*.

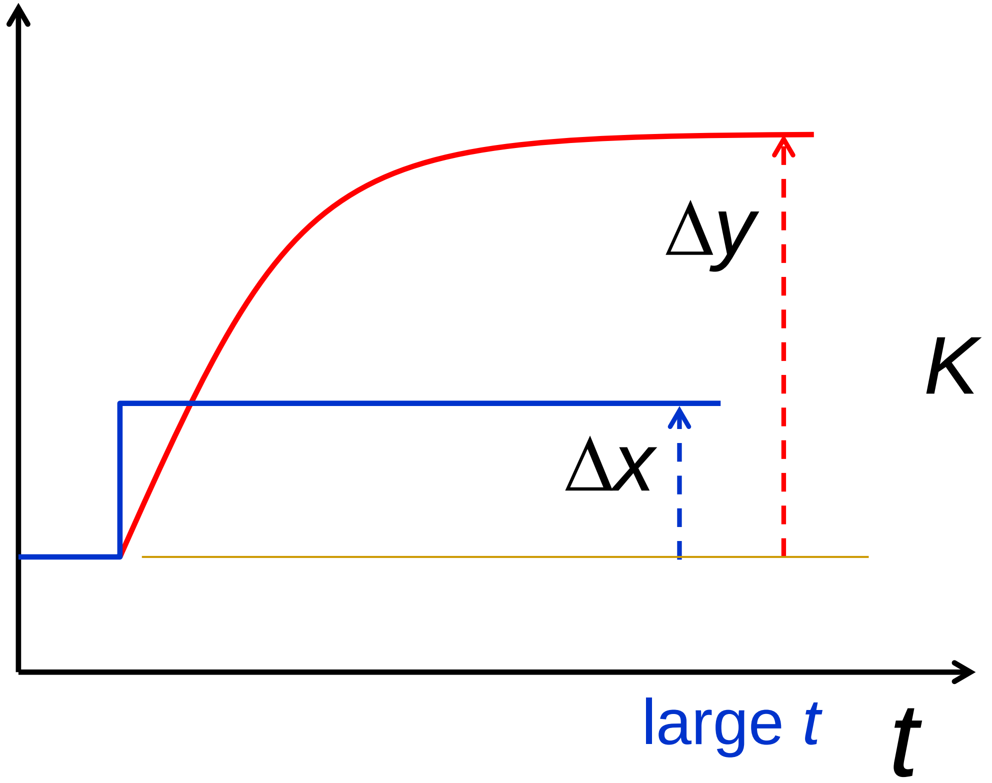
- At t=τ, step response is:

$$y(\tau) = K\Delta x(1 - e^{-\tau/\tau})$$

$$y(\tau) = K\Delta x(0.632)$$

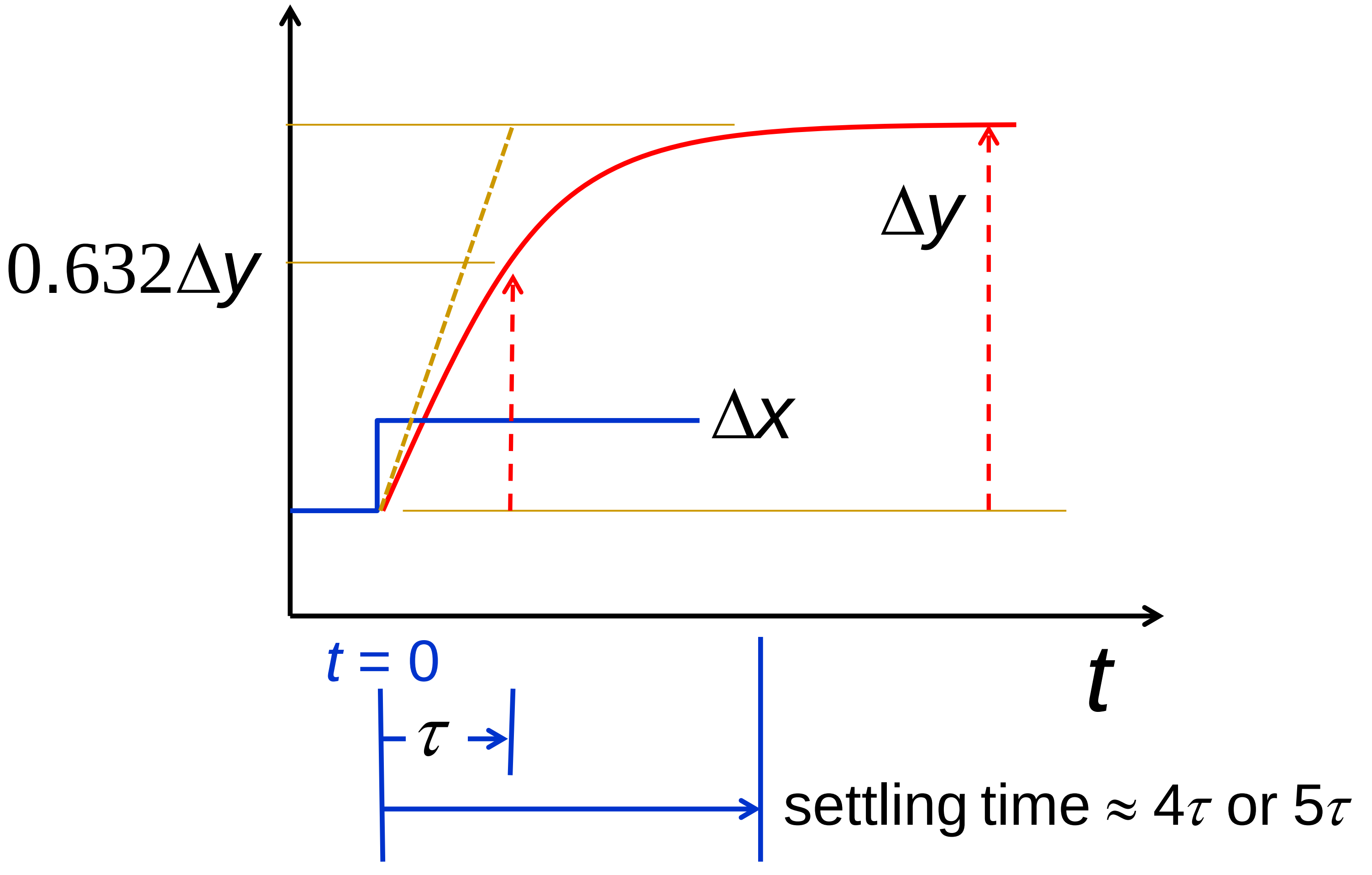
Experimentally measure time at which y(t) = 63.2% of final value to determine time constant, τ.

Finding gain K

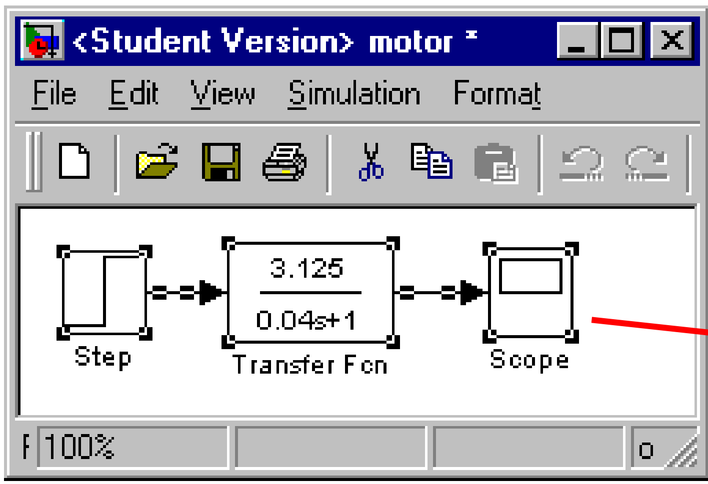


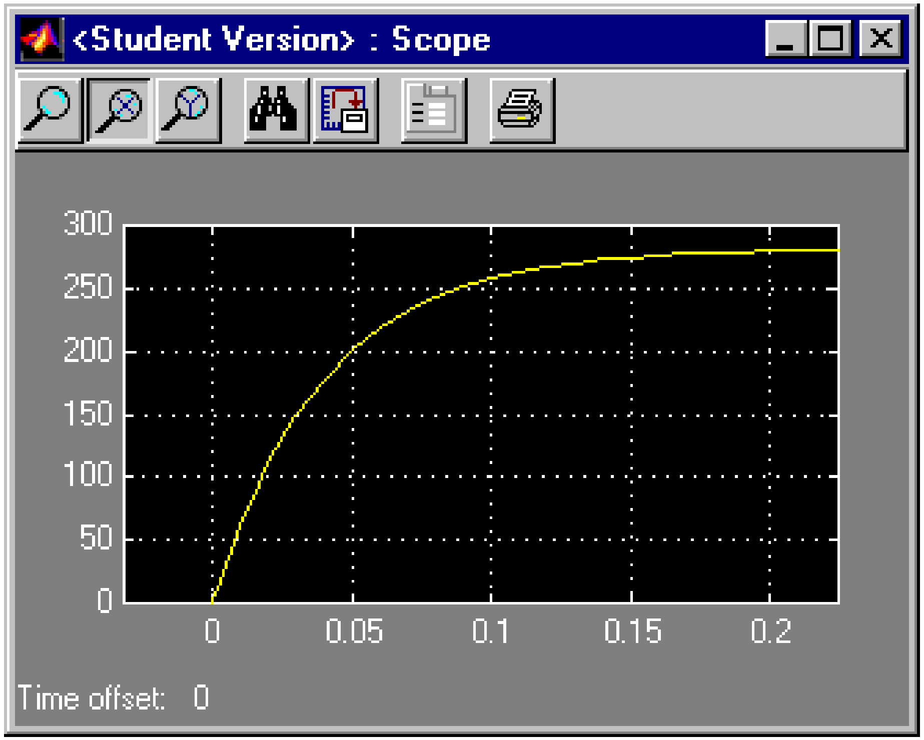
$$K = \frac{\Delta y}{\Delta x}$$

Finding time constant τ



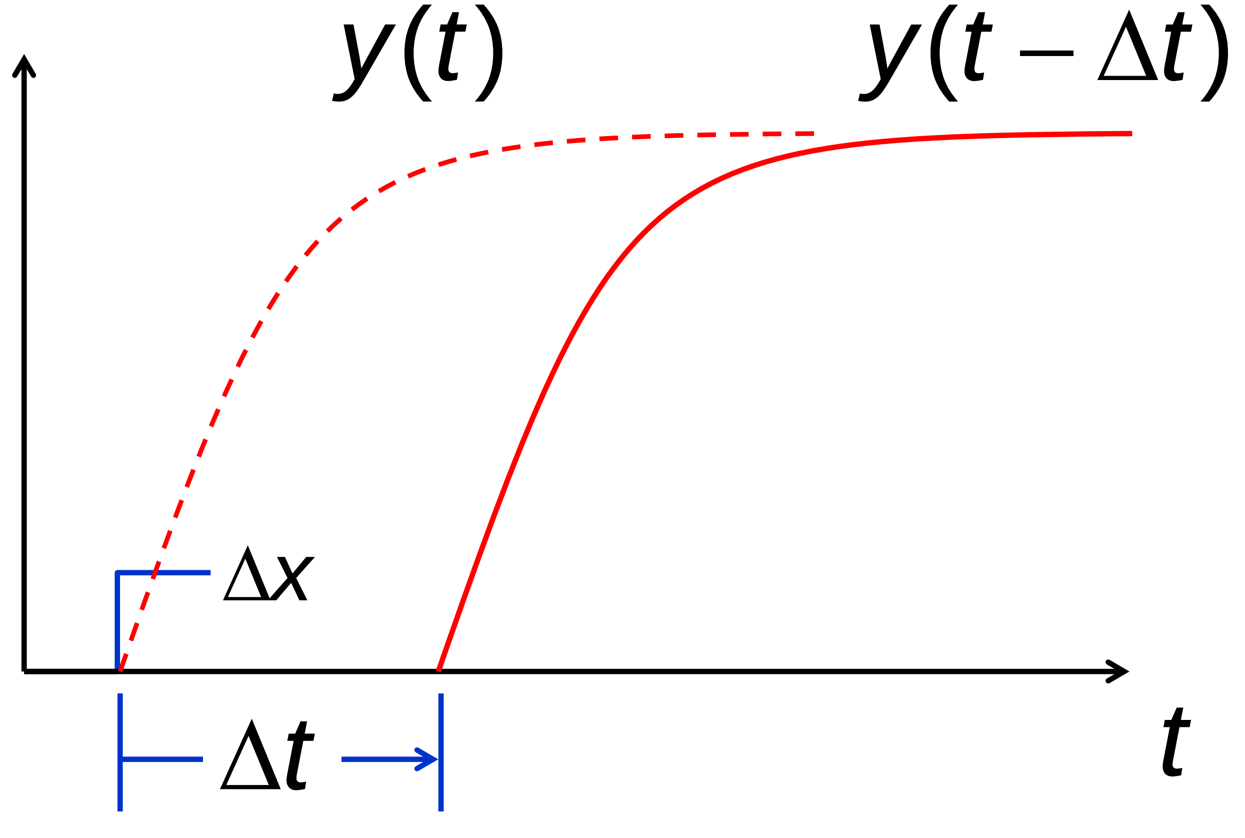
Verify model in MATLAB/Simulink





(Controller to be added to this to compute the controller parameters.)

First-order response with delay

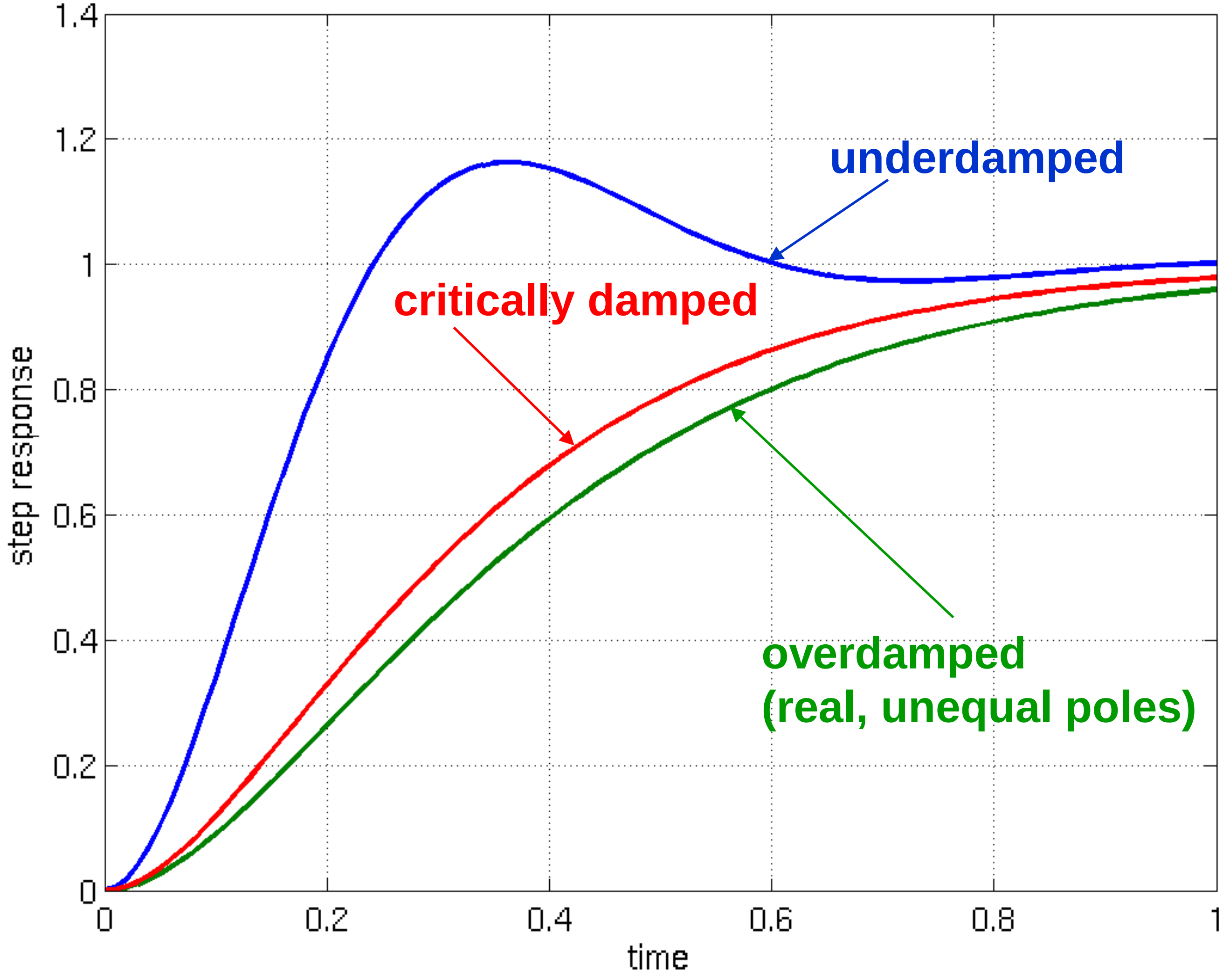


First-order system with delay

$$G(s) = \frac{K}{\tau s + 1} e^{-\Delta t s}$$

$e^{-\Delta t s}$ represents time delay Δt

Second-order step response



Underdamped 2nd-order model

$$G(s)=\frac{Y(s)}{X(s)}=K\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

gain

damping factor

undamped natural frequency

2nd-order model character (a)

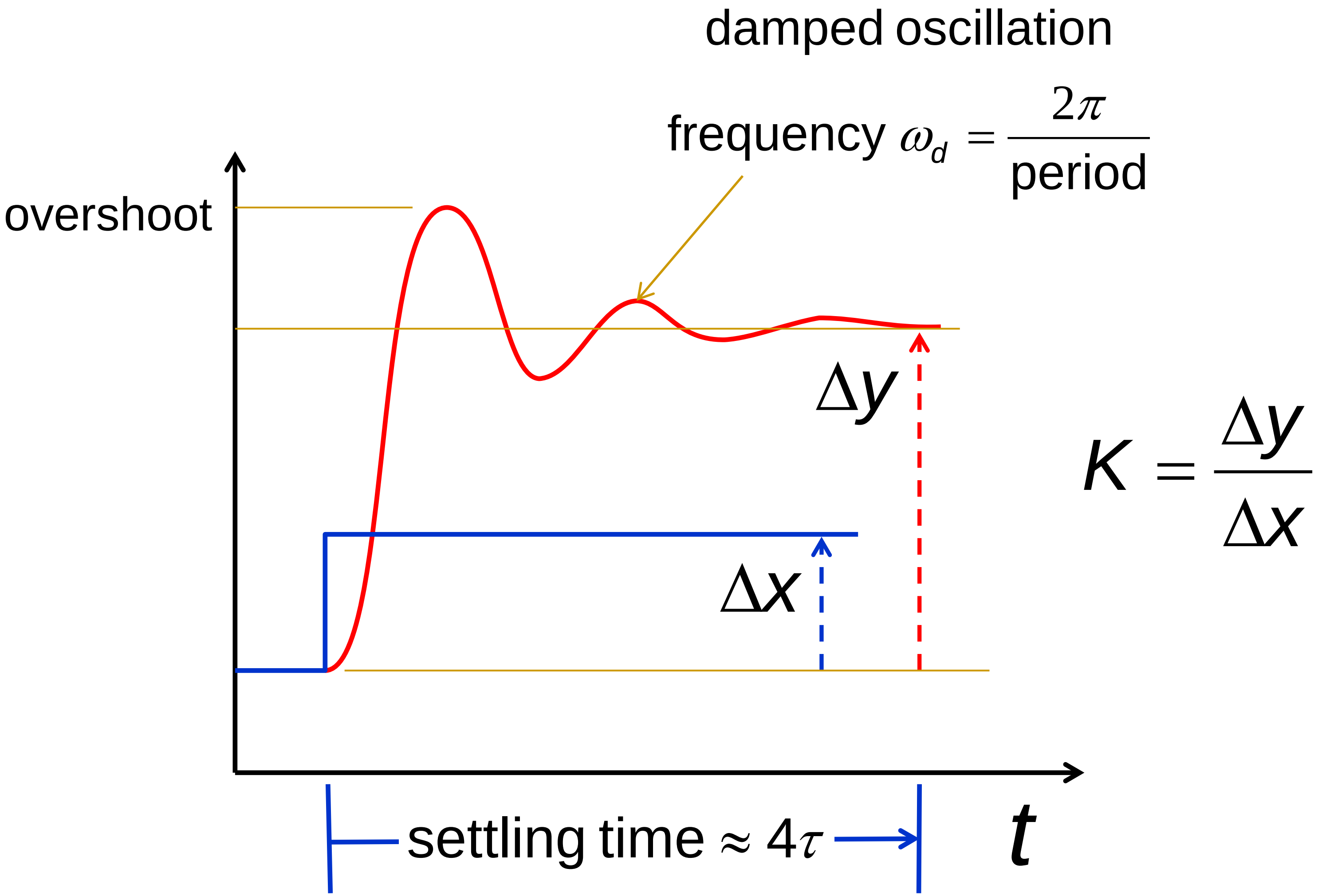
- Underdamped ($0 < \zeta < 1$) model has complex conjugate poles:

$$s_{1,2} = -\underbrace{\zeta\omega_n}_{\text{Re}} \pm j\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\text{Im}}$$

- time constant: inverse of the |Re| part

$$\tau = \frac{1}{\zeta\omega_n}$$

Underdamped step response



2nd-order model character (b)

- oscillation frequency (rad/s): Im part

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

- overshoot (% of final value)

$$\%\ \text{overshoot} = e^{-\left(\frac{\mathrm{Re}}{\mathrm{Im}}\right)\pi} \times 100$$

  - a function only of damping factor

Other 2nd-order forms

- Critically damped model has 2 equal poles

$$G(s) = \frac{K}{(\tau s + 1)^2}$$

- Overdamped model has unequal poles

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Lab Procedure

- Re-verify hardware/software from previous labs
- Modify software to measure the period (or voltage) of the tachometer signal following a step input
  - “Step input” = change in selected speed
  - Save values in an array that can be transferred to the host PC <u>after</u> the motor is stopped
- Plot measured speed vs. time
- Choose a model (1st-order? 2nd-order?)
- Determine model parameters and write the transfer function $G(s)$
- Compare step response of $G(s)$ to the experimental response (suggested tool: MATLAB/Simulink)