Homework Assignment #3 - Due Mon. 2/11/13

Problem 1

Consider the plot in Figure 1.

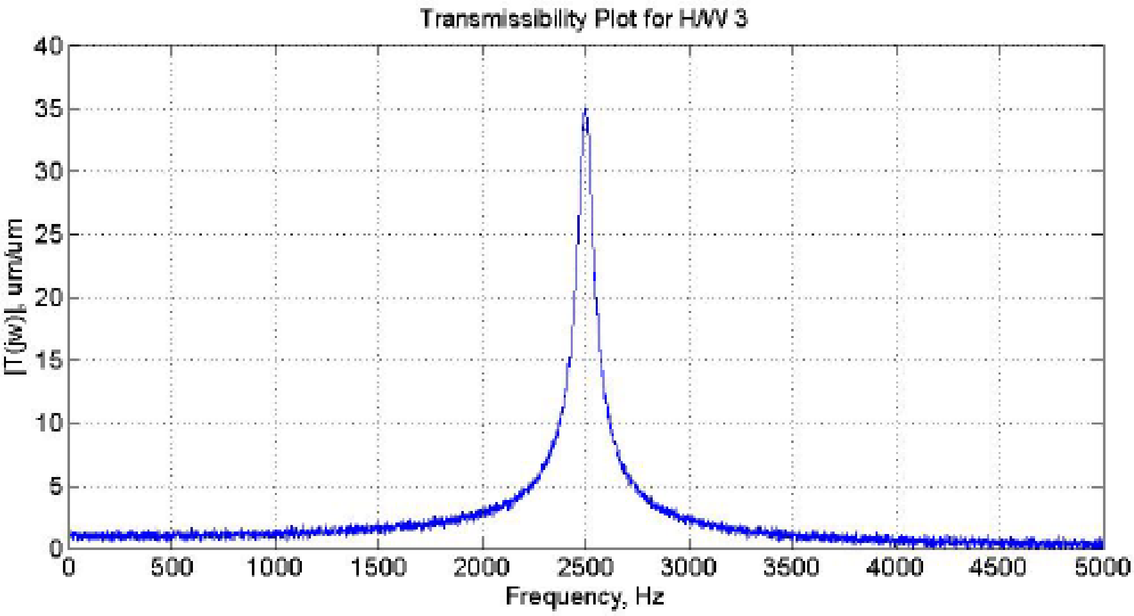

Figure 1: This is the transmissibility plot for a MEMS device with a 100μg proof mass.

a. What is Q?
$\max \|T(j\omega)\| : Q \gg 1 : \omega = \omega_{\mathrm{n}} \Rightarrow Q = 35\mu m/\mu m$

b. What is the damping ratio?
$\zeta = \frac{1}{2Q} = \frac{1}{2*35} = 0.0143 = 14.3 \times 10^{-3}$

c. What is the natural frequency in KHz?
$\max \|T(j\omega)\| : Q \gg 1 : \omega = \omega\mathrm{n} \Rightarrow \frac{\omega_{\mathrm{n}}}{2\pi} = 2.5\mathrm{KHz}$

d. What is the spring constant?
${\omega_{\mathrm{n}}}^2 = \mathrm{K}/m \Rightarrow \mathrm{K} = {\omega_{\mathrm{n}}}^2 * m = (2*\pi*2500\mathrm{rad/s})^2(100\times10^{-9}kg) = 24.67\ kg/s^2 = 24.67\ \mathrm{N}/m$

e. What is the damping coefficient?
$\frac{\omega_n}{Q} = \frac{C}{m} \Rightarrow C = \frac{\omega_n m}{Q} = \frac{(2*\pi*2500\mathrm{HZ})(100\times10^{-9}kg)}{35} = 44.88\times10^{-6}\mathrm{N}\cdot s/m$

f. If the device is excited with a sinusoidal input at its natural frequency with an amplitude of 0.2μm, what is the amplitude of the proof mass displacement at that frequency?
$\omega = \omega_n : y(t) = A\sin(\omega t) : x(t) = B\sin(\omega t)$
$Q = \max\|T(j\omega)\| = X(s)/Y(s) \rightarrow X(s) = Q\times Y(s) \Rightarrow B = Q \times A$
$A = 0.2\mu m \Rightarrow B = 35\times 0.2\mu m = 7\mu m$

g. For the input in (f), what is the maximum accelertion experienced by the proof mass, in G's? [1G= $9.8m/s^2$]
$x(t) = B\sin(\omega t); \dot{x}(t) = B\omega\cos(\omega t); \ddot{x}(t) = -B\omega^2\sin(\omega t); \max\ddot{x} = B{\omega_n}^2$
$\Rightarrow \max\ddot{x} = (7\mu m)(2*\pi*2.5Krad/s)^2 = 1727m/s^s$
$1727m/s^2 \approx 176.24G \Rightarrow \max\ddot{x} = 176.24G$

h. What is the expression for T(s) for this device?
$T(s) = \frac{2\zeta\omega_n s+{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2} : : \zeta = 14.3\times10^{-3} : \omega_n = 2*\pi*2.5\mathrm{Krad/s}$
$\Rightarrow T(s) = \frac{2(14.3\times10^{-3})(2*\pi*2.5Krad/s)(s)+(2*\pi*2.5Krad/s)^2}{s^2+2(14.3\times10^{-3})(2*\pi*2.5Krad/s)(s)+(2*\pi*2.5Krad/s)^2}$
$T(s) = \frac{449.24\mathrm{Krad}/ss+246.7\times10^6\mathrm{rad/s}^2}{s^2+449.24\mathrm{Krad}/ss+246.7\times10^6\mathrm{rad/s}^2}$

i. Using Matlab with an m-file, plot $\|T(j\omega)\|$. Turn in your plot (in a similar format to the one above (it should look very similar, but with less noise)) AND your m-file.

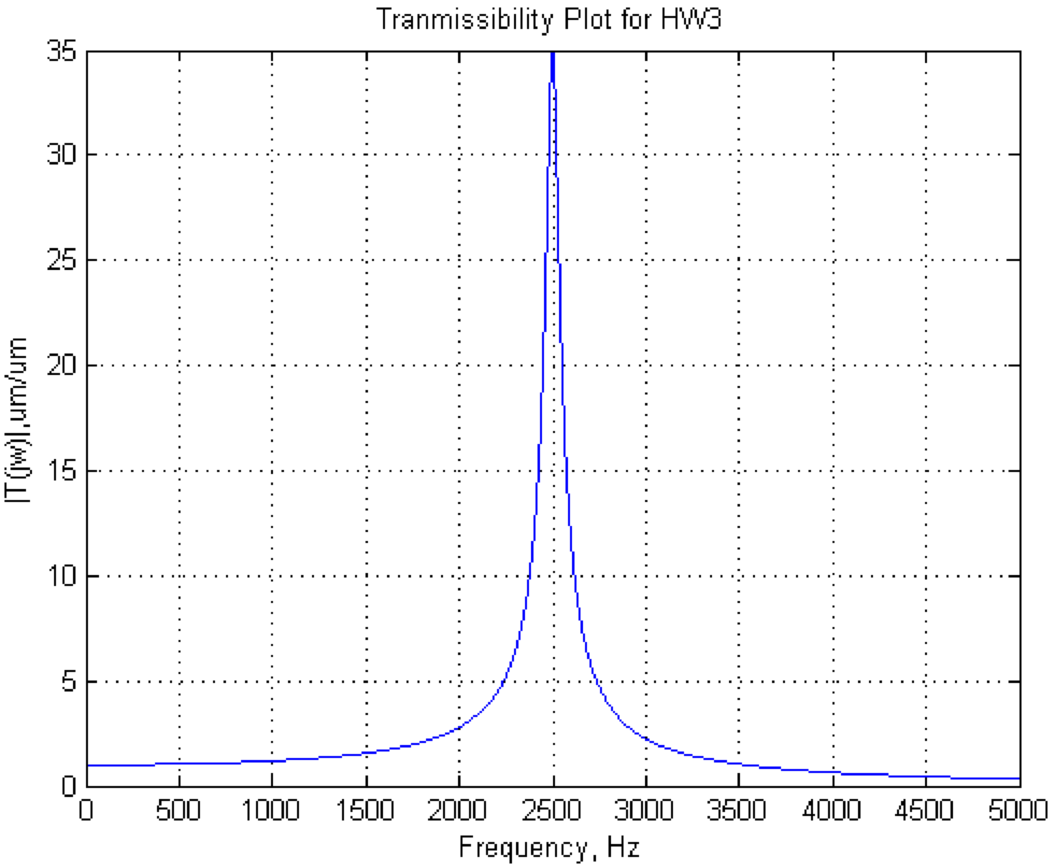

Figure 2: This is the plot of $\|T(j\omega)\|$ produced in MATLAB.

HW3_1_i.m

```matlab
% HW 3, problem 1 part (i)
% Plot |T(jw)|
%% Define Variables
z = 14.3e-3;
wn = 2*pi*2500; % 2.5KHz
f = 0:5000;
w = 2*pi.*f;
%% Compute |T(jw)|
T = ((1 + ((2*z.*w)./wn).^2)./((1 - (w./wn).^2).^2+((2.*z.*w)./wn).^2)).^(0.5);
%% Plot
figure(1)
plot(f,T)
title('Tranmissibility Plot for HW3')
xlabel('Frequency, Hz')
ylabel('|T(jw)|,um/um')
grid on
```