1. a. For a cantilever : $K = \frac{E w t^3}{4L^3} = \frac{(170\times10^9)(10\times10^{-6})(20\times10^{-6})^3}{4(500\times10^{-6})^3} = 27.2\ N/m$

$$\boxed{K = 27.2\ N/m}$$

b. $\forall$ of the proof mass : $\forall = (20\times10^{-6})(1000\times10^{-6})(1000\times10^{-6}) = 2\times10^{-11}\ m^3$

$$\boxed{\forall = 2\times10^{-11}\ m^3}$$

c. $m = \delta \forall$

$$\delta = (2.35\ g/cm^3)\left(\frac{1\ Kg}{1000\ g}\right)\left(\frac{100\ cm}{m}\right)^3 = 2350\ Kg/m^3$$

$$m = \delta \forall = 2350(2\times10^{-11}) = 4.7\times10^{-8}\ Kg = 4.7\times10^{-5}\ g$$

$$\boxed{m = 4.7\times10^{-5}\ g}$$

d. $\omega_n = \sqrt{\frac{K}{m}} \rightarrow f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{27.2}{4.7\times10^{-8}}} = 3828.7\ Hz$

$$\boxed{f_n = 3828.7\ Hz}$$

e. $F_I = ma$ , $F_k = kd$

$F_I = F_D \Rightarrow ma = kd$

$$d = \frac{ma}{K} = \frac{(4.7\times10^{-8})(500)(9.8)}{27.2} = 8.47\times10^{-6}\ m = 8.47\ \mu m$$

$$\boxed{d = 8.47\ \mu m}$$

2. $K \simeq \frac{N_{Leg}}{N_{Zig}} \frac{E w t^3}{L^3} = \frac{4}{1} \frac{E w t^3}{L^3}$

$$\boxed{K \simeq \frac{4 E w t^3}{L^3}}$$