Homework Assignment #2 - Due Mon. 2/4/13

Problem 1

A MEMS Si cantilevered beam is used as a spring. It is 500μm long, 20μ thick and 10μm wide. At its free end, there is an attached Si rectangular proof mass, 20μm thick, 1000μm long and 1000μm wide. For Si having a Young's Modulus of 170GPA and a density of 2.35g/cm^3:

a. What is the spring constant in N/m?
$K = \frac{Ewt^3}{4L^3} = \frac{170\text{GPa}(10\mu m)(20\mu m)^3}{4(500\mu m)^3} = 27.2\text{N/m}$

b. What is the volume of the proff mass in m^3?
$V = L \times w \times t = (1000\mu m)(1000\mu m)(20\mu m) = 20 \times 10^6(\mu m)^3 = 20 \times 10^{-12}m^3$

c. What is the mass of the proof mass in g?
Density $\equiv \frac{\text{Mass}}{\text{Volume}} \Rightarrow m = V \times \rho = (20 \times 10^{-12}m^3)(2350000g/m^2) = 4.7 \times 10^{-5}g$

d. What is the system's natural frequency in Hz?
$f_n = \omega/_{2\pi} = \frac{1}{2\pi}\sqrt{K/m} = \frac{1}{2\pi}\sqrt{\frac{27.2\text{N/m}}{4.7\times10^{-8}kg}} = 3828.7\text{Hz}$

e. If the mass experiences a 500G acceleration so that the beam bends in the direction of its thickness, what is the displacement of the proof mass in μm (1G $= 9.8m/s^2$)?
$F_s - F_a = 0 \Rightarrow F_s = F_a \Rightarrow Kd = ma \Rightarrow d = ma/K$
$\Rightarrow d = (4.7 \times 10^{-8}kg)(500 \times 9.8m/s^2)/27.2\text{N/m} = 8.47\mu m$

Problem 2

For the MEMS device shown below, what is an approximate expression for the system spring constant for the mode where the proof mass moves perpendicular to the plane of the paper? (*Ans. in Figure 1*).

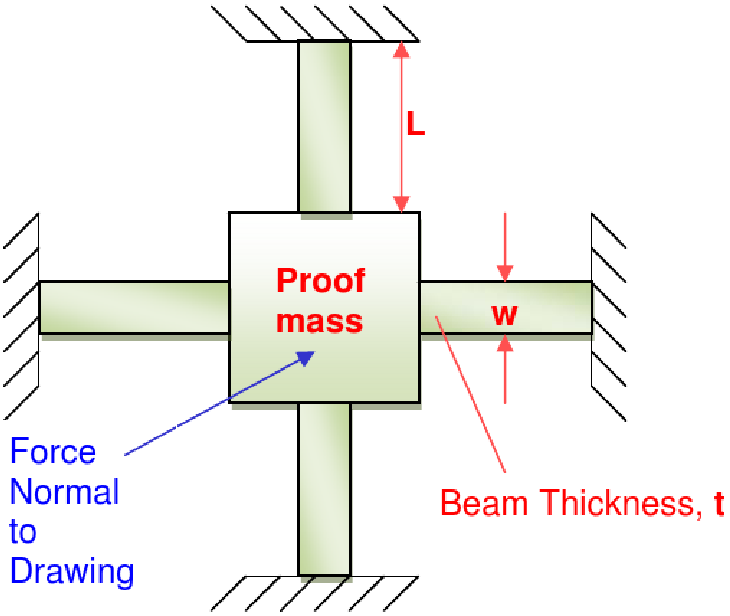

Figure 1: The spring constant can be defined as $K \approx \frac{N_{LEG}}{N_{ZIG}} \times \frac{E\times w\times t^3}{L^3}$. By looking at the picture, one can see that $N_{LEG} = 4$ and $N_{ZIG} = 1$. Therefore, an approximate expression for the system is $K \approx \frac{4}{1}\frac{E\times w\times t^3}{L^3}$.