Homework Assignment #3 - Due Mon. 2/11/13

Problem 1

Consider the plot in Figure 1.

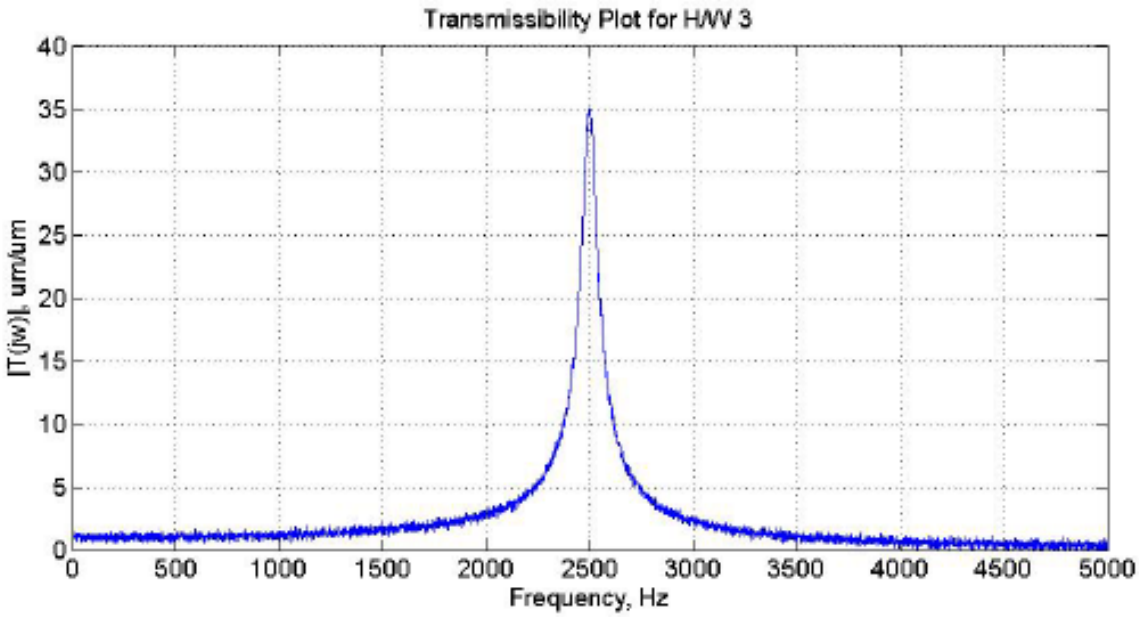

Figure 1: This is the transmissibility plot for a MEMS device with a 100μg proof mass.

a. What is Q?
$\max \|T(j\omega)\|: Q \gg 1: \omega = \omega_{\mathrm{n}} \Rightarrow Q = 35\mu m/\mu m$

b. What is the damping ratio?
$\zeta = \frac{1}{2Q} = \frac{1}{2*35} = 0.0143 = 14.3 \times 10^{-3}$

c. What is the natural frequency in KHz?
$\max \|T(j\omega)\|: Q \gg 1: \omega = \omega\mathrm{n} \Rightarrow \omega_{\mathrm{n}} = 2.5\mathrm{KHz}$

d. What is the spring constant?
${\omega_{\mathrm{n}}}^2 = \mathrm{K}/m \Rightarrow \mathrm{K} = {\omega_{\mathrm{n}}}^2 * m = (2500\mathrm{Hz})^2(100 \times 10^{-9}kg) = 0.625\ kg/s^2 = 0.625\ \mathrm{N}/m$

e. What is the damping coefficient?
$\frac{\omega_n}{Q} = \frac{C}{m} \Rightarrow C = \frac{\omega_n m}{Q} = \frac{(2500\mathrm{HZ})(100\times10^{-9}kg)}{35} = 7.14 \times 10^{-6}\mathrm{N}\cdot s/m$

f. If the device is excited with a sinusoidal input at its natural frequency with an amplitude of 0.2μm, what is the amplitude of the proof mass displacement at that frequency?
$\omega = \omega_n: y(t) = A\sin(\omega t): x(t) = B\sin(\omega t)$
$Q = \max \|T(j\omega)\| = X(s)/Y(s) \rightarrow X(s) = Q \times Y(s) \Rightarrow B = Q \times A$
$A = 0.2\mu m \Rightarrow B = 35 \times 0.2\mu m = 7\mu m$

g. For the input in (f), what is the maximum accelertion experienced by the proof mass, in G's? [1G= $9.8m/s^2$]