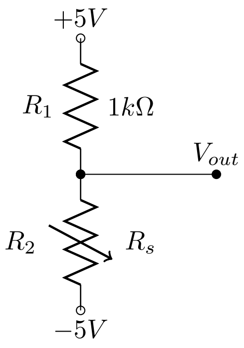
Homework Assignment #4 - Due Fri. 2/15/13

1) An unbuffered resistive sensor, R_s, is in the circuit shown below, where $500\Omega \leq R_s \leq 1.5k\Omega$. What is V_{out} for the minimum, mid range and maximum resistance values of the sensor?



(a) The Circuit

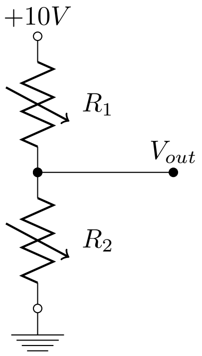
$$V_{out} = -5 + \frac{10R_2}{R_2+R_1}$$

(b) Calculations

R_s, Ω	V_{out}, V
500	-1.67
1k	0
1.5k	1

(c) Voltage vs. Resistance

2) For the differential resistance sensor shown below, where $R_1 = 1k\Omega + \Delta R$, $R_2 = 1k\Omega - \Delta R$, and $0\Omega \leq \Delta R \leq 100\Omega$, calculate the minimum and maximum V_{out}.



(d) The Circuit

$$V_{out} = \frac{10R_2}{R_2+R_1} = \frac{10(1k\Omega-\Delta R)}{1k\Omega-\Delta R+1k\Omega+\Delta R)}$$
$$V_{out} = \frac{10(1k\Omega)-10\Delta R}{2(1k\Omega)}$$
$$V_{out} = 5 - 5\frac{\Delta R}{1k\Omega}$$

(e) Calculations

ΔR	V_{out}	
0Ω	5V	max
100Ω	4.5V	min

(f) Voltage vs. Resistance

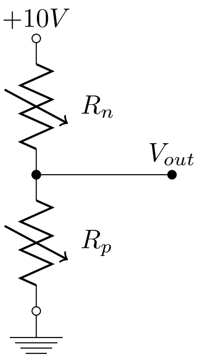
3) A rectangular resistive temperature sensor (5mm long, 50μm wide, and 1μm thick), where current flows through the length of the sensor, is made of a material with a resistivity of 5×10^{-6} Ω-cm at $0°C$, and a TCR of $5 \times 10^{-3}(°C)^{-1}$. What is the approximate resistance at $0°C$ and $100°C$?
$l = 5mm$: $2 = 50\mu m$: $t = 1\mu m$: $\rho_0 = 5 \times 10^{-6}\Omega\text{-cm}|0°C$: $TCR = 5 \times 10^{-3}(°C)^{-1} = \alpha$
$R = \rho\frac{L}{S}$: $\rho = \rho_0(1+\alpha T)$

$$T = 0°| \ R = \rho_0(1+\alpha T)\frac{l}{wt} = (5\times10^{-6}\Omega\text{-cm}\frac{m}{100cm})(1+\frac{5\times10^{-3}}{°C}0°C)\frac{5\times10^{-3}m}{50\times10^{-6}m1\times10^{-6}m} = 5\Omega$$
$$T = 100°| \ R = \rho_0(1+\alpha T)\frac{l}{wt} = (5\times10^{-6}\Omega\text{-cm}\frac{m}{100cm})(1+\frac{5\times10^{-3}}{°C}100°C)\frac{5\times10^{-3}m}{50\times10^{-6}m1\times10^{-6}m} = 7.5\Omega$$

4) A certain metal strain gauge has a nominal resistance of $10k\Omega$ and a gauge factor of 1.8. If it experiences a 1% axial strain, what does the resistance become?
$GF = \frac{\delta R}{\delta L} \Rightarrow \Delta R = GF\frac{\Delta L}{L}R$
$\Delta R = 1.8(0.01)(10k\Omega) = 180\Omega$
$R_{new} = R + \Delta R = 10k + 180 = 10.18k\Omega$

5) If the strain gauge in (4) experiences a -1% axial strain, what does the resistance become?
$GF = \frac{\delta R}{\delta L} \Rightarrow \Delta R = GF\frac{\Delta L}{L}R$
$\Delta R = 1.8(-0.01)(10k\Omega) = -180\Omega$
$R_{new} = R + \Delta R = 10k - 180 = 9.82k\Omega$

6) A polysilicon differential piezoresistive sensor is connected to a 10V source as shown below, where R_n is a N-type piezoresistor and R_p is a P-Type piezoresistor. With no strain on the piezoresistors, $R_n = R_p = 1k\Omega$. Calculate V_{out} for no strain. If R_n has a GF of -30 and R_p has a GF of +30, and both piezoresistors experience a 0.2% axial strain, calculate V_{out}.



(g) The Circuit

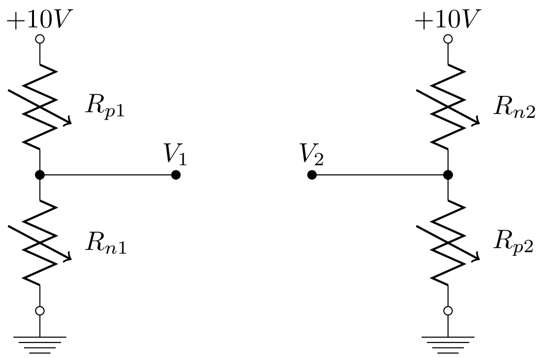
$$V_{out} = 10\frac{R_p}{R_p+R_n}$$
$$V_{out} = 10\frac{1k}{1k+1k}$$
$$V_{out} = 5V$$

(h) Calculation of V_{out} with no strain.

$$R_n = R_n + (GF_n)(\varepsilon_l)(R_n)$$
$$R_n = 1k - 30(.002)(1k) = 940\Omega$$
$$R_p = R_p + (GF_p)(\varepsilon_l)(R_p)$$
$$R_p = 1k + 30(.002)(1k) = 1060\Omega$$
$$V_{out} = \frac{10R_p}{R_p+R_n}$$
$$V_{out} = \frac{10(1060)}{1060+940}$$
$$V_{out} = 5.3V$$

(i) Calculation of V_{out} with an axial strain of 0.2%.

7) If four piezoresistors from problem (6) are connected in a Wheatstone bridge configuration, as shown below, calculate $V_{out} = V_2 - V_1$ for all resistors experiencing a 0.1% axial strain.



(j) The Circuit

$$R_{p1} = R_{p2} = R_p + (GF_p)(\varepsilon_l)(R_p)$$
$$R_{p1} = R_{p2} = 1k + 30(.001)(1k) = 1030\Omega$$
$$R_{n1} = R_{n2} = R_n + (GF_n)(\varepsilon_l)(R_n)$$
$$R_{n1} = R_{n2} = 1k - 30(.001)(1k) = 970\Omega$$

(k) Calculate Resistances.

$$V_1 = \frac{10R_{n1}}{R_{n1}+R_{p1}}$$
$$V_1 = \frac{10(970)}{970+1030}$$
$$V_1 = 4.85V$$

(l) Calculate V_1.

$$V_2 = \frac{10R_{p2}}{R_{n2}+R_{p2}}$$
$$V_2 = \frac{10(1030)}{970+1030}$$
$$V_2 = 5.15V$$

(m) Calculate V_2.

$$V_{out} = V_2 - V_1$$
$$V_{out} = 0.3V$$

(n) Calculate V_{out}.