# Homework Assignment #5 - Due Wed. 2/20/13

1) A MEMS device consists of a proof mass attached to a frame with a suspension system. The bottom of the proof mass is 1mm by 1mm in size and serves as an electrode. Another electrode of the same size is located 2$\mu$m beneath it. If the proof mass can move up and down $\pm 1\mu$m from its nominal distance to the bottom electrodes, calculate the nominal, maximum and minimum capacitance between the two electrodes. Assume that the device is in a vacuum.

Figure 1: Values of Capacitance for (1).
$d = 2\mu m\colon \Delta d = \pm 1\mu m\colon \varepsilon = 8.854pF/m$

$C = \frac{\varepsilon \times w \times L}{d - \Delta d}$
$C = \frac{8.854pF/m \times 1mm \times 1mm}{2\mu m - 1\mu m}$
$C = 8.854pF$
(a) Maximum: $d = d - \Delta d$

$C = \frac{\varepsilon \times w \times L}{d}$
$C = \frac{8.854pF/m \times 1mm \times 1mm}{2\mu m}$
$C = 4.427pF$
(b) Nominal: $d = d$

$C = \frac{\varepsilon \times w \times L}{d - \Delta d}$
$C = \frac{8.854pF/m \times 1mm \times 1mm}{2\mu m + 1\mu m}$
$C = 2.951pF$
(c) Minimum: $d = d + \Delta d$

2) A certain MEMS capacitance has a rest (i.e. nominal) value of 3pF, a minimum value of 2pF and a maximum value of 5pF. Place it in a charge amplifier circuit that has an input voltage of 10V and a feedback capacitor (C2) of 10pF. Calculate the amplifier output voltage (at the end of the $\phi_2$ cycle) for the nominal, minimum and maximum capacitance values.

Figure 2: Amplifier Output Voltages for (2).
$V_{in} = 10V\colon C_2 = 10pF\colon$ After $\phi_2$, $V_{out} = \frac{-C_s V_{in}}{C_2}$

$C_s = 2pF$
$V_{out} = \frac{-C_s V_{in}}{C_2} = \frac{-2pF \times 10V}{10pF}$
$V_{out} = -2V$
(a) $V_{out}$ for $C_{min}$

$C_s = 3pF$
$V_{out} = \frac{-C_s V_{in}}{C_2} = \frac{-3pF \times 10V}{10pF}$
$V_{out} = -3V$
(b) $V_{out}$ for $C_{nom}$

$C_s = 5pF$
$V_{out} = \frac{-C_s V_{in}}{C_2} = \frac{-3pF \times 10V}{10pF}$
$V_{out} = -5V$
(c) $V_{out}$ for $C_{max}$

3) For the MEMS capacitance in (2) place it in a 5V "fast" CMOS ring oscillator circuit with both resistors being $100k\Omega$. What is the output frequency for $C_{min}$, $C_{nom}$ and $C_{max}$?

Figure 3: Output Frequencies for (3).
$t = -RC \ln {}^1\!/_3\colon\ f = {}^1\!/_T\colon T = 2t\colon f = \frac{0.455}{RC}$

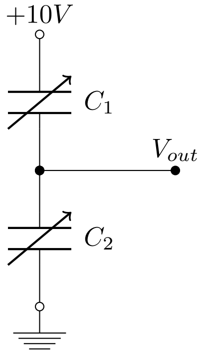
$C = 2pF\colon R = 100k\Omega$
$f = \frac{0.455}{RC} = \frac{0.455}{100k\Omega 2pF}$
$f = 2.275$MHz
(a) $f_o$ for $C_{min}$

$C = 3pF\colon R = 100k\Omega$
$f = \frac{0.455}{RC} = \frac{0.455}{100k\Omega 3pF}$
$f = 1.517$MHz
(b) $f_o$ for $C_{nom}$

$C = 5pF\colon R = 100k\Omega$
$f = \frac{0.455}{RC} = \frac{0.455}{100k\Omega 5pF}$
$f = 910$kHz
(c) $f_o$ for $C_{max}$

4) If two MEMS capacitances from (2) are placed in a capacitive AC voltage divider to realize a differential capacitive sensor configuration, with the input voltage having an amplitude of 10V, what is the output voltage amplitude for each case?

Figure 4: Output Voltage Amplitude for each case in (4).



(a) AC Voltage Divider

$$Q = VC\colon C_{eq} = \frac{C_2 C_1}{C_1 + C_2}\colon$$
$$V_{out} = \frac{Q}{C_2} = \frac{V_i n C_{eq}}{C_2}$$
$$V_{out} = V_{in} \frac{C_1}{C_1 + C_2}$$

(b) Calculations

| $C_1$, pF | $C_2$, pF | $V_{out}$, V |
|---|---|---|
| 2 (min) | 5 (max) | 2.86 |
| 3 (nom) | 3 (nom) | 5.00 |
| 5 (max) | 2 (min) | 7.14 |

(c) Voltage outputs for each case.

5) If the MEMS capacitance from (2) is placed in a switched-capacitor circuit that is switched at 250KHz, what is the value of the equivalent resistance for the nominal, minimum and maximum capacitance values?

Figure 5: Equivalent Resistances for (5).

$$f_{switch} = 250kHz$$
$$i = \frac{\delta q}{\delta t} = \frac{Q}{T} = \frac{CV_{in}}{T} = Cf_{switch}V_{in}$$
$$\Rightarrow R = \frac{V}{i} = \frac{1}{Cf_{switch}}$$

$C = 2pF$

$R = \frac{1}{Cf_{switch}} = \frac{1}{2pF \times 250kHz}$

$R = 2M\Omega$

(a) $R$ for $C_{min}$

$C = 3pF$

$R = \frac{1}{Cf_{switch}} = \frac{1}{3pF \times 250kHz}$

$R = 1.333M\Omega$

(b) $R$ for $C_{nom}$

$C = 5pF$

$R = \frac{1}{Cf_{switch}} = \frac{1}{5pF \times 250kHz}$

$R = 800k\Omega$

(c) $R$ for $C_{max}$

6) If the MEMS device in (2) is placed in an RC phase delay circuit, where R = 250k$\Omega$, what is the phase delay in $\mu s$ for the nominal, minimum and maximum capacitance values?

Figure 6: Phase Delay for (6).

$$V_{out} = \frac{V_{in}}{s}\frac{1/sC}{R + 1/sC} = \frac{V_{in}}{s} - \frac{V_{in}}{s + 1/RC}$$
$$V_{out}(t) = V_{in}(1 - e^{-t/RC})$$
$$t\colon V_{out} = V_{in}/2 \rightarrow V_{in}/2 = V_{in}(1 - e^{-t/RC})$$
$$1/2 = 1 - e^{-t/RC} \rightarrow t = -RC \ln 1/2$$
$$t = 0.693RC$$

$C = 2pF\colon R = 250k\Omega$

$t = 0.692(2pf)(250k\Omega)$

$t = 0.346\mu s$

(a) Phase delay for $C_{min}$

$C = 3pF\colon R = 250k\Omega$

$t = 0.692(3pf)(250k\Omega)$

$t = 0.520\mu s$

(b) Phase delay for $C_{nom}$

$C = 5pF\colon R = 250k\Omega$

$t = 0.692(5pf)(250k\Omega)$

$t = 0.866\mu s$

(c) Phase delay for $C_{max}$