

a. $Q = 35$

b. $\zeta = \frac{1}{2Q} = \frac{1}{70} = 0.0143$

c. $f_n = 2.5 \text{ KHz}$

d. $m = 100 \mu\text{g} = (100 \times 10^{-6})(1 \times 10^{-3}) = 1 \times 10^{-7} \text{ Kg}$

$$\omega_n = 2\pi f_n$$

$$\omega_n^2 = \frac{k}{m}$$

$$k = m \omega_n^2 = m (2\pi f_n)^2 = 1 \times 10^{-7} (2\pi(2500))^2 = 24.67 \text{ N/m}$$

e. $\frac{c}{m} = \frac{\omega_n}{Q}$

$$c = \frac{m \omega_n}{Q} = \frac{m 2\pi f_n}{Q} = \frac{(1 \times 10^{-7}) 2\pi(2500)}{35} = 4.49 \times 10^{-5} \text{ Kg/s}$$

f. input amplitude = $0.2 \mu\text{m}$

$$\begin{aligned} \text{output amplitude} &= (\text{input amplitude}) \left| \frac{T_c(j\omega)}{\omega_n} \right| \\ &= (\text{input amplitude}) Q \\ &= (0.2 \mu\text{m})(35) \\ &= 7 \mu\text{m} \end{aligned}$$

g. if $x(t) = X_0 \sin(\omega t) \rightarrow \text{displacement}$

$$\dot{x}(t) = X_0 \omega \cos(\omega t) \rightarrow \text{velocity}$$

$$\ddot{x}(t) = -X_0 \omega^2 \sin(\omega t) \rightarrow \text{acceleration}$$

$$\ddot{y}(t) = Q \ddot{x}(t)$$

$$\omega = \omega_n$$

$$\ddot{y}(t)_{\text{max}} = Q X_0 \omega_n^2 = 35(2 \times 10^{-7})(2\pi 2500)^2 = 1727 \text{ m/s}^2$$

$$\ddot{y}(t)_{\text{max}} = \frac{1727}{9.8} = 176 \text{ g's}$$

$$h. \frac{\omega_n}{Q} = \frac{2\pi f_n}{Q} = \frac{2\pi(2500)}{35} = 448.8 \text{ s}^{-1}$$

$$\omega_n^2 = (2\pi f_n)^2 = (2\pi 2500)^2 = 2.47 \times 10^8 \text{ s}^{-2}$$

$$T(s) = \frac{\frac{\omega_n}{Q}s + \omega_n^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2} = \frac{448.8s + 2.47 \times 10^8}{s^2 + 448.8s + 2.47 \times 10^8}$$

Sample M-file:

```
%Transmissibility M-file for HW #3
```

```
clear all;  
format compact;  
format long;
```

```
Q=35  
fn=2500
```

```
for i=1:5000,  
    f(i)=i;  
    T(i)=sqrt((1+(i/(Q*fn))^2)/((1-(i/fn)^2)^2+(i/(Q*fn))^2));  
end
```

```
plot(f,T)
```

