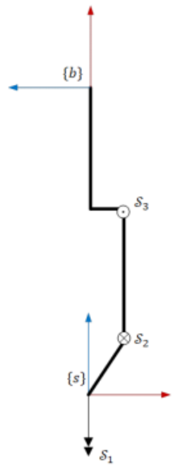
Comau

09 March 2025 22:06



Forward kinematics using screws $\mathcal{S} = \begin{bmatrix}\mathbf{w}\\ \mathbf{v}\end{bmatrix}$, $\|\mathbf{w}\| = 1$, $\mathbf{v} = -\mathbf{w} \times \mathbf{p}$

The vector $\mathbf{w}$ is a unit vector along the screw axis.

The vector $\mathbf{p}$ is a position vector from $\{s\}$ to any any point on the screw axis.

Note that $\{s\} = \{\mathbf{0}, \hat{\mathbf{x}}_s, \hat{\mathbf{y}}_s, \hat{\mathbf{z}}_s\}$ is the fixed base frame and $\{b\} = \{\mathbf{o}_b, \hat{\mathbf{x}}_b, \hat{\mathbf{y}}_b, \hat{\mathbf{z}}_b\}$ is the moving end-effector frame.

Note the directions of the screws, where $\otimes$ denotes a vector into the page and $\odot$ denotes a vector out of the page.

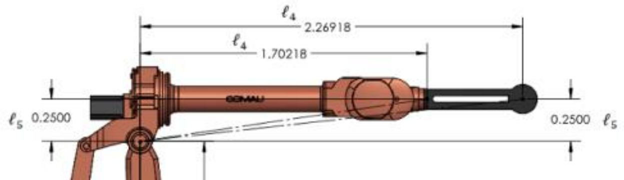
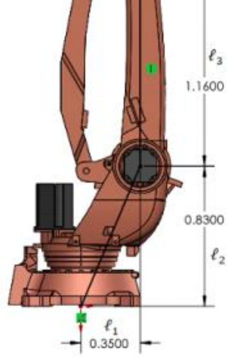
$$\mathcal{S}_1 = \begin{bmatrix}-\hat{\mathbf{z}}_s\\ \mathbf{v}_1\end{bmatrix}, \qquad \mathbf{v}_1 = \mathbf{0}$$

$$\mathcal{S}_2 = \begin{bmatrix}\hat{\mathbf{y}}_s\\ \mathbf{v}_2\end{bmatrix}, \qquad \mathbf{v}_2 = -\hat{\mathbf{y}}_s \times (\ell_1 \hat{\mathbf{x}}_s + \ell_2 \hat{\mathbf{z}}_s)$$

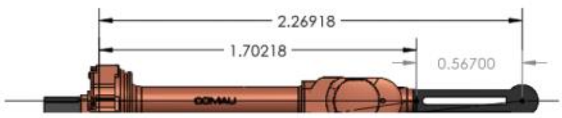
$$\mathcal{S}_3 = \begin{bmatrix}-\hat{\mathbf{y}}_s\\ \mathbf{v}_3\end{bmatrix}, \qquad \mathbf{v}_3 = \hat{\mathbf{y}}_s \times (\ell_1 \hat{\mathbf{x}}_s + (\ell_2 + \ell_3)\hat{\mathbf{z}}_s)$$

The zero pose of the robot

$$\mathbf{H}_b^s(\mathbf{0}) = \begin{bmatrix}\mathbf{R}_b^s(\mathbf{0}) & \mathbf{r}_b^s(\mathbf{0})\\ \mathbf{0} & 1\end{bmatrix} = \begin{bmatrix}\mathbf{R}_y(-\pi/2) & \begin{bmatrix}(\ell_1 - \ell_5)\\ 0\\ \ell_2 + \ell_3 + \ell_4\end{bmatrix}\\ \mathbf{0} & 1\end{bmatrix}$$

On the robot at the motionlab, this part is appended:



Forward kinematics can be written

$$\mathbf{H}_b^s(\mathbf{q}) = \exp(\tilde{\mathcal{S}}_1 q_1)\exp(\tilde{\mathcal{S}}_2 q_2)\exp(\tilde{\mathcal{S}}_3 q_3)\,\mathbf{H}_b^s(\mathbf{0})$$

where the $\tilde{\square}$ (tilde) operator denotes a Lie algebra

$$\tilde{\mathcal{S}} = \begin{bmatrix}\tilde{\mathbf{w}} & \mathbf{v}\\ \mathbf{0} & 0\end{bmatrix} \in se(3), \qquad \tilde{\mathbf{w}} = \begin{bmatrix}0 & -w_3 & w_2\\ w_3 & 0 & -w_1\\ -w_2 & w_1 & 0\end{bmatrix} \in so(3)$$

Table when using $L_4 = 1492.18$

x	z	q2	q3
700.63	1483.7	-75	-86
-757.56	2643.2	-75	-10
-33.831	3433	-20	-10
2871.3	77.808	95	-125
649.95	-518.9	95	-223.95
649.97	151.55	52.94	-213.94
1842.2	2240	0	-90

Table from doc

Pos	X	Z	Ax.2	Ax.3
	[mm]	[mm]	[deg]	[deg]
1	700,63	1483,71	-75°	-86°
2	-757,56	2643,15	-75°	-10°
3	-33,82	3432,97	-20°	-10°
4	2871,3	77,81	95°	-125°
5	650	-518,93	95°	-223,95°
6	650	151,56	52,94°	-213,94°
7	1842,18	2240	0°	-90°

Velocity and acceleration using twist $\mathcal{V} = \begin{bmatrix}\boldsymbol{\omega}_b^s\\ \mathbf{v}_{b/s}^s\end{bmatrix}$, where $\boldsymbol{\omega}_b^s$ is the angular velocity of $\{b\}$ in $\{s\}$.

$$\mathcal{V} = \begin{bmatrix}\boldsymbol{\omega}_b^s\\ \mathbf{v}_{b/s}^s\end{bmatrix} = \mathbf{J}(\mathbf{q})\,(\dot{\mathbf{q}} + \dot{q}_2 \hat{\mathbf{z}}_s)$$

$$\dot{\mathcal{V}} = \frac{d\mathcal{V}}{dt} = \begin{bmatrix}\dot{\boldsymbol{\omega}}_b^s\\ \dot{\mathbf{v}}_{b/s}^s\end{bmatrix} = \frac{\partial \mathbf{J}}{\partial \mathbf{q}}\,\dot{\mathbf{q}}\,(\dot{\mathbf{q}} + \dot{q}_2 \hat{\mathbf{z}}_s) + \mathbf{J}(\mathbf{q})(\ddot{\mathbf{q}} + \ddot{q}_2 \hat{\mathbf{z}}_s)$$

The Jacobian is built using screws mapped to adjacent joints (the previous screw axis)

$$\mathbf{J}(\mathbf{q}) = [\mathcal{S}_1 \quad \mathrm{Ad}_{\mathbf{H}_1(\mathbf{q})}(\mathcal{S}_2) \quad \mathrm{Ad}_{\mathbf{H}_2(\mathbf{q})}(\mathcal{S}_3)] \in \mathbb{R}^{6\times 3}$$

where

$$\mathrm{Ad}_{\mathbf{H}(\mathbf{q})} = \begin{bmatrix}\mathbf{R}(\mathbf{q}) & \mathbf{0}\\ \tilde{\mathbf{r}}(\mathbf{q})\mathbf{R}(\mathbf{q}) & \mathbf{R}(\mathbf{q})\end{bmatrix}, \qquad \mathbf{H}_i(\mathbf{q}) = \begin{bmatrix}\mathbf{R}_i(\mathbf{q}) & \mathbf{r}_i(\mathbf{q})\\ \mathbf{0} & 1\end{bmatrix} \in SE(3)$$

The elements in $SE(3)$, i.e. $\mathbf{H}_1$ and $\mathbf{H}_2$ are elements mapping joints to the base frame $\{s\}$.

Note that the vector $\mathbf{v}_{b/s}^s$ is <u>not</u> the translational velocity of $\{b\}$ in $\{s\}$. That would just be $\mathbf{v}_b^s = \dot{\mathbf{r}}_b^s$.

Rather, it is the velocity of $\{b\}$ when mapped back to the origin of $\{s\}$. Hence, to calculate the velocity and acceleration:

$$\dot{\mathbf{r}}_b^s = \mathbf{v}_{b/s}^s + \boldsymbol{\omega}_b^s \times \mathbf{r}_b^s$$

$$\ddot{\mathbf{r}}_b^s = \dot{\mathbf{v}}_{b/s}^s + \dot{\boldsymbol{\omega}}_b^s \times \mathbf{r}_b^s + \boldsymbol{\omega}_s \times \dot{\mathbf{r}}_b^s$$

To calculate the orientation of $\{b\}$, we assume an order of rotation: XYZ, (roll-pitch-yaw)

$$\boldsymbol{\varphi} = \begin{bmatrix}\phi_x\\ \phi_y\\ \phi_z\end{bmatrix}, \qquad \mathbf{R} = \mathbf{R}_x(\phi_x)\mathbf{R}_y(\phi_y)\mathbf{R}_z(\phi_z)$$

We can use the elements of $\mathbf{R}$ to determine consistent values of $\boldsymbol{\varphi}$, but with a catch whenever $\phi_y = \pm 90°$.

At that point, $\mathbf{R}$ gets the form $\mathbf{R} = \begin{bmatrix}0 & 0 & \pm 1\\ c_x s_z \pm s_x c_z & c_x c_z \mp s_x s_z & 0\\ s_x s_z \mp c_x c_z & c_x s_z \pm s_x c_z & 0\end{bmatrix}$ and we cannot determine a unique ϕ_x, ϕ_z.

$$2(c_x s_z + s_x c_z) = r_{21} + r_{32}$$

If we need the rates of change of $\boldsymbol{\varphi}$, $\dot{\boldsymbol{\varphi}}$, then we can find a relation $\boldsymbol{\omega}_b^s = \mathbf{T}(\mathbf{q})\dot{\boldsymbol{\varphi}}$, using XYZ order of rotation.