CONTENIDOS

MÓDULO 2: MÉTODO MATRICIAL DE RIGIDEZ

OBJETIVO ESPECÍFICO: Resolver cualquier tipo de sistema estructural, mediante el ensamble de la matriz global de rigidez de la estructura.

CONTENIDO:

1. Definir la matriz de rigidez local de cualquier elemento estructural
2. Deducir la matriz de transformación
3. Obtener la matriz global de rigidez
4. Obtener la solución de sistemas estructurales

MÓDULO 2: MÉTODO MATRICIAL DE RIGIDEZ

2.3. Matriz de rigidez en coordenadas globales

En coordenadas locales: $\{f\} = [k] * \{u\}$

En coordenadas globales: $\{F\} = [K] * \{U\}$

$$\{F\} = [T] * \{f\} \Rightarrow \{U\} = [T] * \{u\}$$

$$\{U\} = [T] * \{u\}$$

$$[T]^{-1} * \{U\} = \underbrace{[T]^{-1}[T]}_{[I]} * \{u\} = \{u\}$$

$$\{f\} = [k] * \{u\} = [k] * [T]^{-1} * \{U\}$$

$$\{F\} = \underbrace{[T] * [k] * [T]^{-1}}_{[K]} * \{U\}$$

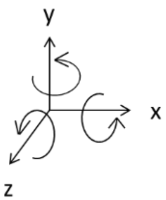
$$\boxed{\begin{gathered}\{F\} = [K] * \{U\} \\ [K] = [T] * [k] * [T]^{-1}\end{gathered}}$$

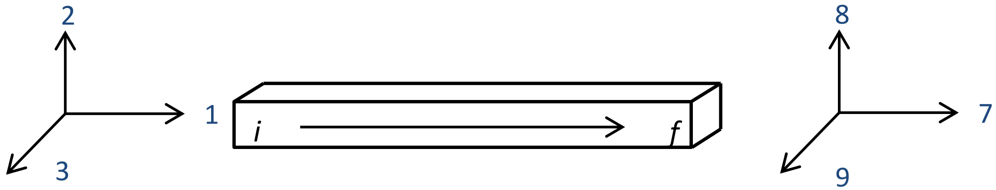
CONVENCIONES:

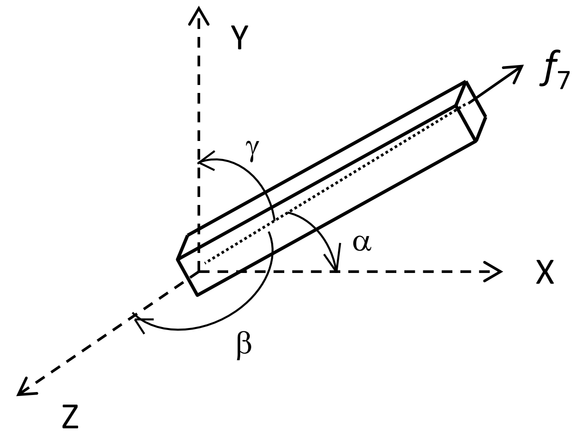
minúsculas $\longrightarrow$ Sistema local

MAYÚSCULAS $\dashrightarrow$ Sistema global

2.3.1. Matriz de rigidez global – cerchas espaciales







$$[T] = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 7 & 8 & 9 \end{matrix} \\ \left[\begin{array}{ccc|ccc} CX & * & * & 0 & 0 & 0 \\ CY & * & * & 0 & 0 & 0 \\ CZ & * & * & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & CX & * & * \\ 0 & 0 & 0 & CY & * & * \\ 0 & 0 & 0 & CZ & * & * \end{array}\right] \end{array}$$

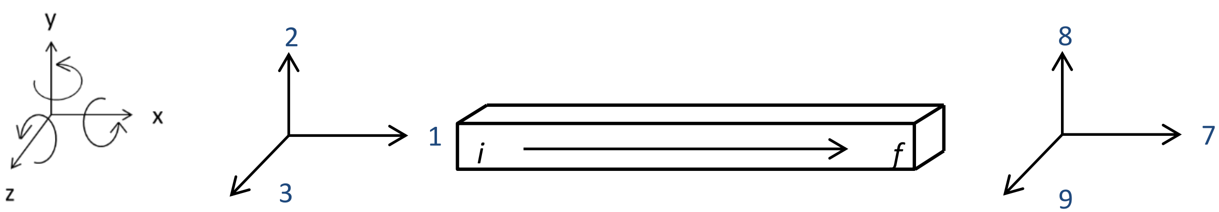
$$[K] = [T] * [k] * [T]^{-1}$$

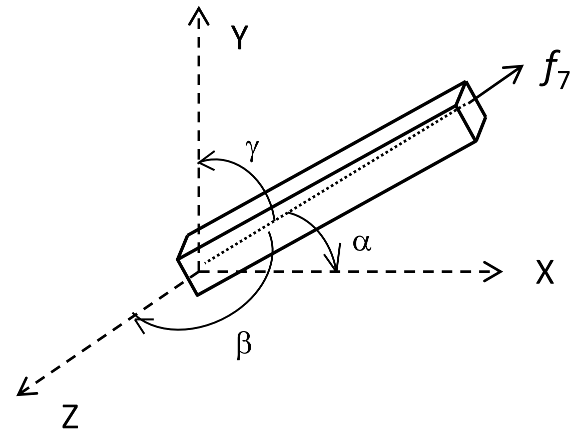
$$k = \frac{AE}{L} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 7 & 8 & 9 \end{matrix} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$[T] * [k] = \frac{AE}{L} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 7 & 8 & 9 \end{matrix} \\ \begin{bmatrix} CX & 0 & 0 & -CX & 0 & 0 \\ CY & 0 & 0 & -CY & 0 & 0 \\ CZ & 0 & 0 & -CZ & 0 & 0 \\ -CX & 0 & 0 & CX & 0 & 0 \\ -CY & 0 & 0 & CY & 0 & 0 \\ -CZ & 0 & 0 & CZ & 0 & 0 \end{bmatrix} \end{array}$$

$$[T]^{-1} = [T]^T = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 7 & 8 & 9 \end{matrix} \\ \left[\begin{array}{ccc|ccc} CX & CY & CZ & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & CX & CY & CZ \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \end{array}\right] \end{array}$$

2.3.1. Matriz de rigidez global – cerchas espaciales



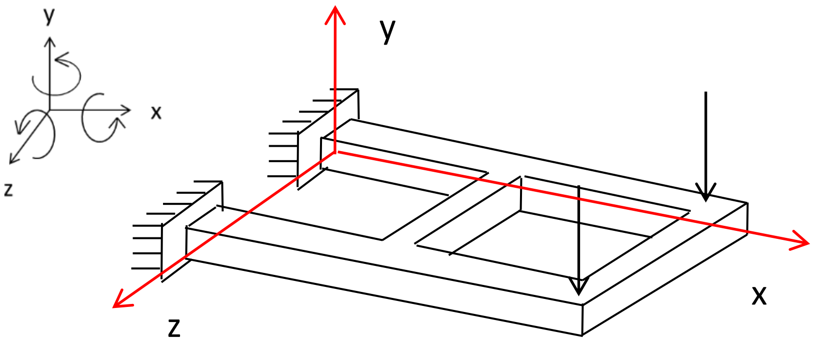


$$
\begin{array}{c}
\qquad\qquad\qquad 1 \qquad\qquad\qquad 2 \qquad\qquad\qquad 3 \qquad\qquad\qquad 7 \qquad\qquad\qquad 8 \qquad\qquad\qquad 9 \\
[K] = \dfrac{AE}{L}\begin{bmatrix}
CX^2 & CX\cdot CY & CX\cdot CZ & -CX^2 & -CX\cdot CY & -CX\cdot CZ \\
CX\cdot CY & CY^2 & CY\cdot CZ & -CX\cdot CY & -CY^2 & -CY\cdot CZ \\
CX\cdot CZ & CY\cdot CZ & CZ^2 & -CX\cdot CZ & -CY\cdot CZ & -CZ^2 \\
-CX^2 & -CX\cdot CY & -CX\cdot CZ & CX^2 & CX\cdot CY & CX\cdot CZ \\
-CX\cdot CY & -CY^2 & -CY\cdot CZ & CX\cdot CY & CY^2 & CY\cdot CZ \\
-CX\cdot CZ & -CY\cdot CZ & -CZ^2 & CX\cdot CZ & CY\cdot CZ & CZ^2
\end{bmatrix}
\end{array}
$$

2.3.1. Matriz de rigidez global – cerchas planas

$$
\begin{array}{c}
\qquad\qquad\qquad 1 \qquad\qquad\qquad 2 \qquad\qquad\qquad 7 \qquad\qquad\qquad 8 \\
[K] = \dfrac{AE}{L}\begin{bmatrix}
CX^2 & CX\cdot CY & -CX^2 & -CX\cdot CY \\
CX\cdot CY & CY^2 & -CX\cdot CY & -CY^2 \\
-CX^2 & -CX\cdot CY & CX^2 & CX\cdot CY \\
-CX\cdot CY & -CY^2 & CX\cdot CY & CY^2
\end{bmatrix}
\end{array}
$$

2.3.3. Matriz de rigidez global – entramados

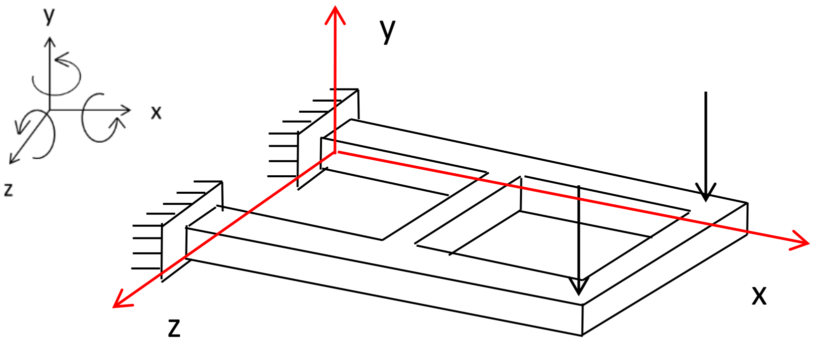


$$[T] = \begin{array}{c} \begin{array}{cccccc} 2 & 4 & 6 & 8 & 10 & 12 \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & CX & -CZ & 0 & 0 & 0 \\ 0 & CZ & CX & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & CX & -CZ \\ 0 & 0 & 0 & 0 & CZ & CX \end{array}\right] \end{array}$$

$$[T]^{-1} = [T]^T = \begin{array}{c} \begin{array}{cccccc} 2 & 4 & 6 & 8 & 10 & 12 \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & CX & -CZ & 0 & 0 & 0 \\ 0 & CZ & CX & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & CX & -CZ \\ 0 & 0 & 0 & 0 & CZ & CX \end{array}\right] \end{array}$$

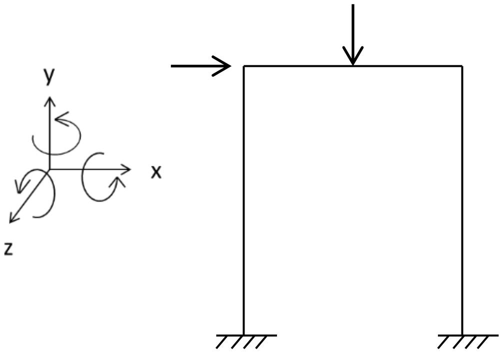
$$k = \begin{array}{c} \begin{array}{cccccc} 2 & 4 & 6 & 8 & 10 & 12 \end{array} \\ \left[\begin{array}{ccc|ccc} \frac{12EI_z}{L^3} & 0 & \frac{6EI_z}{L^2} & -\frac{12EI_z}{L^3} & 0 & \frac{6EI_z}{L^2} \\ 0 & \frac{GI_p}{L} & 0 & 0 & -\frac{GI_p}{L} & 0 \\ \frac{6EI_z}{L^2} & 0 & \frac{4EI_z}{L} & -\frac{6EI_z}{L^2} & 0 & \frac{2EI_z}{L} \\ \hline -\frac{12EI_z}{L^3} & 0 & -\frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & 0 & -\frac{6EI_z}{L^2} \\ 0 & -\frac{GI_p}{L} & 0 & 0 & \frac{GI_p}{L} & 0 \\ \frac{6EI_z}{L^2} & 0 & \frac{2EI_z}{L} & -\frac{6EI_z}{L^2} & 0 & \frac{4EI_z}{L} \end{array}\right] \end{array}$$

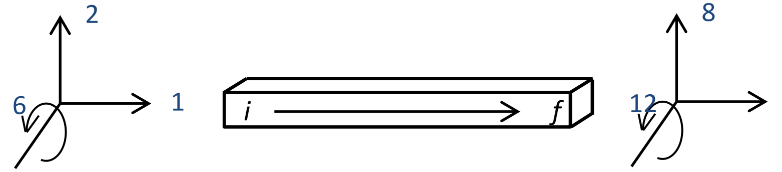
2.3.3. Matriz de rigidez global – entramados



$$
[K] = \begin{array}{c}
\begin{array}{cccccc} 2 & 4 & 6 & 8 & 10 & 12 \end{array} \\
\left[\begin{array}{ccc|ccc}
\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2}CZ & \frac{6EI_z}{L^2}CX & -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2}CZ & \frac{6EI_z}{L^2}CX \\
-\frac{6EI_z}{L^2}CZ & \frac{GI_p}{L}CX^2+\frac{4EI_z}{L}CZ^2 & \left(\frac{GI_p}{L}-\frac{4EI_z}{L}\right)\cdot CX\cdot CZ & \frac{6EI_z}{L^2}CZ & -\frac{GI_p}{L}CX^2+\frac{2EI_z}{L}CZ^2 & -\left(\frac{GI_p}{L}+\frac{2EI_z}{L}\right)\cdot CX\cdot CZ \\
\frac{6EI_z}{L^2}CX & \left(\frac{GI_p}{L}-\frac{4EI_z}{L}\right)\cdot CX\cdot CZ & \frac{GI_p}{L}CZ^2+\frac{4EI_z}{L}CX^2 & -\frac{6EI_z}{L^2}CX & -\left(\frac{GI_p}{L}+\frac{2EI_z}{L}\right)\cdot CX\cdot CZ & -\frac{GI_p}{L}CZ^2+\frac{2EI_z}{L}CX^2 \\
\hline
-\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2}CZ & -\frac{6EI_z}{L^2}CX & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2}CZ & -\frac{6EI_z}{L^2}CX \\
-\frac{6EI_z}{L^2}CZ & -\frac{GI_p}{L}CX^2+\frac{2EI_z}{L}CZ^2 & -\left(\frac{GI_p}{L}+\frac{2EI_z}{L}\right)\cdot CX\cdot CZ & \frac{6EI_z}{L^2}CZ & \frac{GI_p}{L}CX^2+\frac{4EI_z}{L}CZ^2 & \left(\frac{GI_p}{L}-\frac{4EI_z}{L}\right)\cdot CX\cdot CZ \\
\frac{6EI_z}{L^2}CX & -\left(\frac{GI_p}{L}+\frac{2EI_z}{L}\right)\cdot CX\cdot CZ & -\frac{GI_p}{L}CZ^2+\frac{2EI_z}{L}CX^2 & -\frac{6EI_z}{L^2}CX & \left(\frac{GI_p}{L}-\frac{4EI_z}{L}\right)\cdot CX\cdot CZ & \frac{GI_p}{L}CZ^2+\frac{4EI_z}{L}CX^2
\end{array}\right]
\end{array}
$$

2.3.4. Matriz de rigidez global – pórticos planos





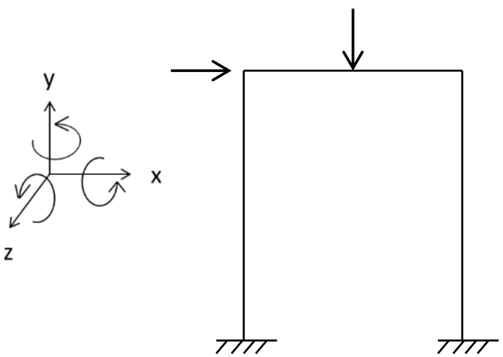
Pórtico plano

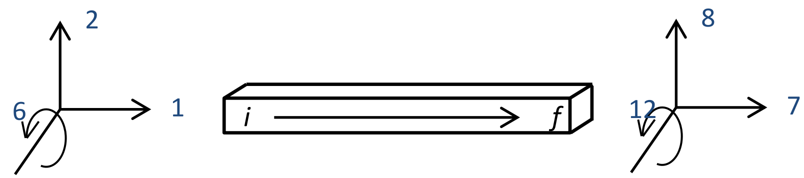
$$
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 6 & 7 & 8 & 12 \end{array} \\
[T] = \left[\begin{array}{ccc|ccc}
CX & -CY & 0 & 0 & 0 & 0 \\
CY & CX & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & CX & -CY & 0 \\
0 & 0 & 0 & CY & CX & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 6 & 7 & 8 & 12 \end{array} \\
[T]^{-1} = [T]^{T} = \left[\begin{array}{ccc|ccc}
CX & CY & 0 & 0 & 0 & 0 \\
-CY & CX & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & CX & CY & 0 \\
0 & 0 & 0 & -CY & CX & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 6 & 7 & 8 & 12 \end{array} \\
k = \left[\begin{array}{ccc|ccc}
\frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\
0 & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & 0 & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\
0 & \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\
\hline
-\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\
0 & -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\
0 & \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L}
\end{array}\right]
\end{array}
$$

2.3.4. Matriz de rigidez global – pórticos planos





Pórtico plano

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \color{red}{1} & \color{red}{2} & \color{red}{6} & \color{red}{7} & \color{red}{8} & \color{red}{12} \end{array} \\
[K] = & \left[\begin{array}{ccc|ccc}
\frac{AE}{L}CX^2+\frac{12EI_z}{L^3}CY^2 & \left(\frac{AE}{L}-\frac{12EI_z}{L^3}\right)\cdot CX\cdot CY & -\frac{6EI_z}{L^2}CY & -\frac{AE}{L}CX^2-\frac{12EI_z}{L^3}CY^2 & \left(-\frac{AE}{L}+\frac{12EI_z}{L^3}\right)\cdot CX\cdot CY & -\frac{6EI_z}{L^2}CY \\
\left(\frac{AE}{L}-\frac{12EI_z}{L^3}\right)\cdot CX\cdot CY & \frac{AE}{L}CY^2+\frac{12EI_z}{L^3}CX^2 & \frac{6EI_z}{L^2}CX & \left(-\frac{AE}{L}+\frac{12EI_z}{L^3}\right)\cdot CX\cdot CY & -\frac{AE}{L}CY^2-\frac{12EI_z}{L^3}CX^2 & \frac{6EI_z}{L^2}CX \\
-\frac{6EI_z}{L^2}CY & \frac{6EI_z}{L^2}CX & \frac{4EI_z}{L} & \frac{6EI_z}{L^2}CY & -\frac{6EI_z}{L^2}CX & \frac{2EI_z}{L} \\
\hline
-\frac{AE}{L}CX^2-\frac{12EI_z}{L^3}CY^2 & \left(-\frac{AE}{L}+\frac{12EI_z}{L^3}\right)\cdot CX\cdot CY & \frac{6EI_z}{L^2}CY & \frac{AE}{L}CX^2+\frac{12EI_z}{L^3}CY^2 & \left(\frac{AE}{L}-\frac{12EI_z}{L^3}\right)\cdot CX\cdot CY & \frac{6EI_z}{L^2}CY \\
\left(-\frac{AE}{L}+\frac{12EI_z}{L^3}\right)\cdot CX\cdot CY & -\frac{AE}{L}CY^2-\frac{12EI_z}{L^3}CX^2 & -\frac{6EI_z}{L^2}CX & \left(\frac{AE}{L}-\frac{12EI_z}{L^3}\right)\cdot CX\cdot CY & \frac{AE}{L}CY^2+\frac{12EI_z}{L^3}CX^2 & -\frac{6EI_z}{L^2}CX \\
-\frac{6EI_z}{L^2}CY & \frac{6EI_z}{L^2}CX & \frac{2EI_z}{L} & \frac{6EI_z}{L^2}CY & -\frac{6EI_z}{L^2}CX & \frac{4EI_z}{L}
\end{array}\right]
\end{array}
$$