BINOMIAL DISTRIBUTION (GRAPHICAL ANALYSIS): RELATION BETWEEN PROBABILITY OF SUCCESS (p) & FAILURE (q) WITH THE SKEWNESS

Binomial Distribution Function

$$b(x\,;n\,,p\,) = C_x^n\, p^x\, q^{n-x}\ ; \quad x = 0, 1, 2, 3 \ldots$$

$n =$ Total number of events
$x =$ Total number of successfull events
$p =$ Probability of success in a single trial
$q =$ Probability of failure

Formulas & Concepts

Moment about mean	Skewness
$\mu_1 = \text{Mean} = \mu_1' = n\,p$	$\beta_1 = 0$; Symmetric Distribution
$\mu_2 = \text{Variance} = n\,p$	$\beta_1 > 0$; Positively Skewed
$\mu_3 = n\,p\,q\,(q - p)$	$\beta_1 < 0$; Negitively Skewed

Cases to Analyse the Skewness Variation

Case (1)

(Probability of Success = Probability of Failure) $p = q$

Case (2)

(Probability of Success > Probability of Failure) $p > q$ $\quad \vdots\ q = 1 - p$

Case (3)

(Probability of Success < Probability of Failure) $p < q$

Case (1) $p = q$

A coin is tossed12 times. What is the probability of getting exactly 7 heads?

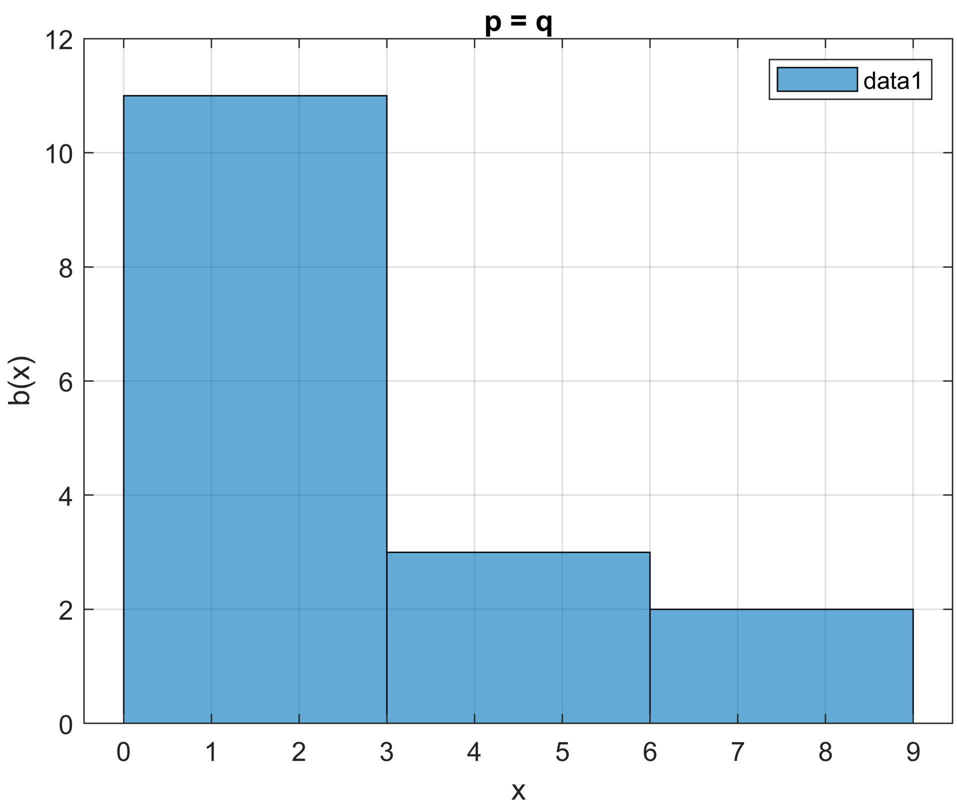
```
p = 0.5;
q = 1-p;
n = 12;
for k = 0:7
    com = nchoosek(n,k);
bin = com*p^k*q^(n-k);
X = [k, bin]
disp(X)
end
```

```
X = 1×2
10^-3 ×
         0    0.2441
   1.0e-03 *

         0    0.2441
X = 1×2
    1.0000    0.0029
    1.0000    0.0029
X = 1×2
    2.0000    0.0161
    2.0000    0.0161
X = 1×2
    3.0000    0.0537
    3.0000    0.0537
X = 1×2
    4.0000    0.1208
    4.0000    0.1208
X = 1×2
    5.0000    0.1934
    5.0000    0.1934
X = 1×2
    6.0000    0.2256
    6.0000    0.2256
X = 1×2
    7.0000    0.1934
    7.0000    0.1934
```

```
load ('c01.mat')
histogram(c01)

xlim([-0.45 9.45])
ylim([0.0 12.0])
grid on
legend('show')
title('p = q')
xlabel('x')
ylabel('b(x)')
```




```
mu = n*p
```

```
mu = 6.3000
```

```
var = n*p*q
```

```
var = 0.6300
```

```
std = sqrt(var)
```

```
std = 0.7937
```

Case (2) $p > q$

On average, every one out of 10 telephones is found busy. Seven telephone numbers are selected at random. Find the probability that 5 of them will not found busy.

```
p = 0.9;
```

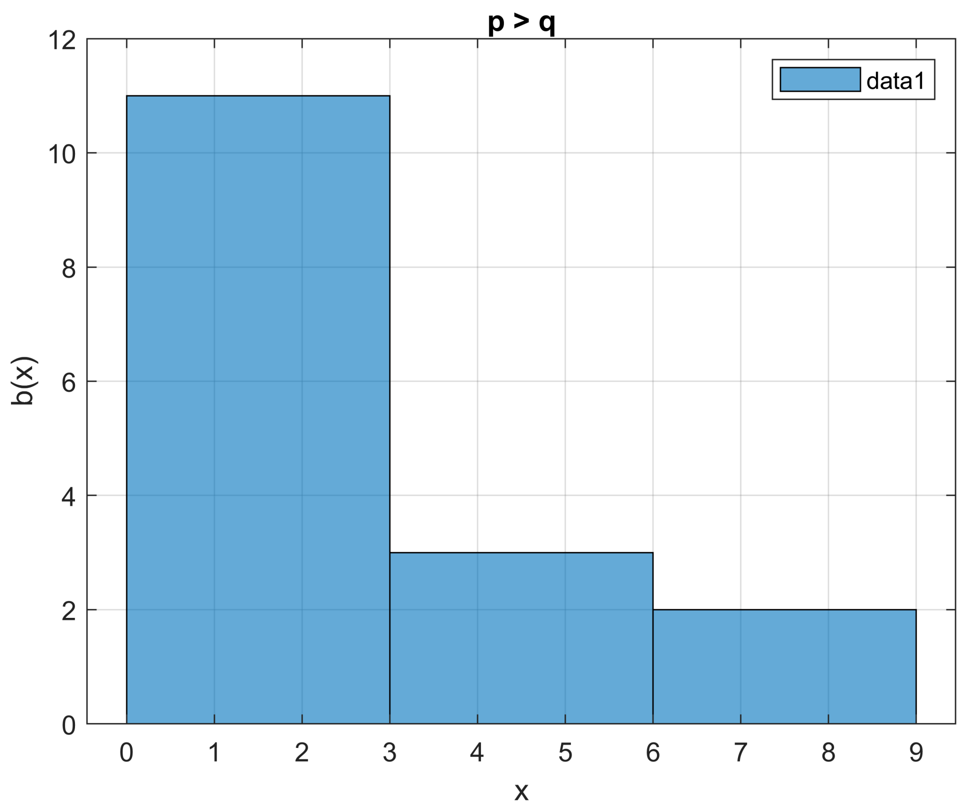
```
q = 1-p;
n = 7;
for k = 0:7
    com = nchoosek(n,k);
bin = com*p^k*q^(n-k);
X = [k, bin]
disp(X)
end
```

```
X = 1×2
10^-7 ×
         0    1.0000
   1.0e-07 *

         0    1.0000
X = 1×2
    1.0000    0.0000
    1.0000    0.0000
X = 1×2
    2.0000    0.0002
    2.0000    0.0002
X = 1×2
    3.0000    0.0026
    3.0000    0.0026
X = 1×2
    4.0000    0.0230
    4.0000    0.0230
X = 1×2
    5.0000    0.1240
    5.0000    0.1240
X = 1×2
    6.0000    0.3720
    6.0000    0.3720
X = 1×2
    7.0000    0.4783
    7.0000    0.4783
```

```
load ('c02.mat')
histogram(c02)

xlim([-0.45 9.45])
ylim([0.0 12.0])
grid on
legend('show')
title('p > q')
xlabel('x')
ylabel('b(x)')
```




```
mu = n*p
```

```
mu = 6.3000
```

```
var = n*p*q
```

```
var = 0.6300
```

```
std = sqrt(var)
```

```
std = 0.7937
```

Case (3) $p < q$

On average, every one out of 10 telephones is found busy. Six telephone numbers are selected at random. Find the probability that four of them will be busy.

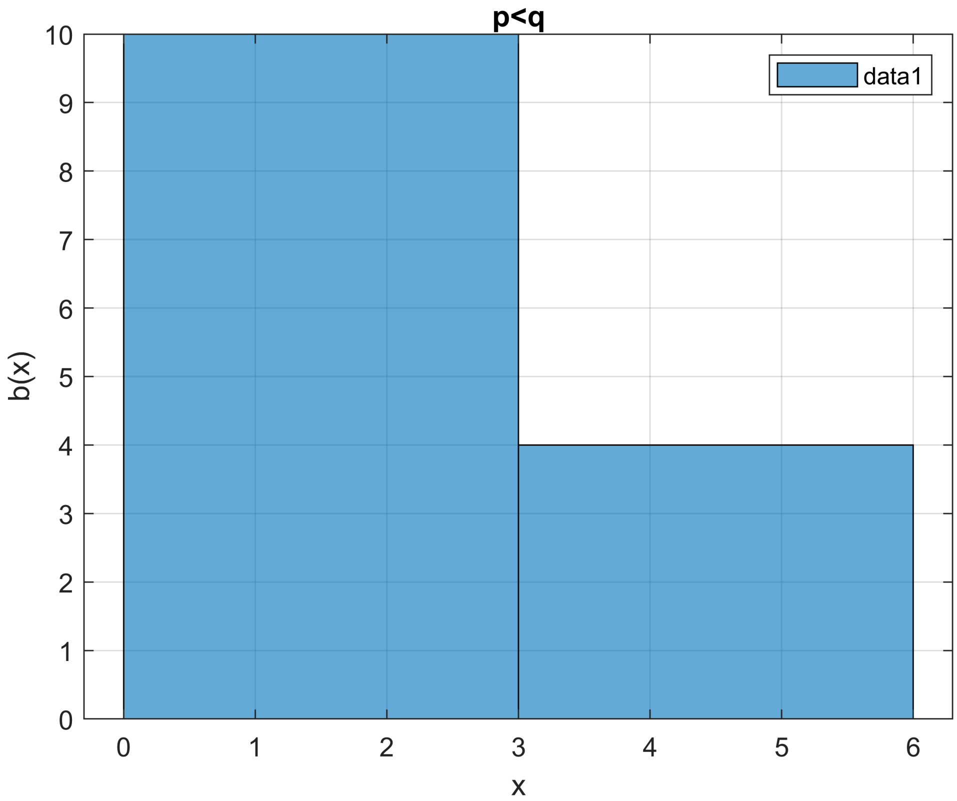
```
p = 0.1;
q = 1-p;
n = 6;
for k = 0:6
    com = nchoosek(n,k);
bin = com*p^k*q^(n-k);
X = [k, bin]
```

```
disp(X)
end
```

```
X = 1×2
         0    0.5314
         0    0.5314
X = 1×2
    1.0000    0.3543
    1.0000    0.3543
X = 1×2
    2.0000    0.0984
    2.0000    0.0984
X = 1×2
    3.0000    0.0146
    3.0000    0.0146
X = 1×2
    4.0000    0.0012
    4.0000    0.0012
X = 1×2
    5.0000    0.0001
    5.0000    0.0001
X = 1×2
    6.0000    0.0000
    6.0000    0.0000
```

```
load ('c03.mat')
histogram(c03)

xlim([-0.30 6.30])
ylim([0.00 10.00])
grid on
legend('show')
title('p<q')
xlabel('x')
ylabel('b(x)')
```




```
mu = n*p
```

```
mu = 0.6000
```

```
var = n*p*q
```

```
var = 0.5400
```

```
std = sqrt(var)
```

```
std = 0.7348
```