1 DIMENSION HEAT CONDUCTION

A UNIFORM ROD WITH DIRICHLET'S BOUNDARY CONDITIONS

Separation of Variable(SoV)

Introduction

Heat conduction in a uniform rod with Dirichlet's boundary conditions is a significant problem in the field of thermal sciences and engineering. Understanding the behavior of heat transfer in such systems is crucial for designing efficient thermal systems and predicting temperature distributions. In this study, we explore the problem of 1D heat conduction in a uniform rod with Dirichlet's boundary conditions and investigate its properties, solution techniques, and applications.

Nature of the Equation

The governing equation for heat conduction in a uniform rod is a parabolic partial differential equation known as the heat equation. It describes the time evolution of temperature distribution in the rod based on the rate of heat transfer and the thermal properties of the material. This equation captures the complex interplay between conduction, convection, and radiation mechanisms and forms the basis for studying heat conduction phenomena.

$$\frac{\partial u}{\partial t} = k\,\frac{\partial^2 u}{\partial x^2}$$

$u(x,t)$ is the temperature at position (spatial variable) and time.

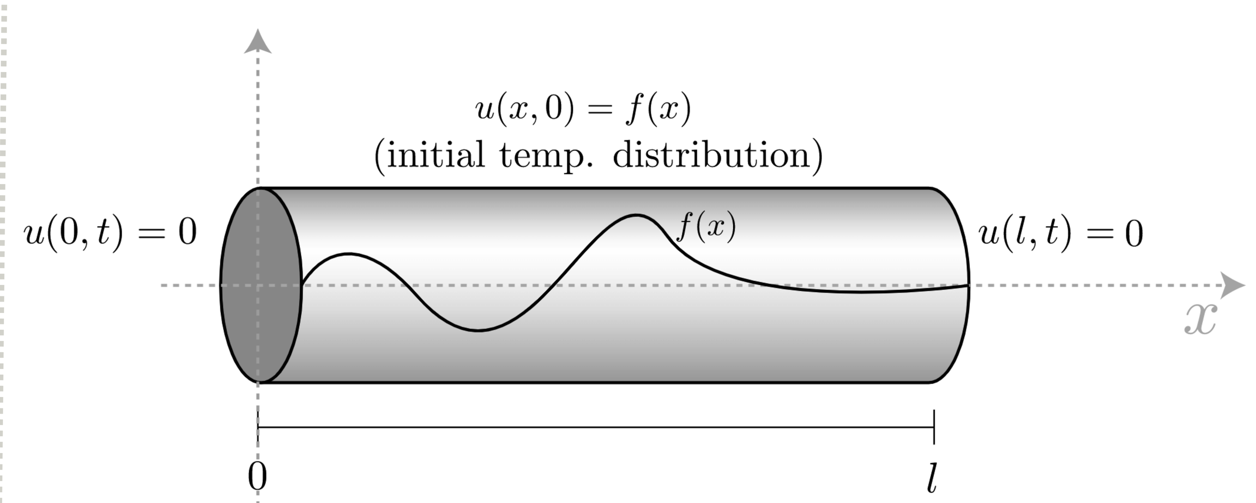
$\frac{\partial u}{\partial t}$ is the partial derivative with respect to time

k is the rate of heat transfer in a material

$\frac{\partial^2 u}{\partial x^2}$ is the second partial derivaitve of u with respect ot spatial variable x

Problem

The problem involves a uniform rod with specified temperatures at its ends, known as Dirichlet's boundary conditions. Our objective is to determine the temperature distribution within the rod over time. By solving this problem, we gain insights into the thermal behavior and energy transfer mechanisms in solid materials, enabling us to optimize various engineering systems.



$$0 \le x \le L \text{ and } t \ge 0$$

Boundary Conditions

$$u(0,t) = 0 \quad ; \quad u(L,t) = 1$$

$$u(x,0) = T_0$$

Solution

Separation of variables:

This technique assumes a separable solution in terms of spatial and temporal variables and reduces the partial differential equation into a set of ordinary differential equations.

$$u(x, t) = X(x) * T(t).$$

$$(1/\alpha) * T'(t)/T(t) = X''(x)/X(x) = -\lambda^2$$

$$T'(t)/T(t) = -\alpha\lambda^2$$

$$T(t) = C_2 * \exp(-\alpha\lambda^2 t)$$

$$X''(x)/X(x) = -\lambda^2$$

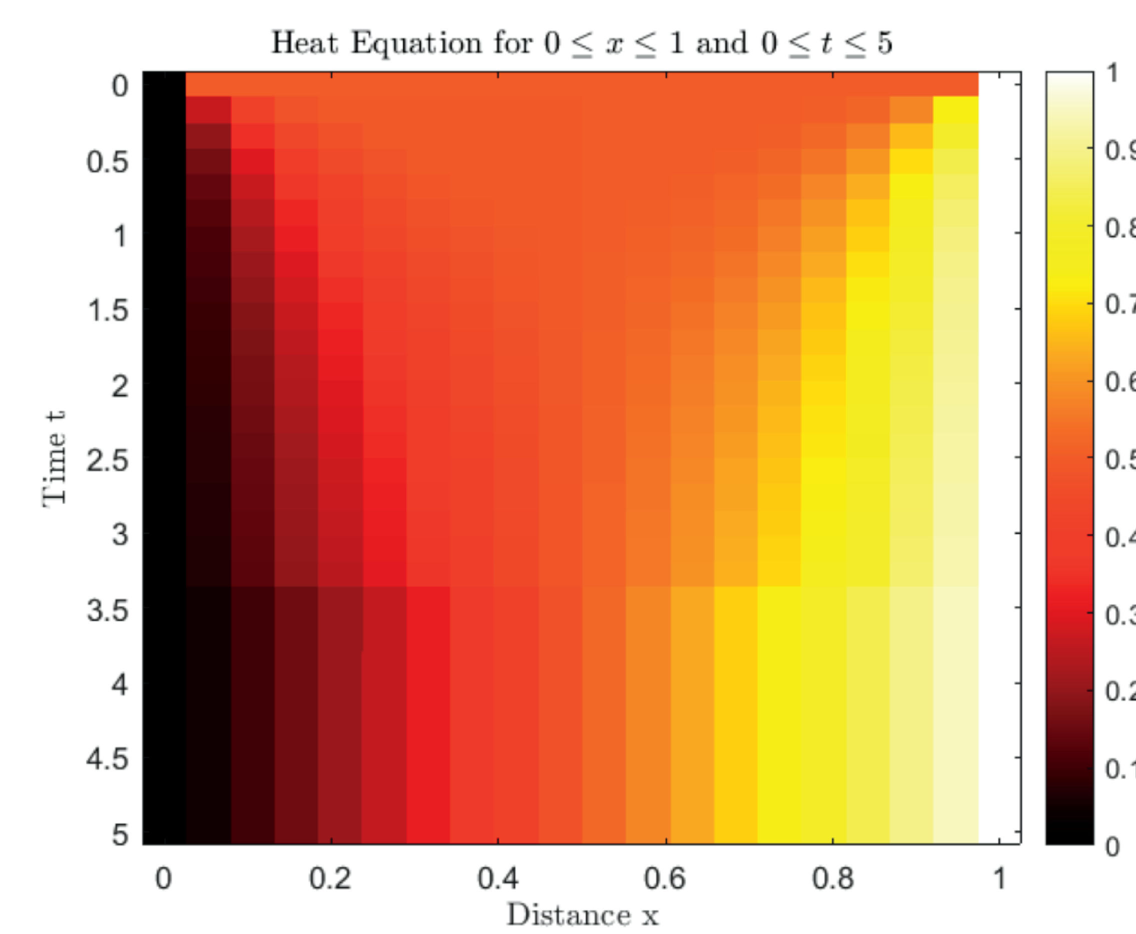
$$X(x) = A * \cos(\lambda x) + B * \sin(\lambda x)$$

By solving these equations with appropriate boundary and initial conditions, we obtain the complete solution for the temperature distribution in the rod.

$$X(x) = B * \sin(\lambda x)$$

$$T(t) = C_2 * \exp(-\alpha\lambda^2 t)$$

$$u(x, 0) = f(x) = B * \sin(\lambda x) * C_2$$



Properties

- Parabolic equation, indicating that it possesses diffusion-like behavior, where heat propagates through the rod due to thermal gradients.
- Linear equation, allowing for the superposition principle and simplifying the solution process.
- Boundary and initial conditions determines the unique solution for this specific problem.

Applications

Various applications in engineering disciplines

- It is extensively used in thermal system design, such as heat exchangers, electrical circuitry, and cooling systems.
- Understanding temperature distributions is essential for optimizing the performance, efficiency, and reliability of these systems.
- It is also relevant to material science, allowing for the characterization of thermal properties and behavior of different materials.

Conclusion

In conclusion, the investigation of 1D heat conduction in a uniform rod with Dirichlet's boundary conditions provides valuable insights into the thermal behavior of solid materials.

The heat equation serves as a powerful tool to model heat conduction phenomena, and the method of separation of variables offers an effective approach in solving and governing equations.

The knowledge gained from this study enables us to design efficient thermal systems, predict temperature distributions, and understand the behavior of materials under thermal conditions.

Submitted By

- Muhammad Arslan (201408)
- Sawera Nadeem (201436)

Submitted To

- Dr. Iffat Zehra