PDEs in 2 dim space:

(1) **Elliptic:** $\nabla(c\,\nabla u) + au - f$

$\nabla^2 u = 0$. Laplace eqn

$AU_{xx} + BU_{xy} + CU_{yy} + DU_x + EU_y + FU - G$

$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$

$\vec{\nabla}.\vec{A}$ = Divergence of vector A

∇f = Gradient of scalar f.

(2) **Parabolic:** $du_t - \nabla.(c\vec{\nabla}u) + au = f$

$\nabla^2 u - u_t = 0 \Rightarrow$ Heat eqn

(3) **Hyperbolic:** $du_{tt} - \nabla.(c\nabla u) + au = f$

$\Rightarrow$ Wave eqn

PDE : Matlab to built in ftn.

Example of elliptical PDE

Consider Poisson's eqn on a rectangle.

$u(x,y) \in [0,2] \times [0,1]$

$u_{xx} + u_{yy} = x^2 + y^2$

$u(x,0) = x$; $u(x,1) = 1$

$u(0,y) = y$; $u(2,y) = 1$

Dirichlet → on u.
N-lect → norm

On command window

Type: pdetool

Turning grid on: Select options, Grid

Increase domain window: options, Axes limits on
x-axes $[-0.5, 2.5]$

Tricks edges ⇒
y-axis $[-0.5, 1.5]$ apply and close.

Specify to draw rectangle: click on rectangle icon at top left of menu options left click on the point $(0,0)$ and keeping left mouse button pressed drag rectangle to point $(2,1)$ and release it.
Convert it by double clicking, if wrong done then change by double clicking on that and write specific value of given order

Specify boundary Conditions:
Click on symbol $\partial\Omega$ or select boundary, Boundary mode or type control-b.
Color of boundary indicates type of condition
i.e red = Dirichlet, blue = Neumann, green = mixed
Double click on button horizontal line = click on over written PDE and on left there is type of PDE i.e. elliptic

we get formula $hU = r$
our case ⇒ $h = 1,\ r = x$ } $u(x,0)$

Continue around boundary specifying remaining cond; then

Save as and save matlab file for future use.

For elliptic PDE:

$$-\nabla\cdot(c\nabla u) + au = f$$

our case $c = -1$, $a = 0$, $f(x,y) = x^2 + y^2$

click on: PDE icon = elliptic

Create a triangular finite element mesh:
select Δ icon or choose mesh, Initialize mesh
we can refine mesh by choosing icon of
Δ inside a triangle.
Finally solve: choose '=' icon or solve,
solve PDE.
{ click on plot and select ——— ?

Elliptic pdes no time involvement

Criterian for parabollic b^2 - 4ac > 0

Heat equation U_t =

Previous Task discussion Steady case no change in time eloptic pde

Boundary condition are dritchelet boundary conditions

no invelvement of derivatice

Dritchlet B.C's :(Matlab specify)

$$h\,u = r$$

ANeuman boundry condition

Mixed boundary condition (Combinition of both)

Noslip dritchlet = 0

flux = 0

neuman = 0

inflow = outflow

Pipe

boundary drich = 0

tab, drich = 1 neuman = 0, open (inflow = outflow)

Parabollic PDE: Lab_Task 14_Dec_23

Date: 14 Dec 2023 Thursday

Example: Consider the heat equation

$$U_t = U_{xx} + U_{yy} + \sin(t)$$

$$U(t,0,y) = 0; U_x(t,\pi,y) = 1$$

$$U_y(t,x,0) = 0; U(t,x,2\pi) = x$$

$$U(0,x,y) = 0$$

Note: For unique solution because 2 derivative of space variable then we need 2 boundary conditions (left & right)

$$x = 0; x = \pi$$

Same goes for y

$$y = 0; y = 2\pi$$

For t we have initial condition

$$t = 0$$

Solution:

$$[0,\pi]x[0,2\pi]$$

For Neumann B.C's

Matlab specify:

$$n * C * \text{grad}(U) + qU = g$$

$$\vec{n} \cdot (C(x,y)\nabla U) + q(x,y)U = g(x,y)$$

where $\vec{n}$ represents a unit vector normal to domain

For case $U_y(t,x,0)$, we have,

$$\vec{n} = (0,-1), C(x,0) = 1, q(x,0) = 0, g(x,0) = 0$$

For case $U_x(t,\pi,y) = 1$ we have

$$\vec{n} = (1,0), C(\pi,y) = 1, q(\pi,y) = 0, g(x,0) = 1$$

Note: Derechilt = Red , Neuman b.cs = Blue Color

General Parabolic Pde:

$$d\,U_t - \nabla \cdot (C\nabla U) + \mathrm{aU} = f$$

In our case,

$$d = 1, C(t, x, y) = 1, f(t, x, y) = \sin(t)$$

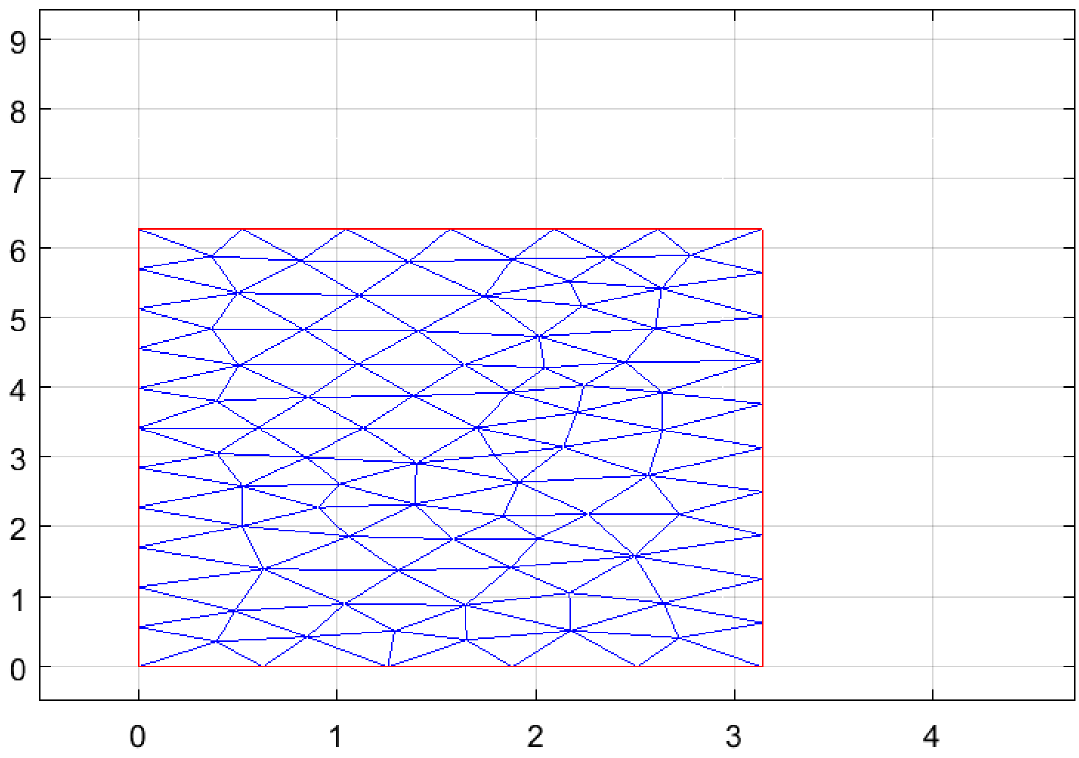
For Initial condition:

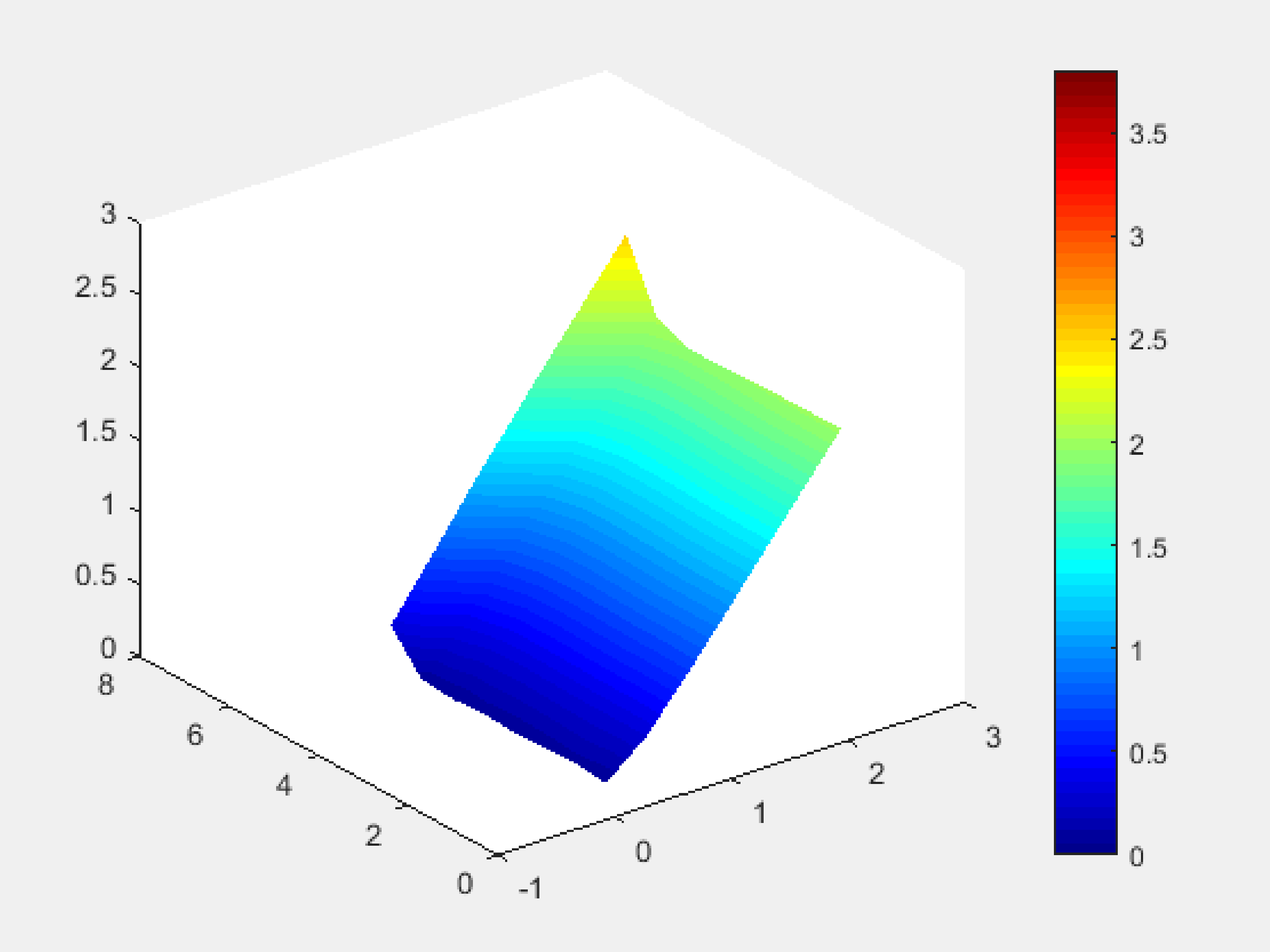
Solve, parameters, Time: U(t0)

Let type:

$$\text{linspace}(0, 5, 10)$$

for the time to run till 5 sec






```
% write pdetool in command window
% make the domain
% option, grid on (for easy scaling)
% opetion axes limit
% x  = [-0.5 1.5*pi] for visualize actual [0, pi]
% y = [-0.5 3*pi] for visualization purpose actual [0, 2*pi]
% select the rectangle icon from the top(1) and drag
% make this exact by double clicking the rectangular domain
% alternative usign draw tab
% partial sign for boundary condition drichlet / neuman
% for pde click on PDE, select the type and give the general values
% give the initial condition by click solve, parameter and put values by comparison
% plot, parameters, check animations and plot
% check plot in x, y grid, 3d height
%(Autamitacally triangularization jet color is encouraged)
```