ADOMIAN DECOMPOSITION METHOD

LINEAR NON-HOMOGENEOUS FREDHOLM INTEGRAL EQUATION OF SECOND KIND

Adomian Decomposition Method

Decomposition method for short. we used Adomian decomposition method (decomposition method) for solving linear non-homogeneous Fredholm integral equations of second kind because this method provides the solution in a rapidly convergent series with components that are elegantly computed.

The aim of this work is to establish exact solutions for linear non-homogeneous Fredholm integral equations of second kind using Adomian decomposition method without large computational work

Nature of the Equation

The Fredholm integral equation can be derived from boundary value problem with give boundary conditions.

Many inverse problems in science and engineering lead to the Fredholm integral equations.

Introduction

Linear non-homogeneous Freholm integral equation of second kind was given by

$$u(x) = f(x) + \lambda \int_a^b K(x,t)u(t)\mathrm{d}t \; ...(1)$$

a and b : constants

$f(x)$: given real-valued functions

$u(x)$: unknown function

$K(x,t)$: kernel

λ: parameter

Solution of linear non-homogeneous Fredholm integral equations of second kind:

The solution arises in any of the following two types:

Exact solution:

A solution is called exact if it has a closed form such as a polynomial, trigonometric function, exponential function or the combination of two or more of these elementary funcitons.

Series solution:

Sometimes we can not obtain exact solution for concrete problems. In this case, we have solution in the series form that may converge to exact solution if such a solution exists. Otherwise series may not give exact solution and in this case we have approximate solution for numerical purpose. The more terms in the series give the higher accuracy in the solution

Problem

Consider linear non-homogeneous Fredholm integral equations of second kind with

$$f(x) = e\,x - x, \; \lambda = 1,$$

$$K(x,t) = xt, \; a = 0, \; b = 1$$

$$u(x) = e^x - x + \int_0^1 x\,t\,u(t)\,\mathrm{d}t \; \ldots\ldots (1)$$

Now using (2), the series solution of (1) is given by

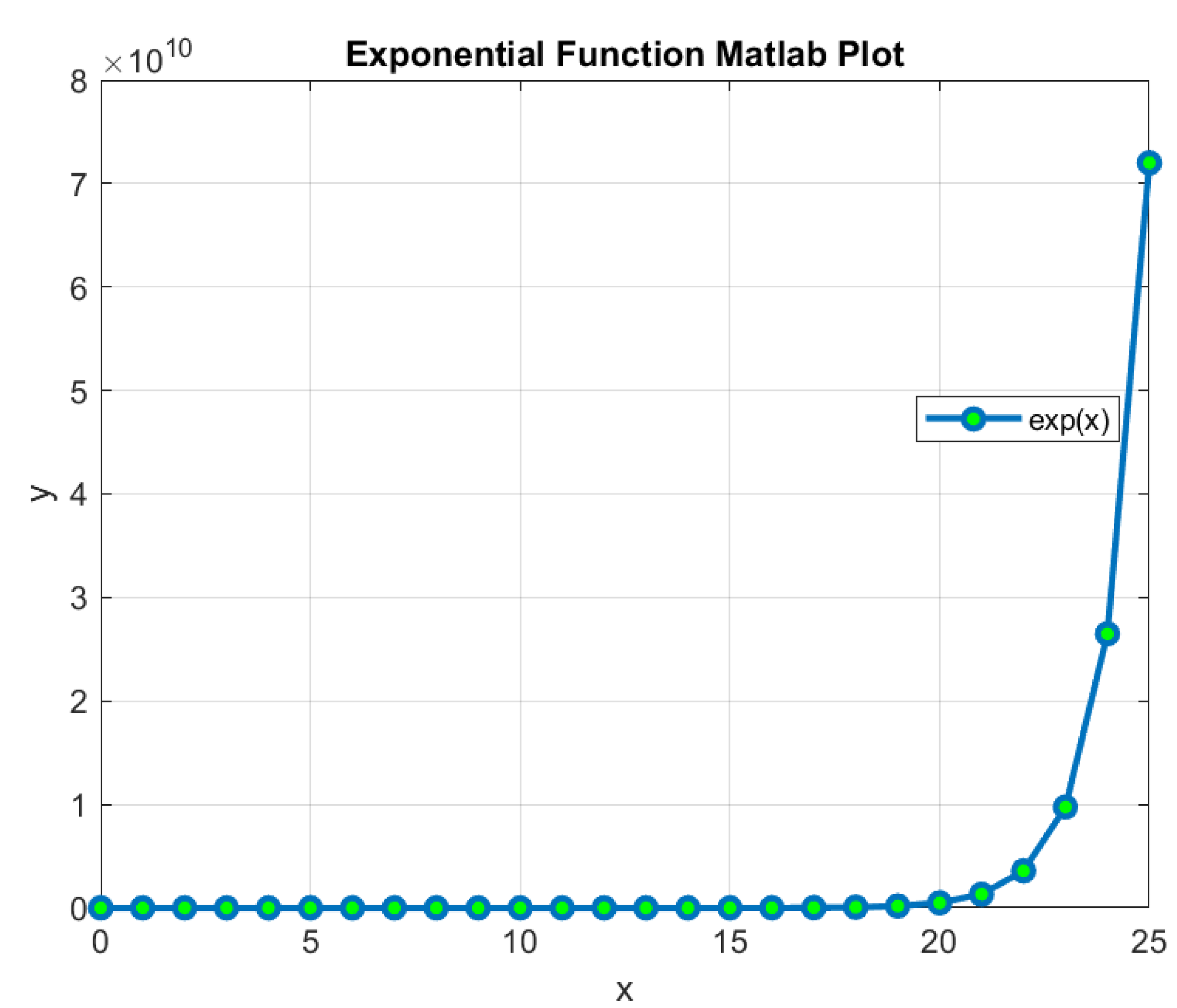
$$u_n(x) = u_0(x) + u_1(x) + u_3(x) + \ldots\ldots$$

$$= e^x - x + \frac{2x}{3} + \frac{2x}{9} + \frac{2x}{27} + \ldots\ldots$$

$$= e^x - x + \frac{2x}{3}\left[1 + \frac{1}{3} + \frac{1}{9} + \ldots\ldots\right]$$

$$= e^x - x + \frac{2x}{3}\left[\frac{1}{1-\frac{1}{3}}\right] = e^x \; \ldots\ldots (9)$$

Which is the exact solution of (1)



Applications

This type of equations generally appears in many physical models such as stereology, cosmic radiations, electromagnetic fields, radiography spectroscopy and image processing.

Conclusion

We have successfully developed the Adomian decomposition method for solving linear non-homogeneous Fredholm integral equations of second kind. The given applications showed that the exact solution have been obtained using very less computational work and spending a very little time. The proposed scheme can be applied for other non-homogeneous Fredholm integral equations of second kind and their system.

Submitted By

Muhammad Arslan (201408)
Muhammad Izaan Ullah Khan (201409)

Submitted To

- Dr. Nabeela Kousar

IE Project Poster Presentation

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