Applications of Adomian Decomposition Method

Solving Linear Non-Homogeneous Fredholm Integral Equations of Second Kind

Problem:

Consider linear non-homogeneous Fredholm integral equations of second kind with

$$f(x) = ex - x, \lambda = 1,$$

$$K(x,t) = xt, a = 0, b = 1$$

$$u(x) = e^x - x + \int_0^1 x\, t\, u(t)\, \mathrm{dt} \; \ldots \ldots (1)$$

The Adomian decomposition method assumes the unknown function $u(x)$ in the form of an infinite series as

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \; \ldots \ldots (2)$$

By substituting (2) in(1), we have

$$\sum_{n=0}^{\infty} u_n(x) = e^x - x + x\int_0^1 t \sum_{n=0}^{\infty} u_n(t)\, \mathrm{dt} \; \ldots \ldots (3)$$

From(3), our required recursive relation is given by

$$u_0(x) = e^x - x \; \ldots \ldots (4)$$

$$u_{n+1}(x) = x\int_0^1 t\, u_n(t)\mathrm{dt}, n \geq 0 \ldots \ldots (5)$$

The first few components of R.H.S. of (2) by using (4) and (5) are given by

$$u_1(x) = x\int_0^1 t\, u_0(t)\mathrm{dt}$$

$$= x\int_0^1 t\,(e^t - t)^1 \mathrm{dt} = \frac{2x}{3} \ldots \ldots (6)$$

$$u_2(x) = x\int_0^1 t\, u_1(t)\mathrm{dt}$$

$$= x\int_0^1 \frac{2t^2}{3}\mathrm{dt} = \frac{2x}{9} \ldots . . (7)$$

$$u_3(x) = x\int_0^1 t\, u_2(t)\mathrm{dt}$$

$$= x\int_0^1 \frac{2t^2}{9}\mathrm{dt} = \frac{2x}{27} \ldots .. . (8)$$

and so on.

Now using (2), the series solution of (1) is given by

$$u_n(x) = u_0(x) + u_1(x) + u_3(x) + \ldots\ldots$$

$$= e^x - x + \frac{2x}{3} + \frac{2x}{9} + \frac{2x}{27} + \ldots\ldots$$

$$= e^x - x + \frac{2x}{3}\left[1 + \frac{1}{3} + \frac{1}{9} + \ldots\ldots\right]$$

$$= e^x - x + \frac{2x}{3}\left[\frac{1}{1-\frac{1}{3}}\right] = e^x \quad \ldots\ldots (9)$$

Which is the exact solution of (1)

Solution Ploting:

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x = 0:25
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```
x = 1×26
     0     1     2     3     4     5     6     7     8     9    10    11    12 ···
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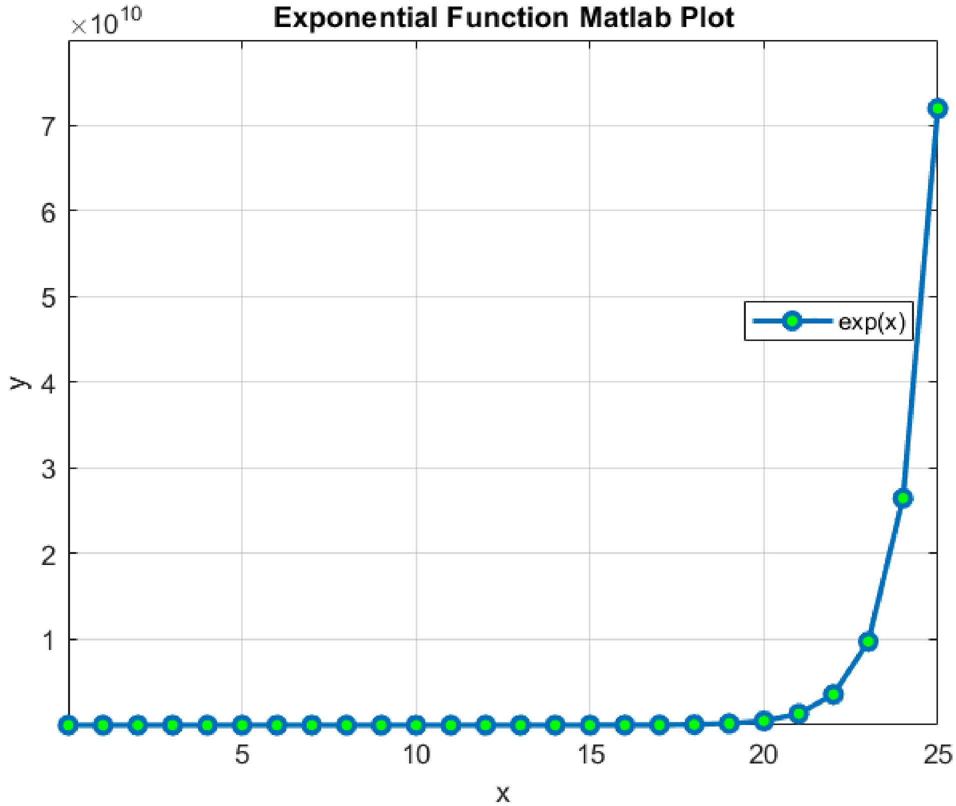
```
y = exp(x);
plot (x, y,'-o','LineWidth',2 ,'MarkerSize', 6,'MarkerFaceColor','g')
grid on;
xlabel('x')
ylabel('y')
title('Exponential Function Matlab Plot')
legend('exp(x)','Location','Best')
```


Exponential Function Matlab Plot
×10^10
7
6
5
4
3
2
1
y
exp(x)
5
10
15
20
25
x