Expressions du gradient, de la divergence, du rotationnel et du laplacien dans les différents systèmes de coordonnées

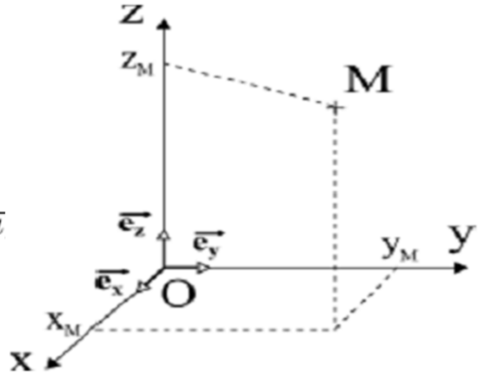
Coordonnées cartésiennes

$$\vec{\text{grad}}\ V = \frac{\partial V}{\partial x}\vec{\mathrm{u_x}} + \frac{\partial V}{\partial y}\vec{\mathrm{u_y}} + \frac{\partial V}{\partial z}\vec{\mathrm{u_z}}$$

$$div\ \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\text{rot}}\ \vec{A} = (\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z})\vec{u_x} + (\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})\vec{u_y} + (\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})\vec{u}$$

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$



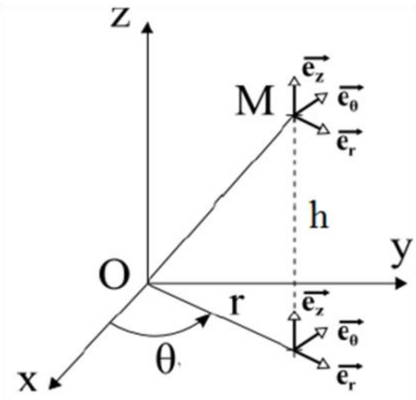
Coordonnées cylindriques

$$\vec{\text{grad}}\ V = \frac{\partial V}{\partial r}\vec{\mathrm{u_r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{\mathrm{u_\theta}} + \frac{\partial V}{\partial z}\vec{\mathrm{u_z}}$$

$$\left[div\ \vec{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}\right]$$

$$\left[\vec{\text{rot}}\ \vec{A} = (\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z})\vec{u_r} + (\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r})\vec{u_\theta} + \frac{1}{r}(\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta})\vec{u_z}\right]$$

$$\left[\Delta V = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial V}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}\right]$$



Coordonnées sphériques

$$\vec{\text{grad}}\ V = \frac{\partial V}{\partial r}\vec{\mathrm{u_r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{\mathrm{u_\theta}} + \frac{1}{r\ sin\theta}\frac{\partial V}{\partial \varphi}\vec{\mathrm{u_\varphi}}$$

$$\left[div\ \vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\ sin\theta}\frac{\partial (sin\theta\ A_\theta)}{\partial \theta} + \frac{1}{r\ sin\theta}\frac{\partial A_\varphi}{\partial \varphi}\right]$$

$$\left[\vec{\text{rot}}\ \vec{A} = \frac{1}{r\ sin\theta}(\frac{\partial (sin\theta\ A_\varphi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi})\vec{u_r} + \frac{1}{r}(\frac{1}{sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial (rA_\varphi)}{\partial r})\vec{u_\theta} + \frac{1}{r}(\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta})\vec{u_\varphi}\right]$$

$$\left[\Delta V = \frac{1}{r}\frac{\partial^2 (rV)}{\partial r^2} + \frac{1}{r^2\ sin\theta}\frac{\partial}{\partial \theta}(sin\theta\frac{\partial V}{\partial \theta}) + \frac{1}{r^2\ sin^2\theta}\frac{\partial^2 V}{\partial \varphi^2}\right]$$