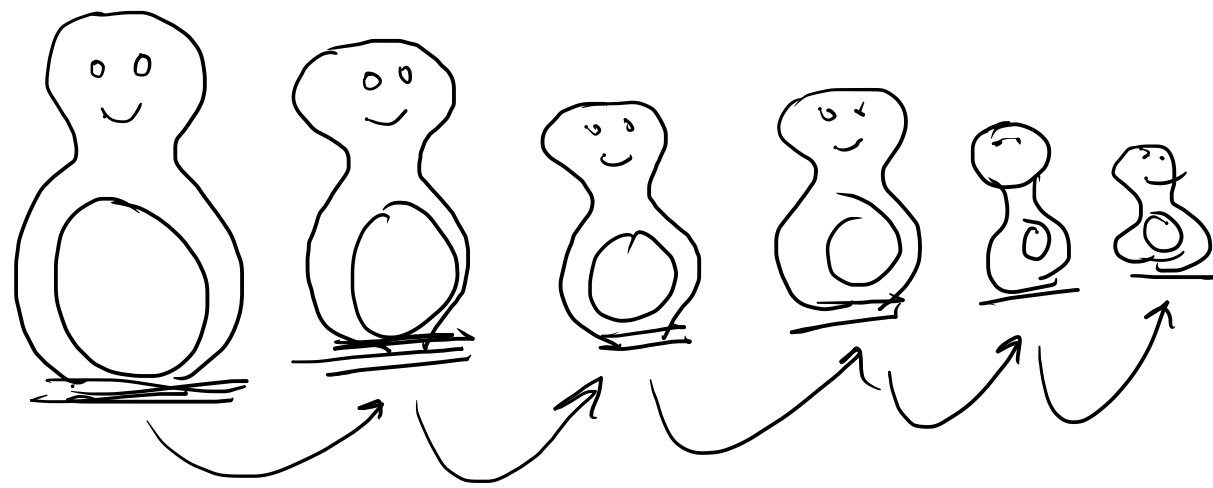


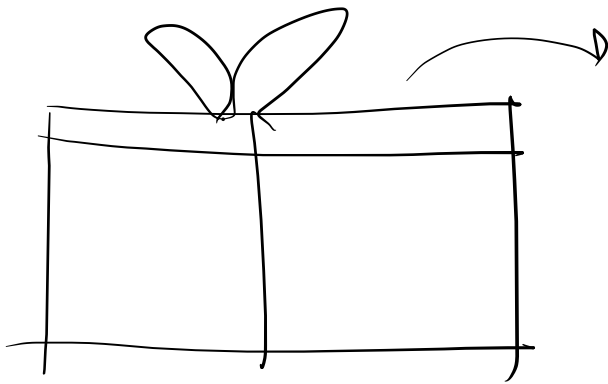
Recursion

↳ process of func calling itself

↳ recursive function
function that calls itself

Recursion





Solve bigger problem



Solve smaller sub-problems

Q.) Sum of numbers from n to 1

$$\underline{n = 5}$$

$$\text{Sum} = 5 + 4 + 3 + 2 + 1 = \underline{\underline{15}}$$

Sum of 1 to n

$$\text{formula} = \frac{n(n+1)}{2}$$

$$\underline{\underline{n=5}}$$

$$\text{sum} = \frac{5 \times 6}{2} = \underline{\underline{15}}$$

sum of (5 to 1)

→ = 5 + sum of (4 to 1)

→ 4 + sum of (3 to 1)

→ 3 + sum of (2 to 1)

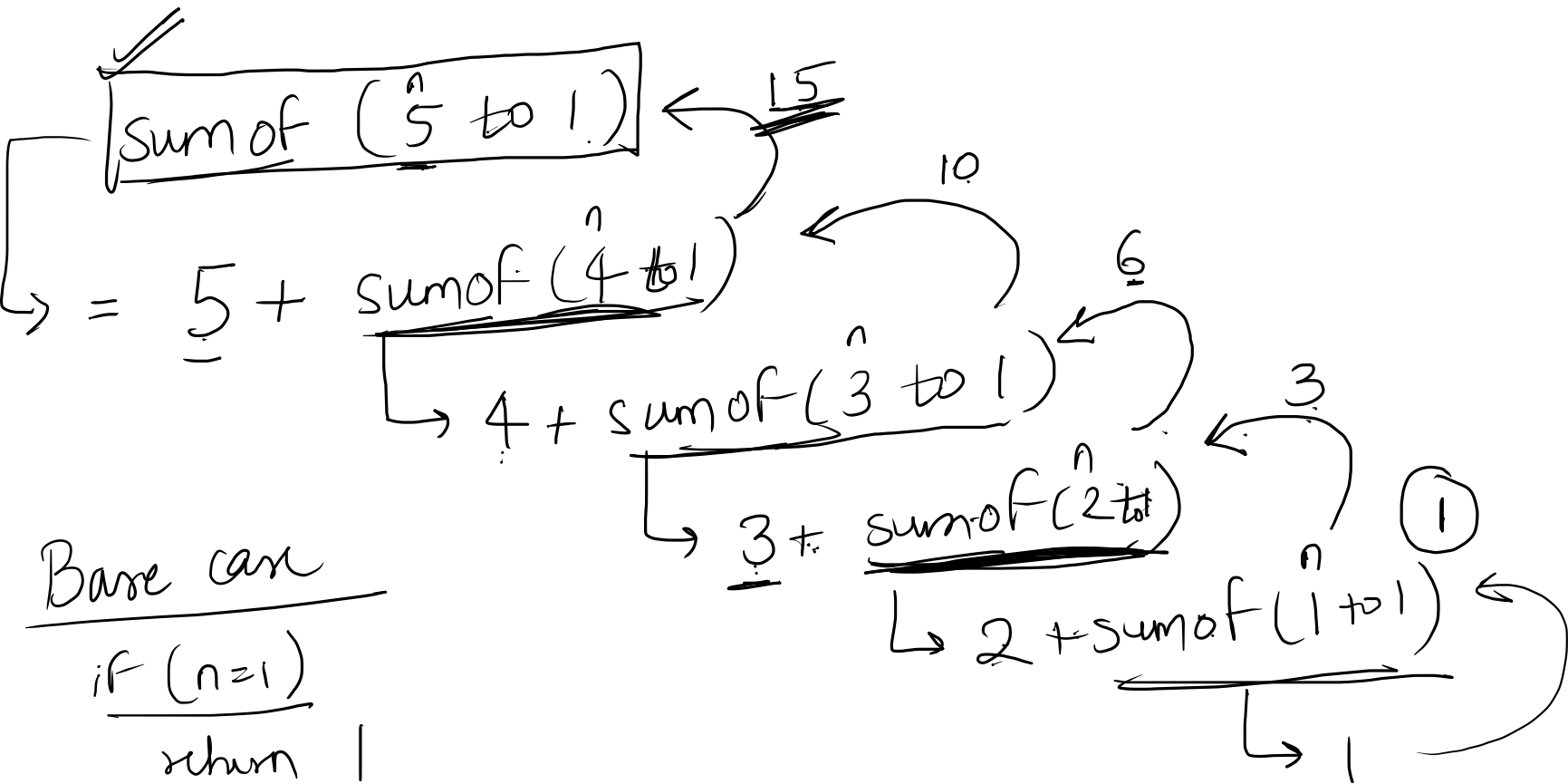
→ 2 + sum of (1 to 1)

→ 1

Base case

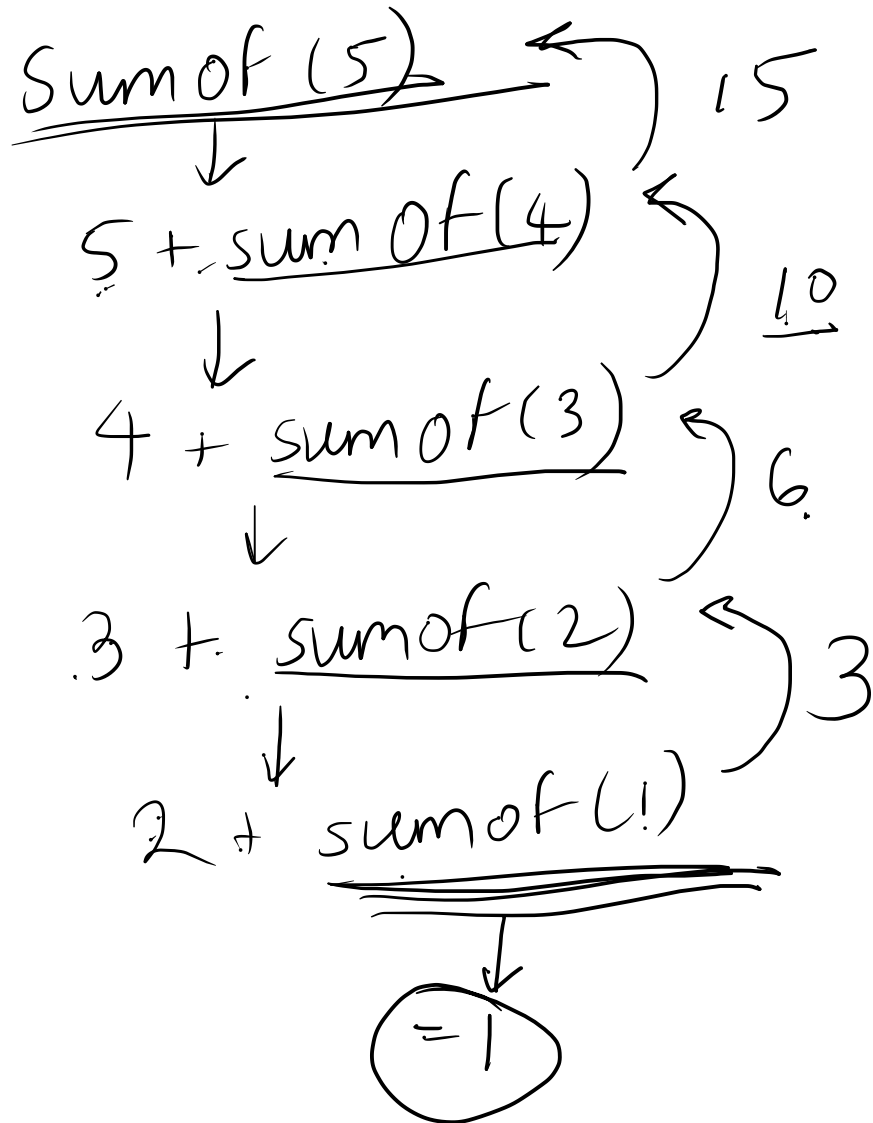
if (n=1)

return 1



$$\text{sumof}(n) = n + \text{sumof}(n-1)$$

$$\text{sumof}(5) = 5 + \text{sumof}(4)$$



Fibonacci Series

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

0, 1, ~~1~~, ~~2~~, ~~3~~, 5, 8, 13, 21, 34, 55

seed
values

$$F(0) = 0$$

$$F(1) = 1$$

$$F(0) = 0$$

$$F(1) = 1$$

$$F(2) = 1$$

$$F(3) = 2$$

$$F(4) = 3$$

$$F(5) = 5$$

$$F(6) = 8$$

$$\underline{F(25)}$$

$$F(0) = 0$$

$$F(1) = 1$$

$$F(2) = 1$$

$$F(3) = 2$$

$$F(4) = 3$$

$$F(5) = 5$$

$$F(6) = 8$$

$$\underline{f(25)}$$

~~$$f(3) = 1 + 2$$~~

$$\underline{f(3) = f(1) + f(2)}$$

Sum of previous two fibo numbers

$$\begin{array}{lcl} \times & F(6) = \underline{5} + 4 & \times \\ \checkmark & F(6) = \underline{F(5)} + \underline{F(4)} & \checkmark \end{array}$$

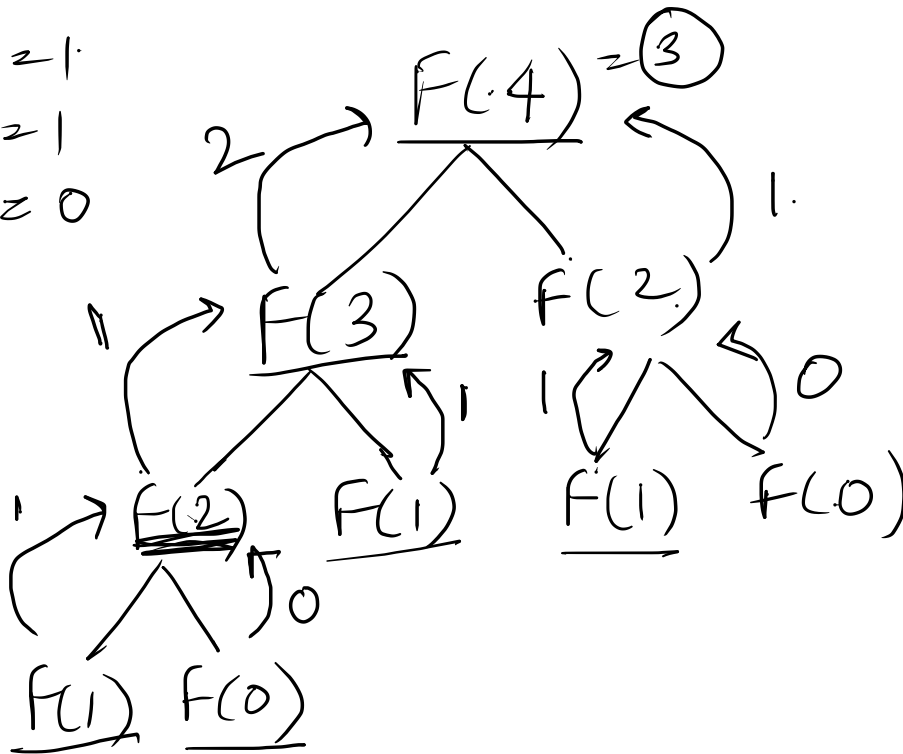

$$F(6) = F(5) + F(4)$$

$$F(n) = F(n-1) + F(n-2)$$

$$F(2) = 1$$

$$F(1) = 1$$

$$F(0) = 0$$

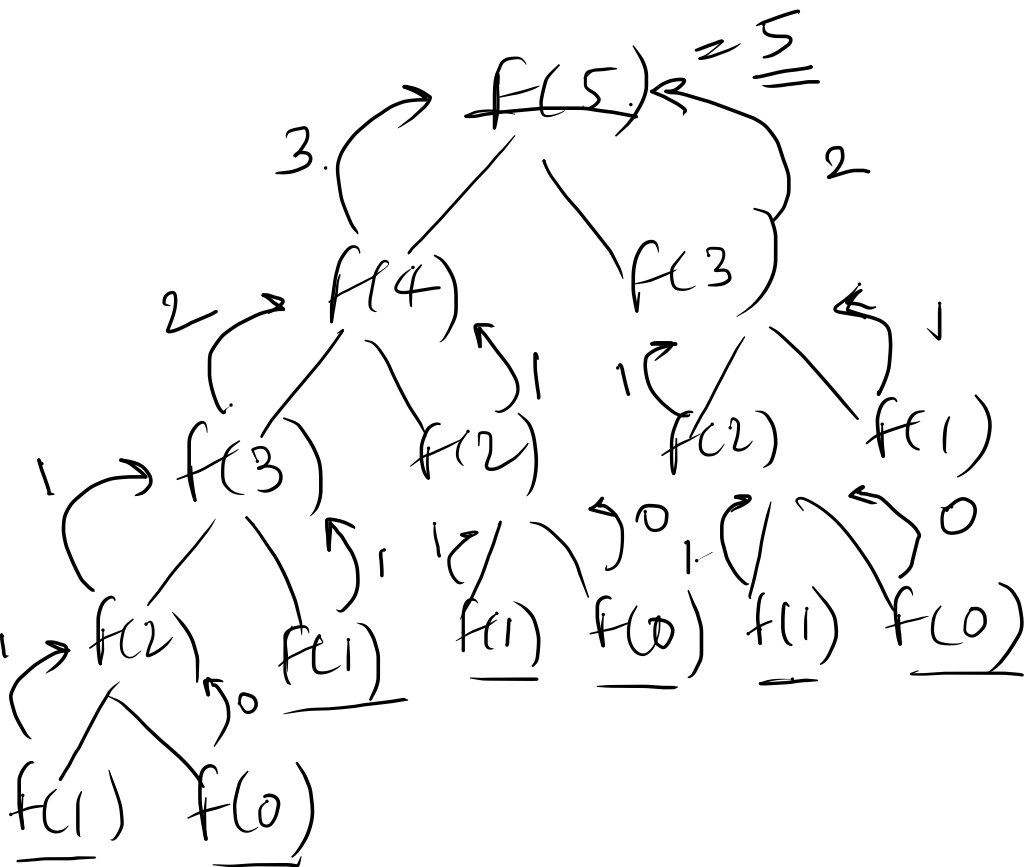


Base case

$$F(1) = 1$$

$$F(0) = 0$$

n	0	1	2	3	4
$F(n)$	0	1	1	2	<u>3</u>



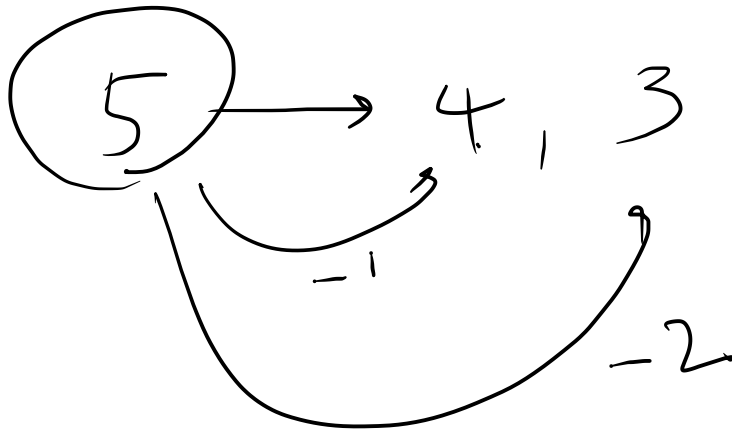
$$f(0) = 0$$

$$f(1) = 1$$

$$f(\underline{n}) = f(\underline{n-1}) + f(n-2)$$

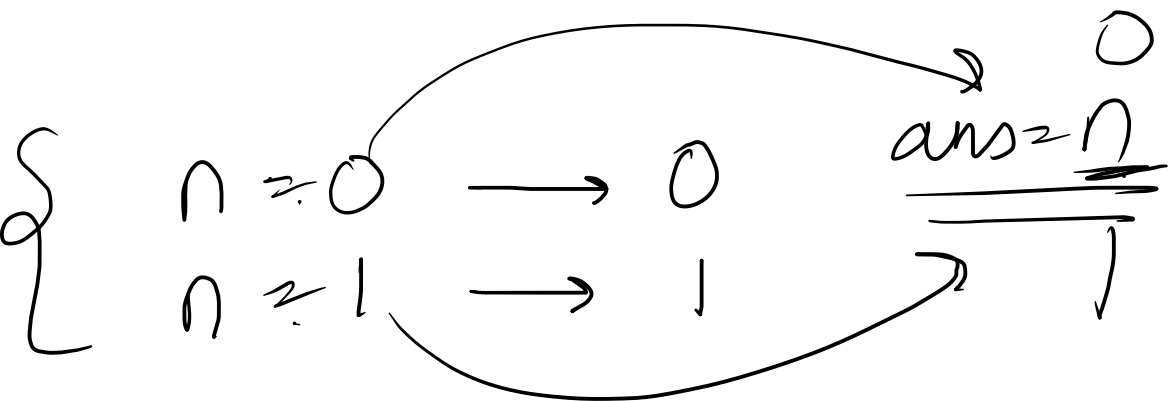
$$f(\underline{5}) = f(\underline{4}) + f(\underline{3})$$

$\xrightarrow{\quad \quad \quad}$
 $\xrightarrow{\quad \quad \quad}$



$$\left\{ \begin{array}{l} f(10) = f(9) + f(8) \\ f(15) = f(14) + f(13) \\ f(7) = f(6) + f(5) \end{array} \right.$$

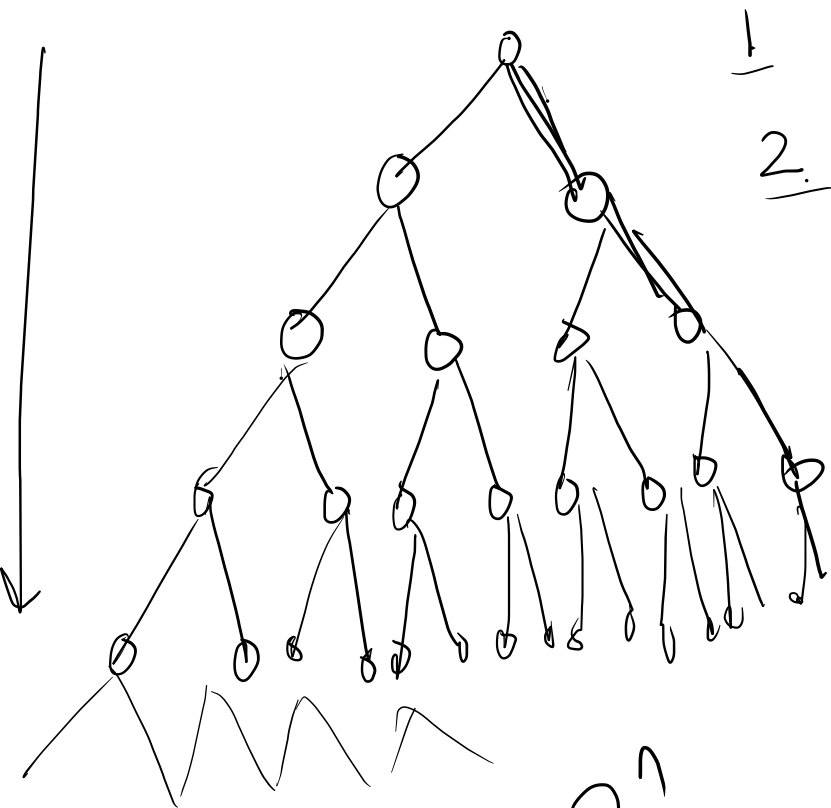
$$\checkmark f(n) = \underline{f(n-1)} + \underline{f(n-2)}$$



ans = n

$f(0) = 0$

$f(1) = 1$



$$\begin{array}{r} 1 \\ \hline 2 \\ \hline \end{array}$$

$$= 2^1$$

$$\begin{array}{r} 4 \\ \hline \end{array} = 2^2$$

$$\begin{array}{r} 8 \\ \hline \end{array} = 2^3$$

$$\begin{array}{r} 16 \\ \hline \end{array} = 2^4$$

$$32$$

$$\underline{\underline{2^5}}$$

Factorial

Power of 4

$$4^3 = 64 \quad \checkmark$$

$$4^2 = 16 \quad \checkmark$$

$$4^0 = 1$$

$$4^1 = 4$$

$$4^0 = 1$$

$$4^1 = 4 = 4$$

$$4^2 = 16 = 4 \times 4$$

$$4^3 = 64 = 4 \times 4 \times 4$$

$$4^4 = 256 = 4 \times 4 \times 4 \times 4$$

$$4^5 = \underbrace{4 \times 4 \times 4 \times 4 \times 4}_{\text{five 4s}} = \underline{1024}$$

$4^0 \geq 1$

if ($n \leq 0$)
return false

if ($n == 1$)
return true

n
4
16
64
256

$$n \% 4 = 0$$

$$\underline{\underline{8}} \rightarrow \underline{8 \% 4 = 0}$$

~~3~~

~~5~~

~~7~~

~~9~~

~~4~~

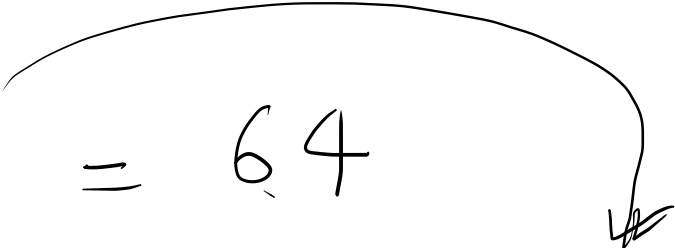
~~8~~


+2

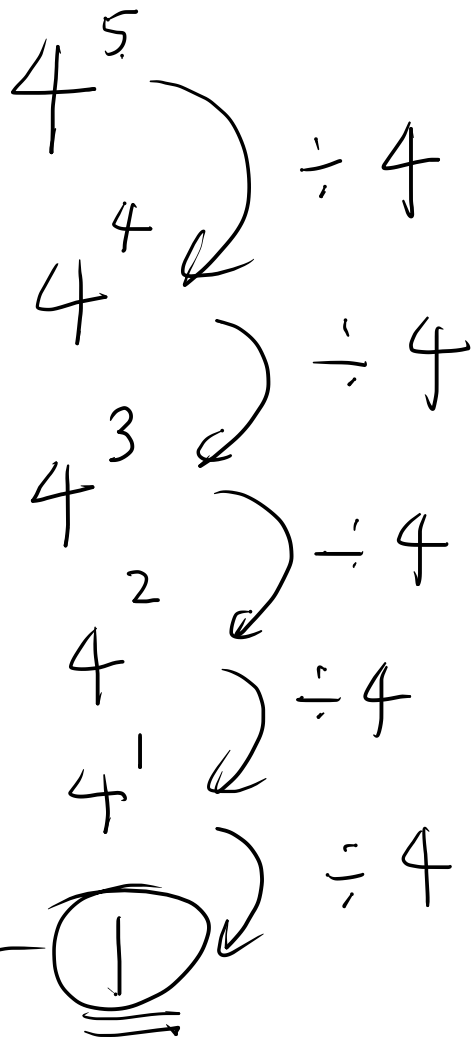
~~16~~

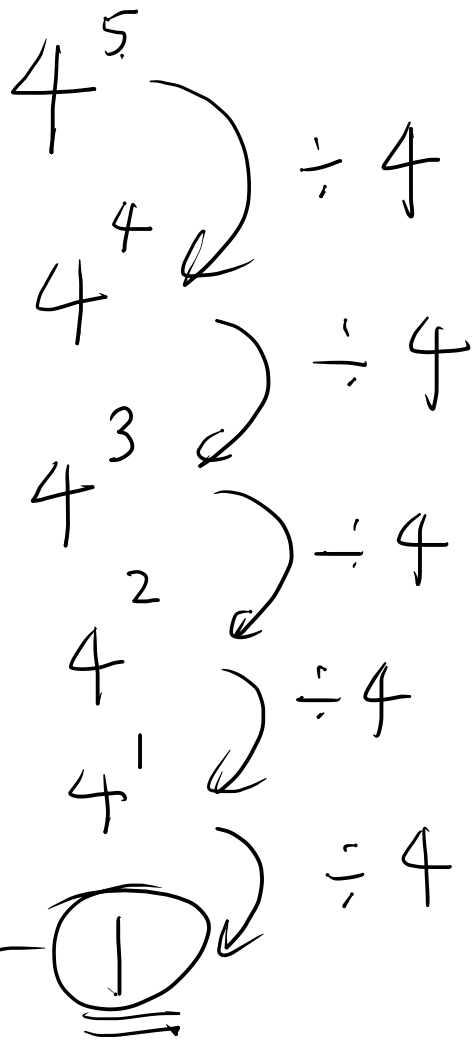
20

24

$$4^3 = 64$$


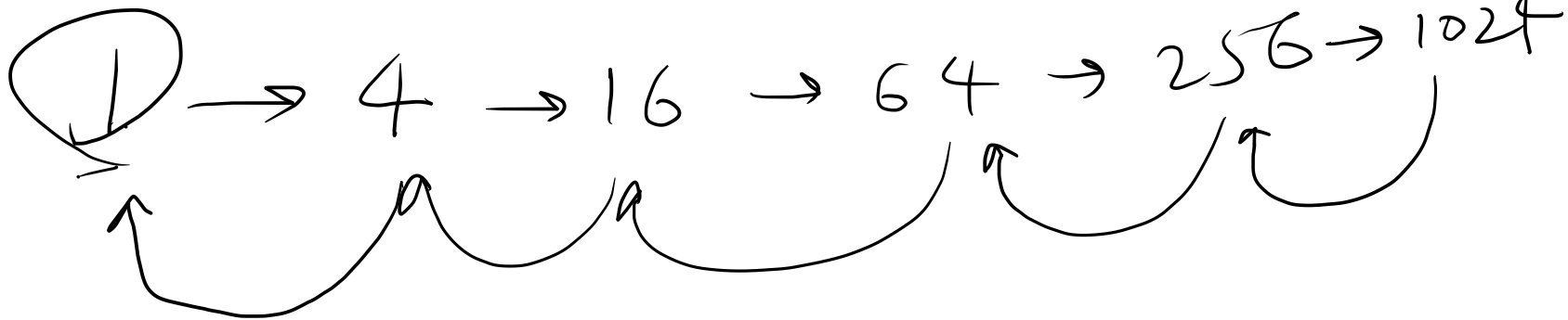
$$\begin{array}{lcl} 64/4 & = & 16 = 4^2 \\ 16/4 & = & 4 = 4^1 \\ 4/4 & = & 1 = 4^0 \end{array}$$






true

base



$$\begin{array}{r} 2.4 \\ \hline 6 \end{array}$$

x not zero or negative
 $x = 21?$

~~$$n \% 4 = 20$$~~

$$f(n/4)$$


```
if (n <= 0)
```

return false

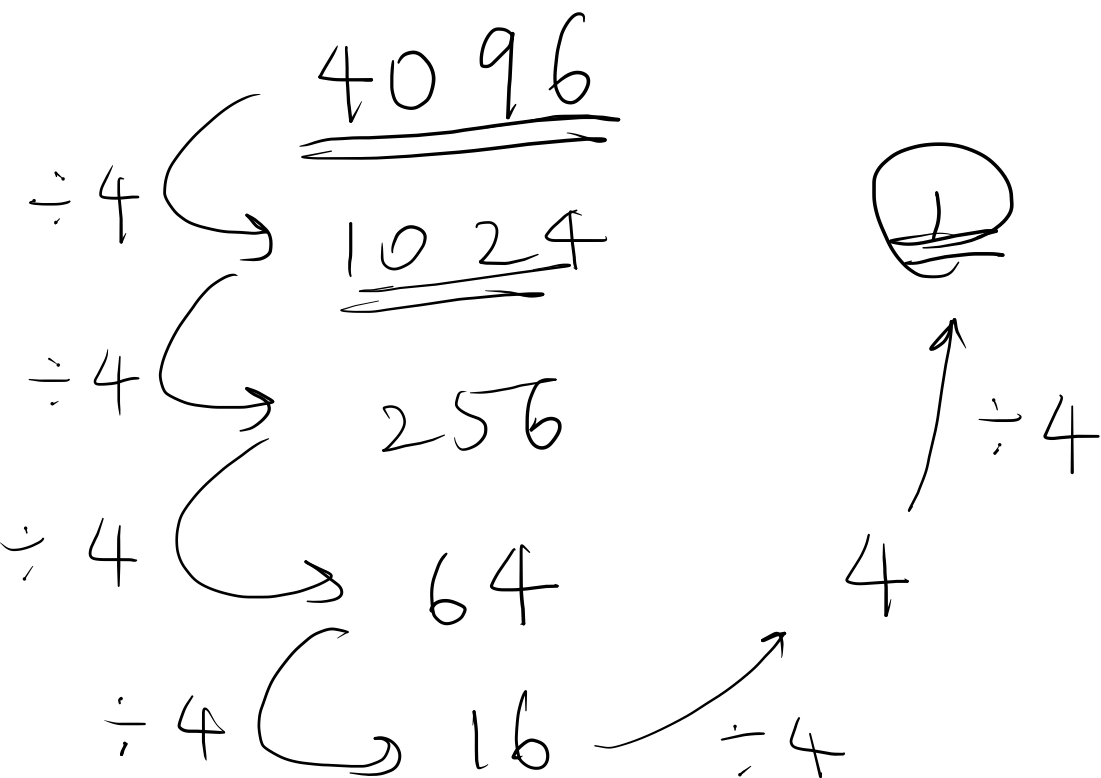
```
if (n == 1)
```

return true

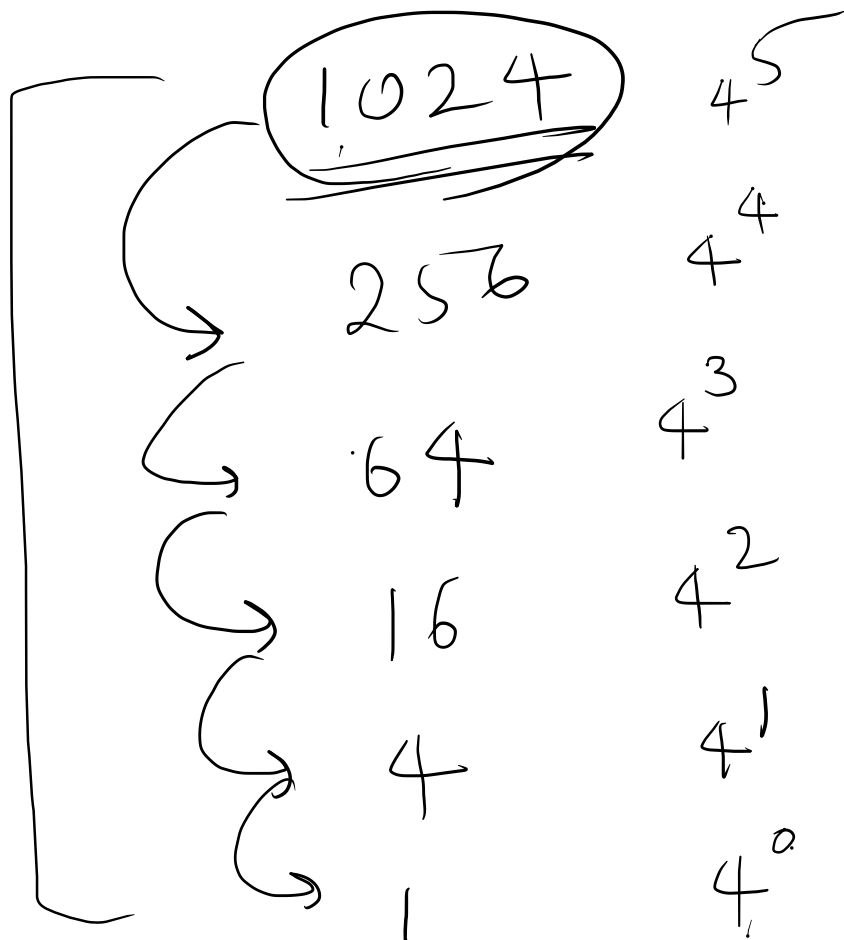
```
isPowerOfFour (n / 4)
```

```
if (n % 4 != 0)
```

return false



$$4^n$$



1024

→ 5

256

→ 4

64

→ 3

16

→ 2

4

→ 1

1

→ 0

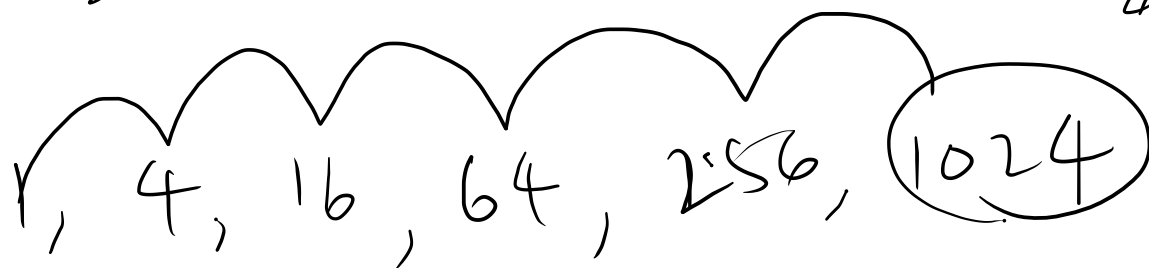
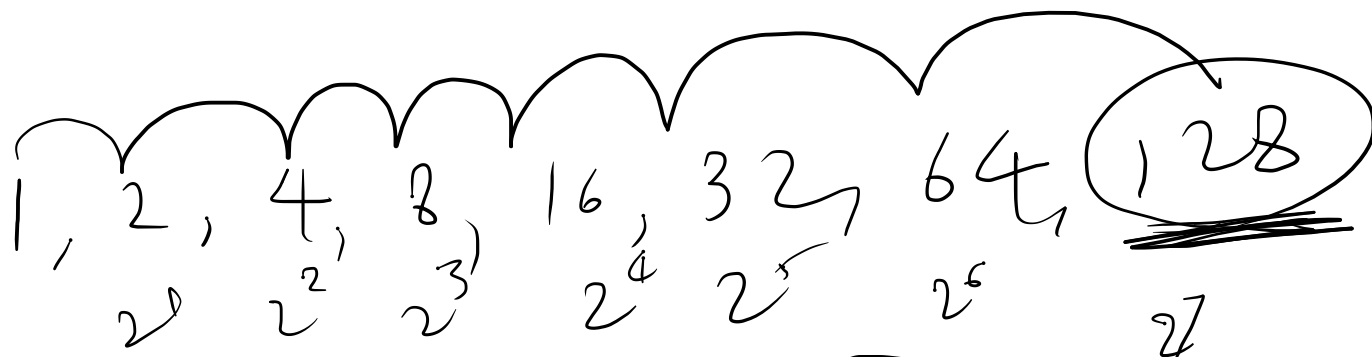
0(1)

$$\log_4 n$$

$$\log_4 1024 = 5$$

$$\underline{1024} = 4^{\textcircled{?}}$$

$$\underline{4^5 = 1024}$$



$$\log_4 1024 \approx 5$$