PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

Algorithm Exam - Medium Difficulty
Instructions: Answer all questions to the best of your ability. Show your work where applicable.

Section 1: Multiple Choice (4 points each, 40 points total)

Instructions: Choose the best answer for each question.

Question 1: In the given Insertion Sort example, what is the value of 'key' when j = 3?
A) 2
B) 4
C) 8
D) 9

Question 2: In the Bellman-Ford algorithm example, what does d[v] represent?
A) The predecessor of vertex v in the shortest path tree.
B) The estimated shortest distance from the source vertex to vertex v.
C) The actual shortest distance from the source vertex to vertex v.
D) The weight of the edge connecting the source to vertex v.

Question 3: According to the provided merging arrays, what are the first five elements after merging two sorted arrays?
A) 20, 12, 13, 11, 7
B) 1, 2, 7, 9, 11
C) 7, 9, 11, 12, 13
D) 11, 13, 20, 12, 7

Question 4: In the given examples of Recursion Tree, what does $T(n) = 2T(n/2) + \blacksquare(n)$ represent?
A) An upper bound to the problem.
B) A lower bound to the problem.
C) A guess to both upper and lower bounds.
D) The time complexity of a divide-and-conquer algorithm.

Section 2: Short Answer (6 points each, 30 points total)

Instructions: Answer each question in 2-3 sentences.

Question 5: Briefly explain the purpose of the 'while' loop within the Insertion Sort algorithm.

Question 6: In the Bellman-Ford algorithm, what is the significance of 'relaxing' a vertex?

Question 7: Describe how the guess "T (n) $\leq$ cn lg n" is verified using the substitution method in the Recursion Tree examples.

Section 3: Problem Solving (10 points each, 30 points total)

Instructions: Provide detailed solutions for each problem. Show all steps.

Question 8: Using the given Insertion Sort code example, trace the algorithm's execution on the following array: A = [5, 1, 4, 2, 8]. Show the state of the array after each iteration of the outer loop (for j = 2 to n).

Question 9: Apply the Bellman-Ford algorithm on a graph with nodes S, A, B, C and edges (S,A,4), (S,B,2), (A,C,2), (B,A,1), (B,C,4), (C,B, -1), with S as the source node. Run only two iterations. Show the d[v] and p[v] values for each node after each iteration.

Question 10: Consider the recurrence relation T(n) = T(n-1) + ■(n). Using the substitution method, prove that T(n) = O(n^2). Show all steps of your proof, including the initial guess, substitution, and inductive step.