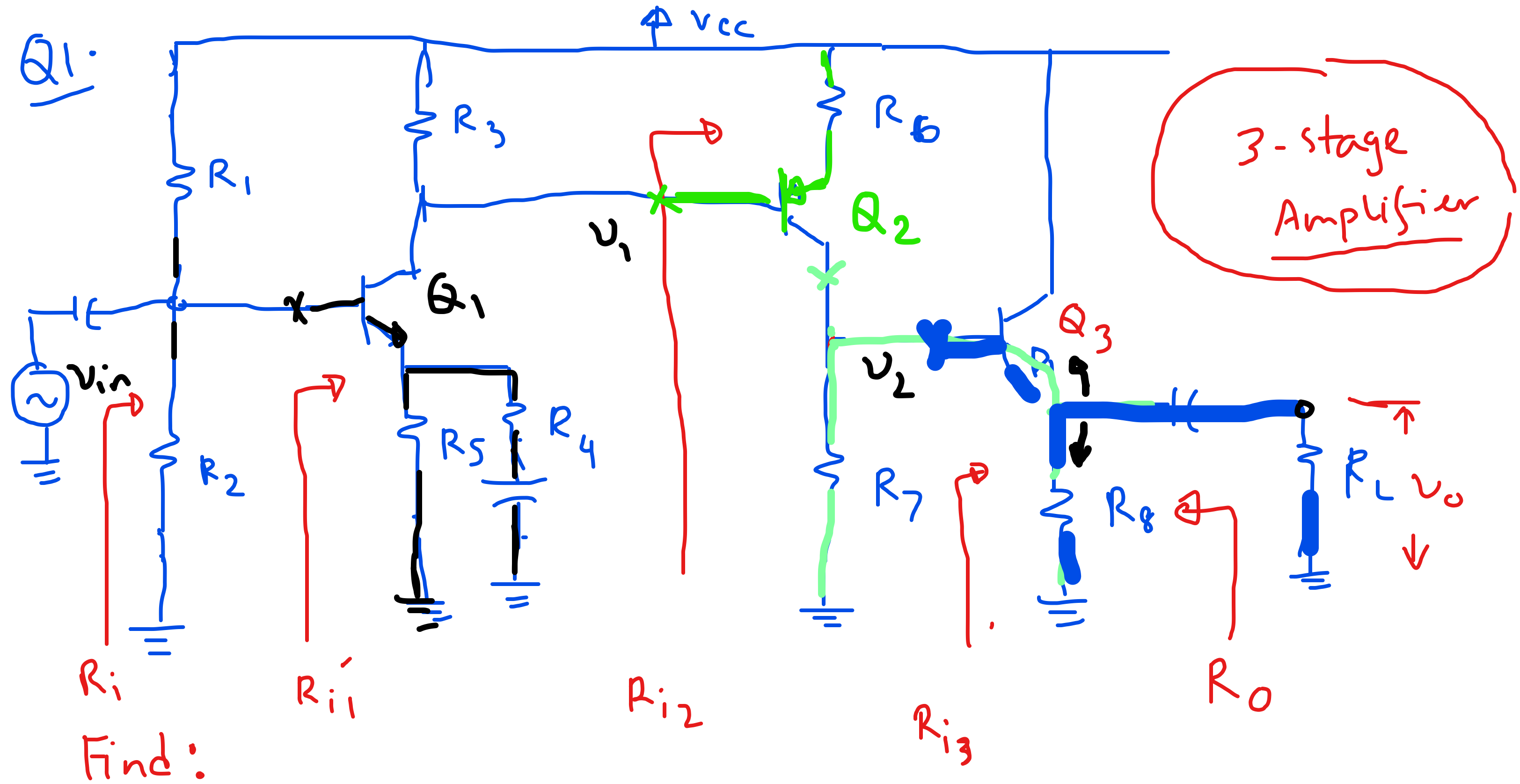
Q1:



Find:

$\beta_1 = \beta_2 = \beta_3 = \beta$

$$R_o = R_8 \parallel \frac{(r_{\pi 3} + R_7)}{(\beta+1)}$$

$Q_3 = \{ g_{m3},\ r_{\pi 3},\ r_{e3}$

$$R_{i3} = r_{\pi 3} + (R_8 \parallel R_L)(\beta+1)$$

$$R_{i2} = r_{\pi 2} + R_6(\beta+1)$$

$Q_2 = \{ g_{m2},\ r_{\pi 2},\ r_{e2}$

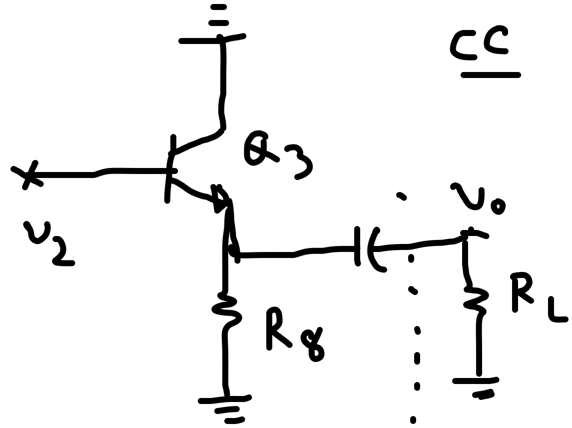
$$R_{i1}' = r_{\pi 1} + (R_4 \parallel R_5)(\beta+1)$$

$$R_i = R_1 \parallel R_2 \parallel R_{i1}'$$

$Q_1 = \{ g_{m1},\ r_{\pi 1},\ r_{e1}$

Volt. Gain for Q_3

CC



$$A_{v_3} = \frac{v_o}{v_2} = \frac{R_8}{r_{e_3} + R_8}$$

$$\cong \frac{g_{m_3} R_8}{1 + g_{m_3} R_8}$$

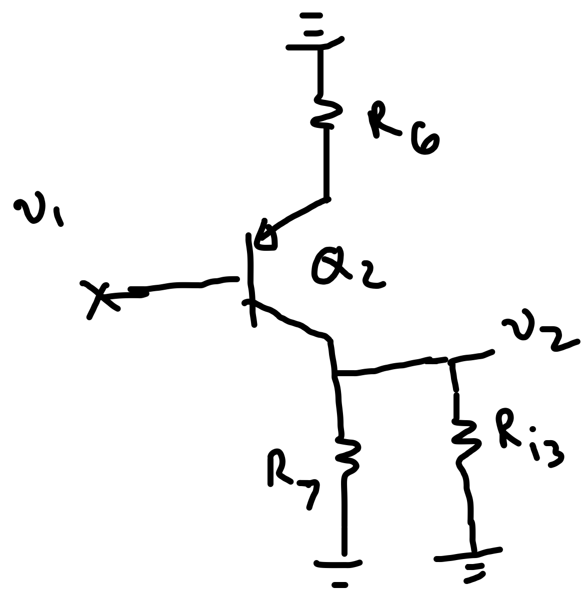
$$r_{e3} = \frac{\alpha}{g_{m3}}$$

Voltage gain for Q_2

CE

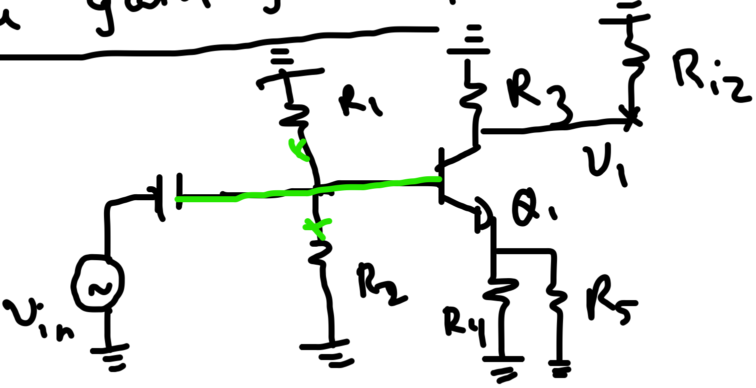
$$A_{v_2} = -\frac{g_{m_2}(R_7 \| R_{i_3})}{1 + g_{m_2} R_6}$$

$$= \frac{v_2}{v_1}$$

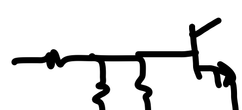


$$A_{v_{CE}} = -\frac{g_m R_C}{1 + g_m R_E}$$

Voltage gain for Q_1



CE



$$A_{v1} = -\frac{g_{m1}(R_3 \| R_{i2})}{1+g_{m1}(R_4 \| R_5)} = \frac{v_1}{v_{in}}$$

$$A_{vo} = A_{v1} \times A_{v2} \times A_{v3}$$

$$= \frac{v_1}{v_{in}} \times \frac{v_2}{v_1} \times \frac{v_o}{v_2} = \frac{v_o}{v_{in}}$$

$$= (\quad)(\quad)(\quad)$$

with - load

$$A_v = A_{vo} \times \frac{R_L}{R_L + R_o}$$

or

with load

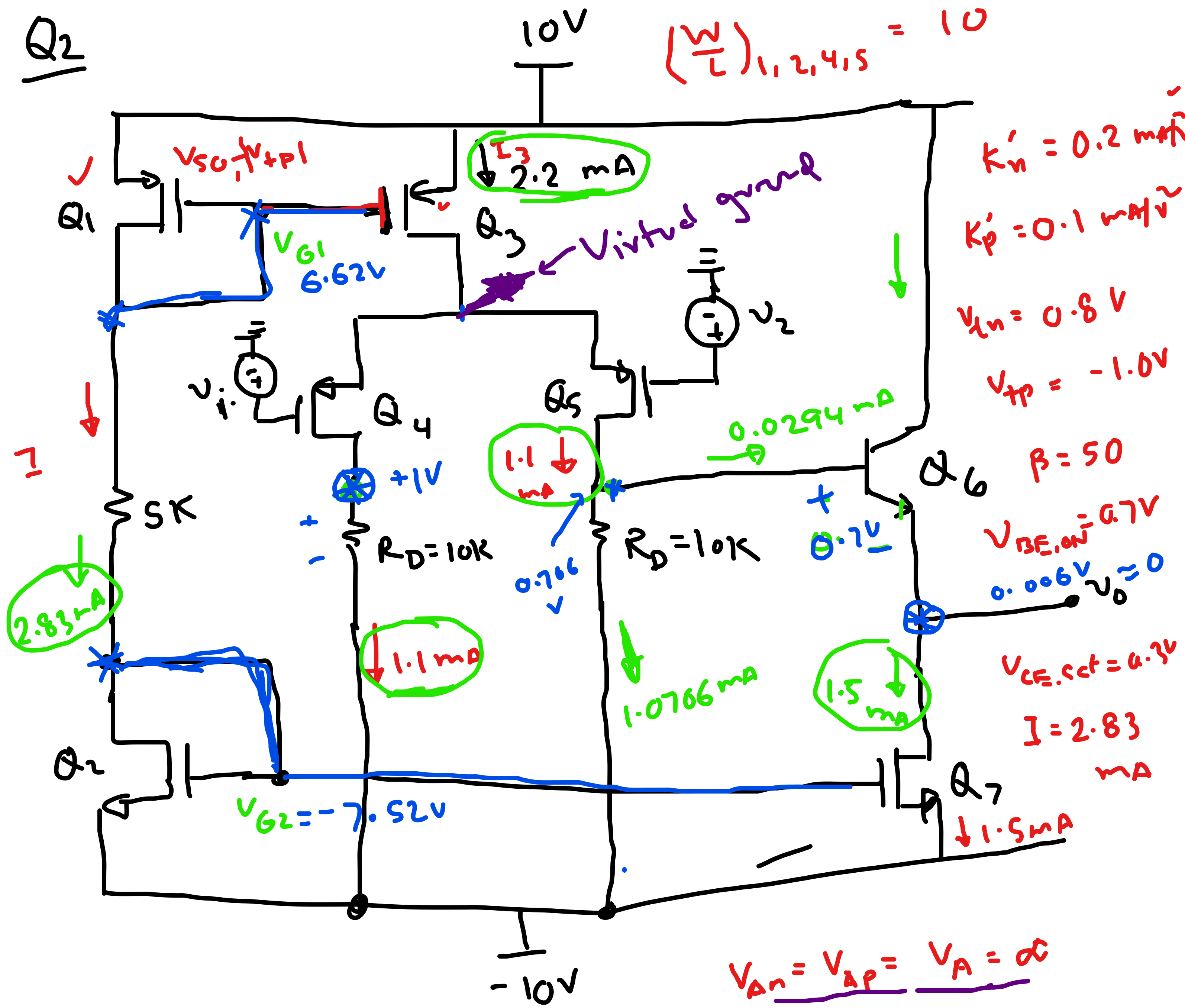
$$A_{v3-L} = \frac{R_8 \| R_L}{r_{e3} + R_8 \| R_L}$$

$$A_v = A_{v1} \times A_{v2} \times A_{v3\_w-l}$$

$$g_m = 40\, I_C \quad (\text{DC analysis})$$

$$r_\pi = \frac{\beta}{g_m}$$

$$r_e = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1}$$



$V_{An} = V_{Ap} = V_A = \infty$

1) Find the V_{ov} (overdrive voltage)

$$V_{ov1} = \sqrt{\frac{2I}{k_p' \times \left(\frac{W}{L}\right)_1}} = \sqrt{\frac{2 \times 2.83}{0.1 \times 10}} = 2.38\ V = V_{ov3}$$

$$V_{ov2} = V_{ov7} = \sqrt{\frac{2I}{k_n' \times \left(\frac{W}{L}\right)_2}} = \sqrt{\frac{2 \times 2.83}{0.2 \times 10}} = 1.68\ V$$

$$V_{ov4} = V_{ov5} = \sqrt{\frac{2 \times 1.1}{0.1 \times 10}} = 1.48\ V$$

$$I_{D3} = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_3 V_{ov3}^2$$

$$\frac{I_3}{I_{ref}} = \frac{(W/L)_3}{(W/L)_{ref}}$$

(II) Find $\left(\frac{W}{L}\right)_3$ and $\left(\frac{W}{L}\right)_7$

$$\left(\frac{W}{L}\right)_3 = \frac{I_3}{I_{ref}} \times \left(\frac{W}{L}\right)_1 = \frac{2.2}{2.87} \times 10 = 7.77$$

$$\left(\frac{W}{L}\right)_7 = \frac{I_7}{I_{ref}} \times \underbrace{\left(\frac{W}{L}\right)_{ref}}_{Q_2} = \frac{1.5}{2.83} \times 10 = 5.31$$

(III) Find the branch current and node voltage

$$I_{B.6} = \frac{1.5}{50+1} = 0.0294 \text{ mA}$$

$$I_{C.6} = I_{B.6} \times 50 = 1.47 \text{ mA}$$

$$V_{SG1} - |V_{tp}| = V_{ov1}$$

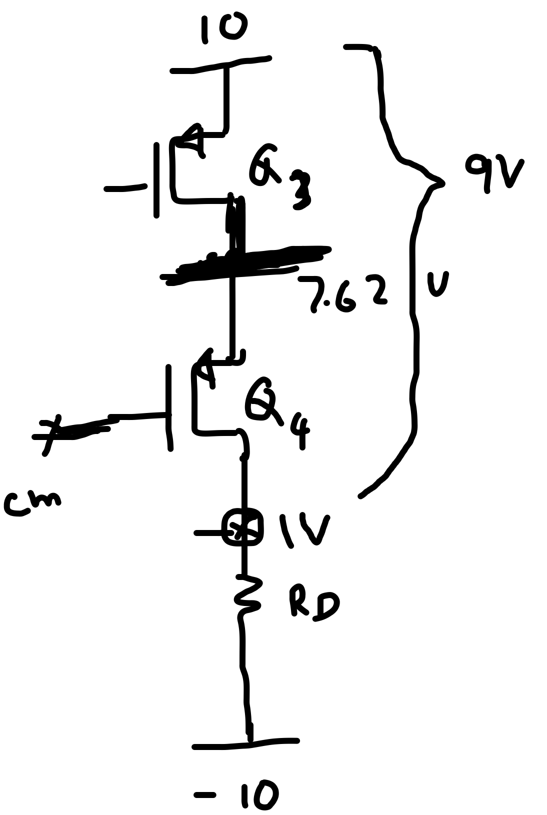
$$V_{SG1} = 2.38 + 1 = 3.38 \text{ V}$$

$$V_{S1} - V_{G1} = 3.38 \text{ V} \qquad V_{G1} = 10 - 3.38 = 6.62 \text{ V}$$

$$V_{GS2} - V_{tn} = V_{ov2}$$

$$V_{G2} = V_{S2} + V_{tn} + V_{ov2}$$
$$= -10 + 0.8 + 1.68 = -7.52\ V$$

(c) Common mode input voltage range



Q₃ at margin to find $V_{cm\,max}$

$$\Rightarrow v_{SD3} = V_{ov3}$$

$$10 - V_{D3} = 2.38$$

$$V_{D3} = 10 - 2.38 = 7.62\ V$$

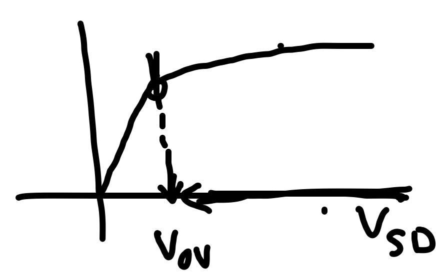
$$V_{SG4} - |V_{tp}| = V_{ov4}$$

$$V_{S4} - V_{G4} - |V_{tp}| = 1.48$$

$$7.62 - V_{G4} - 1 = 1.48$$

$$V_{G4} = 7.62 - 1 - 1.48$$
$$= 5.14$$

$$V_{cm\,max} \leq 5.14\ V$$



v_{cm} – min

Q_4 at margin

$v_{SD4} = V_{ov4}$

$V_{S4} - \underline{V_{D4}} = 1.48$

$V_{S4} = 2.48\ V$

$V_{SG4} - |V_{tp}| = V_{ov4}$

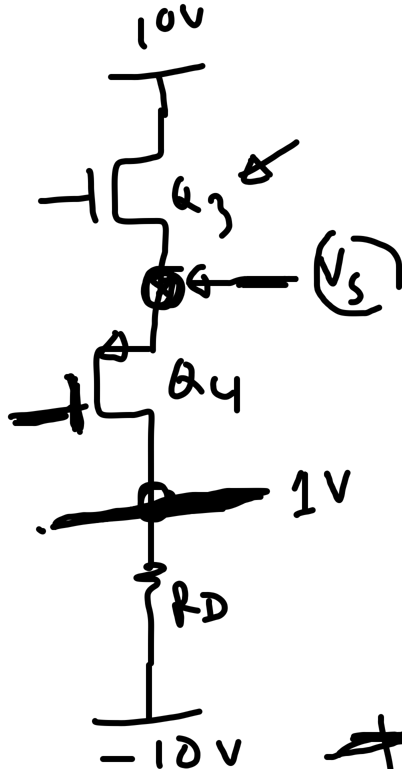
$V_{S4} - V_{G4} - |V_{tp}| = V_{ov4}$

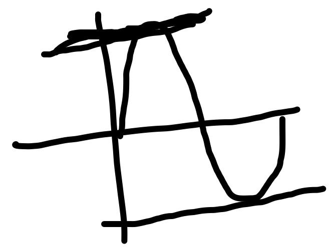
$V_{G4} = V_{S4} - |V_{tp}| - V_{ov4}$

$= 2.48 - 1 - 1.48 = 0$

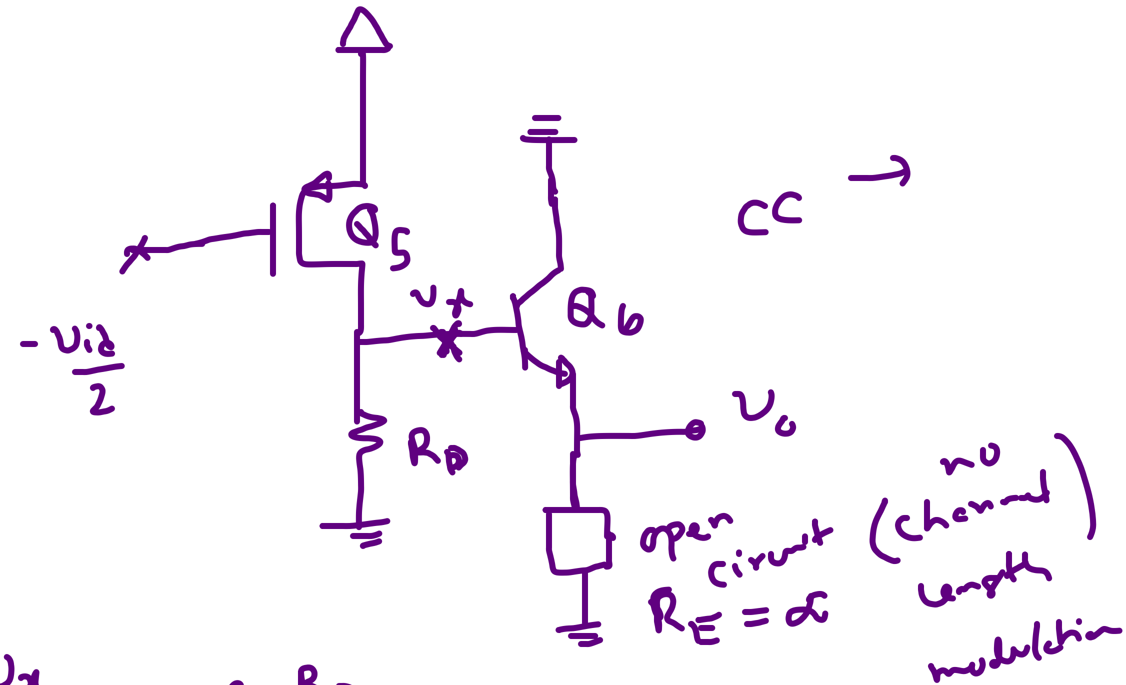
$v_{cm\,min} = 0\ V$

$0 \leq v_{cm} \leq 5.14\ V$





(d) Differential mode gain



$$\frac{v_x}{-\frac{v_{id}}{2}} = -g_{m5} R_D$$

$$\frac{v_x}{v_{in}} = \frac{1}{2} g_{m5} R_D \qquad \frac{v_o}{v_x} \approx \text{unity} = \frac{g_m R_E}{1 + g_m R_E} \simeq 1$$

$$A_{v,d} = \frac{v_o}{v_x} \cdot \frac{v_x}{v_{in}} = \frac{1}{2} g_{m5} R_D$$

$$g_{m5} = \frac{2 I_{D5}}{V_{ov5}} = \frac{2 \times 1.1}{1.48} = 1.48 \text{ mS}$$

$$A_{v,d} = \frac{1}{2} \times 1.48 \times 10 = 7.48 \text{ V/V}$$

(d) Repeat part (c) if $V_{An} = 40V$, $V_{AP} = -20V$, $V_A = 80V$

$$r_{o5} = \frac{20}{1.1} = \frac{|V_{AP}|}{I_{DS}}$$

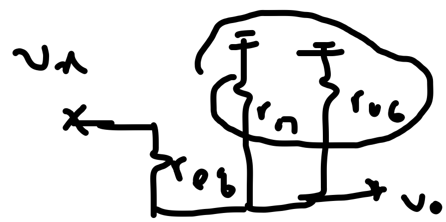
$$= 18.18\ k\Omega$$

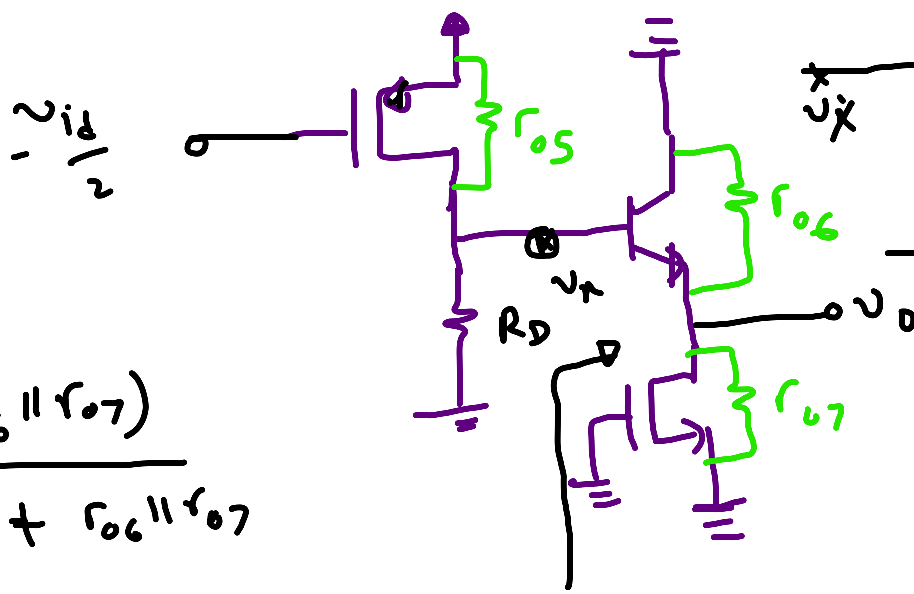
$$r_{\pi 6} = \frac{\beta}{g_{m6}}$$

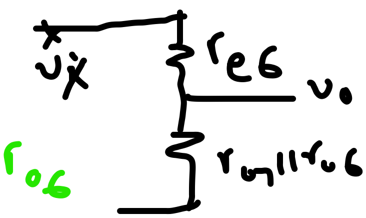
$$g_{m6} = 40 I_{C6} = 40 \times 1.47$$

$$r_{o6} = \frac{V_A}{I_C} = \frac{80}{1.47} = 54.42\ k\Omega$$

$$r_{o7} = \frac{40}{1.5} = 26.67\ k\Omega$$







$$\frac{v_o}{v_x} = \frac{(r_{o6} \| r_{o7})}{r_{e6} + r_{o6} \| r_{o7}}$$

$$R_i' = r_{\pi 6} + (\beta+1)(r_{o6} \| r_{o7})$$

or $r_{e6} = \frac{r_{\pi 6}}{\beta+1}$

$$\frac{v_o}{v_x} = \frac{g_{m6}(r_{o6} \| r_{o7})}{1 + g_{m6}(r_{o6} \| r_{o7})} \approx 1$$

$$\frac{v_x}{-\frac{v_{id}}{2}} = -g_{m5}(R_D \| R_i')$$

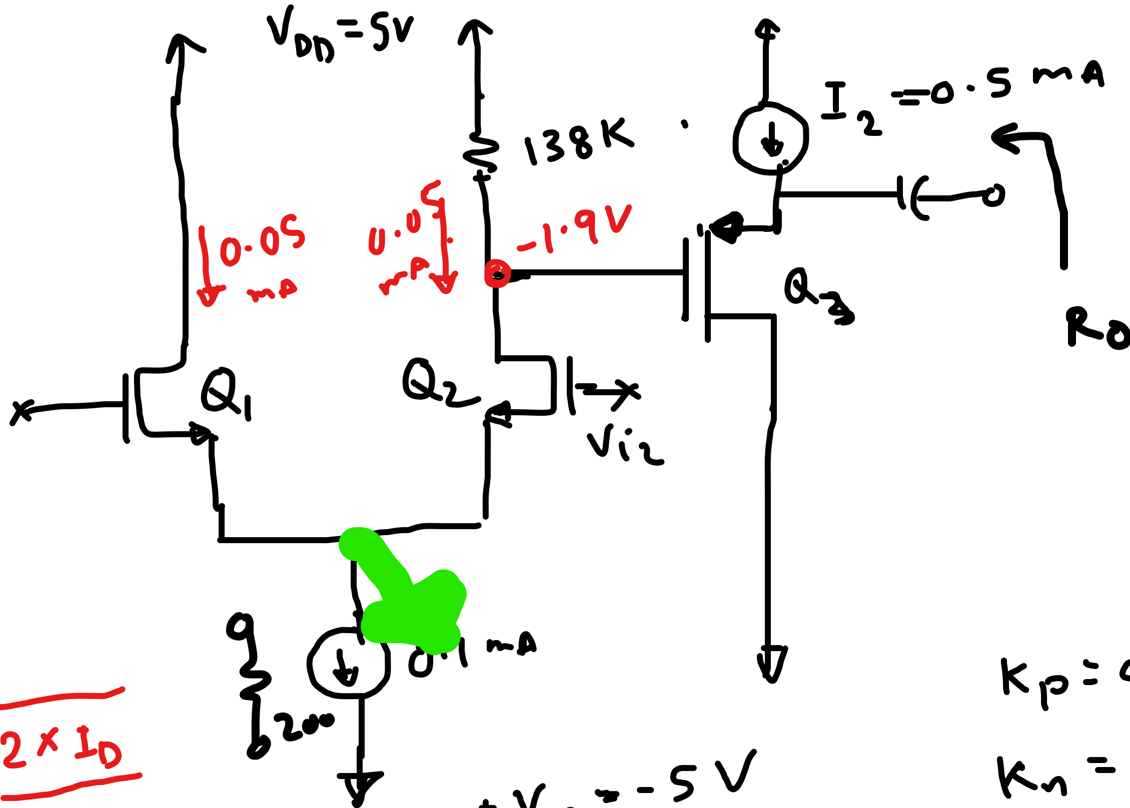
$CC \to$ gain unity

$$\frac{v_x}{v_{id}} = \frac{1}{2} g_{m5} (R_o \parallel R_i')$$

$$A_v = \frac{v_x}{v_{id}} \cdot \frac{v_o}{v_x} = \frac{v_o}{v_{id}}$$

$$= \frac{1}{2} g_{m5} (R_o \parallel R_i') \left(\frac{r_{o6} \parallel r_{o7}}{r_{e6} + r_{o6} \parallel r_{o7}} \right)$$

40·1140 = 200



$K_n = K_n'\left(\frac{W}{L}\right)$

$K_p = 0.39$

$K_n = 1$

$V_{tn} = |V_{tp}| = 0.3$ ✓

$\lambda = 0$

$$V_{ov2} = V_{ov1} = \sqrt{\frac{2 \times I_D}{K_n}}$$

$$= \sqrt{\frac{2 \times 0.05}{1}} = 0.316V$$

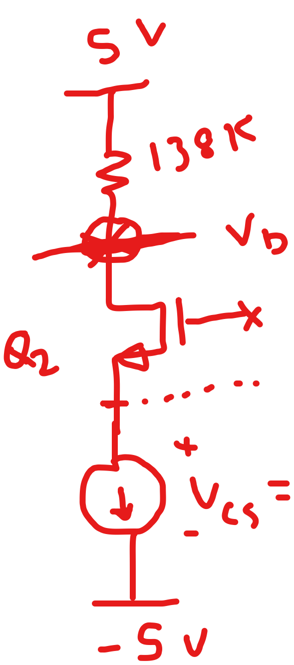
$$V_{o3} = \sqrt{\frac{2 \times 0.5}{0.39}} = 1.601\ V$$

minimum voltage for the current source = 0.35 V

(a) Common mode voltage gain

Maximum V_{cm}

Q_2 will operate at margin



$$v_{DS2} = V_{ov2} = 0.316V$$

$$V_{DS} - V_{S2} = 0.316$$

$$V_{S2} = V_{DS} - 0.316$$

$$= -1.9 - 0.316 = -2.216\ V$$

$$V_{GS2} - V_{tn} = V_{ov2}$$

$$V_{G2} - V_{S2} - V_{tn} = V_{ov2}$$

$$V_{G2} = V_{S2} + V_{tn} + V_{ov2} = -2.216 + 0.3 + 0.316 = -1.6\text{ V}$$

$$V_{O2} = V_{cm,max}$$

Minimum V_{cm}

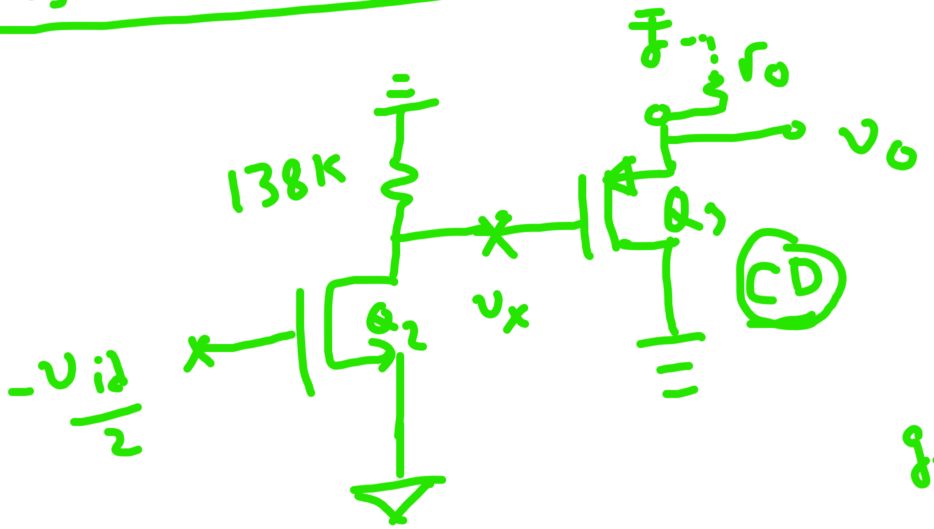
$$V_{S2} = -5 + 0.35 = -4.65\text{ V}$$

$$V_{G2} = V_{S2} + V_{tn} + V_{ov2} = -4.65 + 0.3 + 0.316 = -4.034\text{ V}$$

$$v_{cm\text{-}min}$$

$$\boxed{-4.034 < v_{cm} < -1.6\text{ V}}$$

Differential mode voltage gain



$$g_{m2} = \frac{2I_D}{V_{ov2}} = \frac{2 \times 0.05}{0.316} = 0.3165 \text{ mS}$$

$$\frac{v_x}{-\frac{v_{id}}{2}} = -g_{m2} R_D$$

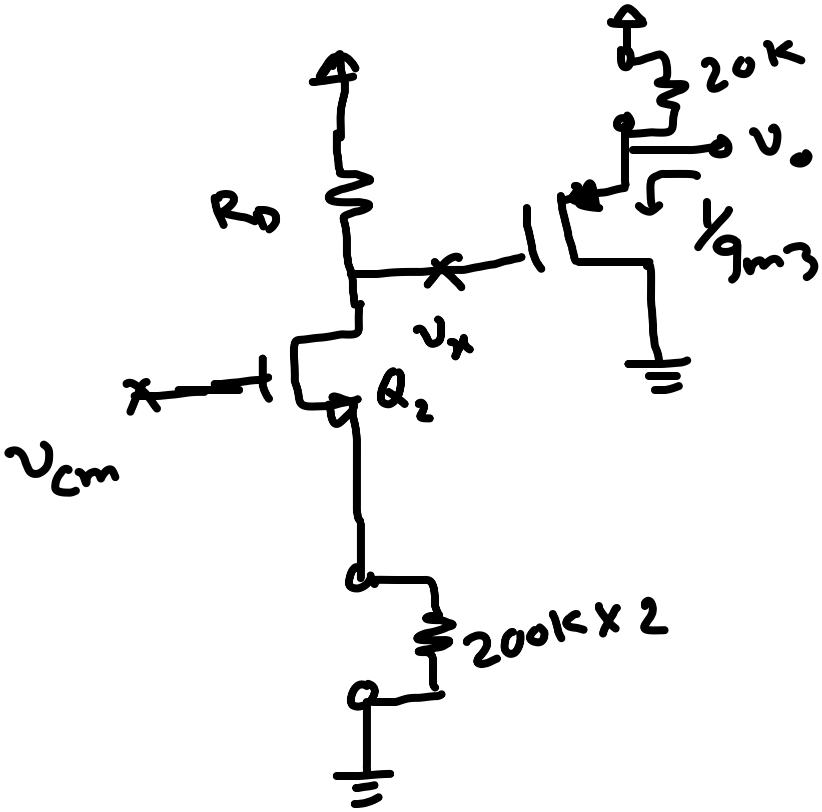
$$\frac{v_x}{v_{id}} = \frac{1}{2} g_{m2} (138) = 21.83 \text{ V/V}$$

$$\frac{v_o}{v_x} \simeq 1 = \frac{g_{m3} r_o}{1 + g_{m3} r_o} \qquad r_o \to \infty$$

If r_o is known (e.g. non-ideal current source) $r_o = 20K$

$$A_{v,d} = \left(\frac{g_{m3} r_o}{1 + g_{m3} r_o}\right)\left(\frac{1}{2} g_{m2} R_D\right) = 0.92588 \text{ V/V} \times 21.83 = 20.22 \text{ V/V}$$

Common mode voltage gain



we must need the internal resistance of the current source

$$\frac{v_x}{v_{cm}} = \frac{g_{m2} R_D}{1 + g_{m2}(2 \times 200)} = -0.679 \text{ V/V}$$

$$\frac{v_o}{v_x} = \frac{20}{20 + \frac{1}{g_{m3}}} = 0.92587 \text{ V/V}$$

$$A_{v\,cm} = -0.629 \text{ V/V}$$

Output resistance

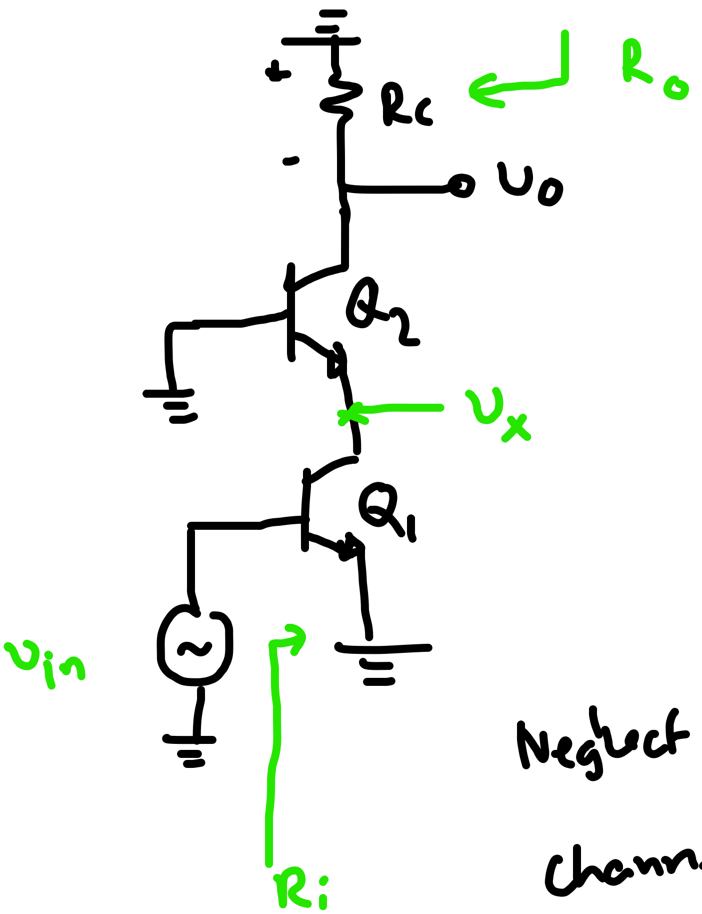
$$R_o = \frac{1}{g_{m3}}$$ if ideal current source

$$R_o = r_o \| \frac{1}{g_{m3}}$$ if current source resistance is r_o

$$= 20 \| \frac{1}{0.6245}$$

$$= 1.48\ \underline{k\Omega}$$

Cascode



Neglect the channel length modulation

$$r_{\pi 1} = \frac{\beta}{g_{m1}}$$

$$R_i = r_{\pi 1}$$

$$R_o = R_C$$

$$\frac{v_x}{v_{in}} = \underline{-g_{m1}(r_{e2})}$$

$$v_o = -i_{c_2} R_C$$
$$= -g_{m_2} v_{be_2} R_C$$
$$= -g_{m2}(v_{b2} - v_{e2}) R_C$$
$$= -g_{m2}(0 - v_x) R_C = g_{m2} v_x R_C$$

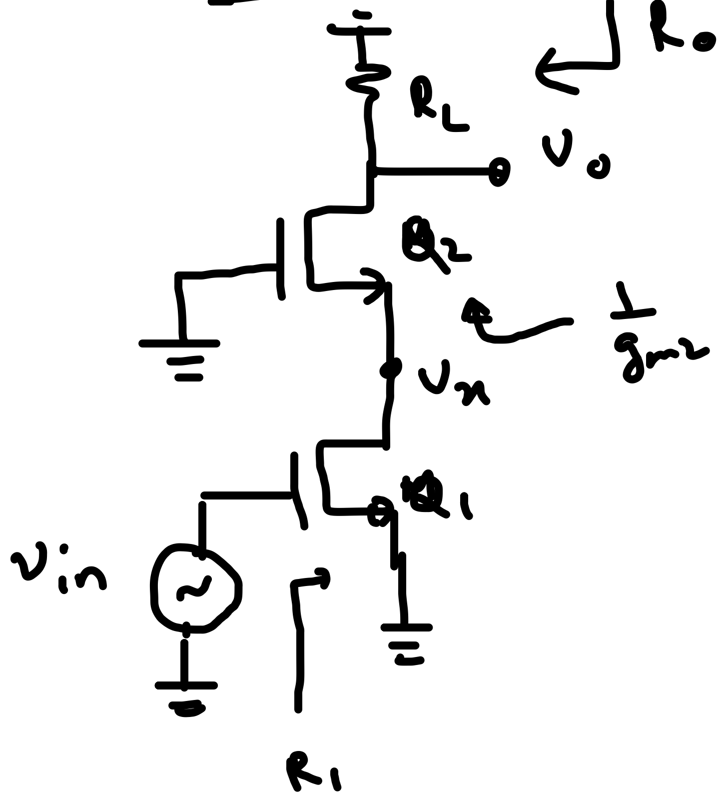
$$\boxed{\frac{v_o}{v_\pi} = g_{m2} R_C}$$

$$A_v = -\left(g_{m1} r_{e2}\right) g_{m2} R_C$$

$$= -g_{m1} \frac{\alpha_2}{g_{m2}} g_{m2} R_C$$

$$= -\alpha_2 g_{m1} R_C \approx -g_{m1} R_C$$

Cascode MOSFET



$A_v = ?$

Neglect channel length modulation

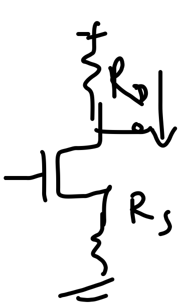
$R_i = \infty$

$R_o = \infty$

$$\frac{v_o}{v_x} = g_{m2} R_L$$

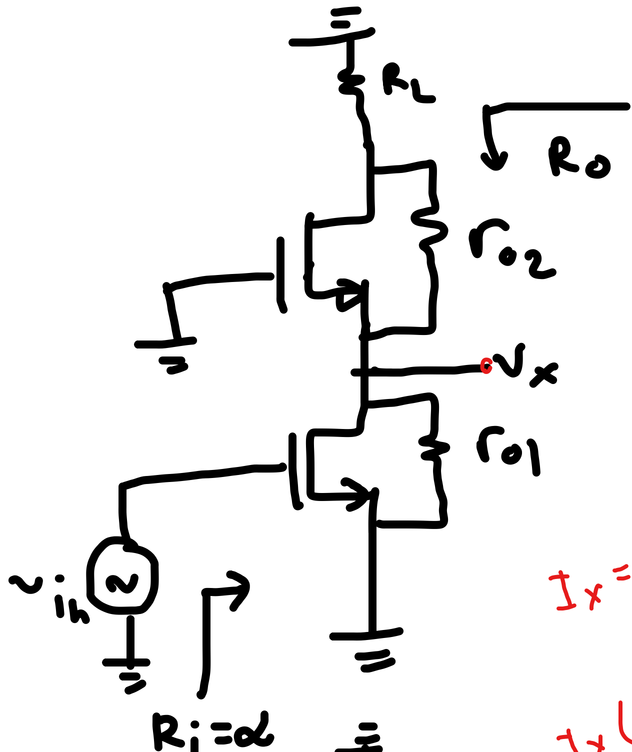
$$\frac{v_x}{v_{in}} = -g_{m1}\frac{1}{g_{m2}} = -\frac{g_{m1}}{g_{m2}}$$

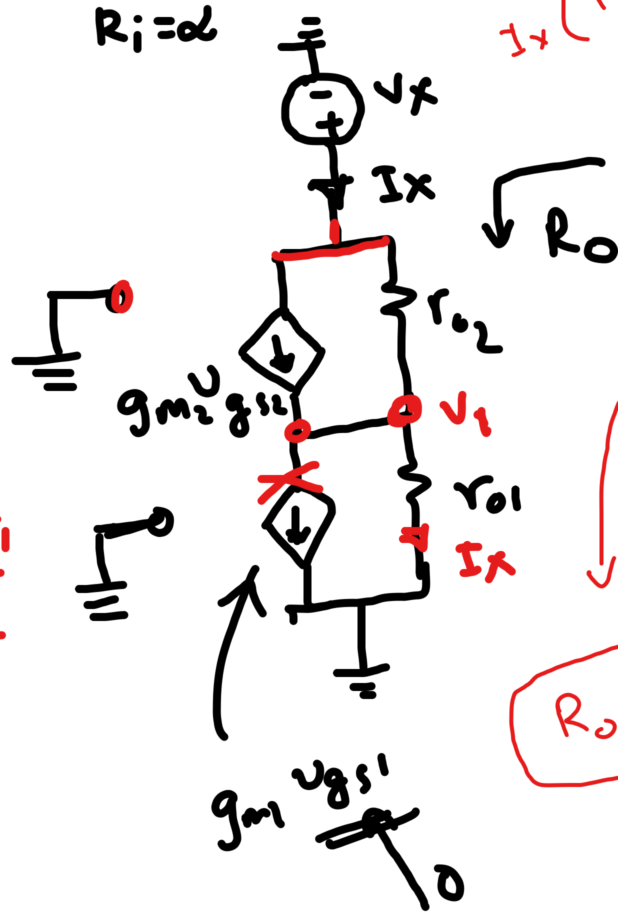
$$\frac{v_o}{v_{in}} = -g_{m1} R_L$$



experience → 15 → Multi
← 40 → Diff.
→ 25 → DC + single
10 → Cascode
10 → ⇓
10

Consider channel length modulation





$$I_x = g_{m2} v_{gs2} + \frac{V_x - V_1}{r_{o2}}$$

$$I_x = g_{m2}(0 - V_1) + \frac{V_x - V_1}{r_{o2}}$$

$$V_1 = I_x r_{o1}$$

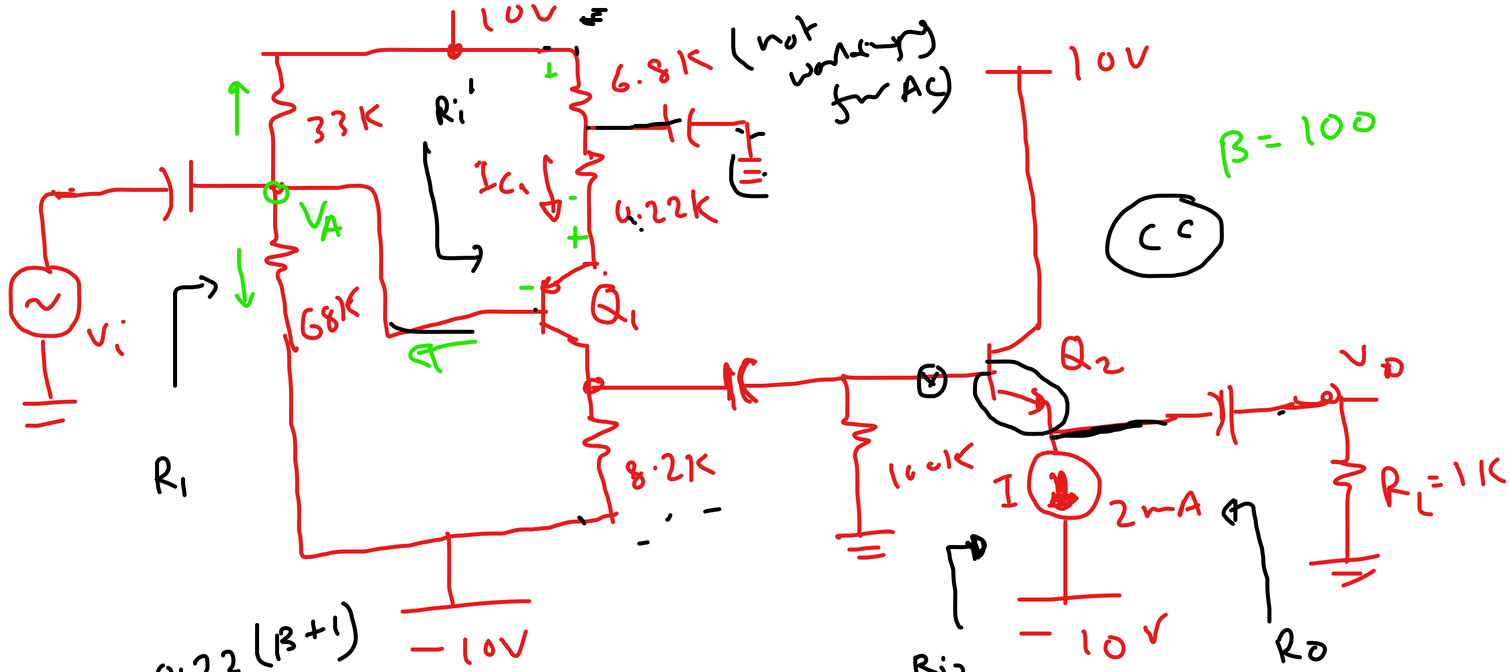
$$I_x = -g_{m2} I_x r_{o1} + \frac{V_x}{r_{o2}} - \frac{I_x r_{o1}}{r_{o2}}$$

$$I_x \left(1 + g_{m2} r_{o1} + \frac{r_{o1}}{r_{o2}}\right) = \frac{V_x}{r_{o2}}$$

$$I_x \left(\frac{r_{o2} + g_{m2} r_{o1} r_{o2} + r_{o1}}{r_{ox}}\right) = \frac{V_x}{r_{ox}}$$

$$\frac{V_x}{I_x} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}$$

$$\boxed{R_o \cong g_{m2} r_{o1} r_{o2}}$$



$R_i' = r_{\pi 1} + 0.22(\beta+1)$

$R_i = 33 \| 68 \| R_i'$

$\frac{V_A - 10}{33} + \frac{V_A + 10}{68} - I_B = 0$

$V_A\left(\frac{1}{33} + \frac{1}{68}\right) - I_B = \frac{10}{33} - \frac{10}{68}$ → (I)

$10 - (6.8 + 0.22)(\beta+1) I_B - 0.7 = V_A$

$V_A + 709.02\, I_B = 9.3$ → (II)

$R_o = r_{e2} + \frac{(100 \| 8.2)}{(\beta+1)}$

$V_A = 3.64\,V$

$I_B = 7.98\ \mu A$

$I_C = 100 \times 7.98\ \mu A = 0.798\ mA$

$R_{i2} = r_{\pi 2} + R_L(\beta+1)$

$g_{m1} = 40 I_C$

$$\boxed{A_{v1} = \frac{-g_{m1}(8.2 \| 100 \| R_{i2})}{1 + g_{m1}(0.22)} \qquad A_{v2} = \frac{R_L}{r_{e2} + R_L}}$$

$r_{\pi 1} = \frac{\beta}{g_{m1}}$

$A_v = A_{v1} A_{v2}$

$I_{C2} = I_{e2} \times \frac{\beta}{(\beta+1)} = 2 \times \frac{100}{101}$

$r_{\pi 2} = \frac{\beta}{g_{m2}}$

$g_{m2} = 40 I_{C2}$