

PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

Data Structures and Algorithms Exam

Instructions: Please answer all questions to the best of your ability.

Section 1: Multiple Choice Questions (4 points each, 40 points total)

Instructions: Choose the best answer for each question.

Question 1: Which of the following best describes the primary operation performed in the provided "Merging two sorted arrays" snippets?

- A) Sorting an unsorted array
- B) Searching for a specific element
- C) Combining two sorted arrays into one sorted array
- D) Reversing the order of elements in an array

Question 2: Based on the provided code snippets, what is the purpose of the `while` loop in the `INSERTION-SORT` algorithm?

- A) To iterate through the entire array.
- B) To find the minimum element in the array.
- C) To shift elements greater than the `key` to the right.
- D) To swap the `key` with the element at the current index `i`.

Question 3: In the recurrence relation $T(n) = T(n-1) + \lg n$, what does `lg n` represent?

- A) The size of the input.
- B) The base-10 logarithm of n .
- C) The natural logarithm of n .
- D) The amount of work done at each recursive step.

Question 4: According to the substitution method example, what is the general form of $T(n)$ after expanding it for 'm' times, where $T(n) = T(n-1) + \lg n$?

- A) $T(n) = T(n-m) + m * \lg n$
- B) $T(n) = T(n-m) + \lg(n-m)$
- C) $T(n) = T(n-m) + \lg(n-(m-1)) + \lg(n-(m-2)) + \dots + \lg(n-1) + \lg n$
- D) $T(n) = T(n-m) + \lg(n! - m!)$

Section 2: Short Answer Questions (6 points each, 30 points total)

Instructions: Answer each question in 2-3 sentences.

Question 5: Briefly explain the role of the 'key' variable in the provided insertion sort algorithm.

Question 6: Explain why analyzing algorithms is important, with reference to the provided content.

Question 7: In the context of recurrence relations and complexity analysis, what is the substitution method used for, and what is the general idea behind it?

Section 3: Problem-Solving Questions (10 points each, 30 points total)

Instructions: Provide detailed solutions to each problem, showing your reasoning.

Question 8: Given the initial array [9, 5, 1, 4, 3], trace the execution of the provided `INSERTION-SORT` algorithm. Show the state of the array after each iteration of the outer loop ($j=2$ to n).

Question 9: Consider the recurrence relation $T(n) = 2T(n/2) + \Theta(n)$. The attempt to prove $T(n) \leq cn \lg n$ using substitution results in the condition $-cn + n \leq 0$. Explain what this condition implies about the relationship between c and n , and describe how one would typically proceed to find a suitable value for c .

Question 10: Using the recurrence tree method, analyze the recurrence $T(n) = T(n-1) + n$. Draw the recursion tree for the first three levels. Based on the tree, derive a possible upper bound for $T(n)$ in Big O notation.