

# **MEC511**

## **Thermodynamics & Fluids**

### **Chapter03**

## **Evaluating Properties**



**Lecture 10**

## **NOTE!**

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### 3.9 Introducing Specific Heat $C_v$ , and $C_p$

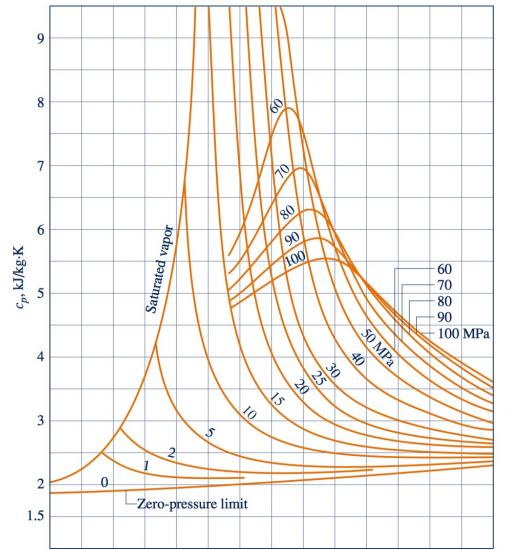
Two other quantities which are related to internal energy and enthalpy:

$$c_v = \left( \frac{\partial u}{\partial T} \right)_v$$

kJ/kg.K

$$c_p = \left( \frac{\partial h}{\partial T} \right)_p$$

Specific heat ratio ,  $k$        $k = \frac{c_p}{c_v}$



$c_p$  of water vapor as a function of temperature and pressure.

### 3.10 Evaluating Properties of Liquids and Solids

Special methods often can be used to evaluate properties of liquids and solids. These methods provide *simple, yet accurate, approximations* that do not require exact compilations like the compressed liquid tables for water, Tables A-5. Two such special methods are:

- 1- approximations using saturated liquid data
- 2- the incompressible substance model.

### 3.10.1 Approximations for Liquids Using Saturated Liquid Data

From Table A-5 it can be seen that specific volume and internal energy change very little with pressure at a fixed temperature.

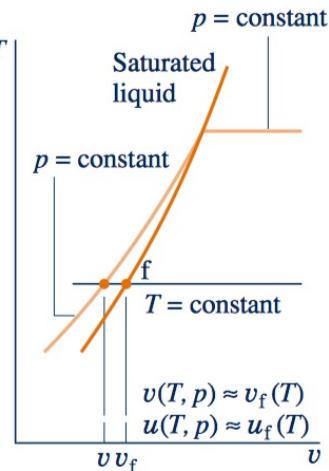
$$v(T, p) \approx v_f(T)$$

$$u(T, p) \approx u_f(T)$$

$$h(T, p) \approx u_f(T) + p v_f(T)$$

$$h(T, p) \approx h_f(T)$$

$$h(T, p) \approx h_f(T) + v_f(T)[p - p_{\text{sat}}(T)]$$



### 3.10.2 Incompressible Substance Model

To simplify evaluations involving liquids or solids, the specific volume (density) is often assumed to be constant

Also the specific internal energy assumed to vary only with temperature.

A substance idealized in this way is called *incompressible*.

For substances modeled as incompressible:

$V = \text{constant}$

$$U = u(T) \quad c_v(T) = \frac{du}{dT} \quad h(T, p) = u(T) + pV \implies \left. \frac{\partial h}{\partial T} \right)_p = \frac{du}{dT}$$

$$c_p = c_v \quad c_p = c_v = c \quad u_2 - u_1 = \int_{T_1}^{T_2} c(T) dT$$

$$h_2 - h_1 = u_2 - u_1 + v(p_2 - p_1) \\ = \int_{T_1}^{T_2} c(T) dT + v(p_2 - p_1)$$

$$u_2 - u_1 = c(T_2 - T_1)$$

$$h_2 - h_1 = c(T_2 - T_1) + v(p_2 - p_1)$$

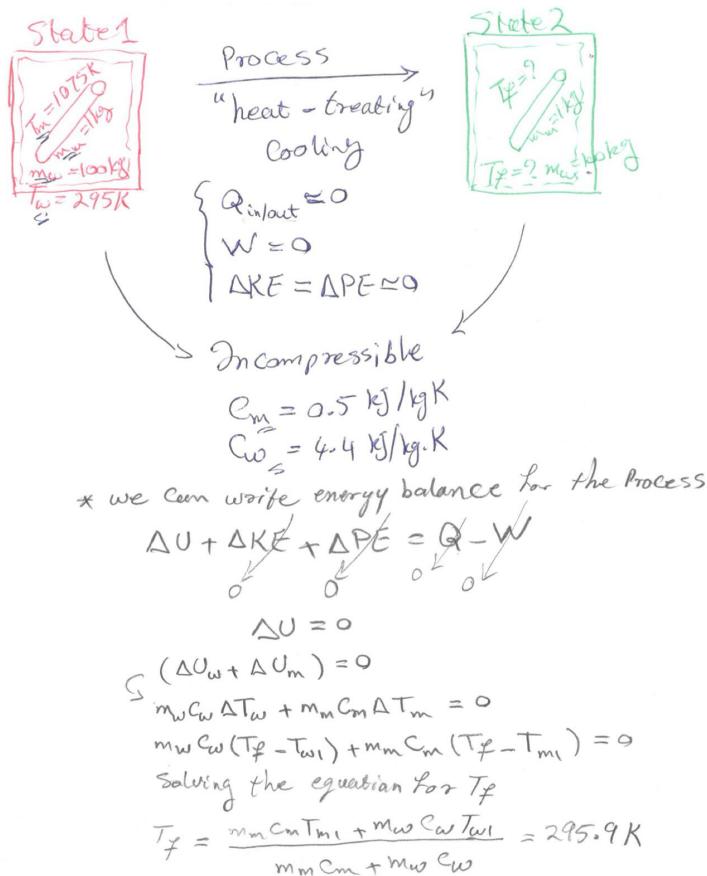
Small magnitude

- Over limited temperature intervals the variation of  $c$  with temperature can be small.
- In such instances, the specific heat  $c$  can be treated as constant without a serious loss of accuracy.

### Example 3.88:

In a heat-treating process, a 1-kg metal part, initially at 1075 K, is quenched in a closed tank containing 100 kg of water, initially at 295 K. There is negligible heat transfer between the contents of the tank and their surroundings. Modeling the metal part and water as incompressible with constant specific heats 0.5 kJ/kg.K and 4.4 kJ/kg.K, respectively, determine the final equilibrium temperature after quenching, in K.

### Solution:



### 3.12 Introducing the Ideal Gas Model

At states where the pressure  $p$  is small relative to the critical pressure  $p_c$  and/or the temperature  $T$  is large relative to the critical temperature  $T_c$ , the compressibility factor,  $Z = pv/RT$ , is approximately 1. At such states, we can assume with reasonable accuracy that  $Z = 1$ , or:

ideal gas model of state  $\downarrow$   $pv = RT$       the internal energy of air at low density (large specific volume) depends primarily on temperature.  $\rightarrow u = u(T)$

$$h = u(T) + pv \quad h = h(T) = u(T) + RT$$

Note:

- The ideal gas equation of state does not provide an acceptable approximation at all states.
- Accordingly, whether the ideal gas model is used depends on the error acceptable in a given calculation.
- To verify that a gas can be modeled as an ideal gas, the states of interest can be located on a compressibility chart to determine how well  $Z = 1$  is satisfied.
- Still, *gases often do approach ideal gas behavior*, and a particularly simplified description is obtained with the ideal gas model.

#### 3.13.1 Internal Energy, Enthalpy, and Specific Heats of Ideal Gases

For a gas obeying the ideal gas model, specific internal energy depends only on temperature.

$$c_v(T) = \frac{du}{dT} \quad du = c_v(T) dT$$

$$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T) dT$$

On integration, the change in specific internal energy is

### 3.13.1 Internal Energy, Enthalpy, and Specific Heats of Ideal Gases

Similarly, for a gas obeying the ideal gas model, the specific enthalpy depends only on temperature:

On integration, the change in specific enthalpy is:

$$c_p(T) = \frac{dh}{dT} \quad dh = c_p(T) dT$$
$$h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T) dT$$

- An important relationship between the ideal gas specific heats can be developed by differentiating ideal gas model  $h = h(T) = u(T) + RT$  with respect to temperature

$$\frac{dh}{dT} = \frac{du}{dT} + R \quad c_p(T) - c_v(T) = R$$

Specific heat ratio  $k = \frac{c_p(T)}{c_v(T)}$

Since  $c_p > c_v$ , it follows that  $k > 1$ .

$$c_p(T) = \frac{kR}{k-1}$$
$$c_v(T) = \frac{R}{k-1}$$

(Ideal gas)

### 3.14.2 Applying the Energy Balance Using Constant Specific Heat

In equations;

$$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T) dT \quad (\text{ideal gas}) \quad \text{and} \quad h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T) dT \quad (\text{ideal gas})$$

when the specific heats are taken as constants, these equations reduce, respectively, to

$$u(T_2) - u(T_1) = c_v (T_2 - T_1)$$

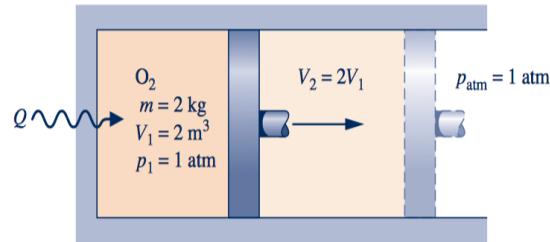
$$h(T_2) - h(T_1) = c_p (T_2 - T_1)$$

And, mean values of  $c_v$  and  $c_p$  are calculated as:

$$c_v = \frac{\int_{T_1}^{T_2} c_v(T) dT}{T_2 - T_1} \quad c_p = \frac{\int_{T_1}^{T_2} c_p(T) dT}{T_2 - T_1}$$

### 3.14.2 Applying the Energy Balance Using Constant Specific Heat

**Example 3.116:** Two kg of oxygen fills the cylinder of a piston–cylinder assembly. The initial volume and pressure are  $2 \text{ m}^3$  and 1 bar, respectively. Heat transfer to the oxygen occurs at constant pressure until the volume is doubled. Determine the heat transfer for the process, in kJ, assuming the specific heat ratio is constant,  $k=1.35$ . Kinetic and potential energy effects can be ignored.



**Solution:**

$\text{State 1}$ $O_2 \text{ gas}$ $P_1 = 1 \text{ bar}$ $V_1 = 2 \text{ m}^3$ $T_1 = ?$	$\xrightarrow{\text{Process}}$ "Const. Pressure" $P_1 = P_2 = \text{Const}$ $\Delta P/E = \Delta K/E \approx 0$ $m = \text{Const} = 2 \text{ kg}$ $\hookrightarrow \text{Closed System}$	$\text{State 2}$ $O_2 \text{ gas}$ $P_2 = P_1 = 1 \text{ bar}$ $V_2 = 2V_1 = 4 \text{ m}^3$ $T_2 = ?$
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$$K = 1.35 \text{ Const} \quad \left. \begin{array}{l} \Rightarrow C_V \text{ and } C_P = \text{Const} \\ \text{"Ideal gas"} \end{array} \right\}$$

\* writing energy balance.

$$\Delta U + \Delta P/E + \Delta K/E = Q - W$$

$$m(u_2 - u_1) = Q - \int_{V_1}^{V_2} P dV$$

$$mC_V(T_2 - T_1) + P(V_2 - V_1) = Q \quad (*)$$

\* To find  $T_2$  and  $T_1$ , we use ideal gas eq'n

$$P_1 V_1 = n R T_1 \rightarrow T_1 = \frac{P_1 V_1}{m \left( \frac{R}{M_{O_2}} \right)} \quad (M_{O_2} = 32 \text{ g/mol})$$

$$T_1 = \frac{(10^5 \text{ Pa})(2 \text{ m}^3)}{(2 \text{ kg}) \left( \frac{8.314 \text{ J/mol K}}{32 \times 10^3 \text{ kg/mol}} \right)} = 384.9 \text{ K}$$

to find  $T_2$  we can write

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow T_2 = \left( \frac{V_2}{V_1} \right) T_1 = 769.8 \text{ K}$$

to find  $C_V$  we can write

$$C_V = \frac{R}{K-1} = \frac{\bar{R}/M_{O_2}}{K-1} = 0.742 \text{ kJ/kg.K}$$

now plug in all values in eq'n (x)

$$\rightarrow Q = (2 \text{ kg}) (0.742 \times 10^3 \text{ J/kg.K}) (769.8 - 384.9 \text{ K}) +$$

$$(10^5 \text{ Pa})(4 - 2 \text{ m}^3) = + 771.2 \underbrace{\text{kJ}}_{\text{Q}_{\text{in}}} \quad \underline{\underline{}}$$

### 3.15 Polytropic Process Relation

A polytropic process is a quasiequilibrium process described by:

$$pV^n = \text{constant} \quad p_1 V_1^n = p_2 V_2^n \quad \frac{p_2}{p_1} = \left( \frac{V_1}{V_2} \right)^n$$

$$\int_1^2 p dV = \frac{p_2 V_2 - p_1 V_1}{1-n} \quad (n \neq 1) \quad \int_1^2 p dV = p_1 V_1 \ln \frac{V_2}{V_1} \quad (n = 1)$$

When the additional idealization of ideal gas behavior is appropriate, further relations can be derived. Thus, when the ideal gas equation of state ( $pV=RT$ ) is introduced into the above equations, the following expressions are obtained,

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{(n-1)/n} = \left( \frac{V_1}{V_2} \right)^{n-1} \quad \int_1^2 p dV = \frac{mR(T_2 - T_1)}{1-n} \quad (n \neq 1)$$

$$\int_1^2 p dV = mRT \ln \frac{V_2}{V_1} \quad (n = 1)$$

- When  $n=0$ , the process is an isobaric (constant-pressure) process
- when  $n=\pm\infty$  the process is an isometric (constant-volume) process.
- For an ideal gas, the case  $n=1$  corresponds to an isothermal (constant-temperature) process

# **MEC511**

## **Thermodynamics & Fluids**

### **Chapter04**

# **Control Volume Analysis using Energy**



## **4 Control volume analysis using energy**

- 4.1 Conservation of mass for a control volume**
- 4.2 Forms of the mass rate balance**
- 4.4 Conservation of energy for a control volume**
- 4.5 Analyzing control volumes at steady state**
- 4.6 Nozzles and diffusers**
- 4.7 Turbines**
- 4.8 Compressors and Pumps**
- 4.9 Heat Exchangers**
- 4.10 Throttling devices**

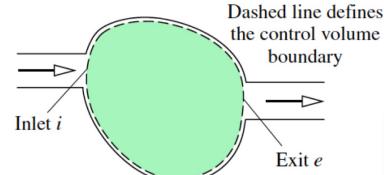
## 4.1 Conservation of mass for a control volume

The mass rate balance for control volumes is introduced by reference to the below figure:

The **conservation of mass principle** states:

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

Mass flow rates



$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

In general, there may be several locations on the boundary through which mass enters or exits.

$$\left[ \begin{array}{l} \text{time rate of change of} \\ \text{mass contained within the} \\ \text{control volume at time } t \end{array} \right] = \left[ \begin{array}{l} \text{time rate of flow of} \\ \text{mass in across} \\ \text{inlet } i \text{ at time } t \end{array} \right] - \left[ \begin{array}{l} \text{time rate of flow} \\ \text{of mass out across} \\ \text{exit } e \text{ at time } t \end{array} \right]$$

## 4.2 Forms of the mass rate balance

When a flowing stream of matter entering or exiting a control volume adheres to idealizations in which the flow is said to be one-dimensional:

$$\dot{m} = \rho V A$$

Density [kg/m<sup>3</sup>]  
Surface area [m<sup>2</sup>]  
Fluid velocity [m/s]  
Mass flow rate [kg/s]

$$\dot{m} = \frac{AV}{v}$$

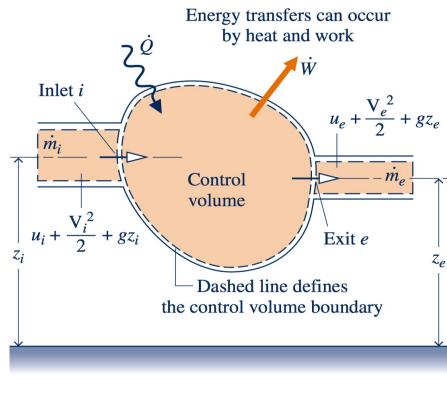
Volumetric flow rate, m<sup>3</sup>/s  
Specific volume, m<sup>3</sup>/kg

$$\frac{dm_{cv}}{dt} = \sum_i \frac{A_i V_i}{v_i} - \sum_e \frac{A_e V_e}{v_e}$$

For steady-state case

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

## 4.4 Conservation of energy for a control volume



1- work associated with the fluid pressure as mass is introduced at inlets and removed at exits.

2- The other contribution includes all other work effects, such as those associated with rotating shafts, displacement of the boundary, and electrical effects.

The energy rate balance equation:

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \dot{m}_i \left( u_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left( u_e + \frac{V_e^2}{2} + gz_e \right)$$

$$\begin{bmatrix} \text{time rate of change} \\ \text{of the energy} \\ \text{contained within} \\ \text{the control volume} \\ \text{at time } t \end{bmatrix} = \begin{bmatrix} \text{net rate at which} \\ \text{energy is being} \\ \text{transferred in} \\ \text{by heat at} \\ \text{time } t \end{bmatrix} - \begin{bmatrix} \text{net rate at which} \\ \text{energy is being} \\ \text{transferred out} \\ \text{by work at} \\ \text{time } t \end{bmatrix} + \begin{bmatrix} \text{net rate of energy} \\ \text{transfer into the} \\ \text{control volume} \\ \text{accompanying} \\ \text{mass flow} \end{bmatrix}$$

### 4.4.2 Evaluating Work for a Control Volume

It is convenient to separate the work term  $\dot{W}$  in the former equation into two contributions:

- One contribution the work associated with the fluid pressure as mass is introduced at inlets and removed at exits.
- The other contribution, denoted by  $\dot{W}_{cv}$ , includes all other work effects, such as those associated with rotating shafts, displacement of the boundary, and electrical effects.

$$\begin{bmatrix} \text{time rate of energy transfer} \\ \text{by work from the control} \\ \text{volume at exit } e \end{bmatrix} = (p_e A_e) V_e$$

$$\dot{W} = \dot{W}_{cv} + (p_e A_e) V_e - (p_i A_i) V_i \quad \text{Also; } \mathbf{A} \mathbf{V} = \dot{m} \mathbf{v}$$

$$\dot{W} = \dot{W}_{cv} + \dot{m}_e (p_e v_e) - \dot{m}_i (p_i v_i) \quad \text{Substituting in the energy rate balance:}$$

*Flow Work*

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i \left( u_i + p_i v_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left( u_e + p_e v_e + \frac{V_e^2}{2} + gz_e \right)$$

## 4.4.2 Evaluating Work for a Control Volume

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i \left( u_i + p_i v_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left( u_e + p_e v_e + \frac{V_e^2}{2} + gz_e \right)$$

$$h = u + pv$$

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)$$

In practice there may be several locations on the boundary through which mass enters or exits. So the energy rate balance is:

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)$$

- The above equation is an accounting balance for the energy of the control volume.
- It states that the rate of energy increase or decrease within the control volume equals the difference between the rates of energy transfer in and out across the boundary.

## 4.5 Analyzing control volumes at steady state

For a control volume at steady state, the conditions of the mass within the control volume and at the boundary do not vary with time. So,

$$dm_{cv}/dt = 0$$

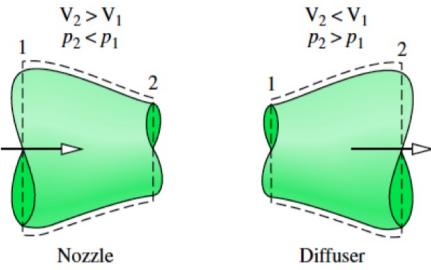
$$\begin{aligned} \sum_i \dot{m}_i &= \sum_e \dot{m}_e \\ 0 &= \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) \\ \dot{Q}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) &= \dot{W}_{cv} + \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) \\ \dot{m}_1 &= \dot{m}_2 \quad 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left( (h_1 - h_2) + \left( \frac{V_1^2}{2} - \frac{V_2^2}{2} \right) + g(z_1 - z_2) \right) \end{aligned}$$

So, the energy rate balance is:

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \left( \frac{V_1^2}{2} - \frac{V_2^2}{2} \right) + g(z_1 - z_2)$$

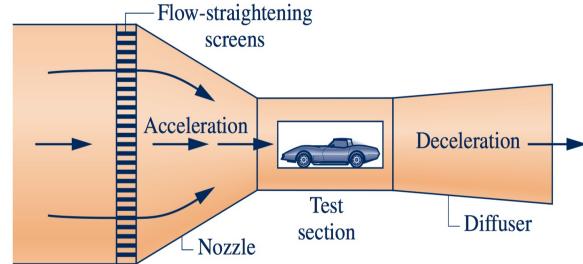
## 4.6 Nozzles and diffusers

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left( (h_1 - h_2) + \left( \frac{V_1^2}{2} - \frac{V_2^2}{2} \right) + g(z_1 - z_2) \right) \quad = 0$$



$$0 = \dot{Q}_{cv} + \dot{m} \left( (h_1 - h_2) + \left( \frac{V_1^2}{2} - \frac{V_2^2}{2} \right) \right)$$

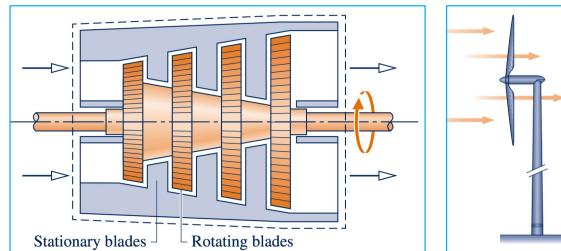
$$0 = (h_1 - h_2) + \left( \frac{V_1^2}{2} - \frac{V_2^2}{2} \right)$$



Wind-tunnel test facility.

## 4.7 Turbines

A **turbine** is a device in which power is developed as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate.



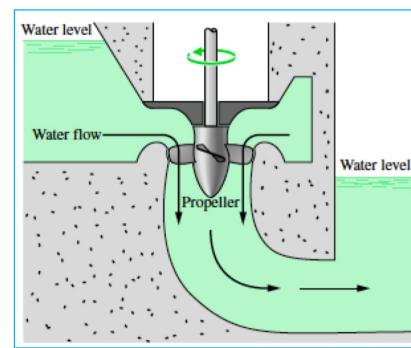
$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left( (h_1 - h_2) + \left( \frac{V_1^2}{2} - \frac{V_2^2}{2} \right) + g(z_1 - z_2) \right)$$

Negligible

$\dot{Q}_{cv}$  The heat transfer between turbine and environment is small

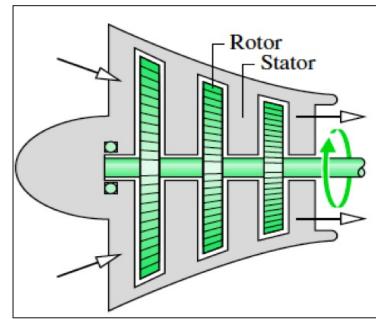
$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$



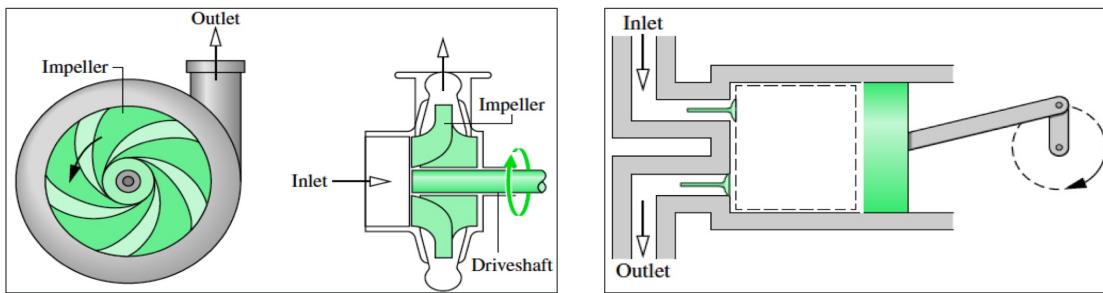
## 4.8 Compressors and Pumps

Compressors are devices in which work is done on a gas passing through them in order to raise the pressure. In pumps, the work input is used to change the state of a liquid passing through.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$



$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$



## 4.9 Heat Exchangers

*Heat exchangers transfer energy between fluids at different temperatures by heat transfer modes.*

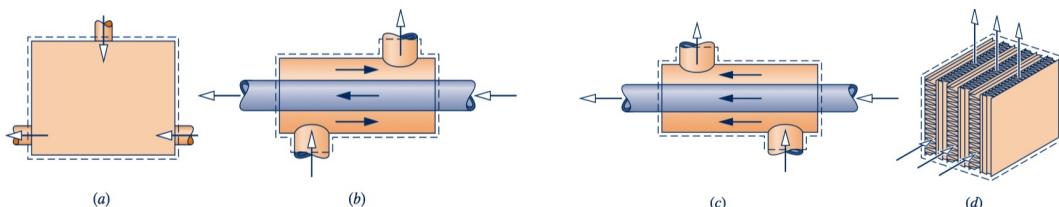
For a control volume enclosing a heat exchanger;

- the only work is flow work at the places where matter enters and exits, so the term  $\dot{W}_{cv}$  drops out of the energy rate balance.
- In addition, the kinetic and potential energies of the flowing streams usually can be ignored at the inlets and exits.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)$$

$= 0$       Negligible

$$0 = \dot{Q}_{cv} + \sum_i \dot{m}_i h_i - \sum_e \dot{m}_e h_e$$



(a) Direct-contact heat exchanger. (b) Tube-within-a-tube counterflow heat exchanger. (c) Tube-within-a-tube parallel-flow heat exchanger. (d) Cross-flow heat exchanger.

## 4.10 Throttling devices

A significant reduction in pressure can be achieved simply by introducing a restriction into a line through which a gas or liquid flows. This is commonly done by means of a partially opened valve or a porous plug.

For a control volume enclosing a throttling device,

- the only work is flow work at locations where mass enters and exits the control volume.
- the term  $\dot{W}_{cv}$  drops out of the energy rate balance.
- there is usually no significant heat transfer with the surroundings.
- the change in potential energy from inlet to exit is negligible.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left( (h_1 - h_2) + \left( \frac{V_1^2}{2} - \frac{V_2^2}{2} \right) + g(z_1 - z_2) \right)$$

$$0 = (h_1 - h_2) + \left( \frac{V_1^2}{2} - \frac{V_2^2}{2} \right)$$

Although velocities may be relatively high in the vicinity of the restriction imposed by the throttling device on the flow through it, measurements show in most cases that the change in the specific kinetic energy of the flowing substance between these locations can be neglected.

$$h_1 = h_2 \quad p_2 < p_1$$

