PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

Algorithm Exam - Medium Difficulty

Instructions:

Answer all questions to the best of your ability. Show your work where applicable.

Section 1: Multiple Choice (4 points each, 40 points total)

Instructions: Choose the *one* best answer for each question.

Question 1: What is the purpose of the `key` variable in the provided INSERTION-SORT algorithm?
A) To store the index of the element being inserted.
B) To temporarily hold the value of the element being inserted.
C) To determine the size of the array being sorted.
D) To mark elements that have already been sorted.

Question 2: In the Bellman-Ford algorithm example, what does `d[v]` represent?
A) The predecessor of vertex `v` in the shortest path.
B) The distance of vertex `v` from the starting vertex `S`.
C) The weight of the edge connecting vertex `v` to the next vertex.
D) The maximum distance from vertex `v` to any other vertex in the graph.

Question 3: Based on the provided merge sort steps, what is the primary operation during the merging of two sorted arrays?
A) Swapping elements within the same array.
B) Selecting the smaller element from the two arrays and placing it in the correct position.
C) Dividing the arrays into smaller sub-arrays.
D) Comparing the lengths of the two arrays.

Question 4: In the recursion tree examples, what is the purpose of the "Guess" when analyzing the time complexity T(n)?
A) To define the base case of the recursion.
B) To estimate the upper or lower bound of the time complexity.
C) To determine the exact number of recursive calls.
D) To simplify the calculation of the constant factors in the time complexity.

Question 5: Based on the provided code for INSERTION-SORT, what is the correct placement of the key?

A) A[i] <- key
B) A[i+1] <- A[i]
C) A[i+1] <- key
D) key <- A[i+1]

Section 2: Short Answer (6 points each, 30 points total)

Instructions: Answer each question in 2-3 complete sentences.

Question 6: Briefly explain how the `while` loop in the INSERTION-SORT algorithm works to find the correct position for the `key`.

Question 7: What is the significance of relaxing edges in the Bellman-Ford algorithm, and how does it help in finding the shortest path?

Question 8: In the context of recursion trees, explain the difference between finding an upper bound and a lower bound for T(n).

Section 3: Problem-Solving (10 points each, 30 points total)

Instructions: Provide detailed solutions to the following problems, showing all steps.

Question 9: Using the provided INSERTION-SORT algorithm, trace the execution for the input array: `[5, 1, 4, 2, 8]`. Show the array's state after each iteration of the outer loop (the `for j` loop).

Question 10: Given the recurrence relation $T(n) = T(n-1) + \Theta(1)$ (similar to the assignment recursion tree example), prove by substitution that $T(n) = O(n)$. Explicitly show your guess, substitution, and inductive step.