

PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

Algorithms and Data Structures Exam

Instructions: Answer all questions to the best of your ability.

Section 1: Multiple Choice (40 points total, 10 points per question)

Instructions: Choose the best answer for each of the following questions.

Question 1: In the provided INSERTION-SORT algorithm, what is the purpose of the 'key' variable?

- A) To store the index of the current element being compared.
- B) To store the value of the current element being inserted into its correct position.
- C) To store the length of the array being sorted.
- D) To track the number of comparisons made during sorting.

Question 2: Based on the example provided for merging two sorted arrays, what is the key concept behind merging?

- A) Randomly assigning elements to a new array.
- B) Comparing elements from both arrays and placing the smaller element into the merged array.
- C) Sorting each array separately before concatenating them.
- D) Reversing the order of elements in one of the arrays.

Question 3: In the recurrence relation $T(n) = 2T(n/2) + \Theta(n)$, what does $\Theta(n)$ typically represent?

- A) The time complexity of dividing the problem into subproblems.
- B) The time complexity of the base case.
- C) The time complexity of combining the solutions of subproblems.
- D) The space complexity of the algorithm.

Question 4: In the INSERTION-SORT algorithm provided, what does the `while i > 0 and A[i] > key` condition check?

- A) If the index `i` is within the array bounds and the element at index `i` is smaller than the `key`.
- B) If the index `i` is within the array bounds and the element at index `i` is greater than the `key`.
- C) If the index `i` is equal to 0 and the element at index `i` is greater than the `key`.
- D) If the array `A` is not empty and the `key` is positive.

Section 2: Short Answer (30 points total, 15 points per question)

Instructions: Answer the following questions in 2-3 sentences each.

Question 5: Explain the role of the inner `while` loop in the provided INSERTION-SORT algorithm.

Question 6: Briefly describe the relationship between a recursion tree and the analysis of a recursive algorithm's time complexity.

Section 3: Problem Solving (30 points total, 15 points per question)

Instructions: Provide detailed explanations and steps for solving the following problems.

Question 7: Using the given INSERTION-SORT algorithm, trace the steps to sort the following array: [5, 1, 4, 2, 8]. Show the array's state after each iteration of the outer `for` loop.

Question 8: Given the recurrence relation $T(n) = T(n-1) + \Theta(n)$, and the guess $T(n) \leq cn^2$, explain how you would prove this upper bound using the substitution method (as illustrated in the provided text). Show the steps to determine a suitable value for the constant 'c'.