PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

Algorithm Exam

Instructions: Please answer all questions to the best of your ability.

Section 1: Multiple Choice (4 points each, 40 points total)

Instructions: Choose the best answer for each question.

Question 1: The provided text contains examples of merging sorted arrays. Which of the following accurately describes the core operation in merging two sorted arrays?
A) Appending the arrays directly.
B) Interleaving elements based on their original position.
C) Comparing elements from both arrays and placing the smaller one into a new array.
D) Reversing the order of the arrays before combining them.

Question 2: In the provided Insertion Sort example, what is the purpose of the 'key' variable?
A) To store the index of the element being inserted.
B) To store the value of the element being inserted.
C) To store the index of the element being compared to.
D) To store the value of the element being compared to.

Question 3: The recurrence relation $T(n) = T(n-1) + \lg n$, with $T(1) = \Theta(1)$, has a time complexity of $O(n \lg n)$. Which method is demonstrated to prove this complexity?
A) Divide and Conquer.
B) Substitution Method and Recursion Tree Method.
C) Dynamic Programming.
D) Greedy Algorithm.

Question 4: According to the text, which of the following is the relationship between $\lg n!$ and $n \lg n$?
A) $\lg n! > n \lg n$
B) $\lg n! = n \lg n$
C) $\lg n! < n \lg n$
D) $\lg n!$ is not comparable to $n \lg n$

Section 2: Short Answer (6 points each, 30 points total)

Instructions: Answer the following questions in 2-3 sentences.

Question 5: Briefly explain the main idea behind the Substitution Method for solving recurrence relations.

Question 6: Explain how the 'while' loop contributes to the functionality of the insertion sort algorithm provided in the text.

Question 7: What does T(n) represent in the context of the given recurrence relations?

Section 3: Problem Solving (10 points each, 30 points total)

Instructions: Provide a detailed analysis to solve the following problems. Show your work.

Question 8: Consider the recurrence relation $T(n) = 2T(n/2) + \Theta(n)$. Based on the text example, prove whether $T(n) \leq cn \log n$ holds true as an upper bound. Show all steps and clearly indicate the conditions required for the proof to be valid.

Question 9: Using the provided information about insertion sort, trace the execution of the algorithm on the following unsorted array: [5, 2, 8, 1, 9]. Show the array's contents after each iteration of the outer loop (for $j \leftarrow 2$ to n).

Question 10: Suppose you need to merge these two sorted arrays: Array1 = [1, 3, 5, 7] and Array2 = [2, 4, 6, 8]. Apply the merging logic as described in the text. What is the final merged and sorted array? Briefly explain the steps you took.