

# PRACTICE EXAM

**Difficulty: MEDIUM**

**Questions: 10**

# Algorithms and Data Structures Exam

**Instructions: Please answer all questions to the best of your ability.**

## **# Section 1: Multiple Choice Questions (4 points each, 40 points total)**

Instructions: Choose the *\*one\** best answer for each question.

**Question 1:** Which line of code in the provided INSERTION-SORT algorithm is responsible for placing the 'key' element in its correct sorted position?

- A) ``for j ← 2 to n``
- B) ``key ← A[ j]``
- C) ``while i > 0 and A[i] > key``
- D) ``A[i+1] = key``

**Question 2:** In the INSERTION-SORT algorithm, what is the purpose of the 'i' variable?

- A) It represents the index of the element being inserted.
- B) It represents the index of the 'key' element.
- C) It iterates through the unsorted portion of the array.
- D) It iterates backwards through the sorted portion of the array, comparing elements to the 'key'.

**Question 3:** Based on the recurrence relation  $T(n) = 2T(n/2) + \Theta(n)$ , and the provided guess  $T(n) \leq cn \lg n$ , what are we trying to prove?

- A) A lower bound for  $T(n)$
- B) An upper bound for  $T(n)$
- C) That  $T(n)$  is always positive.
- D) The exact solution for  $T(n)$ .

**Question 4:** According to the provided information on merging two sorted arrays, what is the first step in the process?

- A) Dividing each array into smaller sub-arrays.
- B) Sorting each array individually.
- C) Comparing the first elements of each array.
- D) Combining the two arrays into one.

## # Section 2: Short Answer Questions (6 points each, 30 points total)

Instructions: Answer each question in 2-3 complete sentences.

**Question 5:** Briefly explain the role of the 'key' variable in the INSERTION-SORT algorithm.

**Question 6:** What is the purpose of the `while` loop in the provided INSERTION-SORT algorithm?

**Question 7:** Explain in your own words why we are trying to prove  $1 \leq c$  in the “Upper bound” portion of the recurrence relation example.

## # Section 3: Problem-Solving Questions (10 points each, 30 points total)

Instructions: Provide a detailed solution for each problem. Show your work where applicable.

**Question 8:** Using the provided INSERTION-SORT algorithm, trace the execution of the algorithm on the following array: `[5, 1, 4, 2, 8]`. Show the array's contents after each iteration of the outer `for` loop (each time `j` increments).

**Question 9:** The document shows the first few steps of merging two sorted arrays. Given arrays  $A = [1, 3, 5]$  and  $B = [2, 4, 6]$ , show the steps of merging the two arrays until the merged array is complete.

**Question 10:** Consider the recurrence relation  $T(n) = T(n-1) + \Theta(n)$ . The provided text attempts to prove  $T(n) \leq n^2$ . Explain what the result  $1 \leq c$  means in the context of trying to prove the upper bound.