

PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

Algorithm Analysis Exam

Instructions: Please answer all questions to the best of your ability. Show your work where applicable.

Section 1: Multiple Choice (4 points each, 40 points total)

Instructions: Choose the best answer for each question and clearly indicate your choice (A, B, C, or D).

Question 1: What algorithmic concept is primarily illustrated by the provided examples involving the insertion of elements into a sorted array?

- A) Divide and Conquer
- B) Dynamic Programming
- C) Greedy Algorithms
- D) Insertion Sort

Question 2: According to the provided text, what is the algorithmic complexity of $T(n) = \begin{cases} 1 & \text{if } n = 1; \\ T(n-1) + \lg n & \text{if } n > 1 \end{cases}$?

- A) $O(\lg n)$
- B) $O(n)$
- C) $O(n \lg n)$
- D) $O(n^2)$

Question 3: In the provided insertion sort example, what is the purpose of the 'key' variable?

- A) It stores the index of the element being inserted.
- B) It stores the value of the element being inserted.
- C) It represents the number of comparisons made.
- D) It holds the final sorted array.

Question 4: Which method is demonstrated for determining the complexity of a recurrence relation, other than the substitution method?

- A) Divide and Conquer
- B) Recursion Tree Method
- C) Dynamic Programming
- D) Greedy Algorithms

Section 2: Short Answer (6 points each, 30 points total)

Instructions: Answer each question concisely in 2-3 sentences.

Question 5: Briefly explain how the Substitution Method is used to find the complexity of a recursive algorithm.

Question 6: Explain, in your own words, what is meant by "merging two sorted arrays," as it relates to algorithmic efficiency.

Question 7: Describe the difference between finding an upper bound and a lower bound when analyzing the complexity of an algorithm.

Section 3: Problem Solving (10 points each, 30 points total)

Instructions: Provide detailed solutions, showing all steps of your reasoning.

Question 8: Given the recurrence relation $T(n) = 2T(n/2) + \Theta(n)$, and the guess $T(n) \leq cn \lg n$, demonstrate how you would verify the upper bound using substitution, including a step indicating the value of n_0 where the condition holds true.

Question 9: Using the provided example of insertion sort, show the state of the array [9, 5, 1, 4, 3] after the second iteration ($j=3$) of the outer loop. Clearly indicate the values of 'key' and 'i' during the comparison.

Question 10: $T(n) = T(n-1) + \Theta(n)$, and the Guess: $T(n) \leq n^2$
Upper bound: Solve for c .