PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

Data Structures and Algorithms Exam

Instructions:

Answer all questions to the best of your ability. Show your work for problem-solving questions to receive partial credit.

Multiple Choice Questions (4 points each, 40 points total)

Question 1: Which of the following best describes the "Merging two sorted arrays" operation illustrated in the provided data?
A) Arranging the elements of two arrays from smallest to largest into a single array.
B) Finding the common elements between two arrays.
C) Separating elements in an array based on their value.
D) Reversing the order of elements in two arrays.

Question 2: Based on the given text, what does $T(n)$ represent in the recurrence relations?
A) The total number of elements in an array.
B) The time complexity of an algorithm for an input size of n.
C) The space complexity of an algorithm.
D) The time taken to execute a specific operation.

Question 3: According to the examples of Insertion Sort, what is the purpose of the 'key' variable?
A) It holds the index of the element being inserted.
B) It temporarily stores the value of the element being inserted into the sorted portion of the array.
C) It indicates the size of the array.
D) It represents the largest element in the array.

Question 4: In the Bellman-Ford algorithm example, what does $d[v]$ represent?
A) The predecessor of vertex v in the shortest path.
B) The distance from the source vertex to vertex v.
C) The weight of the edge connecting to vertex v.
D) The number of vertices reachable from vertex v.

Short Answer Questions (6 points each, 30 points total)

Question 5: Explain in 2-3 sentences how the 'guess' is verified in the upper bound calculation of T(n) = 2T(n/2) + Θ(n).

Question 6: Describe in 2-3 sentences how the Insertion Sort algorithm works.

Question 7: In the context of the Bellman-Ford algorithm, what is a "relaxed" vertex and why is it important for the algorithm's function?

Problem-Solving Questions (10 points each, 30 points total)

Question 8: Given the initial unsorted array: `[9, 5, 1, 4, 3]`, trace the steps of the Insertion Sort algorithm as demonstrated in the examples, showing the array's state after each iteration of the outer loop (for loop).

Question 9: Using the recurrence relation T(n) = T(n-1) + Θ(n), and following the example provided, show the steps to verify if T(n) $\leq$ n^2 is an upper bound. Clearly indicate the condition that needs to be satisfied.

Question 10: Consider a simplified graph with vertices S, A, and B. The edges are: S->A with weight 2, S->B with weight 5, and B->A with weight -3. Apply the Bellman-Ford algorithm (one iteration only) starting from source S. Show the d[v] and P[v] values for each vertex after this single iteration. Initialize d[S]=0 and d[A] = d[B] = infinity; P[v] = null.