

PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

Data Structures and Algorithms Exam

Instructions:

Please answer all questions to the best of your ability. Show your work for problem-solving questions to receive partial credit.

Section 1: Multiple Choice Questions (4 points each, 40 points total)

Instructions: Choose the best answer for each question.

Question 1: What is the time complexity of the recurrence relation $T(n) = 2T(n/2) + \Theta(n)$?

- A) $\Theta(n)$
- B) $\Theta(n^2)$
- C) $\Theta(n \log n)$
- D) $\Theta(2^n)$

Question 2: Based on the content, which algorithm is demonstrated in the later section of the provided text?

- A) Merge Sort
- B) Quick Sort
- C) Insertion Sort
- D) Bubble Sort

Question 3: In the Bellman-Ford algorithm example, after selecting vertex A, what is the updated distance $d[v]$ for vertex D?

- A) 5
- B) 6
- C) 13
- D) Infinity

Question 4: Regarding $T(n) = 2T(n/2) + \Theta(n)$, which condition must be met for the upper bound proof $T(n) \leq cn \lg n$ to hold true?

- A) $c > 0$
- B) $c < 0$
- C) $c \geq 1$
- D) $c \leq 1$

Question 5: What is the purpose of relaxing an edge in the Bellman-Ford algorithm?

- A) To find a negative weight cycle.
- B) To update the shortest path estimate between two vertices.
- C) To initialize the distances of all vertices.
- D) To sort the edges in the graph.

Section 2: Short Answer Questions (6 points each, 30 points total)

Instructions: Answer each question in 2-3 complete sentences.

Question 6: Briefly explain what is meant by "Merging two sorted arrays" in the context of the provided document.

Question 7: In the context of Insertion Sort, what is the 'key' variable, and what role does it play in the algorithm?

Question 8: Explain the significance of n_0 in the context of proving upper or lower bounds for recurrence relations.

Section 3: Problem-Solving Questions (10 points each, 30 points total)

Instructions: Provide detailed solutions, showing all necessary steps.

Question 9: Using the provided recurrence relation analysis, determine the range of values for 'c' that would satisfy the lower bound proof $T(n) \geq cn \lg n$ for $T(n) = 2T(n/2) + \Theta(n)$. Justify your answer.

Question 10: Trace the first two iterations ($j=2$ and $j=3$) of the Insertion Sort algorithm on the array $A = [9, 1, 5, 3, 7, 2]$, showing the state of the array and the key variables after each step. Be sure to clearly indicate array indices and values.