PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

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# Algorithms and Data Structures Exam

## Section 1: Multiple Choice Questions (4 points each, 40 points total)

Instructions: Choose the *best* answer for each question.

**Question 1:** What is the purpose of the `key` variable in the provided INSERTION-SORT algorithm?
A) To store the index of the current element being compared.
B) To store the value of the current element being inserted into the sorted portion of the array.
C) To store the index of the smallest element in the unsorted portion of the array.
D) To store the size of the array being sorted.

**Question 2:** In the INSERTION-SORT algorithm, when does the `while` loop terminate?
A) When `i` is equal to 0 or `A[i]` is less than `key`.
B) When `i` is less than 0 or `A[i]` is greater than or equal to `key`.
C) When `i` is equal to 0 or `A[i]` is greater than `key`.
D) When `i` is greater than 0 or `A[i]` is less than or equal to `key`.

**Question 3:** According to the merging sorted arrays example, what numbers appear on row 55?
A) 1, 2, 7, 9, 11, 12, 20
B) 1, 2, 7, 9, 11, 20, 12
C) 1, 2, 7, 9, 11, 12
D) 1, 2, 7, 9, 11

**Question 4:** In the recurrence relation `T(n) = 2T(n/2) + Θ(n)`, what does `Θ(n)` represent?
A) The time complexity of dividing the problem into subproblems.
B) The time complexity of solving the subproblems recursively.
C) The time complexity of combining the solutions of the subproblems.
D) The overall time complexity of the algorithm.

## Section 2: Short Answer Questions (6 points each, 30 points total)

Instructions: Answer each question in 2-3 sentences.

**Question 5:** Briefly describe the role of the variable `i` in the INSERTION-SORT algorithm, focusing on its purpose within the `while` loop.
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Question 6: In the context of the INSERTION-SORT algorithm, explain why we iterate from `j = 2` to `n` instead of `j = 1` to `n`.

Question 7: What is the general strategy behind merging two sorted arrays into a single sorted array?

Section 3: Problem-Solving Questions (10 points each, 30 points total)

Instructions: Provide a detailed analysis and explanation for each problem.

Question 8: Trace the execution of the first two iterations (`j = 2` and `j = 3`) of the INSERTION-SORT algorithm on the following array: `[5, 1, 4, 2, 8]`. Show the state of the array after each step (i.e., after each assignment or comparison).

Question 9: The recurrence relation `T(n) = T(n-1) + Θ(n)` has an upper bound of `T(n) ≤ n^2`. Briefly explain how you would show that using the substitution method with appropriate substitution, assumption and proof by induction.

Question 10: Consider the array `[10, 5, 8, 2, 1]`. After two complete iterations of the outer loop in the INSERTION-SORT algorithm (i.e., when `j = 3`), what would the array look like? Show each iteration.
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