PRACTICE EXAM

Difficulty: MEDIUM

Questions: 10

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# Algorithms and Data Structures Exam
Instructions: Please answer all questions to the best of your ability.
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Section 1: Multiple Choice Questions (4 points each, 40 points total)

Instructions: Choose the best answer for each question.

Question 1: What is the purpose of the `key` variable in the INSERTION-SORT algorithm?
A) To store the index of the element being inserted.
B) To store the value of the element being inserted.
C) To store the index of the element being compared.
D) To store the value of the element being compared.

Question 2: In the INSERTION-SORT algorithm, the `while` loop continues as long as which condition is true?
A) `i > 0` and `A[i] < key`
B) `i < n` and `A[i] > key`
C) `i > 0` and `A[i] > key`
D) `i < n` and `A[i] < key`

Question 3: Based on the provided materials, the equation 'T (n) = 2T (n/2) + ■(n)' is related to what concept?
A) Insertion Sort best case runtime.
B) Merging two sorted arrays.
C) Recursion tree runtime analysis.
D) Assignment operations within a loop.

Question 4: If T(n) = T(n-1) + ■(n) and we are trying to prove that T(n) ≤ n^2, what step is involved in the inductive proof?
A) Proving the base case for n = 0.
B) Substituting c(n-1)^2 + n into the equation T(n).
C) Showing that n(1-2c) + c ≥ 0
D) Showing that T(n) = T(n/2) + ■(n^2).

Section 2: Short Answer Questions (6 points each, 30 points total)

Instructions: Answer each question in 2-3 sentences.

Question 5: Explain the role of the index `i` in the inner `while` loop of the INSERTION-SORT algorithm.

Question 6: Describe the main operation performed during the merging of two sorted arrays.

Question 7: Explain the difference between the upper bound and lower bound analysis shown with the equation T (n) = 2T (n/2) + ■(n).

Section 3: Problem-Solving Questions (10 points each, 30 points total)

Instructions: Provide detailed steps and explanations for your solutions.

Question 8: Using the INSERTION-SORT algorithm, show the steps required to sort the following array: \[5, 1, 4, 2, 8]. Show the state of the array after each iteration of the outer loop.

Question 9: Suppose you have two sorted arrays: A = \[2, 4, 6, 8] and B = \[1, 3, 5, 7, 9]. Demonstrate how to merge these two arrays into a single sorted array using the merging technique discussed in the materials. Show the comparisons and movements of elements during the merging process.

Question 10: Given the recurrence relation T(n) = T(n-1) + ■(n), prove that T(n) ≤ cn^2 for some constant c > 0. Show all steps of the inductive proof including the base case.
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