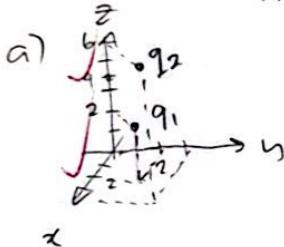


$$R_1 = (2, 2, 2)$$

- Q1. Charge  $q_1 = -6\pi\epsilon_0$  C is located at point (2 m, 2 m, 2 m) in Cartesian coordinates, and charge  $q_2$  is located at point (3 m, 3 m, 6 m).  $R_2 = (3, 3, 6)$

- (a) Draw the locations of the charges in the Cartesian coordinate system. [2] ✓
- (b) Using Coulomb's law, what should charge  $q_2$  be so that the total electric field  $\vec{E}$  at point (0, 0, 4 m) has no x-component? [5] 4
- (c) Find the force  $\vec{F}$  acting on a third charge  $q_3 = 10$  C located the same point (0, 0, 4 m). [3] 1



b)  $E_{1x} = E_{2x} @ (0, 0, 4)$  ~0.5

$$\frac{1}{4\pi\epsilon_0} \left( \frac{q_1(R-R_{1x})}{|R-R_{1x}|^3} \right) = \frac{1}{4\pi\epsilon_0} \left( \frac{q_2(R-R_{2x})}{|R-R_{2x}|^3} \right)$$

$$\frac{-6\pi\epsilon_0(-\hat{x}2)}{2^3} = \frac{q_2(-\hat{x}3)}{3^3}$$

$$q_2 = \frac{-6\pi\epsilon_0(-\hat{x}2)(5^3)}{2^3(-\hat{x}3)}$$

$$= -\frac{6\pi\epsilon_0(54)}{24} = -\frac{27}{2}\pi \left( \frac{1}{36\pi} \times 16^{-9} \right) = -\frac{27}{72} \times 10^{-9}$$

-0.5  $= -0.375 \times 10^{-9}$

c)  $E = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1(R-R_1)}{|R-R_1|^3} + \frac{q_2(R-R_2)}{|R-R_2|^3} \right]$  OK

$$= \frac{1}{4\pi\epsilon_0} \left[ \frac{-6\pi\epsilon_0(-\hat{x}2-\hat{y}2+\hat{z}2)}{(2^2+2^2+2^2)^{3/2}} + \left( -\frac{27}{2}\pi \right) \frac{(-\hat{x}3-\hat{y}3-\hat{z}2)}{(3^2+3^2+2^2)^{3/2}} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \left[ \frac{-6\pi\epsilon_0(-\hat{x}2-\hat{y}2+\hat{z}2)}{(12)^{3/2}} - \frac{27}{2} \frac{\pi(-\hat{x}3-\hat{y}3-\hat{z}2)}{(22)^{3/2}} \right]$$

$$F = q_3 E = -\frac{3}{2(12)^{3/2}} (-\hat{x}2-\hat{y}2+\hat{z}2) - \frac{27}{8(22)^{3/2}} (-\hat{x}3-\hat{y}3-\hat{z}2)$$

= ?  
-2

- Q2. In a rectangular box defined by the region  $0 \leq x \leq 1$  m,  $0 \leq y \leq 2$  m,  $0 \leq z \leq 3$  m, the electric flux density is given by:

$$\vec{D} = 8x\hat{x} - 3y^2\hat{y} + z^3\hat{z} \text{ C/m}^2$$

- (a) Using the differential form of Gauss' Law, find the volume charge density,  $\rho_v$ . [4] 4  
 (b) Use  $\rho_v$  from part (a) to find the total charge  $Q$  contained in the box. [6] 2.5

a)  $Q = \int_V \nabla \cdot D dV = \int_V \rho_v dV$   $\nabla \cdot D = \rho_v$   $(8x, -3y^2, z^3)$

$$\int_V \rho_v dV = \rho_v \int_0^1 dx \int_0^2 dy \int_0^3 dz = 6\rho_v = Q$$

-3

$$\oint_S D_r ds = D_r \cdot 22 = Q$$

$$\nabla \cdot D = \frac{\partial}{\partial x} 8x + \frac{\partial}{\partial y} (-3y^2) + \frac{\partial}{\partial z} z^3 = 8 - 6y + 3z^2 = \rho_v$$

$$D_r \cdot 22 = 6\rho_v$$

$$\vec{D} \cdot \hat{n} = 6\rho_v$$

$$\rho_v = \frac{22}{6R} (8x\hat{x} - 3y^2\hat{y} + z^3\hat{z})$$

$$\rho_v = \frac{1}{R} \left( \frac{88}{3} x\hat{x} - 11y^2\hat{y} + \frac{22}{6} z^3\hat{z} \right)$$

b)  $Q = 6\rho_v = \frac{1}{R} (176x\hat{x} - 66y^2\hat{y} + 22z^3\hat{z})$

$$Q = 6\rho_v = 6(8 - 6y + 3z^2) = 48 - 36y + 18z^2$$

8K

-0.5

**Q1.** The potential distribution in a dielectric medium with permittivity  $\epsilon = 9\epsilon_0$  is given in cylindrical coordinates by

$$V = \frac{1}{r^2} e^{-6z} \cos(\phi) \text{ V}$$

$$\frac{\partial V}{\partial \phi} = -\frac{1}{r^2} e^{-6z} \sin \phi$$

$$\frac{\partial V}{\partial z} = -\frac{6}{r^2} e^{-6z} \cos \phi$$

- (a) Find the electric field  $\vec{E}$  due to this potential distribution. [4]
- (b) Evaluate  $\vec{E}$  at point  $P_1 (r_1, \phi_1, z_1) = (0.1, 2\pi, 0.5)$ . [2]
- (c) Evaluate the electric flux density  $\vec{D}$  at point  $P_1$ . [2]
- (d) Evaluate the electric polarization field  $\vec{P}$  at point  $P_1$ . [2]

$$\begin{aligned} a) \vec{E} &= -\nabla V = -\left(r \frac{\partial V}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial V}{\partial \phi} + \hat{z} \frac{\partial V}{\partial z}\right) & \frac{\partial V}{\partial r} \left(\frac{1}{r^2} e^{-6z} \cos \phi\right) &= -\frac{3}{r^3} e^{-6z} \cos \phi \\ &= -\left(r \left(-\frac{3}{r^3} e^{-6z} \cos \phi\right) + \hat{\phi} \left(-\frac{1}{r^2} e^{-6z} \sin \phi\right) + \hat{z} \left(-\frac{6}{r^2} e^{-6z} \cos \phi\right)\right) \\ \vec{E} &= \hat{r} \frac{3}{r^3} e^{-6z} \cos \phi + \hat{\phi} \frac{1}{r^3} e^{-6z} \sin \phi + \hat{z} \frac{6}{r^2} e^{-6z} \cos \phi \text{ V/m} \end{aligned}$$

$\frac{-1}{2}$

$$\begin{aligned} b) \vec{E}_1 &= \hat{r} \frac{3}{(0.1)^3} e^{-6(0.5)} \cos(2\pi) \hat{r} + \hat{\phi} \frac{1}{(0.1)^3} e^{-6(0.5)} \sin(2\pi) \hat{\phi} + \hat{z} \frac{6}{(0.1)^2} e^{-6(0.5)} \cos(2\pi) \hat{z} \\ &= \hat{r} 149.36 + \hat{z} 29.87 \text{ V/m} \end{aligned}$$

99.8  $\hat{r}$  ...

$$\begin{aligned} c) \vec{D}_1 &= \epsilon \vec{E}_1 = \left(\frac{9\epsilon_0}{\epsilon_0}\right) (149.36, 29.87) \\ &= \hat{r} 11.89 + \hat{z} 2.38 \text{ nC/m}^2 \end{aligned}$$

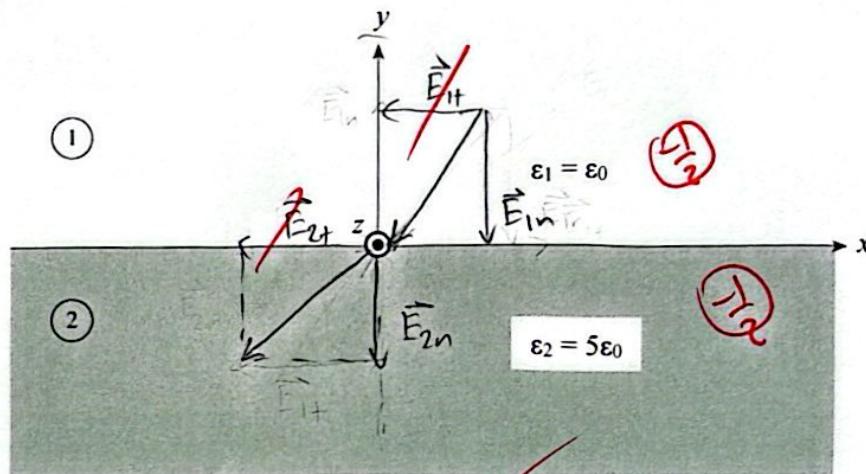
$$d) \epsilon_r = 9 = 1 + \chi_e \quad \chi_e = 9 - 1 = 8$$

$$\begin{aligned} \vec{P}_1 &= \epsilon_0 \chi_e \vec{E}_1 = \left(\frac{10^{-9}}{36\pi}\right) (8) (149.36, 29.87) \\ &= \hat{r} 10.57 + \hat{z} 2.11 \text{ nC/m}^2 \end{aligned}$$

7.0

- Q2. The figure below shows the boundary between two dielectric media that extends along the  $xz$ -plane. Medium 1 above the boundary has a permittivity  $\epsilon_1$ , and Medium 2 below the boundary has a permittivity  $\epsilon_2$ . The electric field in Medium 1 is given by  $\vec{E}_1 = 10\hat{x} - 6\hat{y} + 8\hat{z}$  V/m.

- Find the polarization field in Medium 1,  $\vec{P}_1$ . [3]
- On the diagram below, draw the normal and tangential components of the electric field in Medium 1,  $\vec{E}_{1n}$  and  $\vec{E}_{1t}$ . [1]
- Use the electric boundary conditions to find the electric field in Medium 2,  $\vec{E}_2$ . [5]
- On the diagram below, draw  $\vec{E}_2$  and the normal and tangential components of the electric field in Medium 2,  $\vec{E}_{2n}$  and  $\vec{E}_{2t}$ . [1]



a)  $\epsilon_r = 1 = 1 + \chi_c, \chi_c = 0, \vec{P}_1 = 0 \text{ C/m}^2$  ✓

b)

c)  $\vec{E}_H = \vec{E}_{2t} = 10\hat{x} + 8\hat{z} \text{ V/m}$   
 $\epsilon_1 \vec{E}_{1n} = \epsilon_2 \vec{E}_{2n} \text{ (no } \rho_v \text{)}$

$$\vec{E}_2 = 10\hat{x} - 1.2\hat{y} + 8\hat{z} \text{ V/m}$$

$$\vec{E}_{2n} = \frac{\epsilon_1}{\epsilon_2} \vec{E}_{1n} = \frac{\epsilon_0}{5\epsilon_0} (-6\hat{y}) = -1.2\hat{y} \text{ V/m}$$

**Q1.** A charge with a value of  $q = 10 \text{ mC}$  is travelling with a velocity of  $\vec{u} = 3\hat{x} - 6\hat{y} + 7\hat{z}$  in both an external electric field and an external magnetic field, given by  $\vec{E} = 4xy^2\hat{x} + xz^3\hat{y} - 10yz\hat{z} \text{ V/m}$  and  $\vec{H} = \frac{1}{\mu_0} (2x^2\hat{x} - y^3\hat{y} + 2z^2\hat{z}) \text{ A/m}$ , respectively. At point P (2, 0, 6):

- (a) Find the electric force  $\vec{F}_e$  acting on the charge. [3]
- (b) Find the magnetic force  $\vec{F}_m$  acting on the charge. [3]
- (c) Find the total electromagnetic force  $\vec{F}$  acting on the charge. [2]
- (d) Sketch the force  $\vec{F}$  in the Cartesian coordinate system. [2]

10

$$a) \vec{F}_e = q\vec{E} = (10 \times 10^{-3} \text{ C}) (4xy^2\hat{x} + xz^3\hat{y} - 10yz\hat{z} \text{ V/m})$$

$$= 40xy^2\hat{x} + 10xz^3\hat{y} - 100yz\hat{z} \text{ mN} = 4.32\hat{y} \text{ N}$$

$$b) \vec{F}_m = q\vec{u} \times \vec{B}$$

$$\vec{B} = \mu_0 \vec{H} = 2x^2\hat{x} - y^3\hat{y} + 2z^2\hat{z}$$

$$= (10 \times 10^{-3} \text{ C}) (3\hat{x} - 6\hat{y} + 7\hat{z}) \times (2x^2\hat{x} - y^3\hat{y} + 2z^2\hat{z})$$

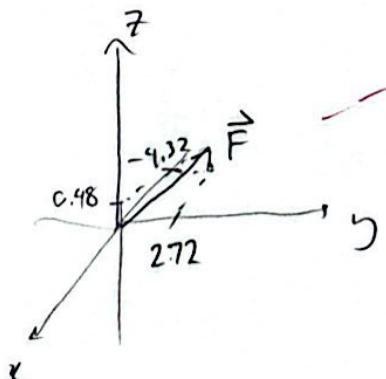
$$= (30\hat{x} - 60\hat{y} + 70\hat{z} \times 10^{-3}) \times (2x^2\hat{x} - y^3\hat{y} + 2z^2\hat{z})$$

$$= (-120z^2 + 70y^3)\hat{x} - (60z^2 - 140x^2)\hat{y} + (-30y^3 + 120x^2)\hat{z} \text{ mN} = -4.32\hat{x} - 1.6\hat{y} + 0.48\hat{z}$$

$$c) \vec{F} = \vec{F}_e + \vec{F}_m = (40xy^2 - 120z^2 + 70y^3)\hat{x} + (10xz^3 - 60z^2 + 140x^2)\hat{y} + (-100yz - 30y^3 + 120x^2)\hat{z} \text{ mN}$$

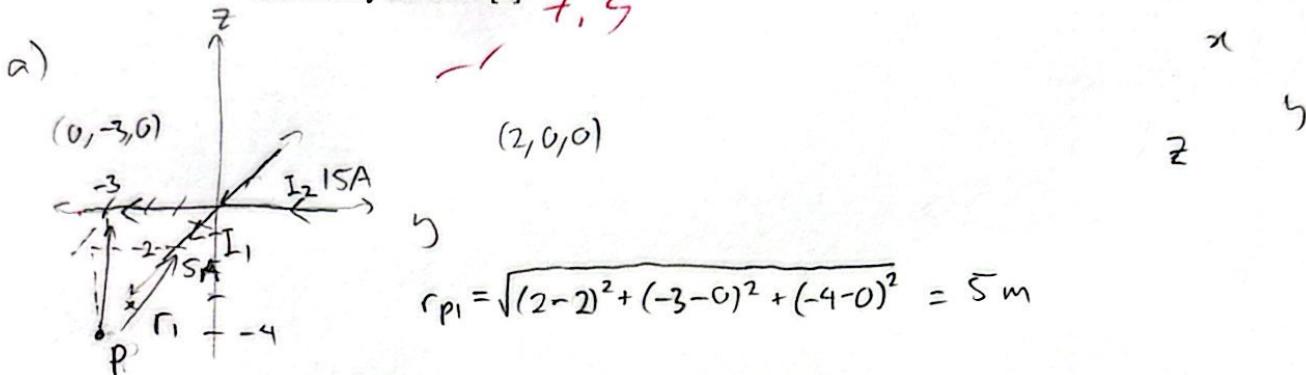
$$= -4.32\hat{x} + 2.72\hat{y} + 0.48\hat{z} \text{ N}$$

d)



Q2. The  $x$ - and  $y$ -axes, respectively, carry filamentary currents of 5 A along  $\hat{x}$  and 15 A along  $-\hat{y}$ .

- (a) Sketch the two filamentary currents in the Cartesian coordinate system. [2] 2  
 (b) Use the Biot-Savart law to find the total magnetic field  $\vec{H}$  at point  $(2, -3, -4)$  m due to both filamentary currents. [8] 7.5



b)  $\vec{B}_1 = \hat{\phi} \frac{\mu_0 I_1}{2\pi r_{p1}}$      $\hat{\phi} = \hat{l} \times \hat{r} = \hat{x} \times \left( \frac{4\hat{x} + 3\hat{y}}{5} \right) = -\frac{4}{5}\hat{y} + \frac{3}{5}\hat{z}$

$$= \frac{(4\pi \times 10^{-7})(5\text{A})}{2\pi (5\text{m})} \left( -\frac{4}{5}\hat{y} + \frac{3}{5}\hat{z} \right)$$

$$= \left( -\frac{8}{5}\hat{y} + \frac{6}{5}\hat{z} \right) \times 10^{-7}$$

$$r_{p2} = \sqrt{(2-0)^2 + (-3-(-3))^2 + (-4-0)^2} = \sqrt{20}$$

$$\vec{B}_2 = \hat{\phi} \frac{\mu_0 I_2}{2\pi r_{p2}}$$

$$\hat{\phi} = \hat{l} \times \hat{r} = (-\hat{y}) \times \left( \frac{9\hat{x} - 2\hat{z}}{\sqrt{20}} \right) = -\frac{4}{\sqrt{20}}\hat{x} - \frac{2}{\sqrt{20}}\hat{z}$$

$$= \frac{(4\pi \times 10^{-7})(15\text{A})}{2\pi \sqrt{20}} \left( -\frac{4}{\sqrt{20}}\hat{x} - \frac{2}{\sqrt{20}}\hat{z} \right)$$

$$= (-6\hat{x} - 3\hat{z}) \times 10^{-7}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \left( -6\hat{x} - \frac{8}{5}\hat{y} - \frac{9}{5}\hat{z} \right) \times 10^{-7} / 4\pi \times 10^{-7}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = -\frac{3}{2\pi}\hat{x} - \frac{2}{5\pi}\hat{y} - \frac{1}{20\pi}\hat{z} \text{ A/m}$$

$$= -0.477\hat{x} - 0.127\hat{y} - 0.143\hat{z} \text{ A/m}$$

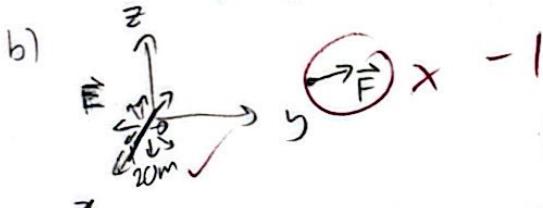
- 0.5.

2.5

**Q1.** A wire extending along  $0 < x < 20$  m has a linear charge density of  $\rho_l = 50x^3 \mu\text{C/m}$ .

- Find the total charge, Q, on the line. [2]
- Draw the wire in the Cartesian coordinate system, and draw vectorially the electric field distribution around the wire. [1]
- Using Coulomb's law, find the electric field  $\vec{E}$  at point  $(0, 0, 100 \text{ km})$  in Cartesian coordinates. [4]
- Find the force  $\vec{F}$  acting on a point charge  $q_1 = 5 \text{ C}$  located the same point  $(0, 0, 100 \text{ km})$ . [2]
- Draw the vector force  $\vec{F}$  on the diagram from part (b). [1]

a)  $Q = \int_L \rho_l dl = \int_{x=0}^{20} 50x^3 \times 10^{-9} dx = 50 \frac{x^4}{4} \times 10^{-9} \Big|_0^{20} = 2 \text{ mC} \quad \text{X} \quad - 1/2$



$$\vec{R} = (x, 0, 0) - (0, 0, 100 \text{ km}) = (x, 0, -100 \text{ km})$$

c)  $\vec{E} = \frac{1}{4\pi\epsilon_0} \int_L \hat{R}' \frac{\rho dl}{R'^2} = \frac{1}{4\pi\epsilon_0} \int_{x=0}^{20} \hat{R}' \frac{50x^3}{(100)^2} dx = \frac{50 \times 10^{-9}}{4\pi\epsilon_0 (100)^2} \left[ \frac{x^4}{4} \Big|_0^{20} \right] \hat{R}$

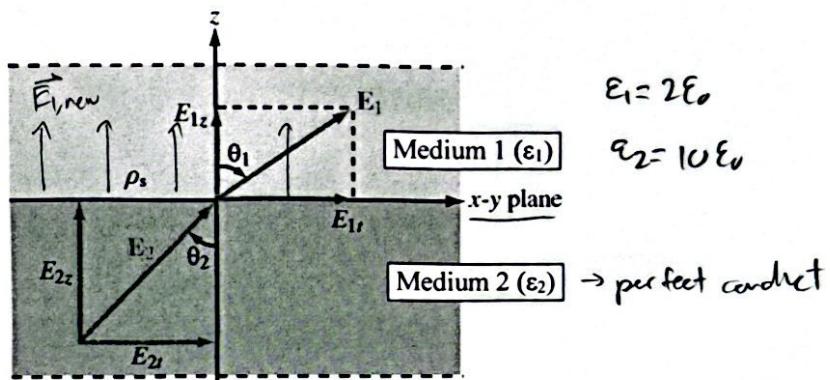
$$= \frac{(50 \times 10^{-9})(40000)}{4\pi(\frac{10^{-9}}{36\pi})(100)^2} \hat{R} = \frac{(50)(9)(40000)}{(100)^2} \hat{R}$$

$$= (\hat{R}) 18 \frac{\mu\text{V/m}}{x} \quad \text{X} \quad - 1/2$$

d)  $\vec{F} = q_1 \vec{E} = (5 \text{ C})(18 \times 10^{-6} \text{ V/m}) = 90 \mu\text{N}$

- Q2.** The figure below shows the boundary between two dielectric media that extends along the  $x$ - $y$  plane. Medium 1 above the boundary has a permittivity  $\epsilon_1$ , and Medium 2 below the boundary has a permittivity  $\epsilon_2$ . The electric field in Medium 2 is given by  $\vec{E}_2 = -10\hat{x} + 5\hat{y} + 20\hat{z}$  V/m, the surface charge density on the boundary is  $\rho_s = 8\epsilon_0$  C/m<sup>2</sup>, and the relative permittivities of the two media are  $\epsilon_{r1} = 2$  and  $\epsilon_{r2} = 10$ .

- (a) Use the electric boundary conditions to find the electric field in Medium 1,  $\vec{E}_1$ . [4]
- (b) Medium 2 is now replaced with a perfect conductor. Thus, now the boundary is between Medium 1, a dielectric with permittivity  $\epsilon_1$ , and Medium 2, a perfect conductor. The surface charge density on the boundary remains at  $\rho_s = 8\epsilon_0$  C/m<sup>2</sup>. Find the new electric field in Medium 1,  $\vec{E}_{1,new}$ . Sketch  $\vec{E}_{1,new}$  in the figure below. [4]
- (c) For the scenario of part (b) above, calculate the energy densities for both Medium 1 and Medium 2. You can leave your answers in terms of  $\epsilon_0$ . [2]



a)  $\vec{E}_4 = \vec{E}_{2t} = -10\hat{x} + 5\hat{y}$

$\epsilon_1 \vec{E}_{1n} - \epsilon_2 \vec{E}_{2n} = \rho_s$

$$\vec{E}_{1n} = \frac{\rho_s + \epsilon_2 \vec{E}_{2n}}{\epsilon_1} = \frac{8\epsilon_0}{2\epsilon_0} + \frac{10\epsilon_0}{2\epsilon_0} (20\hat{z}) = \cancel{100\hat{z} + 4} \\ (100+4)\hat{z}$$

$\vec{E}_1 = -10\hat{x} + 5\hat{y} + 100\hat{z} + 4 \text{ V/m}$

(-1/2)

b)  $\vec{E}_{1t,new} = \vec{E}_{2t} = 0$  ✓

$\epsilon_1 \vec{E}_{1n,new} = \rho_s$

$$\vec{E}_{1n,new} = \frac{\rho_s}{\epsilon_1} = \frac{8\epsilon_0}{2\epsilon_0} = 4$$

$\vec{E}_{1,new} = 4 \text{ V/m } \hat{z}$  ✓

c)  $n_e = \frac{1}{2} \epsilon |\vec{E}|^2$

$n_{e1} = \frac{1}{2} (2\epsilon_0) |4|^2 = 16\epsilon_0 \text{ J/m}^3$

$n_{e2} = \frac{1}{2} \epsilon (0)^2 = 0 \text{ J/m}^3$

Q3. The potential distribution in free space is given in spherical coordinates by:

$$V = \frac{1}{R^3} \cos \phi \sin \theta \quad V$$

$$\frac{\partial V}{\partial R} = -\frac{3}{R^4} \cos \phi \sin \theta$$

$$\frac{\partial V}{\partial \theta} = \frac{1}{R^3} \cos \phi \cos \theta$$

$$\frac{\partial V}{\partial \phi} = -\frac{1}{R^3} \sin \phi \sin \theta$$

- (a) Find the electric field  $\vec{E}$  due to this potential distribution. [6] 6  
 (b) Evaluate  $\vec{E}$  at point  $P_1 (R_1, \theta_1, \phi_1) = (1, \pi/4, \pi/4)$ . [1]  
 (c) Find the electric flux density  $\vec{D}$ . [2] 2  
 (d) Evaluate  $\vec{D}$  at point  $P_1 (R_1, \theta_1, \phi_1) = (1, \pi/4, \pi/4)$ . [1] 0.5

a)  $\vec{E} = -\nabla V = -(\hat{R} \frac{\partial V}{\partial R} + \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi})$   
 $= -(\hat{R} -\frac{3}{R^4} \cos \phi \sin \theta + \hat{\theta} \frac{1}{R^3} \cos \phi \cos \theta + \hat{\phi} -\frac{1}{R^4} \sin \phi)$   
 $= \hat{R} \frac{3}{R^4} \cos \phi \sin \theta - \hat{\theta} \frac{1}{R^4} \cos \phi \cos \theta + \hat{\phi} \frac{1}{R^4} \sin \phi$

b)  $\vec{E} = \hat{R} \frac{3}{R^4} \cos(\frac{\pi}{4}) \sin(\frac{\pi}{4}) - \hat{\theta} \frac{1}{R^4} \cos(\frac{\pi}{4}) \cos(\frac{\pi}{4}) + \hat{\phi} \frac{1}{R^4} \sin(\frac{\pi}{4})$

$= \hat{R} 3(\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) - \hat{\theta} (\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) + \hat{\phi} \frac{1}{\sqrt{2}}$

$= \hat{R} \frac{3}{2} - \hat{\theta} \frac{1}{2} + \hat{\phi} \frac{1}{\sqrt{2}} \quad V/m$

c)  $\vec{D} = \epsilon_0 \vec{E} = \hat{R} \frac{1}{12\pi R^4} \cos \phi \sin \theta + \hat{\theta} \frac{1}{36\pi R^4} \cos \phi \cos \theta$

$+ \hat{\phi} \frac{1}{36\pi R^4} \sin \phi \quad nC/m$

d)  $\vec{D} = \epsilon_0 \vec{E} = \left( \frac{10^{-9}}{36\pi} \right) \vec{E} = \hat{R} 0.1326 - \hat{\theta} 0.0442 + \hat{\phi} 0.0625 \quad \mu C/m^2$

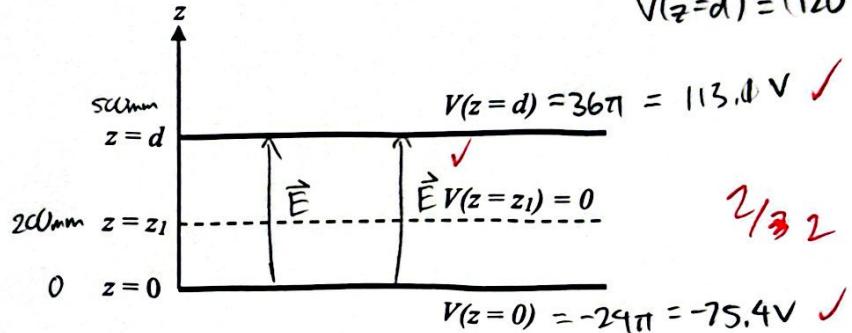
-0.5

Q3

Q4. Two conducting plates are located at  $z = 0$  and  $z = d = 500 \text{ mm}$ , as shown in the figure below. The zero-voltage reference is at  $z = z_1 = 200 \text{ mm}$ , and the electric field is given by

$$\vec{E} = 120\pi \hat{z} \quad \text{V/m}$$

- (a) Using Laplace's equation, calculate the voltage on the top conductor,  $V(z = d)$ . [5] 3.5
- (b) Calculate the voltage on the bottom conductor,  $V(z = 0)$ . [3] 3
- (c) On the diagram below, write down the values for  $V(z = d)$  and  $V(z = 0)$  and sketch the electric fields between the two conducting plates [2]. 2



$$V(z=d) = (120\pi)(0.5) = 36\pi \quad \text{no units}$$

1/2

$$V(z=0) = -24\pi = -75.4 \text{ V} \quad \checkmark$$

2/3 2

a)  $\nabla^2 V = 0$

$$\frac{dV}{dz} = \int \frac{dV^2}{dz^2} dz = \int 0 dz = A \quad V(z=z_1) = 0 \text{ V}$$

$$V = \int \frac{dV}{dz} dz = \int A dz = Az + B \quad \checkmark$$

$$\left. \begin{array}{l} V(z=d) = 36\pi = A(0.5) + B \\ V(z=z_1) = 0 = A(0.2) + B \end{array} \right\} \quad \begin{array}{l} 36\pi = 0.3A \\ A = 120\pi \end{array} \quad \begin{array}{l} B = 36\pi - (120\pi)(0.5) \\ = +24\pi \end{array}$$

$$V = 120\pi z - 24\pi \quad \checkmark$$

b)  $V(z=0) = 120\pi(0) - 24\pi$

$$= -24\pi = -75.4 \text{ V} \quad 3/3$$

c)