Barnes-Hut simulation of a galaxy collision

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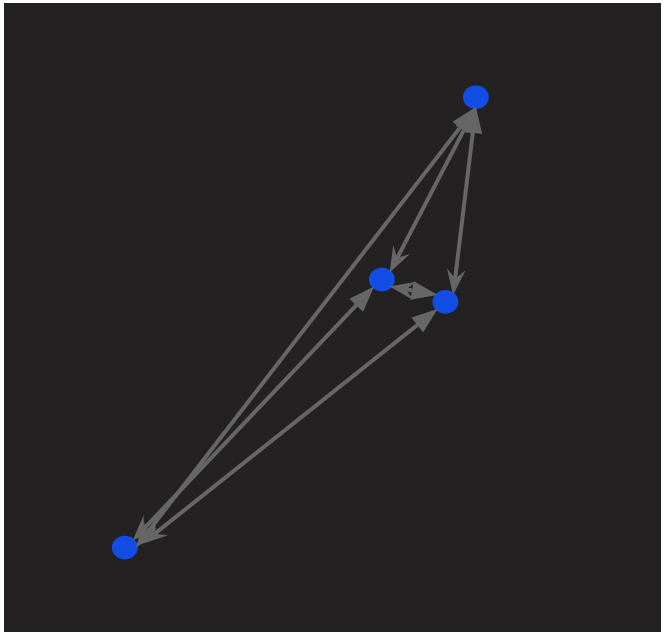
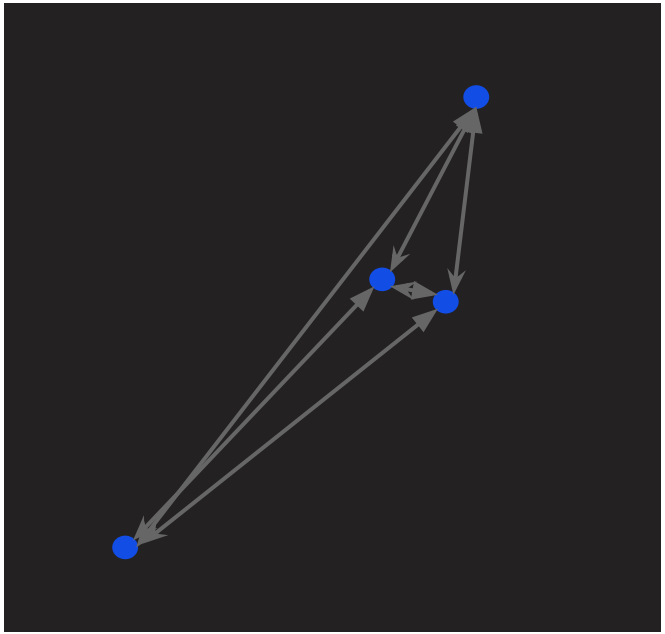
Solving the N-Body Problem

We want to solve the equations of motion for a model galaxy with (>10k) stars



Solving the N-Body Problem

Similiar to Molecular Dynamics, but in this case the forces involved are long-range, and always attractive



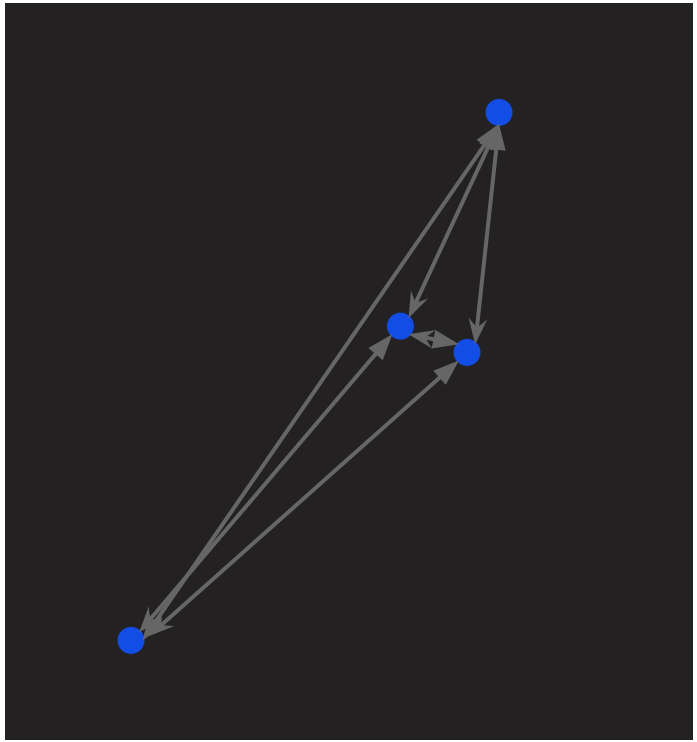
Solving the N-Body Problem

The lack of repulsion causes problems when combined with the singularity in the force at $r = 0$

Therefore a softening parameter $\varepsilon > 0$ is introduced:

$$\mathbf{F} \propto -\frac{\mathbf{r}}{(r^2 + \epsilon^2)^{3/2}}$$

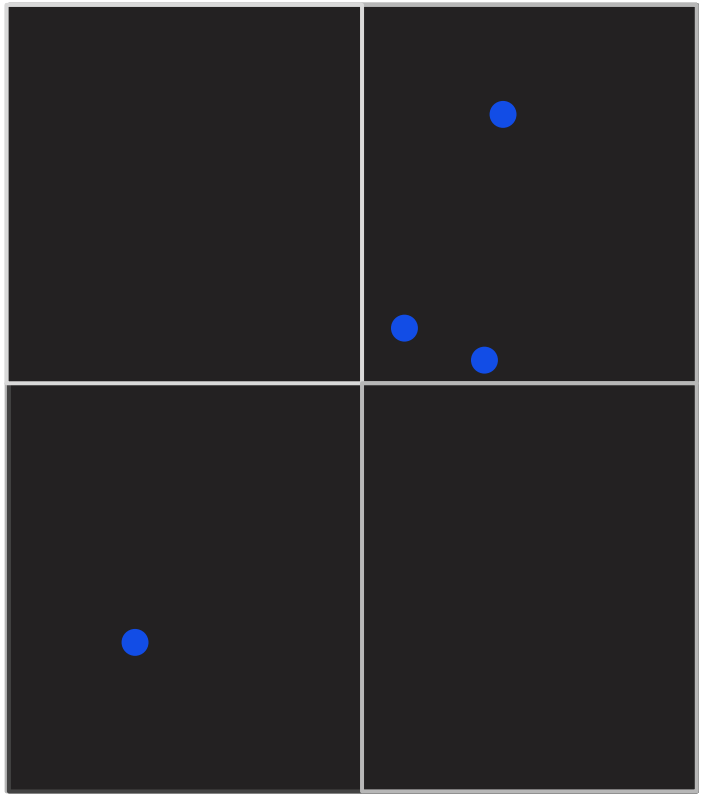
Direct Calculation



> O(N^2) operations

> Long range potential: using a cutoff is impossible

The Barnes-Hut Algorithm (2D)

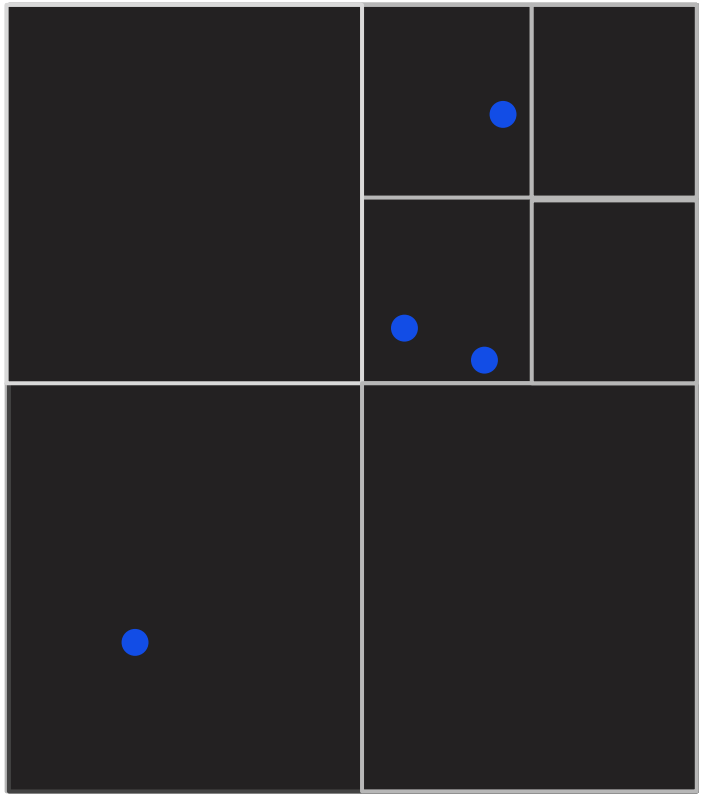


>Create a square cell large enough to contain the entire system

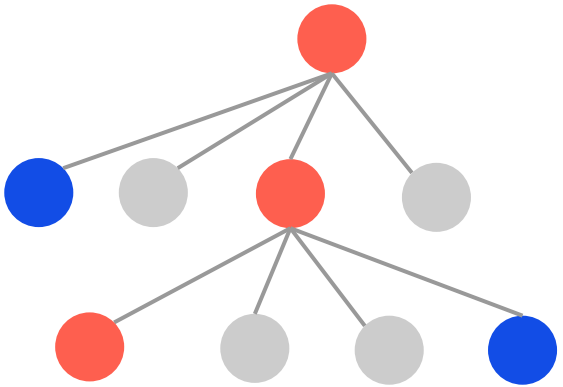
>Divide this cell into its 4 sub cells

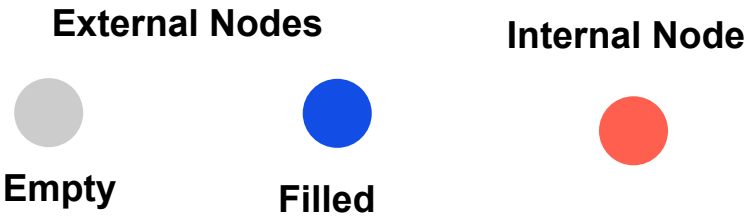
> Continue until each cell contains 1 or 0 particles

The Barnes-Hut Algorithm (2D)

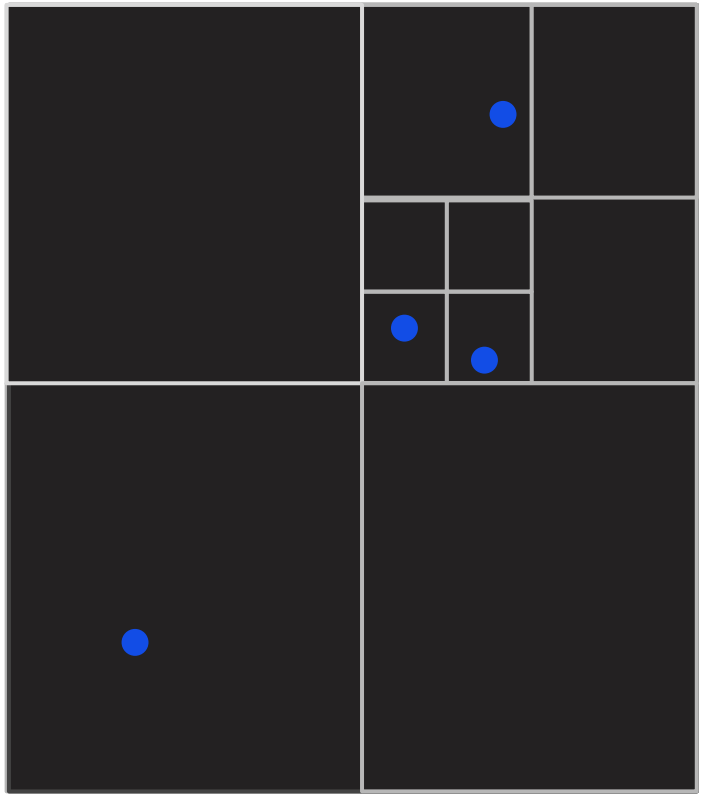


> Each cell is represented by a node

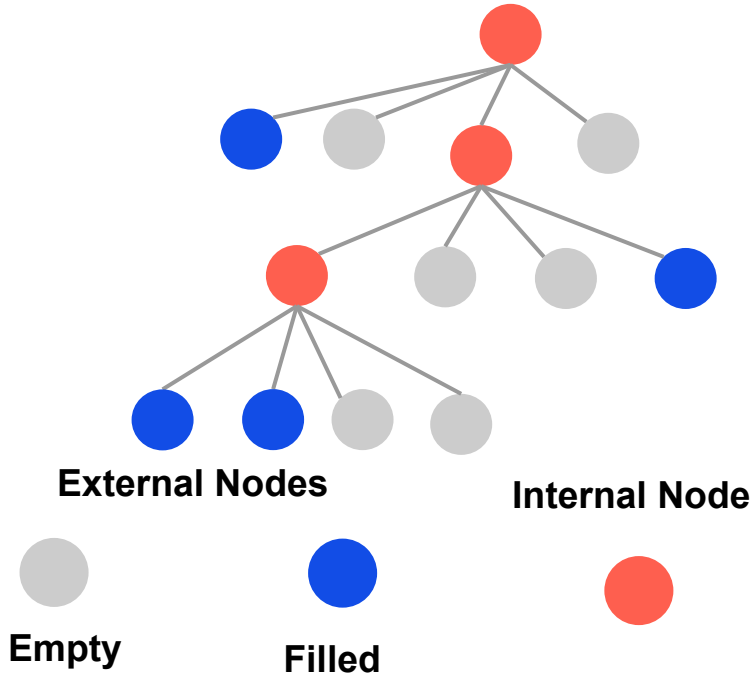




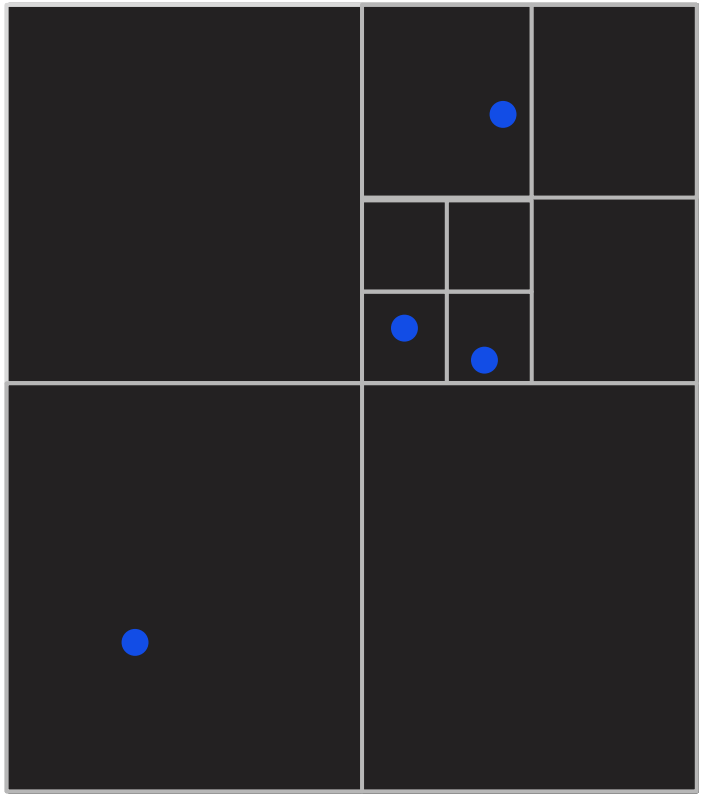
The Barnes-Hut Algorithm (2D)



Together the nodes form a tree



The Barnes-Hut Algorithm (2D)

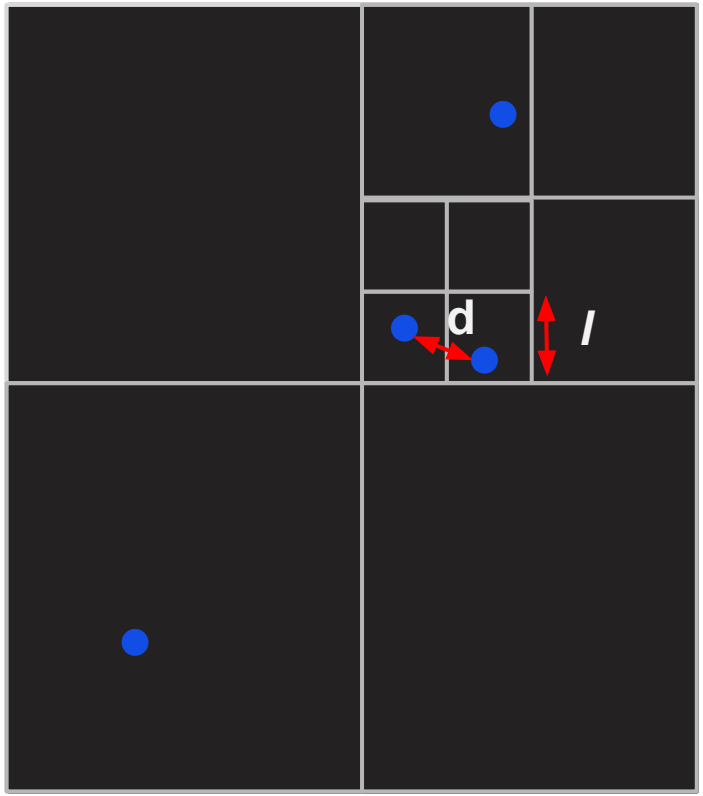


The internal nodes represent virtual particles, with:

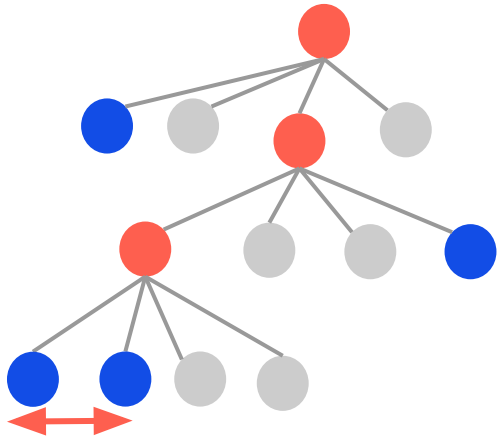
$$M = \sum_{i=1}^{N} m_i$$

$$\mathbf{r} = \frac{1}{M} \sum_{i=1}^{N} m_i \mathbf{r}_i$$

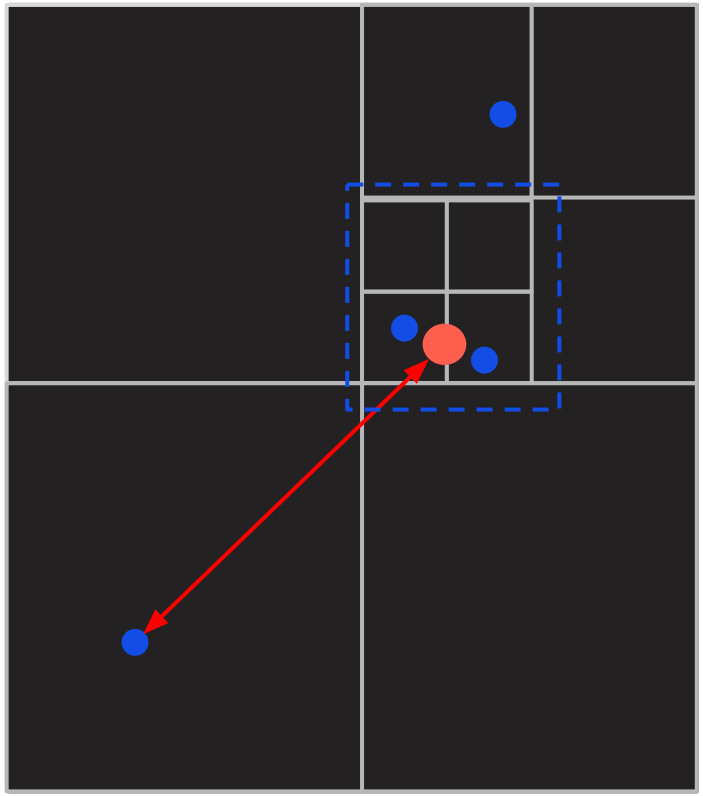
The Barnes-Hut Algorithm (2D)



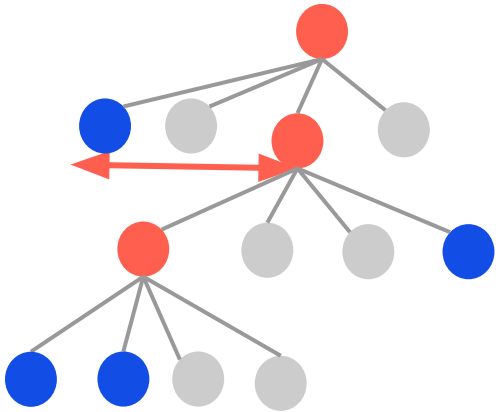
For pair satisfying: $l/d<\theta\sim0.5$
Interactions are calculated directly



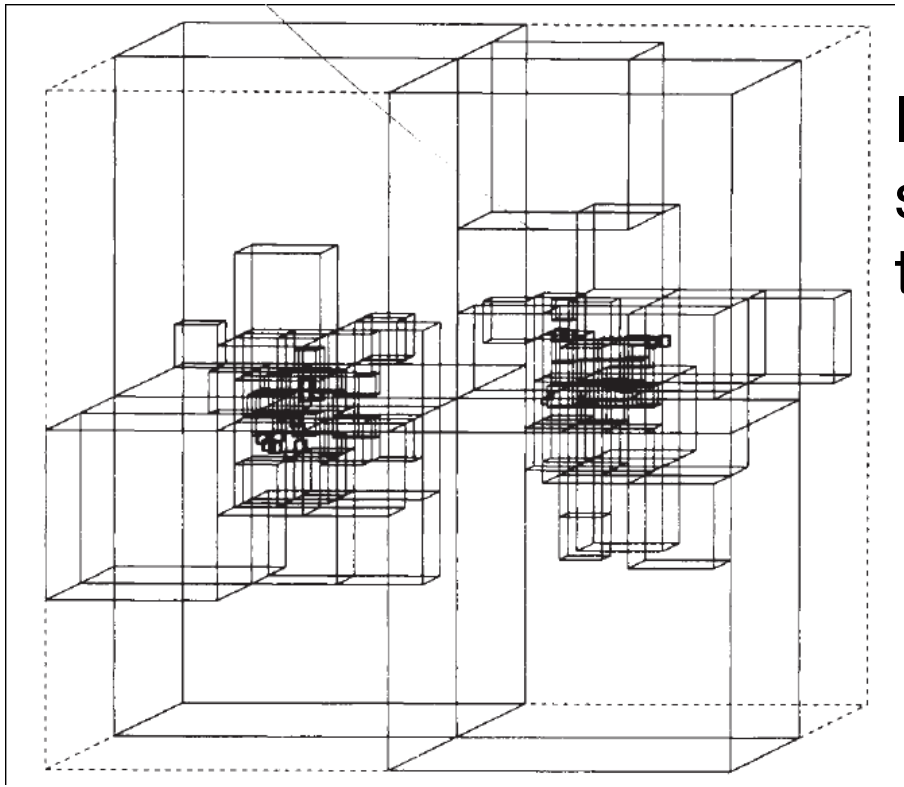
The Barnes-Hut Algorithm (2D)



But when $l/d>\theta$
Particles below ● are represented by the virtual particle at the node



The Barnes-Hut Algorithm (3D)



In 3D the algorithm works the same: Here we have have cubes that are divided into 8 octants

> Image: Barnes and Hut (1986)

The Barnes-Hut Algorithm

This algorithm reduces the number of operations from O (N^2) to O(N log(N)) compared to the direct method

This makes ii possible to calculate the interactions for much larger systems than using the direct method

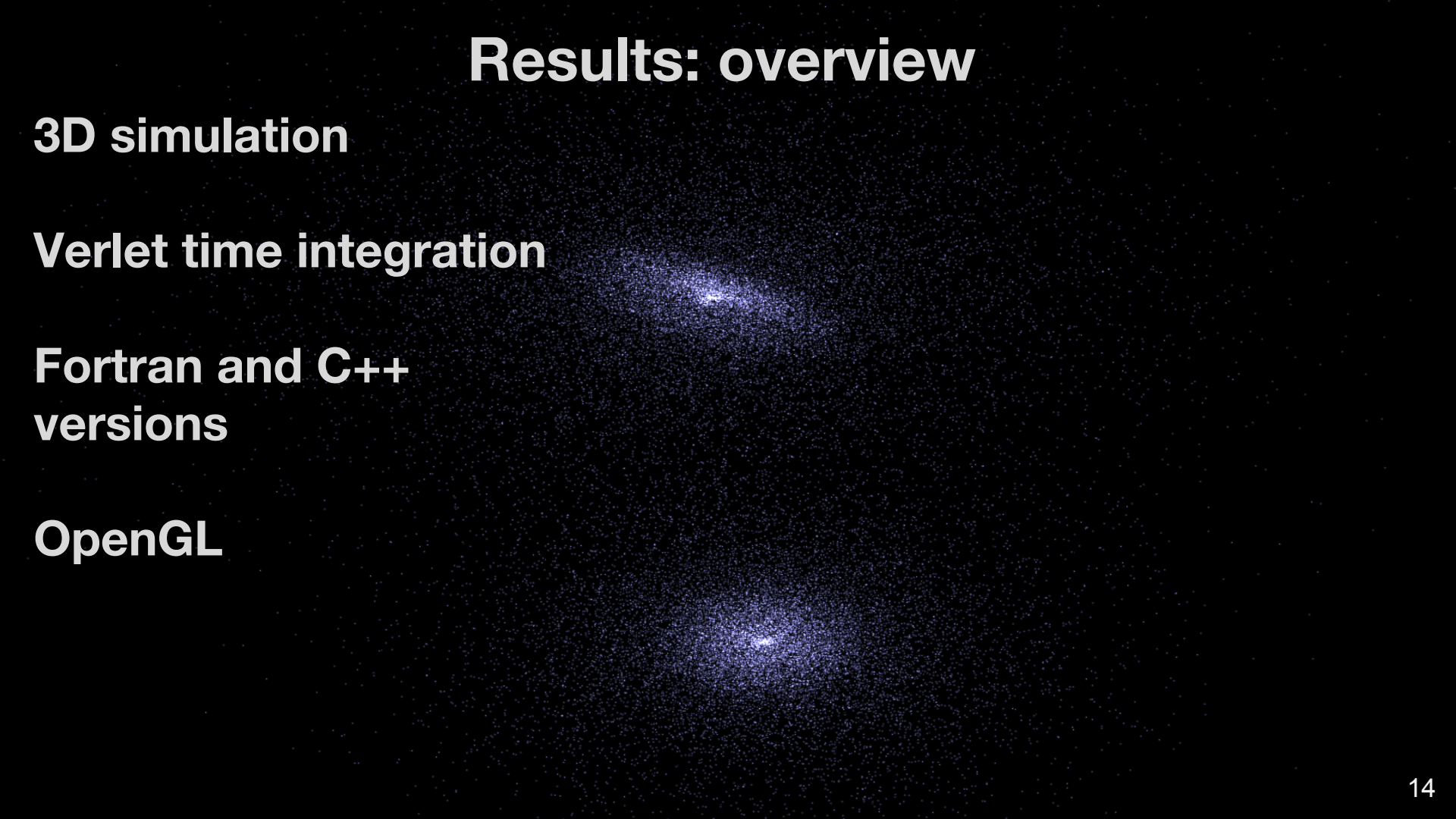
Results: overview

3D simulation

Verlet time integration

Fortran and C++ versions

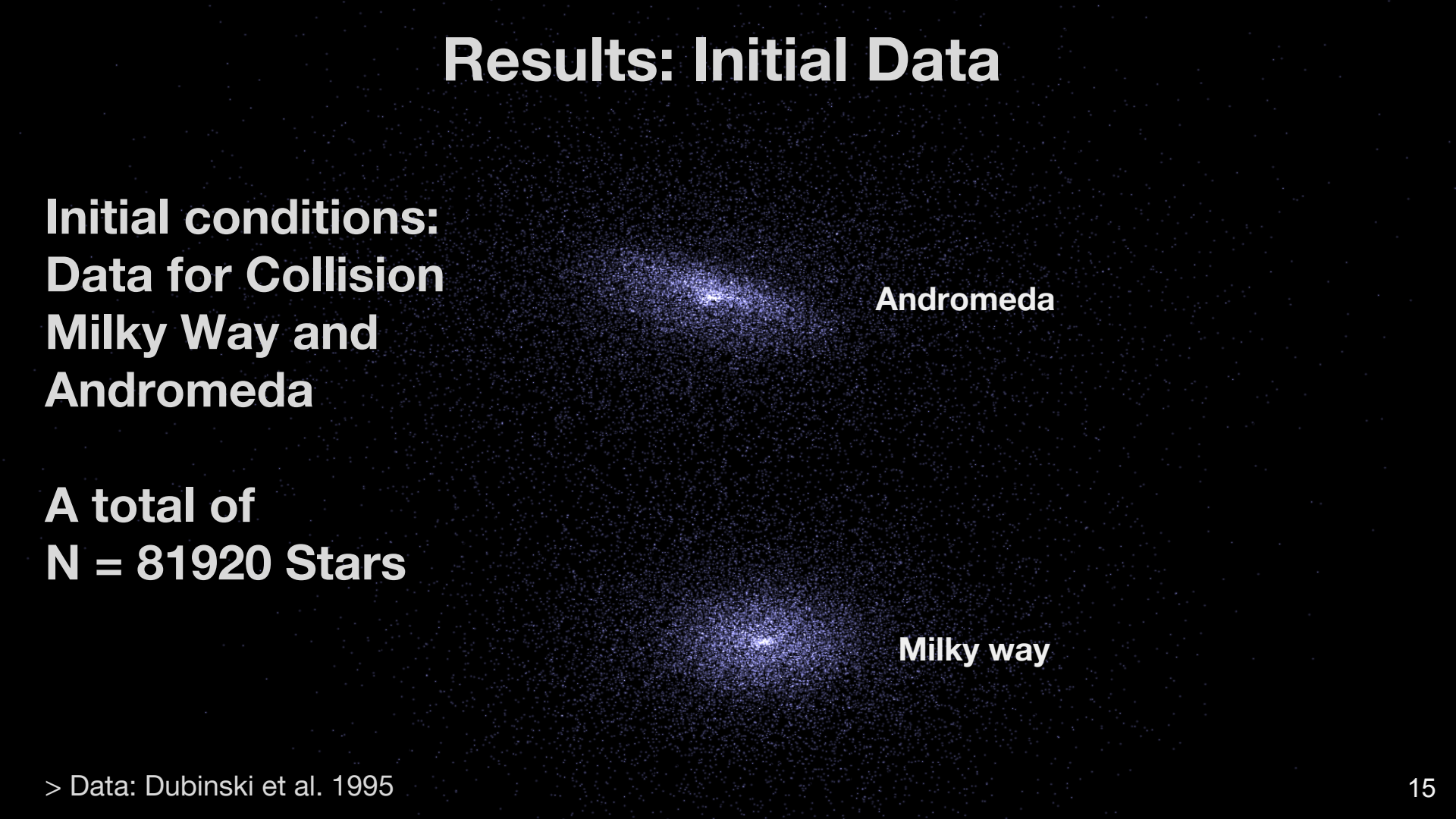
OpenGL



Results: Initial Data

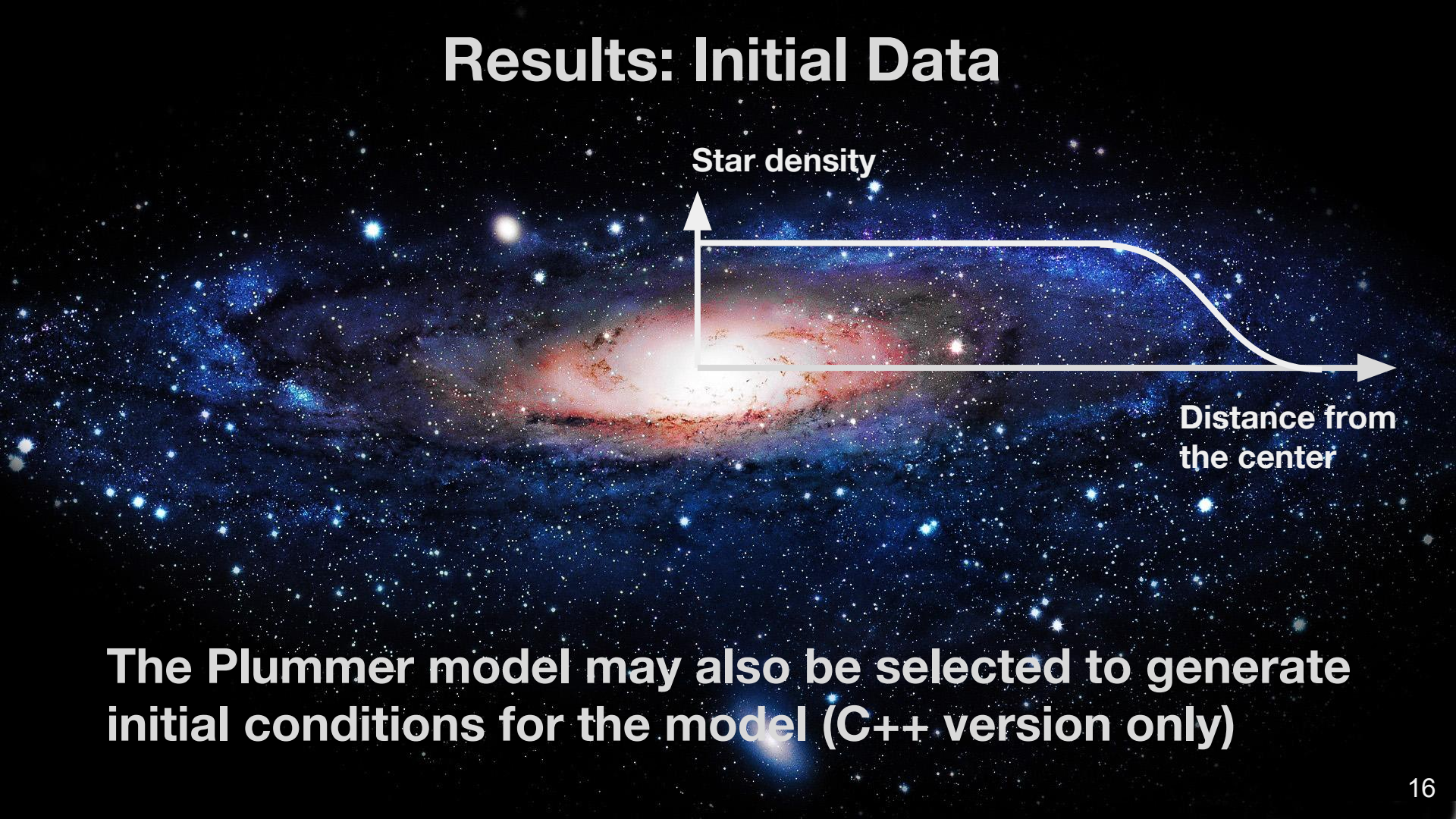
Initial conditions:
Data for Collision Milky Way and Andromeda

A total of
N = 81920 Stars



> Data: Dubinski et al. 1995

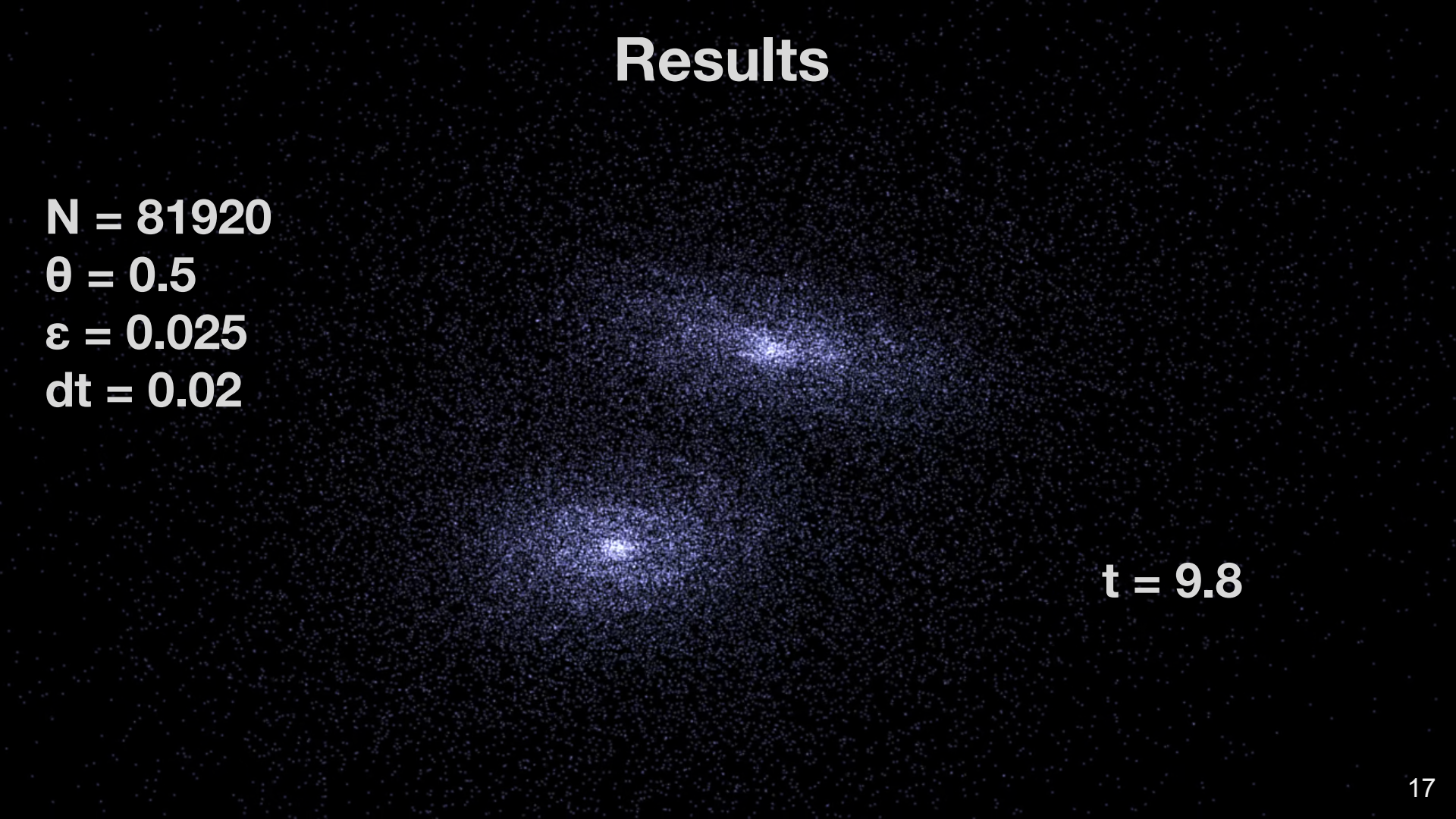
Results: Initial Data



The Plummer model may also be selected to generate initial conditions for the model (C++ version only)

Results

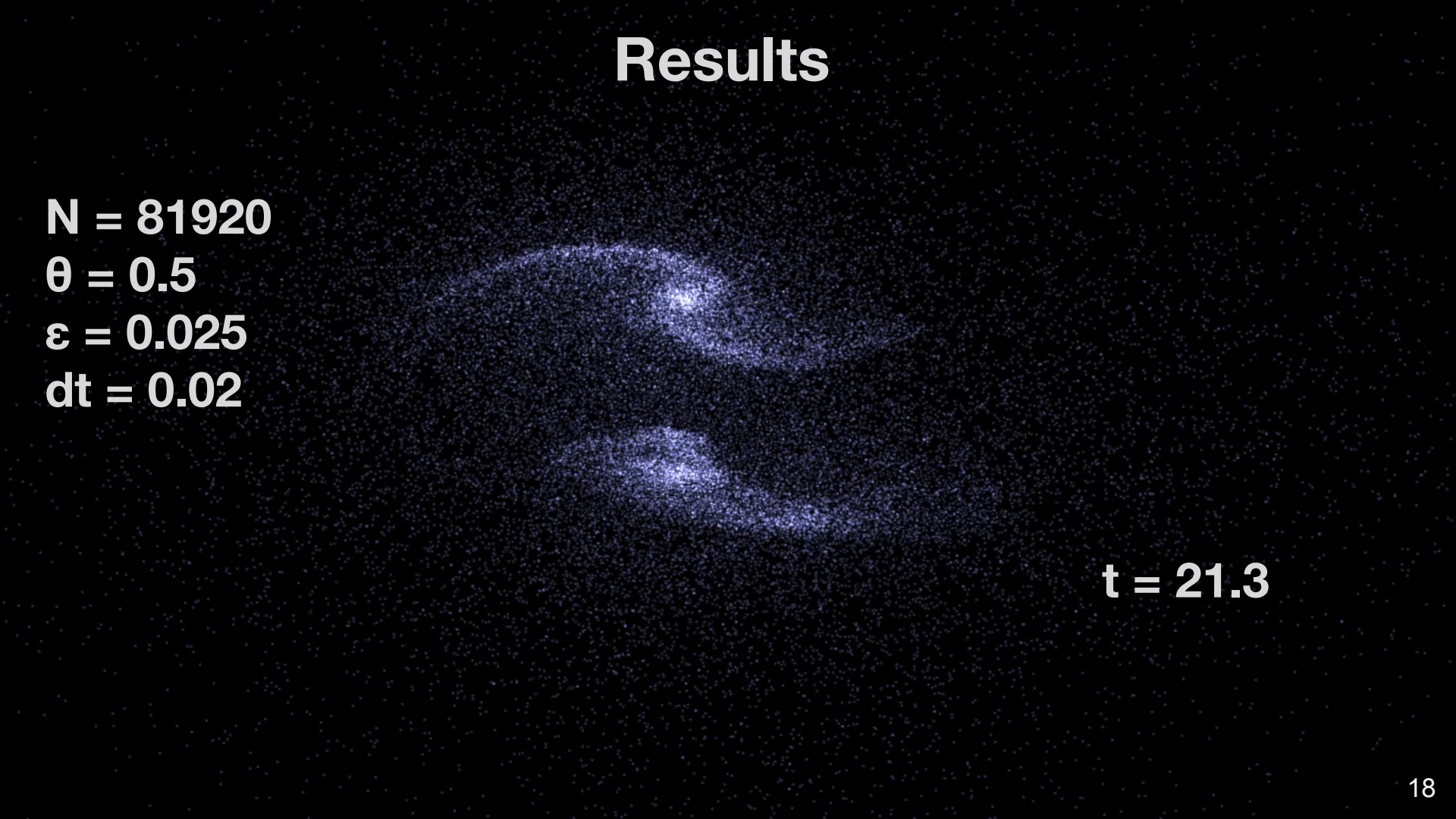
N = 81920
θ = 0.5
ε = 0.025
dt = 0.02



t = 9.8

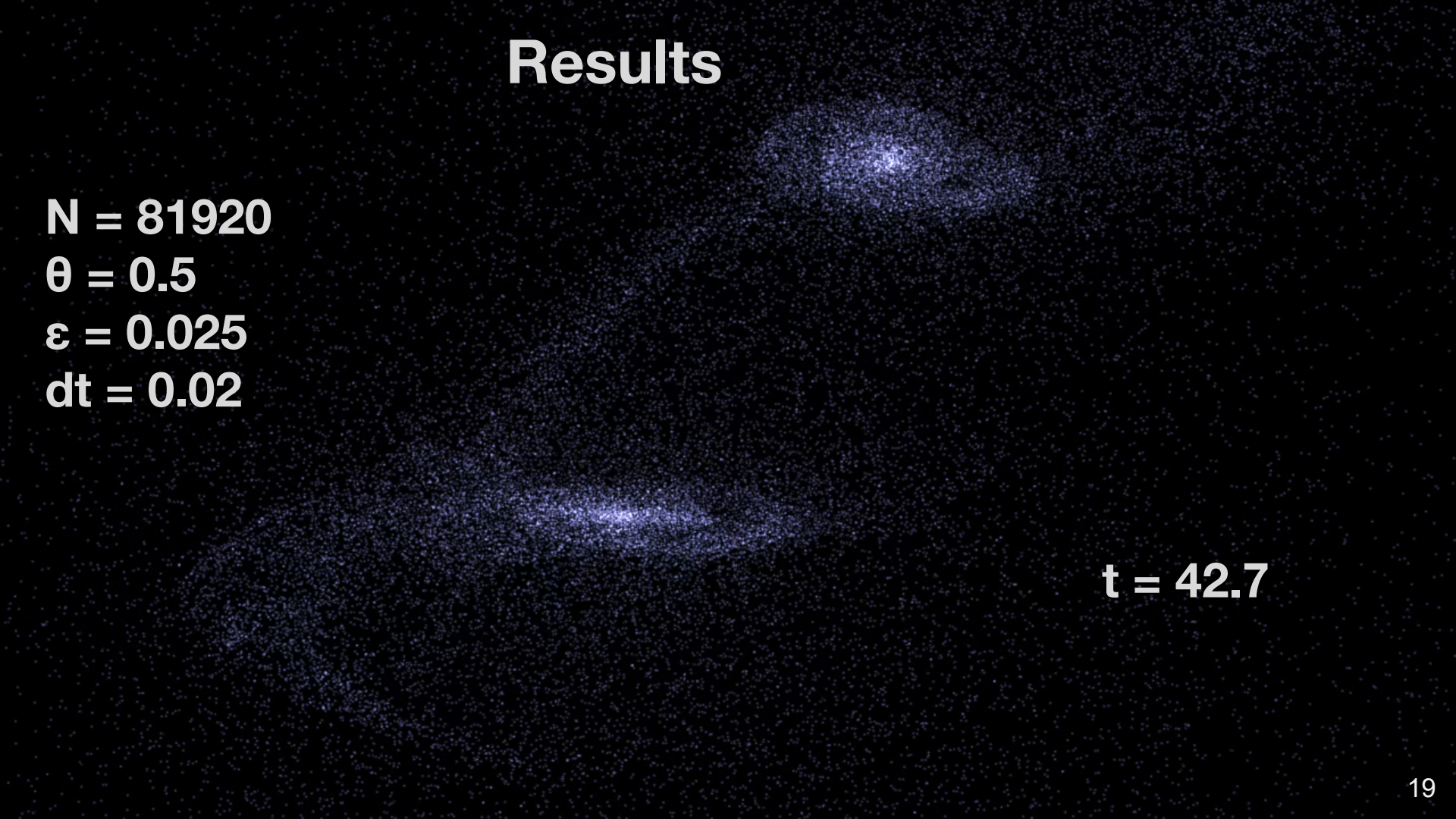
Results

N = 81920
θ = 0.5
ε = 0.025
dt = 0.02



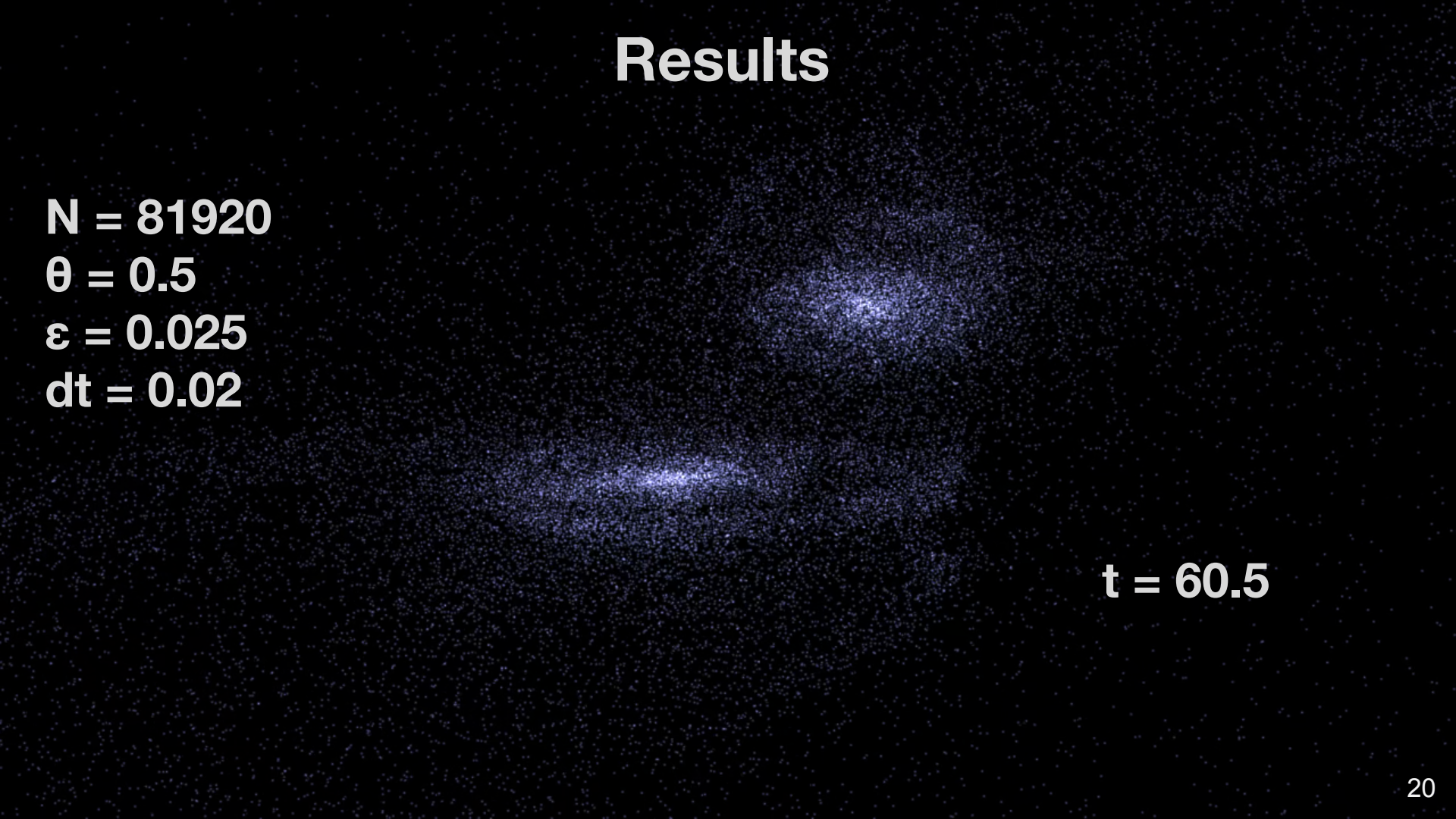
t = 21.3

Results



Results

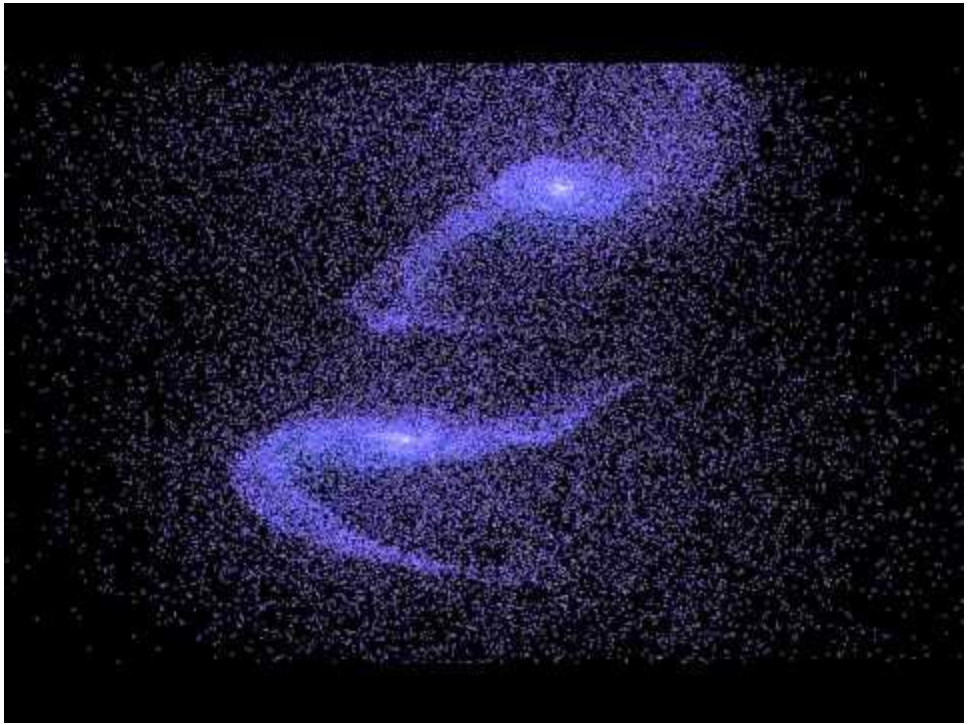
N = 81920
θ = 0.5
ε = 0.025
dt = 0.02



t = 60.5

Results: animation

N = 81920
θ = 0.5
ε = 0.025
dt = 0.02



References

> Barnes, Josh, and Piet Hut. "A hierarchical O (N log N) force-calculation algorithm." Letters to Nature, (1986): 446-449.

> Aarseth, S. J., M. Henon, and Roland Wielen. "A comparison of numerical methods for the study of star cluster dynamics." Astronomy and Astrophysics 37 (1974): 183-187.