Optimization project :

Julien BEUZE, Médéric DE CIBEINS

January 2026

1 Introduction

Healthcare systems around the world are under growing pressure. This is due to rising patient demand, tight budgets, and ongoing staff shortages. Nurses are crucial in maintaining quality care, so their scheduling is a major challenge. Hospitals need to ensure they have enough staff, meet legal and contractual requirements, and support staff well-being, all while keeping costs under control.

Nurse scheduling involves assigning a limited number of nurses to shifts over a specific period. This task is complicated by various constraints, such as minimum coverage needs, mandatory rest periods, limits on consecutive working days, and balancing workloads among nurses. Additional factors like individual preferences, fairness, and skill compatibility add to the problem's complexity. Poorly designed schedules can lead to heavy workloads, staff dissatisfaction, increased absenteeism, and ultimately lower quality of patient care.

These challenges become even more complex in real hospital environments. Patient demand can change due to seasonal trends, emergencies, or other unexpected events. This makes fixed scheduling solutions unstable. Schedules that do not consider such variability often require costly last-minute adjustments, overtime work, or temporary staffing. All of these factors hurt operational efficiency and staff morale.

Traditional scheduling methods, often based on manual planning or simple rules, find it hard to manage this level of complexity and uncertainty. Because of this, there is an increasing interest in optimization methods that can effectively handle large-scale scheduling issues while directly addressing operational constraints and performance trade-offs.

This project focuses on the Nurse Rostering Problem and explores optimization-based approaches for nurse scheduling in hospitals. By using integer linear programming techniques, the goal is to create schedules that balance staffing costs, operational viability, and fairness, while also considering demand uncertainty. This work aims to enhance the understanding of the strengths and weaknesses of mathematical optimization methods in real-world healthcare workforce planning.

2   Related Work

The Nurse Rostering Problem (NRP) has been widely studied in the operations research literature because of its practical importance and its high level of complexity. The problem consists in assigning nurses to work shifts over a given planning horizon while satisfying many operational, legal, and human constraints. Burke et al. [2] provide a well-known overview of nurse rostering problems and show that real hospital settings involve a wide variety of constraints, making the scheduling task difficult to manage.

Exact optimization methods, and in particular integer linear programming (ILP), are commonly used to model and solve nurse scheduling problems. These methods allow a clear and precise representation of hard constraints such as minimum staffing levels and rest regulations. They can also include softer aspects such as fairness or preferences. Ernst et al. [3] show that ILP models are able to generate high-quality schedules and guarantee optimal solutions. However, they also note that these methods can become difficult to solve when the problem size increases.

To overcome these limitations, many researchers have proposed heuristic and metaheuristic approaches, such as tabu search or genetic algorithms. These methods are usually faster and more scalable, but they do not guarantee optimal solutions and often require careful parameter tuning. Burke et al. [1] review these approaches and highlight the trade-off between solution quality and computational efficiency.

More recent studies have focused on introducing uncertainty into nurse rostering models. Some approaches use stochastic or robust optimization techniques to deal with uncertain patient demand or staff availability. The goal is to produce schedules that remain feasible even when conditions change, although this often leads to more conservative solutions.

In this project, we rely on classical ILP formulations of the Nurse Rostering Problem and adopt a simple and transparent optimization approach. Rather than developing a new heuristic, we focus on model clarity, fairness, and robustness. This allows us to clearly analyze the strengths and limitations of exact optimization methods for realistic nurse scheduling problems.

3   Methodology

In this project, we address the Nurse Rostering Problem by formulating it as an Integer Linear Programming (ILP) model. The main objective is to generate a feasible and balanced monthly schedule while strictly respecting hospital operational constraints. The choice of an exact optimization model is motivated by the fact that some constraints, such as minimum staffing levels or rest after night shifts, cannot be violated in real hospital settings.

The planning horizon corresponds to one month, composed of 28 consecutive days. Each day is divided into three shifts: Day, Evening, and Night. This structure is representative of standard hospital organization and allows us to

capture the main scheduling difficulties while keeping the model manageable.

3.1 Decision Variables

To model nurse assignments, we define a binary decision variable:

$$x_{n,d,s} = \begin{cases} 1 & \text{if nurse } n \text{ is assigned to shift } s \text{ on day } d, \\ 0 & \text{otherwise.} \end{cases}$$

This variable directly represents the core decision of the problem: assigning nurses to shifts on specific days. Its binary nature makes the solution easy to interpret and link to real schedules.

3.2 Constraints

Several constraints are introduced to reflect real-world hospital requirements. First, *coverage constraints* ensure that each shift is staffed by a minimum number of nurses every day, guaranteeing continuity and safety of patient care.

Second, *daily workload constraints* limit each nurse to at most one shift per day, preventing unrealistic or unsafe assignments.

Third, *monthly workload limits* impose a maximum number of shifts per nurse over the planning horizon. This constraint promotes fairness and avoids excessive workload differences between nurses.

Availability constraints are also included to model vacations. Nurses cannot be assigned to shifts during their declared days off, ensuring that personal constraints are respected.

Finally, a *rest-after-night* constraint is introduced. Because night shifts are particularly demanding, a nurse who works a night shift cannot be assigned to a day or evening shift on the following day. This improves schedule realism and protects staff well-being.

3.3 Objective Function

The objective is to minimize the total number of assigned shifts:

$$\min \sum_{n,d,s} x_{n,d,s}.$$

This objective helps distribute work evenly while ensuring that all coverage constraints are satisfied. Rather than focusing on explicit cost minimization, the model emphasizes feasibility, balance, and schedule regularity.

3.4 Implementation

The model is implemented in Python using the PuLP library and solved with the CBC solver. The program is interactive: users can either use predefined realistic constraints or manually enter their own constraints through the console.

The final output includes a colored monthly schedule, detailed statistics, and graphical analyses, allowing for both qualitative and quantitative evaluation of the solution.

4   Results and Discussion

The proposed integer linear programming model was tested on a monthly nurse rostering instance with a planning horizon of 28 days and a staff of 15 nurses. Three daily shifts were considered: Day, Evening, and Night. For all tested configurations, the solver was able to find feasible solutions that satisfy all hard constraints, including minimum coverage requirements, workload limits, rest regulations, and vacation constraints.

The results show that the model successfully ensures full coverage of all shifts while avoiding overloading individual nurses. The maximum number of shifts per nurse is strictly respected, and no nurse is assigned more than one shift per day. In addition, an explicit limitation on the number of night shifts per nurse guarantees that demanding night work is not concentrated on a small subset of the staff.

The rest-after-night constraint has a noticeable impact on the structure of the schedules. By preventing nurses from working day or evening shifts immediately after a night shift, the model produces more realistic and acceptable schedules from a human perspective, even though this reduces scheduling flexibility.

The interactive design of the implementation allows users to easily modify constraints such as staffing levels, planning horizon length, workload limits, or vacation periods. This makes it possible to explore different scenarios and observe their impact on feasibility and workload distribution. In all tested cases, the optimization process completed within short computation times, which confirms that exact optimization methods are suitable for nurse rostering problems of moderate size.

The generated schedules are displayed using a color-coded console table, which improves readability and interpretation. This visualization makes it easy to identify working patterns, rest days, and night shift assignments. In addition, several quantitative indicators are computed to support the analysis of solution quality.

Workload statistics per nurse show that the number of assigned shifts is well balanced across the staff. Bar charts illustrating total workload and night shift distribution confirm that no nurse is significantly overworked and that night shifts are fairly shared. A global shift distribution analysis also highlights a reasonable balance between Day, Evening, and Night shifts over the planning horizon.

Compared to manual scheduling or heuristic-based approaches commonly used in practice, the proposed ILP formulation provides stronger feasibility guarantees and clearer constraint handling. While heuristic methods such as tabu search can scale better to very large instances, they do not guarantee op-

timality and often lack transparency. In contrast, the ILP approach offers a structured, interpretable, and reproducible framework that is well suited for academic analysis and decision support.

Nevertheless, the model has some limitations. Individual nurse preferences and skill differences are not explicitly considered, and uncertainty in patient demand is only indirectly handled through conservative coverage constraints. These limitations suggest potential extensions, such as the inclusion of soft constraints, preference modeling, or robust optimization techniques, which could further improve the practical applicability of the approach.

5 Conclusion

In this project, we studied the Nurse Rostering Problem using an optimization-based approach. The goal was to generate feasible and realistic monthly schedules while respecting key operational constraints such as minimum staffing levels, workload limits, rest requirements, and vacation periods.

We proposed an integer linear programming formulation that explicitly models these constraints and guarantees that all generated schedules are feasible. The model was implemented in Python using the PuLP library and was able to produce balanced schedules within short computation times. All shifts were fully covered, workload limits were respected, and rest-after-night constraints led to more realistic working patterns.

Compared to manual scheduling or heuristic approaches, the proposed method provides stronger feasibility guarantees and better transparency, as each constraint has a clear interpretation. This makes the model easy to understand, analyze, and adapt.

Some limitations remain, such as the absence of individual preferences, skill differences, and explicit modeling of uncertainty. These aspects could be addressed in future work. Overall, this project shows that integer linear programming is a suitable and reliable tool for nurse rostering problems of moderate size.

The full implementation of the model as well as the final report are available on GitHub at:

`https://https://github.com/medericdecib/Nurse-rostering-OR-project-Julien-Mederic`

References

[1] Edmund K. Burke, Patrick De Causmaecker, Greet Vanden Berghe, and Hendrik Van Landeghem. A survey of nurse rostering problems and solution methods. *Journal of Scheduling*, 13(1):1–31, 2010.

[2] Edmund K. Burke, Patrick De Causmaecker, Greet Vanden Berghe, and Hendrik Van Landeghem. The nurse rostering problem. *European Journal of Operational Research*, 153(1):1–20, 2004.

[3] Andreas T. Ernst, Houyuan Jiang, Mohan Krishnamoorthy, and Dario Sier. Staff scheduling and rostering: A review of applications, methods and models. *European Journal of Operational Research*, 153(1):3–27, 2004.