1. Update C_i from multinomial distribution, for $k = 1, \ldots, K$

$$\Pr(C_i = k|-) = \frac{\exp(Z_i\beta_k + S_i\alpha_k)\prod_{j_1=1}^{p_1} N(\log Y_{ij_1}; X_i, Z_i, \theta_k^{j_1})\prod_{j_2=1}^{p_2}\psi_{kX_{ij_2}}^{j_2}}{\sum_{k=1}^{K}\exp(Z_i\beta_k + S_i\alpha_k)\prod_{j_1=1}^{p_1} N(\log Y_{ij_1}; X_i, Z_i, \theta_k^{j_1})\prod_{j_2=1}^{p_2}\psi_{kX_{ij_2}}^{j_2}}.$$

2. Update θ_k. Denote $Z_i^* = (X_i, Z_i)$, and introduce normal-InvGamma prior $N(0, \Sigma_0) - IG(1, 1)$

$$f(\theta_k^{j_1}|-) = N\{(\sum_{i:C_i=k} Z_i^{*'} Z_i^* + \Sigma_0^{-1})^{-1}\sum_{i:C_i=k} Z_i^{*'}\log Y_{ij_2}, (\sum_{i:C_i=k} Z_i^{*'} Z_i^* + \Sigma_0^{-1})^{-1}\}$$

$$f(\sigma_k^2|-) = IG(\frac{\sum_i I(C_i = k)}{2} + 1, \frac{\sum_{i:C_i=k}(\log Y_{ij_2} - Z_i^*\theta_k^{j_1})^2}{2} + 1)$$

3. Update ψ_k under prior Dirichlet$(a_1^{j_2}, \ldots, a_{d_{j_2}}^{j_2})$

$$(\psi_k^{j_2}|-) \sim \text{Dirichlet}(a_1^{j_2} + \sum_{i:C_i=k} I(X_{ij_2} = 1), \ldots, a_{d_{j_2}}^{j_2} + \sum_{i:C_i=k} I(X_{ij_2} = d_{j_2}))$$

4. Update (α_k, β_k) under prior $N(0, \Sigma_{0,\alpha})$ and $N(0, \Sigma_{0,\beta})$, with Metropolis-Hasting algorithm, for $k = 1, \ldots, K-1$. Denote $\pi_k(\beta_k, \alpha_k) = \frac{\exp(Z_i\beta_k + S_i\alpha_k)}{\sum_{k=1}^{K}\exp(Z_i\beta_k + S_i\alpha_k)}$, and the multinomial likelihood

$$l(\pi_k) = \prod_{h=1}^{k-1}\pi_h^{\sum_{i=1}^N I(C_i=h)}\pi_k^{\sum_{i=1}^N I(C_i=k)}\prod_{h=k+1}^{K-1}\pi_h^{\sum_{i=1}^N I(C_i=h)}(1 - \sum_{h\neq k}\pi_h - \pi_k)^{\sum_{i=1}^N I(C_i=K)}$$

The new $\alpha_k^* \sim N(\alpha_k^{t-1}, I)$ will be accepted with probability, where α_k^{t-1} is the value at iteration $t-1$,

$$\min\{\frac{l(\pi_k(\beta_k, \alpha_k^*))N(\alpha_k^*; , 0, \Sigma_{0,\alpha})}{l(\pi_k(\beta_k, \alpha_k^{t-1}))N(\alpha_k^{t-1}; , 0, \Sigma_{0,\alpha})}, 1\},$$

which is simplified as

$$\min\{\frac{\pi_k(\beta_k, \alpha_k^*)^{\sum_{i=1}^N I(C_i=k)}(1 - \sum_{h\neq k}\pi_h - \pi_k(\beta_k, \alpha_k^*))^{\sum_{i=1}^N I(C_i=K)}N(\alpha_k^*; , 0, \Sigma_{0,\alpha})}{\pi_k(\beta_k, \alpha_k^{t-1})^{\sum_{i=1}^N I(C_i=k)}(1 - \sum_{h\neq k}\pi_h - \pi_k(\beta_k, \alpha_k^{t-1}))^{\sum_{i=1}^N I(C_i=K)}N(\alpha_k^{t-1}; , 0, \Sigma_{0,\alpha})}, 1\},$$

5. Impute missing Y_{ij_1}

$$(Y_{ij_1}|-) \sim N(\log Y_{ij_1}; X_i, Z_i, \theta_{C_i}^{j_1})$$

6. Impute missing X_{ij_2}

$$(X_{ij_2}|-) = \text{Multinomial}(\psi_{C_i1}^{j_2}, \ldots, \psi_{C_id_{j_2}}^{j_2}),$$