PART - I CONVERGENCE IN PROBABILITY

1. a) $f_Y(y) = \frac{1}{2b} e^{-\left(\frac{|y-\mu|}{b}\right)}$

Given: $\mu = 0, \; b = 5$

$$\Rightarrow \quad f_Y(y) = \frac{1}{10} e^{-\frac{|y|}{5}}$$

$$E(Y) = \int_{-\infty}^{\infty} y \cdot f(y) \, dy$$

$$= \int_{-\infty}^{0} y \cdot \frac{1}{10} e^{y/5} \cdot dy + \int_{0}^{\infty} y \cdot \frac{1}{10} e^{-y/5} \cdot dy$$

$$= \frac{1}{10} \left[\int_{-\infty}^{0} y \cdot e^{y/5} \cdot dy + \int_{0}^{\infty} y \cdot e^{-y/5} \cdot dy \right]$$

$$= \frac{1}{10} \left\{ \left[(5y - 25) e^{y/5} \right]_{-\infty}^{0} + \left[(-5y - 25) e^{-y/5} \right]_{0}^{\infty} \right\}$$

$$= \frac{1}{10} \left[-25 + 25 \right]$$

$$\Rightarrow \boxed{E(Y) = 0}$$

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 \cdot f(y) dy = \int_{-\infty}^{0} y^2 \cdot \frac{1}{10} e^{y/5} dy + \int_{0}^{\infty} y^2 \cdot \frac{1}{10} \cdot e^{-y/5} \cdot dy$$

$$= \frac{1}{10} \left[250 + 250 \right]$$

$$= \frac{1}{10} \times 500$$

$$\boxed{E(Y^2) = 50}$$

$\Rightarrow E(Y^2)$ exists.

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$E(Y) = 0$, $E(Y^2) = 50$, $\Rightarrow$ $Var(Y) = 50$ by variance computing formula

$$L = \frac{1}{n}\sum_{i=1}^{n} Y_i^2$$

By LLN, $\frac{1}{n}\sum_{i=1}^{n} Y_i \xrightarrow{P} E(Y_i)$

By ~~continuity theorem~~ generalization of LLN, $\frac{1}{n}\sum_{i=1}^{n} Y_i^2 \xrightarrow{P} E(Y_i^2) = 50$

$$\Rightarrow \boxed{L \xrightarrow{P} 50}$$

b) $L \xrightarrow{P} 50$

By continuity theorem, $K = \sqrt{L} \xrightarrow{P} \sqrt{50}$

$$\Rightarrow \boxed{K \xrightarrow{P} \sqrt{50}}$$