ST501 Group Project

Our Group

7/20/2018

Introduction

Part 1 Convergences

A

$$f_Y(y) = \frac{1}{2b}e^{-(\frac{|y-\mu|}{b})}$$

Given: $\mu = 0$ and $b = 5$

$$E(Y) = \int_{-\infty}^{\infty} y * f(y)dy$$

$$= \int_{-\infty}^{0} y * \frac{1}{10}e^{\frac{y}{5}} + \int_{0}^{\infty} y * \frac{1}{10}e^{\frac{-y}{5}}$$

$$= \frac{1}{10}\left[\int_{-\infty}^{0} ye^{\frac{y}{5}} + \int_{0}^{\infty} y * e^{\frac{-y}{5}}\right]$$

$$= \frac{1}{10}\left[\left[(5y-25)e^{\frac{y}{5}}\right]_{-\infty}^{0} + \left[(-5y-25)e^{\frac{-y}{5}}\right]_{0}^{\infty}\right]$$

$$= \frac{1}{10}\left[-25+25\right]$$

$$E(Y) = 0 \blacksquare$$

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 * f(y)dy$$

$$= \frac{1}{10}\left[\int_{-\infty}^{0} y^2e^{\frac{y}{5}} + \int_{0}^{\infty} y^2 * e^{\frac{-y}{5}}\right]$$

$$= \frac{1}{10} * \left[250+250\right]$$

$$E(Y^2) = 50 \ \blacksquare$$

Therefor $E(Y^2)$ exists.

Thus:

$E(Y) = 0$, $E(Y^2) = 50$ then $Var(Y) = 50$ by the variance computing formula.

$$L = \frac{1}{n}\sum_{i=1}^{n} Y_i^2$$

By the Law of Large Numbers:

$$\frac{1}{n}\sum_{i=1}^{n} Y_i \underset{\rightarrow}{p} E(Y_i)$$

By generalisation of the Law of Large Numbers:

$$L = \frac{1}{n}\sum_{i=1}^{n} Y_i^2 \underset{\rightarrow}{p} E(Y_i^2) = 50$$

Therefore:

$$L \underset{\rightarrow}{p} 50 \blacksquare$$

B

By the continuity theorem, $K = \sqrt{L} \underset{\rightarrow}{p} \sqrt{50}$

Thus: $K \underset{\rightarrow}{p} \sqrt{50}$.

C

Derrive the CDF of the double exponential distribution.

$$f_Y(y) = \frac{1}{2b} * e^{-(\frac{|y-\mu|)}{b})}$$

This yields two cases for the CDF:

$$F_Y(y) = \begin{cases} \int_{-\infty}^{t} f_Y(Y)dt & \text{for } y \leq \mu \\ \int_{t}^{\infty} f_Y(Y)dt & \text{for } y \geq \mu \end{cases}$$

Solving for these leaves us with the following:

For $y \leq \mu$:

$$\int_{-\infty}^{y} f_Y(Y)dt$$

$$\int_{-\infty}^{y} \frac{1}{2b} * e^{-(\frac{t-\mu}{b})}dt$$

We know that in this case we are solving for when the exponent is positive.

$$\int_{-\infty}^{y} \frac{1}{2b} * e^{(\frac{|t-\mu|)}{b})}dt$$

$$\frac{1}{2b} * e^{(\frac{t-\mu}{b})} * b|_{-\infty}^{y} = \frac{1}{2}e^{(\frac{t-\mu}{b})} - 0 = \frac{1}{2}e^{(\frac{t-\mu}{b})}$$

For the case where $y \geq \mu$

$$\int_{-\infty}^{y} f_Y(Y)dt$$

$$\int_{-\infty}^{y} \frac{1}{2b} * e^{-(\frac{t-\mu}{b})}dt$$

This must be further split into:

$$\int_{-\infty}^{\mu} \frac{1}{2b} * e^{-(\frac{t-\mu}{b})}dt + \int_{\mu}^{y} \frac{1}{2b} * e^{-(\frac{t-\mu}{b})}dt$$

$$\frac{1}{2b} * e^{(\frac{-t+\mu}{b})} * -b|_{-\infty}^{\mu} + \frac{1}{2b} * e^{(\frac{-t+\mu}{b})} * -b|_{\mu}^{y}$$

$$\frac{1}{2} - \frac{1}{2}e^{\frac{\mu-y}{b}} + \frac{1}{2}$$

Thus for this case:

$$1 - \frac{1}{2}e^{\frac{\mu-y}{b}}$$

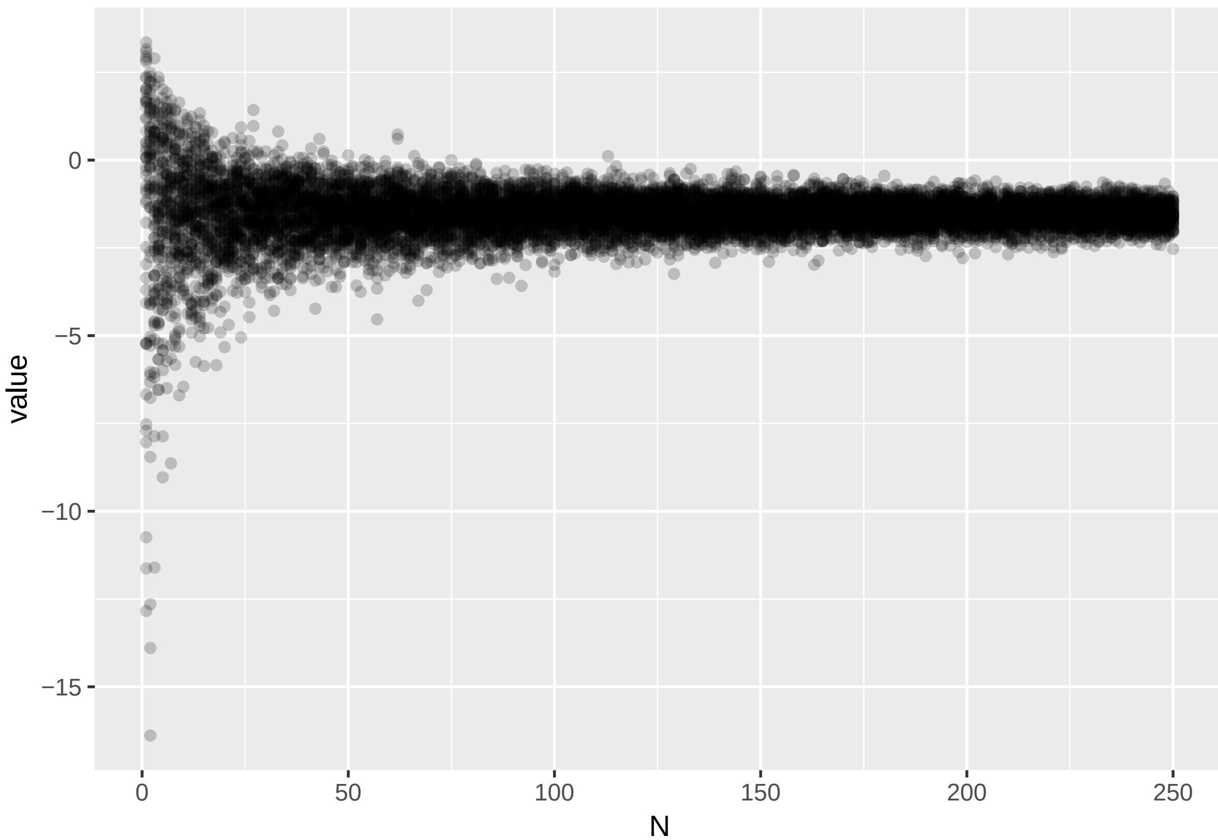
In conclusion the CDF for the double exponential distribution is:

$$F_Y(y) = \begin{cases} \frac{1}{2}e^{(\frac{y-\mu}{b})} & \text{for } y \leq \mu \\ 1 - \frac{1}{2}e^{\frac{\mu-y}{b}} & \text{for } y \geq \mu \end{cases}$$

D

```
set.seed(336)

rdoublex <- function(u, mu = 0, b = 5){
  if( u > 0){
    mu + b*log(2*u)
  }else {
     mu - b*log(2-2*u)
  }
}


output_matrix <- matrix(0, nrow= 250, ncol = 50)

for( j in 1:50){
  for(i in 1:250){
    random_value <- runif(i, 0, 1)
    output_matrix[i, j]<- mean(vapply(random_value, FUN = rdoublex, double(1)))
  }
}

output_matrix %>%
  as.data.frame() %>%
```

```
mutate(N = 1:250) %>%
gather(replication, value, -N) %>%
ggplot(aes(N, value))+
geom_point(alpha=1/5)
```



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