ST501 Group Project

Michael DeWitt

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Introduction

Part 1 Convergences

A

B

C

Derrive the CDF of the double exponential distribution.

$$f_Y(y) = \frac{1}{2b} * e^{-(\frac{|y-\mu|)}{b})}$$

This yields two cases for the CDF:

$$F_Y(y) = \begin{cases} \int_{-\infty}^{t} f_Y(Y)dt & \text{for } y \leq \mu \\ \int_{t}^{\infty} f_Y(Y)dt & \text{for } y \geq \mu \end{cases}$$

Solving for these leaves us with the following:

For $y \leq \mu$:

$$\int_{-\infty}^{y} f_Y(Y)dt$$

$$\int_{-\infty}^{y} \frac{1}{2b} * e^{-(\frac{t-\mu}{b})}dt$$

We know that in this case we are solving for when the exponent is positive.

$$\int_{-\infty}^{y} \frac{1}{2b} * e^{(\frac{|t-\mu|)}{b})}dt$$

$$\frac{1}{2b} * e^{(\frac{t-\mu}{b})} * b|_{-\infty}^{y} = \frac{1}{2}e^{(\frac{t-\mu}{b})} - 0 = \frac{1}{2}e^{(\frac{t-\mu}{b})}$$

For the case where $y \geq \mu$

$$\int_{-\infty}^{y} f_Y(Y)dt$$

$$\int_{-\infty}^{y} \frac{1}{2b} * e^{-(\frac{t-\mu}{b})}dt$$

This must be further split into:

$$\int_{-\infty}^{\mu} \frac{1}{2b} * e^{-(\frac{t-\mu}{b})} dt + \int_{\mu}^{y} \frac{1}{2b} * e^{-(\frac{t-\mu}{b})} dt$$

$$\frac{1}{2b} * e^{(\frac{-t+\mu}{b})} * -b|_{-\infty}^{\mu} + \frac{1}{2b} * e^{(\frac{-t+\mu}{b})} * -b|_{\mu}^{y}$$

$$\frac{1}{2} - \frac{1}{2} e^{\frac{\mu - y}{b}} + \frac{1}{2}$$

Thus for this case:

$$1 - \frac{1}{2} e^{\frac{\mu - y}{b}}$$

In conclusion the CDF for the double exponential distribution is:

$$F_Y(y) = \begin{cases} \frac{1}{2} e^{(\frac{y-\mu}{b})} & \text{for } y \leq \mu \\ 1 - \frac{1}{2} e^{\frac{\mu - y}{b}} & \text{for } y \geq \mu \end{cases}$$

D

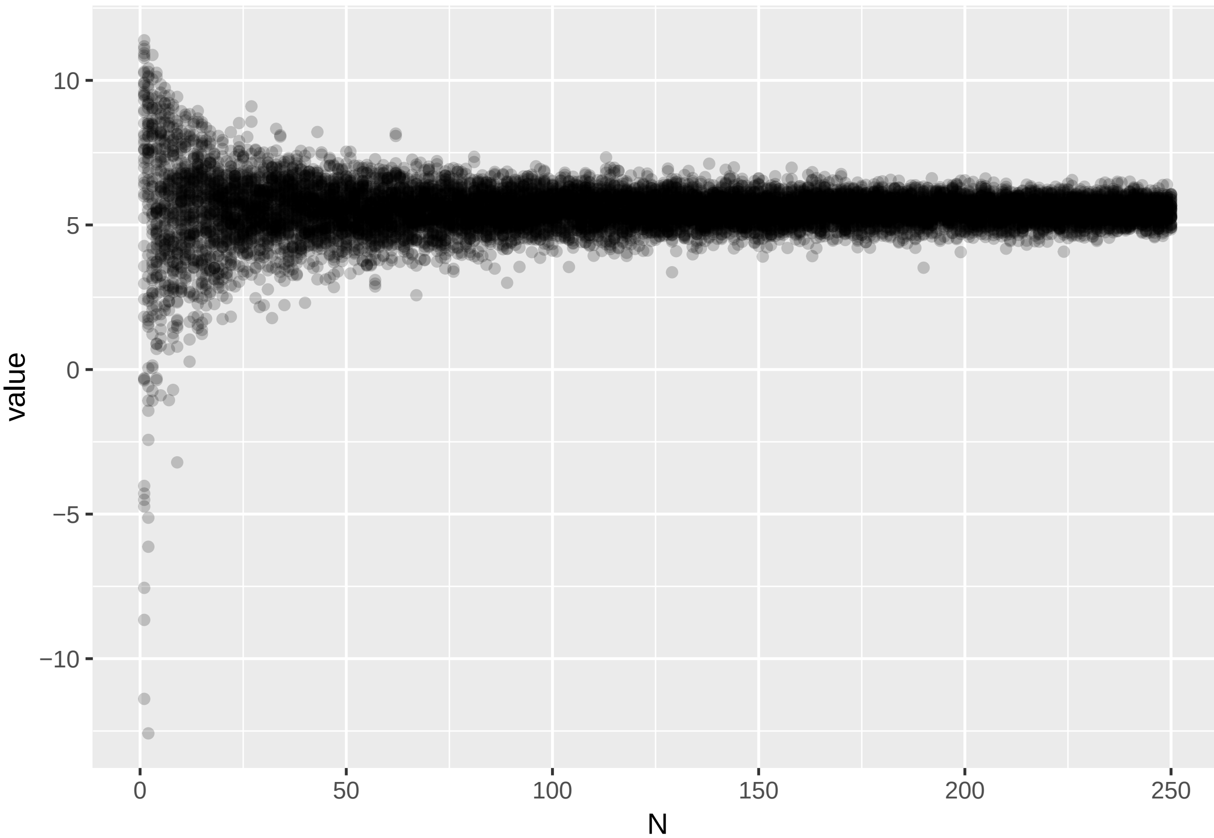
```
set.seed(336)

rdoublex <- function(u, mu = 0, b = 5){
  if( u > 0){
    mu + b*log(2*u)
  }else {
     mu - b*log(2-2*u)
  }
}


output_matrix <- matrix(0, nrow= 250, ncol = 50)

for( j in 1:50){
  for(i in 1:250){
    random_value <- runif(i, -.5, 5)
    output_matrix[i, j]<- mean(vapply(random_value, FUN = rdoublex, double(1)))
  }
}

output_matrix %>%
  as.data.frame() %>%
  mutate(N = 1:250) %>%
  gather(replication, value, -N) %>%
  ggplot(aes(N, value))+
  geom_point(alpha=1/5)
```



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