NOTES ON $S(\vec{Q},\omega)$ FOR MY CODE

FROM DOVE E.25

$$S(\vec{Q},\omega) = \int \underbrace{\langle \rho(\vec{Q},0)\,\rho(-\vec{Q},t)\rangle}_{F(\vec{Q},t)} e^{-i\omega t}\,dt$$

$$F(\vec{Q},t) = \int \rho(\vec{Q},t')\,\rho(-\vec{Q},t'+t)\,dt'$$

WITH $f(\omega) = \int f(t)\,e^{-i\omega t}\,dt$, $f(\omega) = \frac{1}{2\pi}\int f(\omega)\,e^{i\omega t}\,d\omega$

$$F(\vec{Q},t) = \frac{1}{(2\pi)^2}\int\left(\iint \rho(\vec{Q},\omega')\,\rho(-\vec{Q},\omega'')\,e^{i\omega' t'}\,e^{i\omega''(t'+t)}\,d\omega'\,d\omega''\right)dt'$$

$$= \frac{1}{2\pi}\iint \rho(\vec{Q},\omega')\,\rho(-\vec{Q},\omega'')\,e^{i\omega'' t}\underbrace{\left(\frac{1}{2\pi}\int e^{i(\omega''+\omega')t'}\,dt'\right)}_{\delta(\omega''+\omega')\,\to\,\omega''=-\omega}d\omega'\,d\omega''$$

$$F(\vec{Q},t) = \frac{1}{2\pi}\iint \rho(\vec{Q},\omega')\,\rho(-\vec{Q},\omega'')\,e^{i\omega'' t}\,\delta(\omega''+\omega')\,d\omega'\,d\omega''$$

$$= \frac{1}{2\pi}\int \rho(\vec{Q},\omega')\,\rho(-\vec{Q},-\omega')\,e^{-i\omega' t}\,d\omega'$$

WITH $\rho(\vec{Q},\omega) = \int \sum_j b_j\, e^{i\vec{Q}\cdot\vec{r}_j(t)}\,e^{-i\omega t}\,dt$, $\rho(-\vec{Q},-\omega) \equiv \rho^*(\vec{Q},\omega)$

$$S(\vec{Q},\omega) = \frac{1}{2\pi}\iint |\rho(\vec{Q},\omega')|^2\, e^{-i(\omega'+\omega)t}\,dt\,d\omega'$$

$$= \int |\rho(\vec{Q},\omega')|^2 \underbrace{\left(\frac{1}{2\pi}\int e^{-i(\omega'+\omega)t}\,dt\right)}_{\delta(\omega'+\omega)} d\omega'$$

SIGN?

$$\therefore\ S(\vec{Q},\omega) = \int |\rho(\vec{Q},\omega')|^2\,\delta(\omega'+\omega)\,d\omega' = |\rho(\vec{Q},-\omega')|^2$$