

NOTES ON $S(\vec{Q}, \omega)$ FOR MY CODE

FROM DOVE E.25

$$S(\vec{Q}, \omega) = \int \underbrace{\langle p(\vec{Q}, 0) p(-\vec{Q}, t) \rangle}_{F(\vec{Q}, t)} e^{-i\omega t} dt$$

$$F(\vec{Q}, t) = \int p(\vec{Q}, t') p(-\vec{Q}, t' + t) dt'$$

with $f(\omega) = \int f(t) e^{-i\omega t} dt$, $f(\omega) = \frac{1}{2\pi} \int f(\omega') e^{i\omega' t} d\omega'$

$$\begin{aligned} F(\vec{Q}, t) &= \frac{1}{(2\pi)^2} \int \left(\int \int p(\vec{Q}, \omega') p(-\vec{Q}, \omega'') e^{i\omega' t'} e^{i\omega''(t'+t)} d\omega' d\omega'' \right) dt' \\ &= \frac{1}{2\pi} \int \int p(\vec{Q}, \omega') p(-\vec{Q}, \omega'') e^{i\omega'' t} \underbrace{\left(\frac{1}{2\pi} \int e^{i(\omega'' + \omega') t'} dt' \right)}_{\delta(\omega'' + \omega') \rightarrow \omega'' = -\omega} d\omega' d\omega'' \end{aligned}$$

$$\begin{aligned} F(\vec{Q}, t) &= \frac{1}{2\pi} \int \int p(\vec{Q}, \omega') p(-\vec{Q}, \omega') e^{i\omega' t} \delta(\omega'' + \omega') d\omega' d\omega'' \\ &= \frac{1}{2\pi} \int p(\vec{Q}, \omega') p(-\vec{Q}, -\omega') e^{-i\omega' t} d\omega' \end{aligned}$$

with $p(\vec{Q}, \omega) = \int \sum_j b_j e^{i\vec{Q} \cdot \vec{r}_j(t)} e^{-i\omega t} dt$, $p(-\vec{Q}, -\omega) = p^*(\vec{Q}, \omega)$

$$S(\vec{Q}, \omega) = \frac{1}{2\pi} \int \int |p(\vec{Q}, \omega')|^2 e^{-i(\omega' + \omega)t} dt d\omega'$$

$$= \int |p(\vec{Q}, \omega')|^2 \underbrace{\left(\frac{1}{2\pi} \int e^{-i(\omega' + \omega)t} dt \right)}_{\delta(\omega' + \omega)} d\omega'$$

sgn?

$$\therefore S(\vec{Q}, \omega) = \int |p(\vec{Q}, \omega')|^2 \delta(\omega' + \omega) d\omega' = |p(\vec{Q}, -\omega)|^2$$