A Novel Variation of Method of Moments Enables Its Teaching Together with Infinitesimal Dipole

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***Abstract*—Enabling students to devise their own algorithm and write their own codes to simulate a half-wave thin wire dipole antenna as early as such a concept is introduced in the classroom leads to proper understanding of how the current distribution is formed and how the input impedance is actually computed. The conventional method of moments (MoM) relies on more advanced concepts such as Pocklington's Integral and the field of a current which is on the outer surface of a cylinder. In this paper, a variation of the MoM is presented in which the only electromagnetic knowledge required is the near field of an infinitesimal dipole antenna and the fact that the electric tangent to a conductor must be zero. Furthermore, it is explained that the algorithm can be broken down to smaller pieces and ChatGPT or GPT4 can be used to implement those algorithms in the programming language of your choice. So students are provided with a real opportunity to write their own codes to simulate a dipole antenna, helping them obtain a deeper understanding of the problem.**

***Index Terms*—IEEE, IEEEtran, journal, LaTeX, paper, template.**

I. Introduction

The evergrowing comprehensiveness of the antenna simulation tools have slowly decreased the need to understand the electromagnetic theory and computational science behind these tools. Gradually students, graduate students and engineers are getting accustomed to modify the input model and then simply analyze the results. Although great insight can be obtained following this path, however deeper understanding of the theory and computational side can only be achieved by studying numerical methods and writing your own codes. This deeper knowledge will be of immense benefit when taking on unforeseen problems. However, the greatest benefit lies in the fact that as a result of getting familiar with computational electromagnetics, students, engineers, and enthusiasts are then enabled to use open source tools and codes to simulate simpler problems. A development which was previously impossible without having to pay the hefty prices for EM simulators.

Historically there have been two major obstacles when it comes to educating students on the topic of computational electromagnetics. One was the difficulty of the advanced methods such as Pocklington's integral equation. Another was the time and effort required to learn a programming language to an acceptable level. Both obstacles seem to have been resolved.

This paper presents a modification to the conventional method of moments where instead of the rather difficult Pocklington's integral equations and the current being distributed on the outer surfaces of the wire cylinder, the wire dipole is simply considered as many infinitesimally small dipole antennas whose fields are easy to understand and are amongst the first topics taught in an antenna course. As in the conventional MoM, there are limits in terms of the wire diameter and the number segments into which the wire is divided.

The present paper also presents a new approach to the use of programming languages. Until the recent days, it has been necessary to obtain a decent knowledge of and experience in a programming language *prior to* using them in the field of computational electromagnetics. That is no longer the case. With the advent of artificial intelligence (AI) and machine learning (ML) and tools such as ChatGPT and GPT4, one can start using the programming languages without having to master them at first. Although further down the road, certain knowledge of the tool might be required, having no knowledge at the beginning no longer stops you from using it. The author has shown **??** that as long as the algorithm is broken into smaller pieces and described accurately enough, ChatGPT can provide computer codes which are usually an excellent start.

Paper organization is described there.

II. Conventional Method of Moments Applied to a Half-Wave Dipole

The method of moments [**?**] is essentially a way of solving the following equation.

$$\int_{x'=a}^{b} f(x')g(x,x')dx' = h(x); \quad a \leq x \leq b \tag{1}$$

One common and perhaps one of the simplest ways to apply 1 to a half wavelength thin wire antenna with a radius of a and shown in Fig. 1a is presented in [**?**]. They use what [**?**] determined for the z component of the electric field of a current evenly distributed on the surface of a cylinder which is observed at the centre and given by

$$E_z\left(z\right) = \int_{z'=-l/2}^{l/2} \int_{\phi'=0}^{2\pi} J_z\left(z'\right) g\left(z,z\right) a d\phi dz' \tag{2}$$

where $R = \sqrt{a^2 + (z-z')^2}$ and

$$g\left(z,z'\right) = \frac{-je^{-jkR}}{\omega\epsilon 4\pi R^5}\left[\left(1+jkR\right)\left(2R^2-3a^2\right)+\left(kaR\right)^2\right]. \tag{3}$$

Then E_z is assumed to be zero everywhere on the surface of the wire except at the centre which is the excitation point. Consequently, $J_z(z)$ and thus $I_z(z)$ is determined.

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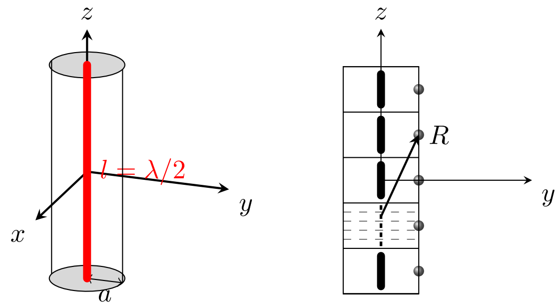

(a) Case I (b) Case II

Fig. 1. Simulation results for the network.

If the intention is to enable students in the antenna engineering course to experience writing their own, however small, simulation codes for a halfwave dipole as soon as possible and perhaps immediately after teaching the infinitesimally small dipole antenna, this method is too advanced.

Ref. [**?**] mentions that the electric field of the surface current described above and observed at the centre is equivalent to the field of a thin Hertzian dipole positioned at the centre and observed on the surface if the line and surface currents are related by $I = 2\pi a J$. However it is clarified that they haven't pursued this idea in determining the current distribution on the wire. This idea is simple and the only electromagnetic knowledge required to teach it is the Hertzian dipole which is perhaps one of the first concepts taught in an antenna course. Using the Hertzian dipole concept instead of (3) is one of the two key points which enables students to start writing their own simulation codes. Let's call this method of forces (MoF).

III. Method of Forces

Fig. 1b shows the lateral view the wire dipole in the yz plane. This halfwave antenna typically has a length of $l = \lambda/2$ and is divided into N segments with the horizontal solid lines. This means that we have N infinitesimally small or Hertzian dipole antennas. Each Hertzian dipole has a length of $\Delta Z = l/N$ and is depicted by thick vertical lines. The goal is determine the electric field tangent to the surface of the wire and equate that to zero. We do so by assuming $E_z = 0$ at the N observation points shown by small dots. Considering that $N = 2k' + 1$ is an odd number, the k'th observation point is where the voltage source is applied and hence $E_z(k'+1)$ is equal to $V_0/\Delta Z$. So we have N unknowns that is the currents and N equations.

So the N hertzian dipoles are located at $\mathbf{r_p} = (0, 0, -l/2 + l/(2N) + pl/N)$ where $p = 0, 1, 2, \ldots, N-1$, the N observation points are at $\mathbf{r_i} = (0, a, -l/2 + l/(2N) + il/N)$ where $i = 0, 1, 2, \ldots, N-1$. The distance between $\mathbf{r_p}$ and $\mathbf{r_i}$ is denoted by $R(p, i)$. In a coordinate system moved and centred on $\mathbf{r_p}$, the electric field of this pth Herztian dipole which is now considered at the centre of the coordinate system at the ith observation point is given by

$$E_r(p, i) = \frac{\eta I_p (l/N) \cos\theta}{2\pi r^2} \left[1 + \frac{1}{jkr}\right] e^{-jkr} \quad (4)$$

$$E_\theta(p, i) = \frac{j\eta k I_p (l/N) \sin\theta}{4\pi r} \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right] e^{-jkr} \quad (5)$$

To satisfy the boundary condition we need to determine the z component of the electric field and ensure its overall value is zero. The tangent component at the ith observation point due to the Hertzian dipole at pth point is determined by

$$E_z(p, i) = \cos\theta E_r - \sin\theta E_\theta \quad (6)$$

The boundary condition at the ith point is satisfied by

$$\sum_{p=1}^{N} E_z(p, i) = 0 \quad (7)$$

which has N unknowns: $I_p, p = 1, 2 \ldots N$. Repeating Eq. (7) for all $i = 1, 2, \ldots, N$ will yield N equations.

At first glance it would appear that we are done and simply need to solve the matrix equation of

$$G \times I = E \quad (8)$$

where G is an N by N matrix and its $G(p, i)$ value is given by $E_z(p, i)/I_p$ and I is an N by 1 matrix whose values, I_p, are to be computed and finally E is also is an N by 1 matrix whose values are zero except the middle value which is $E(k'+1) = V_0/\Delta Z$

However there is a numerical issue with this equation. With the appropriate ratios of l and a and N, it would seem that the above-mentioned description of $E_z(p, i)$ is not a fair representation of the field of the pth current segment at the ith observation point when $p = i$ which will form the largest and thus the most important numbers in the G matrix. When $p = i$, then $R(p, i) = a$. Since a is a very small number, the slightest change in the assumed position of the pth Hertzian dipole will cause change of orders of magnitude. One would naturally increase N by an order of magnitude to overcome this issue, but any large increase in N will lead to two problems. Firstly, the computational cost of solving Eq. (8) increases dramatically by increasing N and secondly more attention must be paid to the accuracy of the solution as with the increase of N, the matrix G will have more and more unevenly distributed values, mostly very small and some very large, making it difficult to solve Eq. (8) accurately.

A solution to a similar issue is deployed in the Fortran codes provided by [**?**]. While we keep the number of unknown currents the same, N, we can consider much smaller Hertzian dipoles. For calculating $E_z(p, i)$, we divide the pth segment, into M smaller subsegments. Each small dipole in these subsegments will have the same current of I_p, causing the electric field $e_z(m, p, i)$ at the ith observation point. So the overall $E_z(p, i)$ is the sum of the effect of all these subsegments at the same point i and is given by

$$E_z(p, i) = \sum_{m=1}^{M} e_z(m, p, i) \quad (9)$$