We have as input a proper level Graph $G = (V, E, l)$ with a level ordering l and an index order and equivalence classes based on the 2-SAT formulation of level planar embedding presented by Randerth et al [RSB+01].

For the second part of the algorithm, we will construct a graph G' from scratch, where G' is a subgraph of G containing all the vertices v processed so far.

We traverse the graph level by level. Let the variable h be the current level. We look at each vertex v in the level h, the algorithm has two main tasks inside the loop of the vertex v, synchronization of adjacent vertices to vertex v and synchronization of the connected components that were merged after adding v to G'.

For the first part, we consider the incident vertices to the vertex v on the level immediatley above h. We look for an ordering $\mathcal{V}$ of the incident vertices such that, for every pair of vertices $(u, w) \in \mathcal{V}$ with $u < w$ all the equivalence classes corresponding to these pairs point in the same direction.

We update each equivalence class of each pair (u, v) of the ordering $\mathcal{V}$ (with $u < w$) with the union of each equivalence class associated to each pair (u, v) of the ordering $\mathcal{V}$ with $u < w$ in the ordering. We update also the equivalence classes of the opposite order for synchronization in the same way. We do the same for the incident vertices to the vertex v in the level immediately below to h.

for the second part, we add v to G' and the edges incident to v with vertices from $G' - \{v\}$. If the vertex v is a cut-vertex in G', we also look for the connected components in $G' - \{v\}$, we pick an arbitrary order of the connected comps (C_1, C_2) we then force the relation of a vertex w_1 of the connected component C_1 with its level $l(w_1) = h - 1$ to each vertex w_2 in the connected component C_2 on the same level $l(w_1)$ to be the same, that's by assiging for each relation (w_1, w_2) the union of all the equivelant classes between a fixed w_1 and all $w_2 \in C_2$ with $l(w_1) = l(w_2) = h$. We also synchronize the equivalence classes of the opposite order (w_2, w_1) with w_1 still fixed. If C_2 encapsulate C_1 then we inverse the pair before forcing the relations.

References

[RSB+01] Bert Randerath, Ewald Speckenmeyer, Endre Boros, Peter Hammer, Alex Kogan, Kazuhisa Makino, Bruno Simeone, and Ondrej Cepek. A satisfiability formulation of problems on level graphs. *Electronic Notes in Discrete Mathematics*, 9:269–277, 2001.