DLQR application

We will work on the example of a car that wants to reach its destination.

To simplify, the car has an initial speed v_0(in m/s) in the direction of the road and an initial distance x_0 (in m) from the destination. The engine can consume 1 unit of energy to change the speed by 1 m/s. In this case, the car loses 2% of its speed per second due to friction.

Time will be divided into units of 1 second. We will denote it as t_i.

The operation will unfold over Ns.

The system must act on the engine in such a way that the penalties P are minimized. The penalties are obtained in the following manner:

Variable	Penaty
x_{t_i}	$a * {x_{t_i}}^2$
x_{t_N}	$b * {x_{t_N}}^2$
v_{t_i}	$c * {v_{t_i}}^2$
v_{t_N}	$d * {v_{t_N}}^2$
u_{t_i}	$e * {u_{t_i}}^2$

such as $a, b, c, d \geq 0; e > 0$

We can easly verify the following equations:

$$x_{t_{i+1}} = x_{t_i} - v_{t_i}$$

$v_{t_{i+1}} = 0.997 * v_{t_i} + u_t$. such that u_t is the amouth of energie consumed by the engine.

$$P = \sum_{i=0}^{N-1}\left(a * {x_{t_i}}^2 + c * {v_{t_i}}^2 + e * {u_{t_i}}^2\right) + b * {x_{t_N}}^2 + d * {v_{t_N}}^2$$

In order to solve the problem, we note $(X_{t_i})_{0\leq i\leq N} \in \mathbb{R}^{1*2}, X_{t_i} = \begin{bmatrix} x_t \\ v_t \end{bmatrix}$;

and the following matrixes:

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 0.98 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, Q = \begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}, Q_f = \begin{bmatrix} b & 0 \\ 0 & d \end{bmatrix}, R = [e]$$

From which we can conclude the following:

$$X_{t_{i+1}} = AX_{t_i} + Bu_{t_i}$$

$$P = X_{t_N}{}^T Q_f X_{t_N} + \sum_{i=0}^{N} X_{t_i}{}^T Q X_{t_i} + u_{t_i}{}^T R u_{t_i}$$

and $Q = Q^T \geq 0 \; et \; R = R^T > 0$

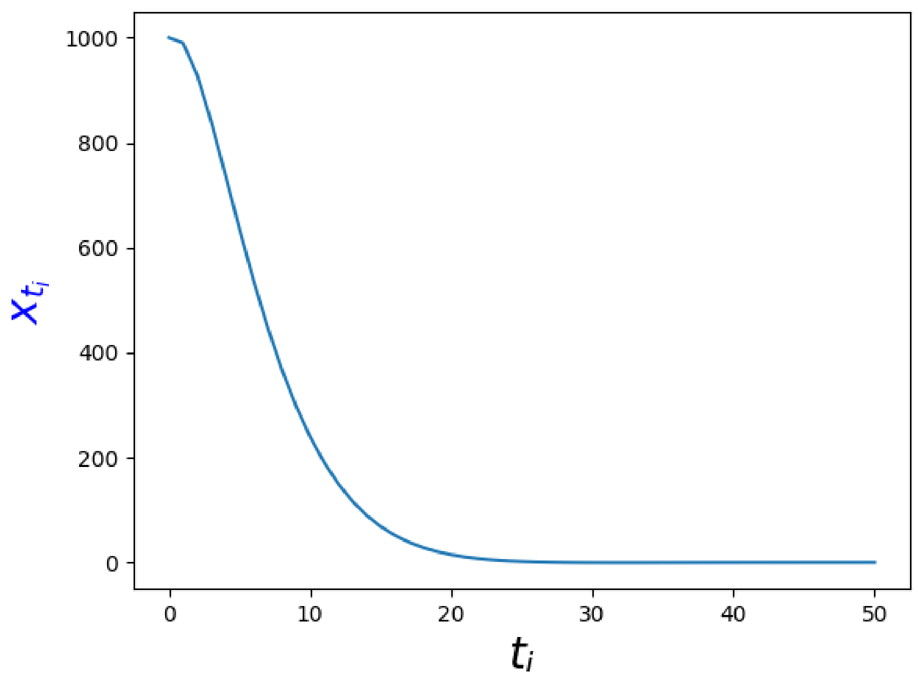
So we can apply DLQR.

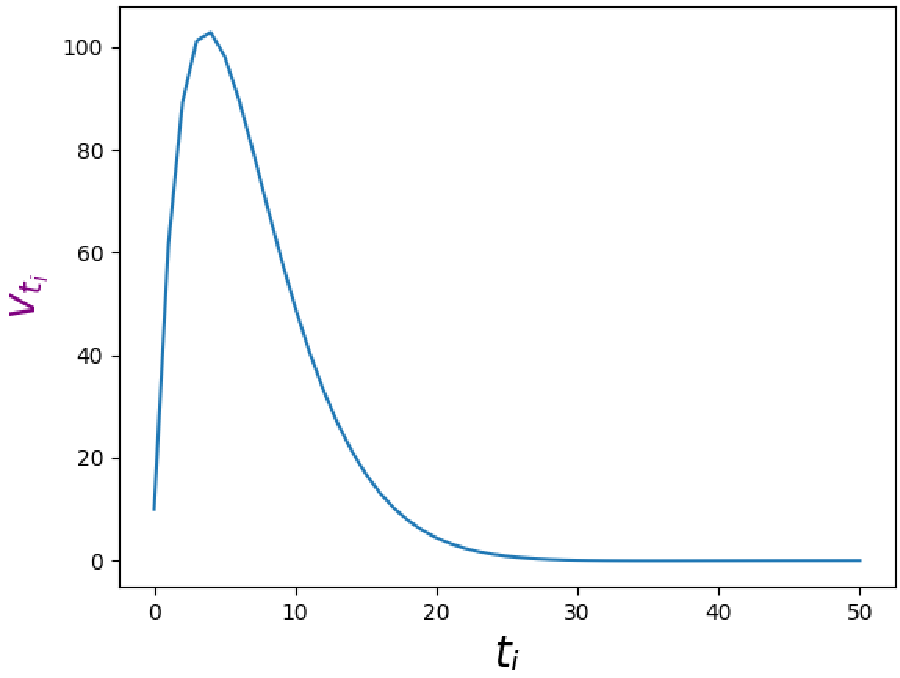
Numerical values:

$$a = 0.05, b = 1000, c = 1, d = 1000000, e = 10$$

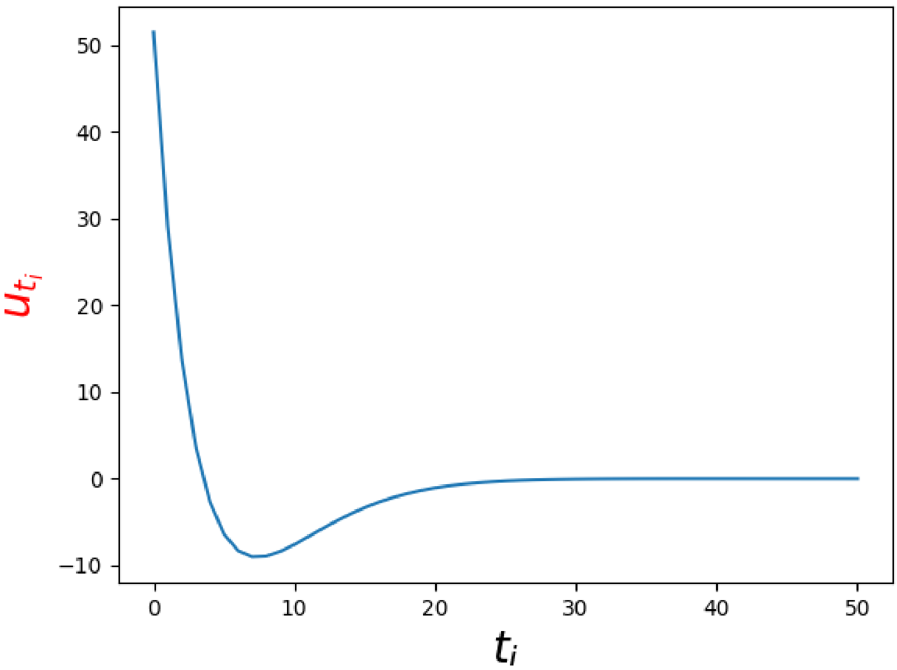
$$x_0 = 1000, v_0 = 10$$

Results:





v_{t_i}
t_i



u_{t_i}
t_i