## SUM Approximation (LSTM) NN, Nano-Level Structure

Since the SUM Approximation NN is the LSTM Network, the Network on -the nano level is interpreted as the LSTM cell see the figure below. The defined uses as inputs the two contemporaneous bits of X1 and X2 and output k-th bit of Y in the Y=X1+X2. The cell inputs and outputs are the variables in R. The detailed description of the LSTM cell shown in the figure below.

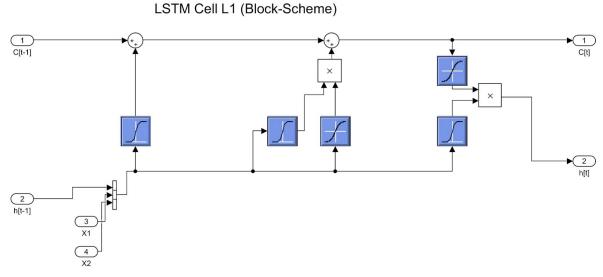


Fig. 1.6 XOR and AND Neural Networks, nano-level

Let us get focused how the higher level is methodologically described.

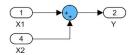
LSTM Network L0 (Block-Scheme)

## SUM Approximation (LSTM) NN, Micro-Level Structure

The structure of the LSTM Network is displayed in the block-schemes below. See the L0 and L1). From the two block-schemes shown below, the 32-bit input of the X1 and X2 are transferred to 32-bit output Y (in this case the bit equivalents in R are considered).

## 4 2 Y[t=31] Y[t=1] (10) C[t] C[t] C[t-1] C[t] h[t] 9 h[t] h[t] LSTM Cell[t=1] h[t] LSTM Cell[t=2] LSTM Cell [t] LSTM Cell [t=31] 7 $\overline{1}$ (3 ( 5 X1[t=0] X1[t=1] X1[t] X1[t=31] 2 6 (4) (8) X2[t=0] X2[t=31]

Fig. 2.1. SUM Neural Networks, LSTM Cells aggregated (micro-level)



SUM (As displayed)

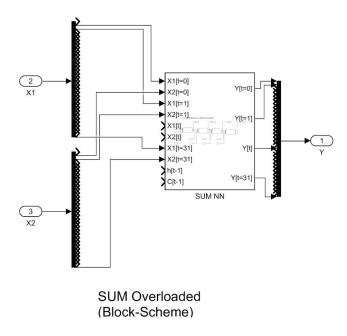


Fig. 2.2. SUM Neural Networks, LSTM Cells aggregated (view outside the block) From the bloch-schemes in Fig. 1.8-1.9 the submission is performed from lower to higher bits using the 32 cell LSTM Neural Network with 1-to-1 relation, same as it is done by using bitwise addition by column method.

## SUM Approximation (LSTM) NN, Results

The LSTM model is trained accordingly methodology: (the higher bits are obtained from the lower bits in the LSTM 1-to-1 sequences, the (two) 32 bit sequences X1 and X2 in the input gives 32 sequence Y in the output. The data is trained over the 10^5 sequences (may be reduced). The perfect accuracy (100%) is obtained on the both training and validation sets.

The (two integer number) SUM operator is overload using the LSTM Neural Network. Perfect accuracy (100%) reached in the both training and test sets on small number of observations.

ос	h   Iteration	Time Elapsed	Mini-batch	Validation	Mini-batch	Validation	Base Learnin	g l			·
	i I	(hh:mm:ss)   Accuracy   Accuracy				Loss	1	Loss	I	Rate	Ι.
	======= 1	 1	00:00:44	======= 51.46%	50.02%	0.6954	0.7041	0.0100			
	1 j	50	00:02:38	59.57%	60.13%	0.6926	0.6926	0.0100 j			
	2 j	100	00:04:33	68.07%	72.43%	0.6145	0.6086	0.0100			
	3	150 j	00:06:32	99.98%	100.00%	0.0521	0.0471	0.0100			
	3	200	00:08:57	100.00%	100.00%	0.0045	0.0044	0.0100			
	4 j	220	00:09:36	100.00% j		1	0.0032 j		1	0.0100	

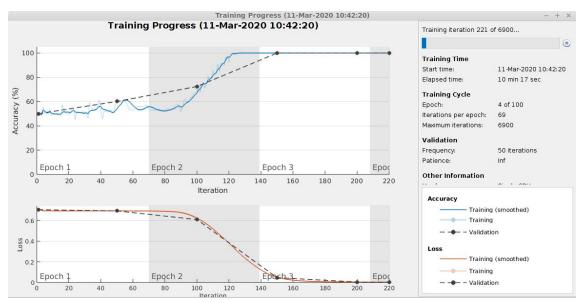


Fig. 2.3. SUM Neural Networks (LSTM), accuracy in the training and validation sets (perfect match)