Ziegler-Nichols Tuning Exercises

Ziegler-Nichols tuning consist of two methods. These methods are followed in order below (1-2 for first method and 3 for second method).

1) Apply Ziegler-Nichols method to tune PID controller parameters for the given plant below:

$$G = \frac{10}{s+10}\,\frac{12}{s+12}$$

<u>Procedure:</u> (note that, in different sources, obtaining the parameters from plots may differ, or the required parameters can be a,K,L, so that PID controller parameter transform tables can show differences, but the results are approximately the same, any method can be used)

- Plot the step response of the plant (open loop system, shown in Figure 1) and find the required parameters below referencing the plot in Figure 2, fill Table 1 (Hint: to find out the time values from plot, it is easier to plot the graph using Matlab script, or the plot values can be transferred to workspace using the block 'to Workspace' in Simulink, then the graph can be plotted by script coding, Simulink Scope plot does not support 'line' feature for tangent line drawing)

  Note: Alternative methods are also supplied in Figure 3 and Figure 4.

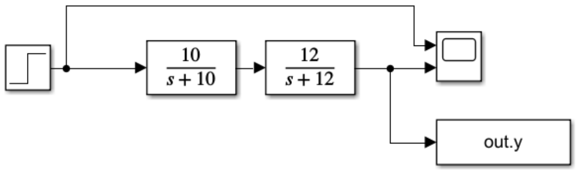

Figure 1 First method plot model

Table 1 First method parameters

L	T

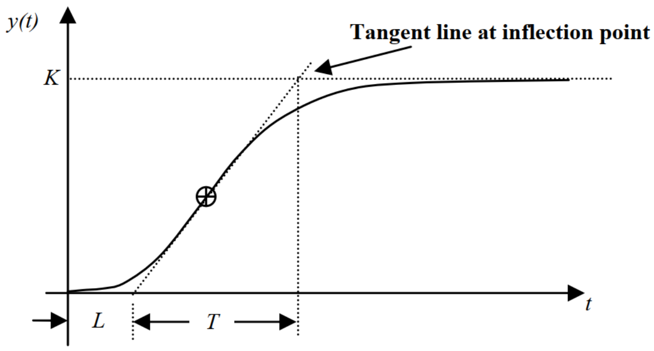

Figure 2 Obtaining L-T parameters from plot in first method

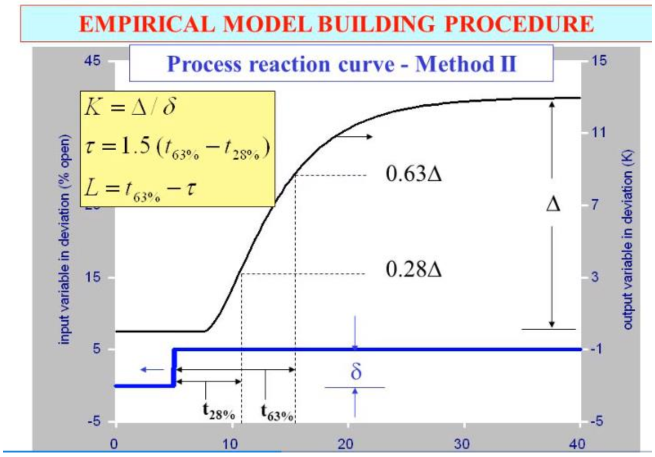

Figure 3 Alternative L-T parameter obtaining method

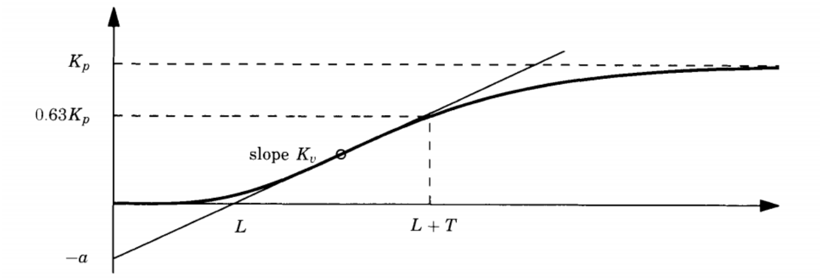

Figure 4 Method using a,L,T parameters

- Use Table 2 to find out the controller parameters (alternatively, Table 3 is also provided)

Table 2 Controller parameter calculations

Controller	Kp	Ti=Kp/Ki	Td=Kd/Kp
P	T/L	-	-
PI	0.9 T/L	L/0.3	-
PID	1.2 T/L	2 L	0.5 L

Table 3 Controller parameter calculations using a,L,T values

Controller	aK	T_i/L	T_d/L	T_p/L
P	1			4
PI	0.9	3		5.7
PID	1.2	2	L/2	3.4

- Design and apply the controller (the controller should be applied to the closed-loop system)

2) This first method can be also applied to first order systems with a time delay. First order systems with a time delay are recognized as one of referencing applications in literature (e.g., tank systems, temperature systems, etc.). The same parameter reading method over the plots can be also applied to these systems and all the steps are the same. Though, looking to the transfer function, the parameters can be also pulled directly.

Apply first Ziegler-Nichols method to tune PID controller parameters for the given plant below:

$$G = \frac{1}{s+1}\, e^{-0.2s}$$

Procedure:

- Find the parameters given above from the plant using the form given below and fill Table 4.

$$G(s) = \frac{Ke^{-sL}}{Ts+1}$$

Table 4 Plot parameters of first order system

K	T	L

- Use Table 5 to find out the controller parameters.

Table 5 Controller parameters

Controller	Kp	Ti=Kp/Ki	Td=Kd/Kp
P	T/L	-	-
PI	0.9 T/L	L/0.3	-
PID	1.2 T/L	2 L	0.5 L

- Design and apply the controller.

3) Second method (also named frequency response method) can be applied both empirically and analytically. Apply Ziegler-Nichols method to tune PID controller parameters

Procedure:

Empirical method (the system is assumed as a black box and the values are obtained experimentally):

- Start with a small Kp, and Ki=Kd=0
- Increase Kp until neutral stability (until the output of the system oscillates)
- Record critical/ultimate gain Ku=Kp at neutral stability
  Record critical/ultimate period oscillation Tu (seconds)
- Look up Kp, Ki, Kd from transform table in Table 6.

Table 6 Second method transform table

Controller type	Kp	Ti	Td	Ki	Kd
P	0.5 Ku	-	-	-	-
PI	0.45 Ku	0.8 Tu	-	0.54 Ku / Tu	-
PD	0.8 Ku	-	0.125 Tu	-	0.1 Ku Tu
PID	0.6 Ku	0.5 Tu	0.125 Tu	1.2 Ku / Tu	0.075 Ku Tu

Analytical method (if the transfer function of the system is known, then it is probable to find out PID parameters in less time, with less cost and more accurately):

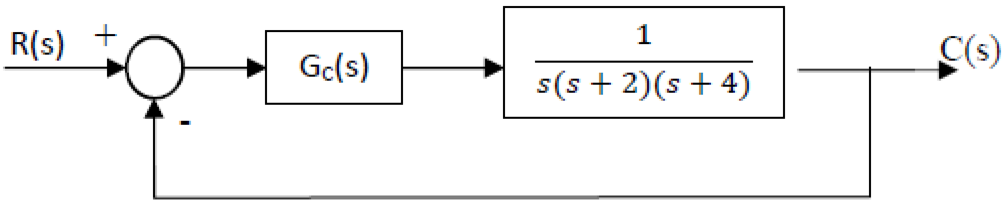

Figure 5 Second method application

- The system characteristic equation is obtained, assuming a P controller with Kcr value as in Figure 6.

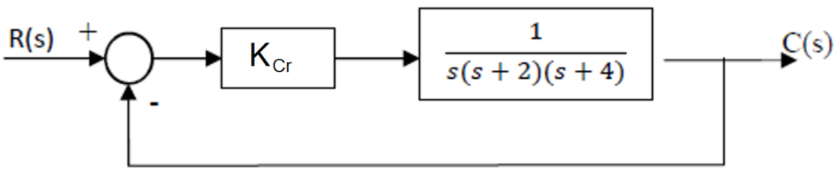

Figure 6 Obtaining critical gain

$$G(s) = \frac{C(s)}{R(s)} = \frac{K_{cr}}{s^3 + 6s^2 + 8s + K_{cr}}$$

- Routh-Hurwitz stable criterion will be used to find marginally stable gain

$$\begin{array}{c|cc} S^0 & B & 0 \\ S^1 & A & 0 \\ S^2 & 6 & K_{cr} \\ S^3 & 1 & 8 \\ \hline \end{array}$$

$A = 8 - \frac{K_{cr}}{6}$ and $B = K_{cr}$ $\rightarrow$ $K_{cr} \geq 0$ and $8 - \frac{K_{cr}}{6} \geq 0$ $\rightarrow$ $K_{cr} \leq 48$ $\rightarrow$ $K_{cr} = 48$

- Critical frequency can be calculated as below:

$$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + 48 = 0$$

$$-\omega^3 j - 6\omega^2 + 8\omega j + 48 = 0$$

$$(-6\omega^2 + 48) + (8\omega - \omega^3)j = 0$$

Considering the value that makes imaginary part

$$8\omega - \omega^3 = 0 \quad \rightarrow \quad \omega_n = 0 \quad or \quad \omega_n = \pm 2{,}8284 \text{ [rad/s]}$$

$$P_{cr} = \frac{2\pi}{\omega_n} \quad \rightarrow \quad P_{cr} = 2{,}2214 \text{ [s]}$$

- Ziegler-Nichols estimation table for PID;

$$K_p = 0{,}6\, K_{cr} \;, \quad T_i = 0{,}5\, P_{cr} \;, \quad T_d = 0{,}125\, P_{cr}$$

where $$K_p = K_p \;, \quad K_i = \frac{K_p}{0{,}5\, P_{cr}} \;, \quad K_d = K_p\, 0{,}125\, P_{cr}$$

$$\rightarrow \quad K_p = 28{,}8 \quad K_i = 25{,}93 \quad K_d = 7{,}99$$

<u>Matlab-Simulink verification:</u>

P Critical;

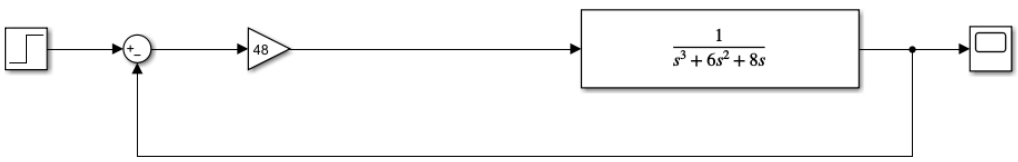

Figure 7 Critical K application

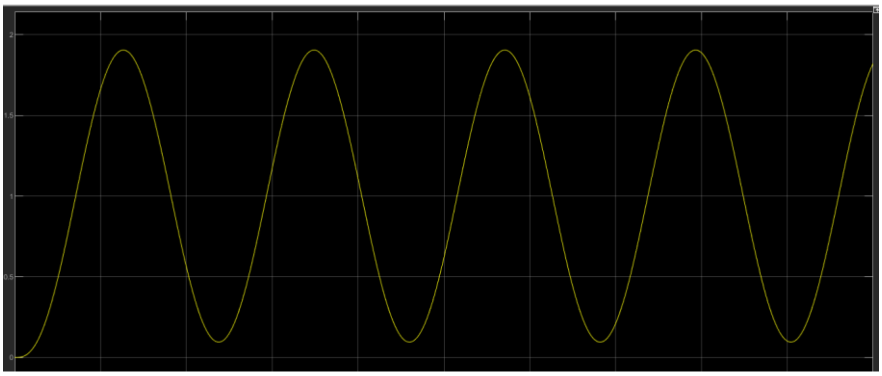

Figure 8 Critical K output

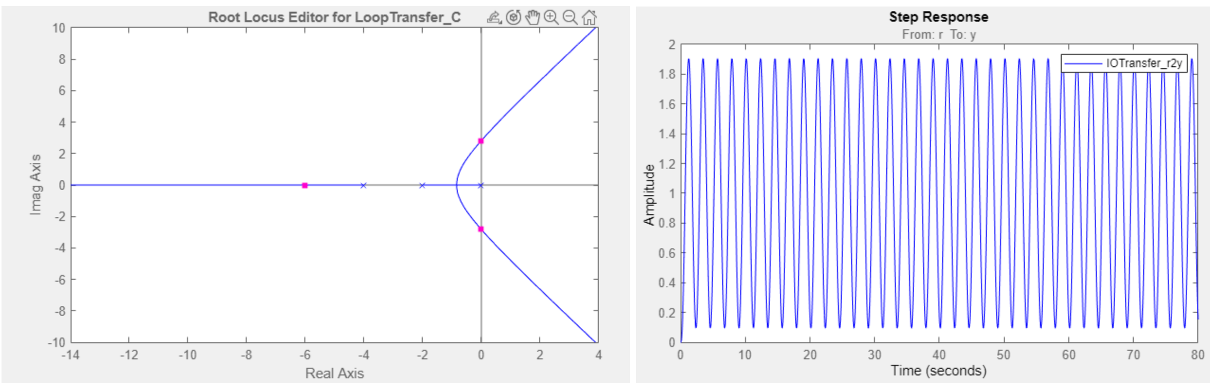

Figure 9 Roots and output of the system at K=48 (obtained using Matlab 'sisotool' function)

Simulink PID controller application;

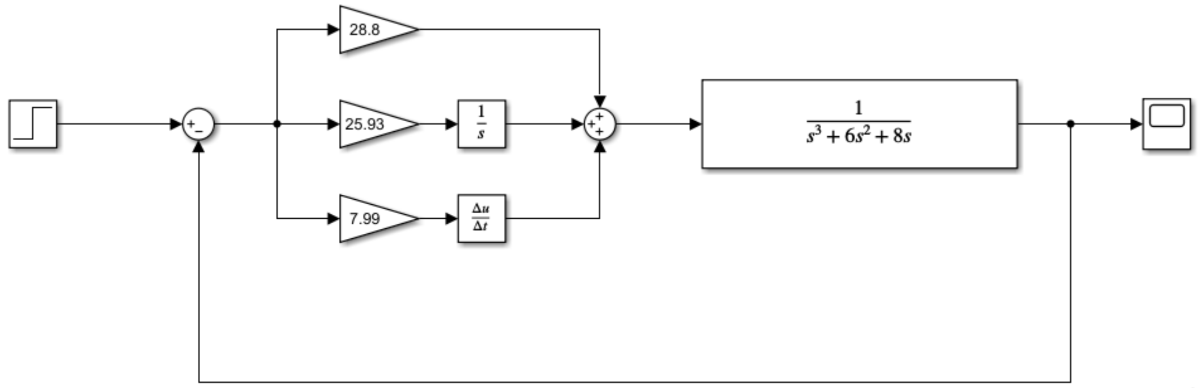

Figure 10 PID control application using the obtained values

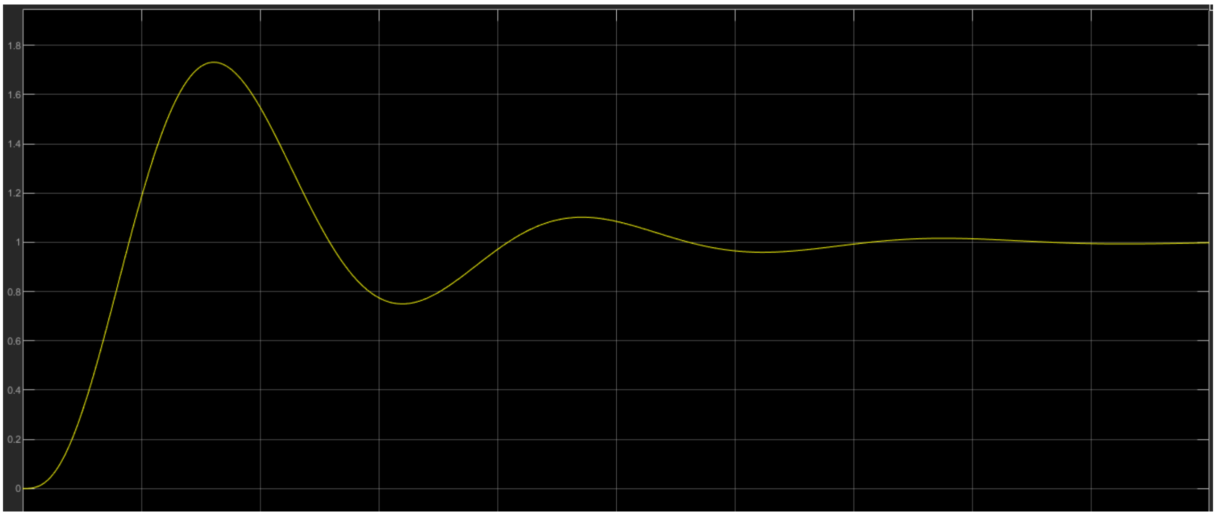

Figure 11 Output of the system using calculated parameters