

Test 1

Total: _____ / 150

Printed Name: _____ Demetrius Johnson _____ 3-8-2020 _____

GRADER/TA: *[CO for Question 1, P v NP, please record statistics]*

I, Demetrius Johnson, have neither given nor received assistance on this examination except
that which is provided by, or approved by, the instructor.

1. [15 pts] If an algorithm A is in the complexity class P, is it necessarily in the complexity class NP?

Explain why or why not.

Well, here is the million dollar question: Is $P \subseteq NP$? But, for this question, we do in fact know that P is a subset of P, it is only a matter of if P is a proper subset of NP. It could be that NP ~~has~~ problems that do not belong to class P.

Short Answer: yes If A is of class P, it is of class NP.

If the case were A is class NP, we could not necessarily say it is a class P problem. So $P \subset NP$, but NP ~~is~~ not necessarily in set P: $NP \subset P$?
If it were so, then $P \subseteq NP \Rightarrow NP \subseteq P$.

2. [15 pts] Given an alphabet $\Sigma = \{a, b, c\}$, list all strings from length 1 to 3 in the language

$L = a(a + b + c)^*$ Assume the empty string λ is length 0.

~~$L = a$~~

~~$L = a(a+b+c)$~~

~~$L = a + a(a+b+c)^1 +$~~

~~$L = a(a+b+c)^0(a+b+c)^1(a+b+c)^2 \dots$~~

~~$(a+b+c)^0 = a$~~

$$L = a[(a+b+c)^0 + (a+b+c)^1 + (a+b+c)^2 \dots]$$

$$(a+b+c)^0 = \lambda$$

$$(a+b+c)^1 = a + b + c$$

$$(a+b+c)^2 = (a+b+c)(a+b+c) = a^2 + ab + ac + ba + b^2 + bc + ca + cb + c^2$$

Solution: $a(a+b+c)^0 = \lambda$ — length = 0

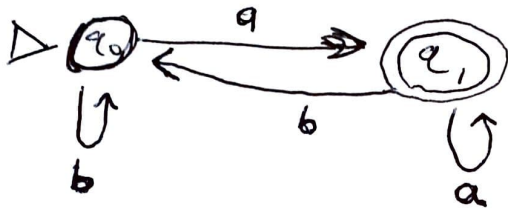
$$a(a+b+c)^1 = \underline{a}^2 + \underline{a}b + \underline{a}c$$

$$a(a+b+c)^2 = \underline{a}^3 + \underline{a}^2b + \underline{a}^2c + \underline{a}b^2 + \underline{a}b^2 + \underline{a}bc + \underline{a}ca + \underline{a}cb + \underline{a}c^2$$

Strings
 length 1-3

3. [15 pts] Draw a state transition diagram for the FA of regular language $L(r)$ where

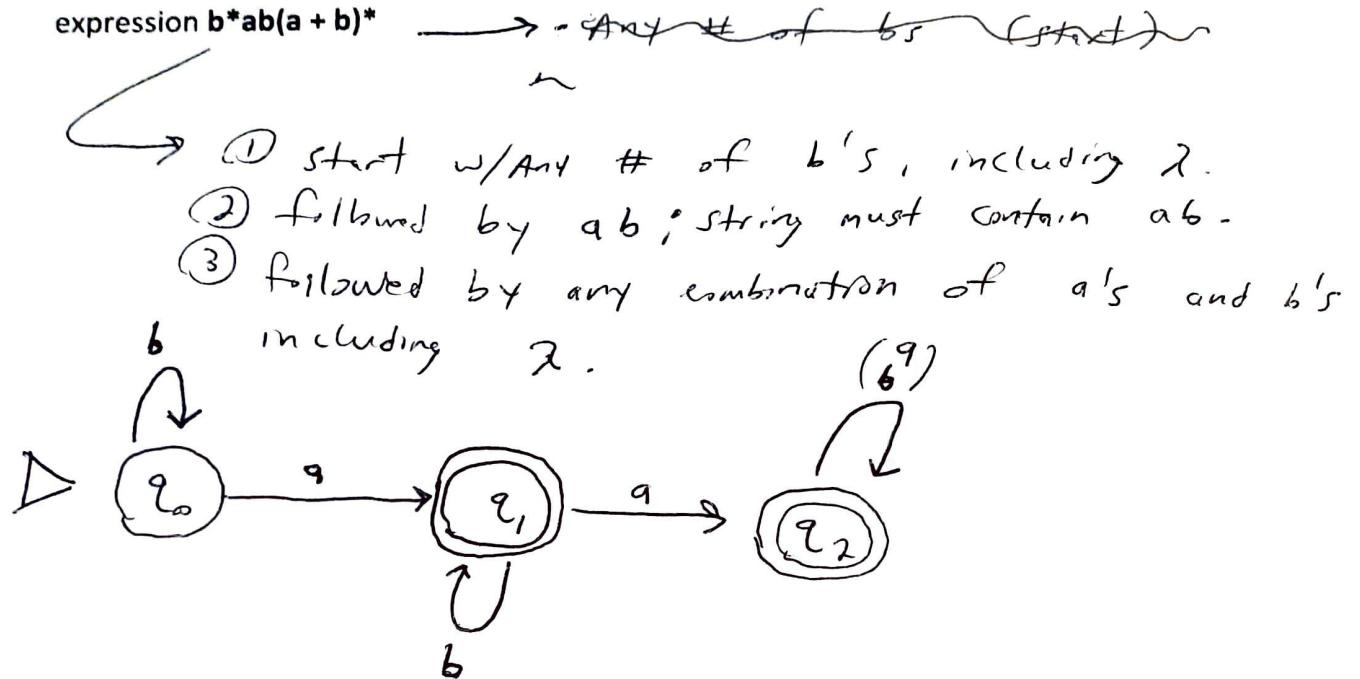
$r = (a + b)^*a$ \longrightarrow Any combinations of a's and b's
including λ , ending with a



4. [15 pts] Write a regular expression for the language L using alphabet {a, b} where all strings in the language start with a single b, followed by any number of a's (including none), and end in a single a.

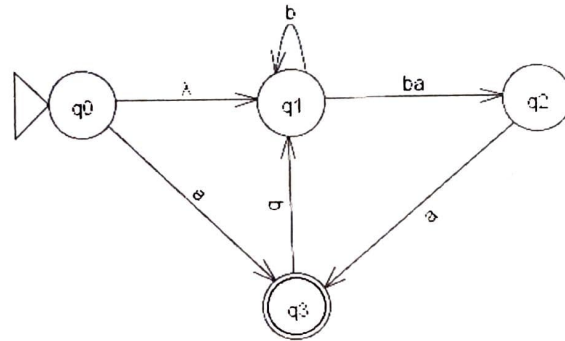
The handwritten regular expression is $L(r) = ba^*a$. It is enclosed in a hand-drawn rounded rectangle. Annotations include: a horizontal line above the text "start with a single b" with an arrow pointing down to the 'b' in the expression; a horizontal line above the text "any number of a's" with an arrow pointing down to the 'a*' in the expression; a horizontal line above the text "end in a" with an arrow pointing down to the final 'a' in the expression; and a curved arrow pointing from the 'a' in the expression back to the 'a' in the 'a*' part, indicating the repetition.

5. [20 pts] Draw the state transition diagram for the finite automaton that models the regular expression $b^*ab(a+b)^*$



6. [20 pts] Convert the following transition graph into a finite automaton by filling in the (final) table and **identifying** the start and accept states. You do not have to draw the state transition diagram for the FA, just fill in the table (note that this is NOT the intermediate table).

FA State	a	b
0	(3)	(1, 1, 2, 1)
(1, 1, 2, 1)	2	1
2	(3)	BH
(3)	BH	(1, 1, 2, 1)



state	a	b
0	(3)	1, 2, 1 1, 2, 1
(3)	crash/BH	1
1	crash/BH	1
(1, 1, 2, 1)	crash/BH	1
2	2	1
	(3)	BH

Combine these states

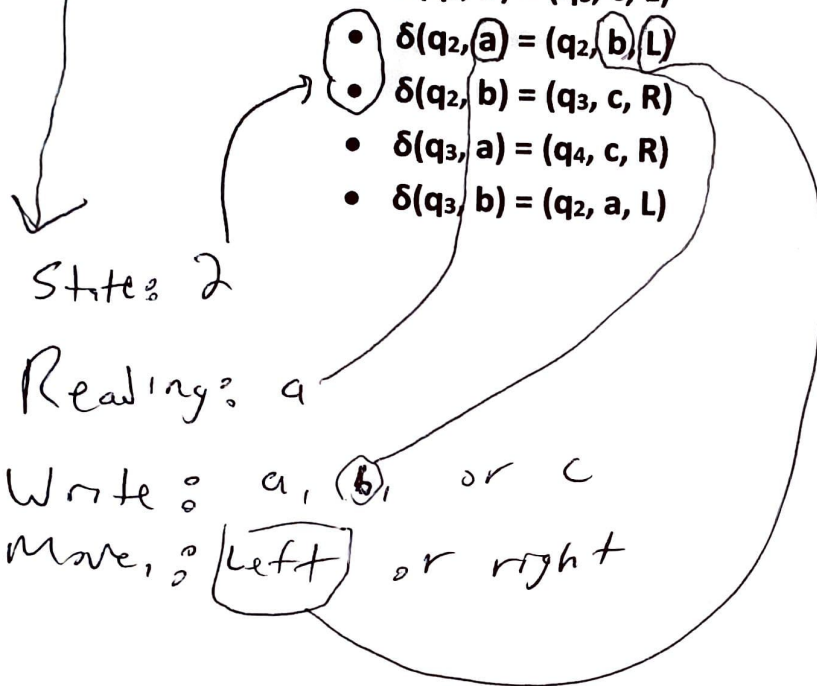
7. [15 pts] Give the configuration after applying the appropriate transition function, using the symbols a, b, c. Only apply the transition function *once*.

Assume the original configuration is: **bbabbq₂aaab**

Available transition functions:

- $\delta(q_1, a) = (q_2, a, R)$
- $\delta(q_1, b) = (q_3, c, L)$
- $\delta(q_2, a) = (q_2, b, L)$
- $\delta(q_2, b) = (q_3, c, R)$
- $\delta(q_3, a) = (q_4, c, R)$
- $\delta(q_3, b) = (q_2, a, L)$

R, W, Move



Appropriate transition Function:

$$\delta(q_2, a) = (q_2, b, L) \rightarrow +b$$

Yields: **bbabq₂bbaab**

move left
 write a "b"
 ↓
 bbabbq₂aaab
 ↓
 bbabq₂bbaab
 stay in state 2.

8. [15 pts] Given sets $A = \{a, b, c, f, g\}$ and $B = \{a, c, d, g\}$:

a. Find $A \cup B$

$$A \cup B = \{a, b, c, d, f, g\}$$

b. Find $A \cap B$

$$A \cap B = \{a, c, g\}$$

c. Find $A - B$

$$A - B = \{a, \textcircled{b}, c, \textcircled{f}, g\} - \{a, c, d, g\}$$

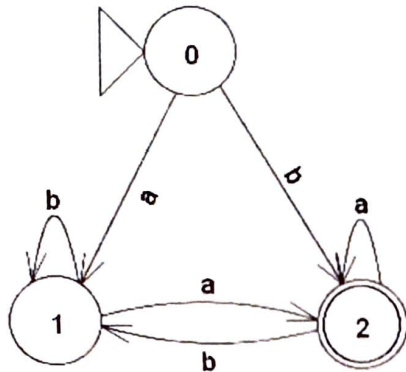
$$A - B = \{b, f\}$$

Final Solution for #9

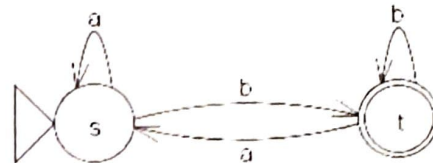
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$L_1' \cup L_2$ Accept states = $(1, t)$

9. [20 pts] Name the accept states (accept group states) in the machine $L_1' \cup L_2$



L_1



L_2

Accept states = $\{2, t\}$
(Both)

Reject states = $\{1, 0, s\}$
(Both)

Start states = $\{0, s\}$
(Both)

$L_1' = \text{NonAccept states} = \{1, 0\}$

$L_2 = \text{Accept states} = \{t\}$

group State	a	b
$(0, s)$	$(1, s)$	$(2, t)$
$(1, s)$	$(2, s)$	$(1, t)$
$(2, t)$	$(2, s)$	$(1, t)$
$(2, s)$	$(2, s)$	$(1, t)$
$(1, t)$	$(2, s)$	$(1, t)$

Solution:

Accept states for

$L_1' \cup L_2 = (1, t)$

No Accept

Accept