Final Exam

Total: ____________ **/ 200**

Printed Name: ___**Demetrius Johnson**_________

GRADER/TA: *[No CO Data]*

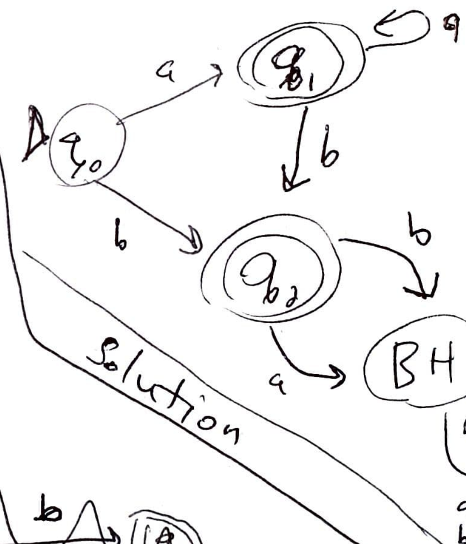
I, ______________________, have neither given nor received assistance on this examination except that which is provided by, or approved by, the instructor.

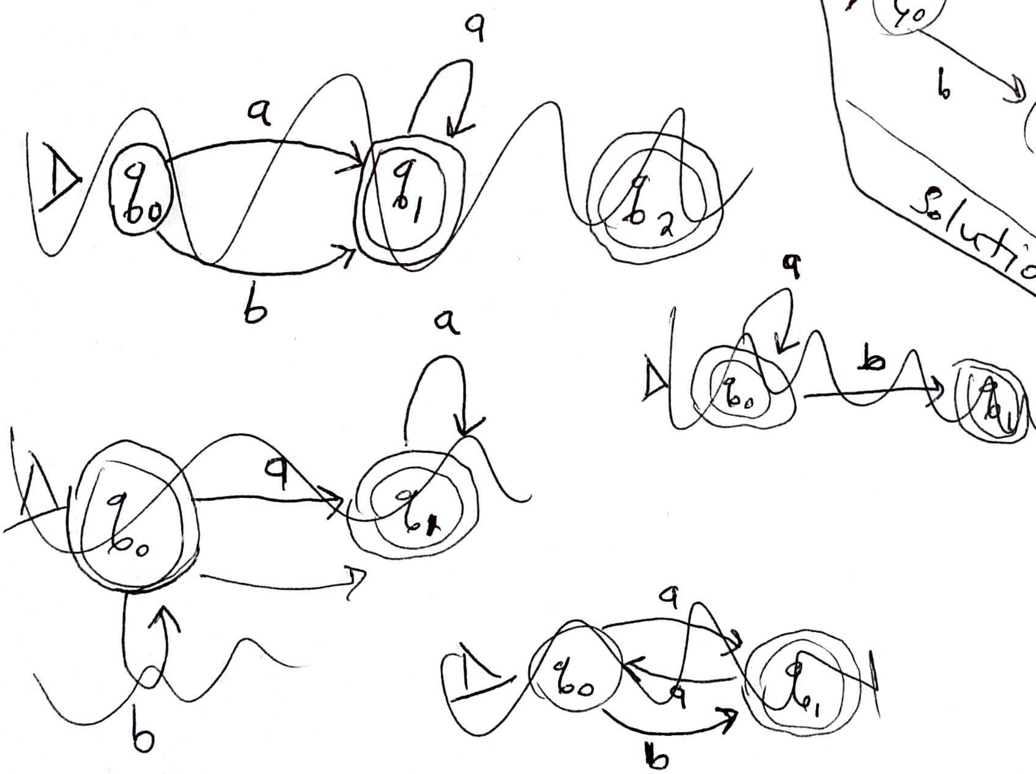
1. [15 pts] Let the alphabet Σ = {a, b}. Given the regular expression r = a*(a + b), assume a language L® is the language defined by the regular language r. Explain in English, what accepted strings in the language are like.

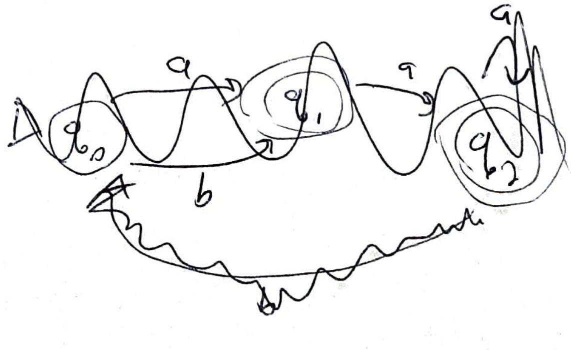
r = a*(a+b) means "any string that starts with any number of a's (including none), and that ends in either a or b.

2. [15 pts] Design and draw the state transition diagram of a finite automaton (FA) for the regular language in question 1, above.

1 start state for TD: → Solution







3. [20 pts] Write a regular expression r, for the language L(r) using the alphabet {a, b, c} where all strings of the language start with at least one a or b, followed by ab, followed by any number of c's.

$$r = (aa^* + bb^*)\,ab\,c^*$$

If it said "start with one a or b

I would do $r = (a+b)\,ab\,c^*$;

Hopefully I didn't misunderstand the wording...

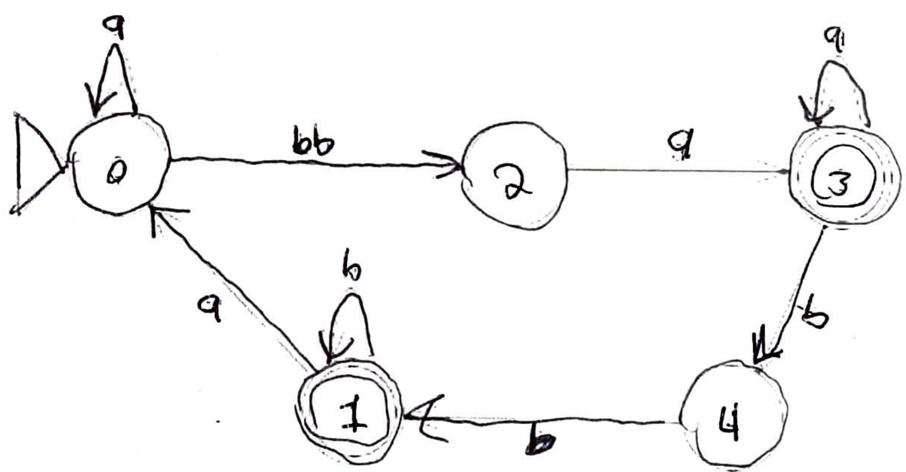
4. [20 pts] Determine whether the following strings are accepted by the transition graph (TG) that is listed.

Stops on q2 a. abb X Not Accepted

Stops on q2 b. aaabbb X Not Accepted

Stops on q1 c. aaabbaaabbb ✓ Accepted

Stops on q0, q2 d. baab X Not accepted



5. [20 pts] Give the configuration after applying the appropriate transition function, using the symbols a, b, c. Only apply the transition function *once*.

Assume the **original configuration is:** **bbabc$q_3$baab** → current state = q_3 Reading "b"

Available transition functions:

- $\delta(\cancel{q_2}, a) = (q_2, \cancel{a}, R)$
- $\delta(\cancel{q_1}, b) = (q_3, c, L)$
- $\delta(\cancel{q_2}, a) = (q_2, \cancel{b}, L)$
- $\delta(\cancel{q_2}, b) = (q_3, c, R)$
- $\delta(\cancel{q_3}, a) = (q_4, \cancel{c}, R)$
- $\delta(q_3, b) = (q_2, \cancel{a}, L)$ → bbabc$q_3$baab → $\boxed{\text{bbab}q_2\text{caaab}}$ Left

6. [20 pts] Solve the following modular arithmetic questions, using the integer representation discussed in class, namely, x = cq + r (find the remainder, r. Hint: r should *always* be non-negative)

a. -25 mod 4

$-25 = (c)4 + r$
Let $c = {}^{-}7$; $-25 = {}^{-}28 + r$
$\boxed{r = 3}$

b. 26 mod 5

$26 = c(5) + r$
Let $c = 5$; $26 = 25 + r$
$r = 1$

c. -42 mod 7

$-42 = c(7) + r$
Let $c = {}^{-}6$; $-42 = -42 + r$
$r = 0$

d. -30 mod 6

$-30 = c(6) + r$
Let $c = -5$; $-30 = -30 + r$
$r = 0$

7. [15 pts] Determine whether the following congruences hold using the modular difference/division property [Hint: (a-b)/c is an integer?]

   a. Is 9 ≡ 7 mod 4?

   $\frac{9-7}{4} = \frac{2}{4} = \frac{1}{2} \neq$ integer; No Congruence

   b. Is 16 ≡ 6 mod 3?

   $\frac{16-6}{3} = \frac{10}{3} = 3.33 \neq$ integer
   No congruence

   c. Is 16 ≡ 6 mod 5?

   $\frac{16-6}{5} = \frac{10}{5} = 2 =$ integer;
   There is congruency

8. [15 pts] Use Euclid's algorithm the find the following greatest common divisors (GCDs)

   a. GCD(15, 48)

   - Let $n = 48$, $m = 15 \rightarrow GCD(48, 15)$ ...
   - $GCD(48\%15, 15) \rightarrow GCD(3, 15)$
   - $GCD(15, 3) \rightarrow GCD(15\%3, 3) \rightarrow GCD(0, 3)$
   - $GCD(3, 0) = 3 \leftarrow$ Solution

   b. GCD(2, 28)

   - Let $n = 28$, $m = 2$
   - $GCD(28, 2) \rightarrow GCD(28\%2, 2) \rightarrow GCD(0, 2)$
   - $GCD(2, 0) = 2 \leftarrow$ Solution

9. [20 pts] Consider the line segments connecting points p_1 and p_2 in each of the following scenarios. Find a vector $\vec{V} = (x, y)$ that represents these line segments.

a. $p_1 = (4, 3)$ and $p_2 = (3, 5)$

$x = 3 - 4$
$y = 5 - 3$
$\boxed{\vec{v}(-1\hat{i}, 2\hat{j})}$

~~$x = 4 - 3 = 1$~~
~~$y = 3 - 5 = -2$~~
~~$\vec{v}(\hat{i}, 1\hat{j} - 2)$~~
~~$\vec{v}(1\hat{i}, -2\hat{i})$~~

b. $p_1 = (-5, 2)$ and $p_2 = (5, 4)$

~~$x = -5 - 5$~~
$y = 4 - 2 = 2$
$x = 5 - -5 = 10$
~~$\vec{v}(2\hat{i}, 10\hat{j})$~~
$\boxed{\vec{v}(10\hat{i}, 2\hat{j})}$

c. $p_1 = (6, 0)$ and $p_2 = (7, 0)$

$x = 7 - 6 = 1$
$y = 0 - 0 = 0$
$\boxed{\vec{v}(1\hat{i}, 0\hat{j})}$

d. $p_1 = (4, -1)$ and $p_2 = (-5, -10)$

$x = -5 - 4 = -9$
$y = -10 - -1 = -9$
$\boxed{\vec{v}(-9\hat{i}, -9\hat{j})}$

10. [20 pts] Given your solutions in question (9) above, find the **magnitudes** of each of the vectors.

a. $|\vec{V}| = \sqrt{-1^2 + 2^2} = \sqrt{1+4} = \boxed{\sqrt{5}}$

b. $|\vec{V}| = \sqrt{10^2 + 2^2} = \sqrt{100+4} = \boxed{\sqrt{104}} = 2\sqrt{26}$

c. $|\vec{V}| = \sqrt{1^2 + 0^2} = \sqrt{1} = 1$

d. $|\vec{V}| = \sqrt{-9^2 + -9^2} = \sqrt{81 + 81} = \sqrt{81 \cdot 2}$

$= \boxed{9\sqrt{2}}$

11. [20 pts] Find the distances from a point to a line, given the following information

a. You are given a point (5, 3) **not** on the line, and two points (0, -2) and (5, 15) through which the line passes

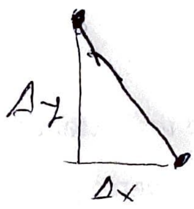
Slope : • $\frac{15 - -2}{5 - 0} = \frac{17}{5} = 3.4$

• $y = 3.4x + b$; $b = 15 - (3.4 \cdot 5)$

• $y = 3.4x - 2$

• $\perp$ to 3.4 is multiplicative inverse and negative: $\frac{17}{5} \rightarrow \frac{-5}{17}$

and a line y = 2x + 4



- $\frac{-5}{17} = m = -0.294$
- pass through point $(5,3)$
- $3 = (-0.294)(5) + b$
- $b = 4.47$
- $y = -0.294x + 4.47$
- Find where $y = 3.4x - 2$ and $y = -0.294x + 4.47$ intersect
- $3.4x - 2 = -0.294x + 4.47$
- $3.694x = 6.47$
- $x = 1.75$; $y = (3.4)(1.75) - 2 = 3.96$
- Intersection is $(1.75, 3.96)$
- Now, Find distance between $(1.75, 3.96)$ and $(5,3)$
- use distance formula: $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
- $\Delta x = |5 - 1.75| = 3.25$
  $\Delta y = |3 - 3.96| = 0.96$ $>$ $d = \sqrt{(3.25)^2 + (0.96^2)}$
- $\boxed{d = 3.39}$ —— Solution

b. You are given a point (2, 3) **not** on the line, and a line y = 2x + 4

$\hookrightarrow \frac{-5}{17}$

- $y = 2x+4$
- $\perp \rightarrow$ inverse • negative $\rightarrow -\frac{1}{2}$
- $y = -\frac{1}{2}x + b$; Find where it passes (2, 3)
- $3 = -\frac{1}{2} \cdot 2 + b$; $b = 4$
- $y = -\frac{1}{2}x + 4$
- Find where $y = -\frac{1}{2}x + 4$ and $y = 2x+4$ intersect
- $-\frac{1}{2}x + 4 = 2x + 4 \rightarrow x = 0 \rightarrow y = 4$
- Now, find distance between (0, 4) and (2, 3)
- • use distance formula: $d = \sqrt{\Delta x^2 + \Delta y^2}$
- $\Delta x = |2-0| = 2$
  $\Delta y = |3-4| = 1$ $\rightarrow d = \sqrt{2^2+1^2} = \sqrt{5}$

$\boxed{d = \sqrt{5}}$ ← Solution