*I wrote this one out to make writing the matrix look easier ___Demetrius Johnson

5. Consider the following transformation of a coordinate frame (CF):
"Translate 3 along X, then Rotate 45^o about Z,"

(a) Find the transformation matrix for the above transformation.

- Translation: (x,y,z) = (3, 0, 0)
- Rotation[about Z-axis] = 45deg.

1. First, we are translating along X.
2. Second, we are rotating about Z 45 deg.

- Note: Every transformation is a transformation of the ORIGIN of the frame with respect to the current position and orientation of the frame after each subsequent transformation.

3. Use:

Translation followed by rotation of a frame can be achieved by:

T = **Trans** · **Rot** (note the order is reversed from the previous slide)

   - This is from the slides.

4. For the Trans(x,y,z), use:

Homogeneous Transformation

- Translation by (a, b, c):

$$\text{Trans}(a,b,c)=\begin{bmatrix}1&0&0&a\\0&1&0&b\\0&0&1&c\\0&0&0&1\end{bmatrix}$$

$$V'=\begin{bmatrix}1&0&0&a\\0&1&0&b\\0&0&1&c\\0&0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}=\begin{bmatrix}x+a\\y+b\\z+c\\1\end{bmatrix}$$

   - C. Lee
   - Trans(3, 0, 0) =

$$\begin{bmatrix}1&0&0&3\\0&1&0&0\\0&0&1&0\\0&0&0&1 \text{ (w = dummy coord.)}\end{bmatrix}$$

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5. For rotation, use:

$$\mathrm{Rot}(z,\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0 & 0\\ \sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

-
- Rot(z, 45deg.)=

$$\begin{bmatrix}\cos(45) & -\sin(45) & 0 & 0\\ \sin(45) & \cos(45) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\text{(w = dummy coord.)}\end{bmatrix}$$

6. Now, for executing a series of homogenous transformations of a Coordinate Frame (CF), we use this formula in order to find the total Transformation:
   - T = Trans. * Rot., which is:

$$\begin{array}{ccc}\text{TRANS.} & & \text{ROT.}\\ \begin{bmatrix}1 & 0 & 0 & 3\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} & \text{X} & \begin{bmatrix}\cos(45) & -\sin(45) & 0 & 0\\ \sin(45) & \cos(45) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\end{array}$$

$$T = \begin{bmatrix}\cos(45) & -\sin(45) & 0 & 3\\ \sin(45) & \cos(45) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

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7.

(b) Identify the position of the origin of the new frame after the transformation with respect to the original coordinate frame:

- Last column of T represents the origin of the new frame with respect to the old frame:
- Thus solution is: Pnew = (3, 0, 0).

(c) Find the old frame coordinate of (1,1,1) in the new frame:

- USE: [coordinates of a fixed point wrt old CF] = **T** · [coord. of the same point wrt new CF].
- Coord. wrt Old Frame = (1,1,1)
- Coord. wrt New Frame = (?,?,?)

1. (1,1,1) = **T** · [coord. of the same point wrt new CF]
2. [coord. of the same point wrt new CF] = (1,1,1,w) / T
3. [coord. of the same point wrt new CF] = T^{-1} * (1,1,1,w)
4. Use:
   - Inverse Transformation (which undoes the transformation)

     Given the inverse transformation is given,

$$\mathbf{T} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{T}^{-1} = \begin{bmatrix} n_x & n_y & n_z & -\mathbf{p}\cdot\mathbf{n} \\ o_x & o_y & o_z & -\mathbf{p}\cdot\mathbf{o} \\ a_x & a_y & a_z & -\mathbf{p}\cdot\mathbf{a} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\mathbf{p}\cdot\mathbf{n} = p_x n_x + p_y n_y + p_z n_z$, $\mathbf{p}\cdot\mathbf{o} = p_x o_x + p_y o_y + p_z o_z$,

C. Lee and $\mathbf{p}\cdot\mathbf{a} = p_x a_x + p_y a_y + p_z a_z$.

In order to solve for T^{-1}.

$$T = \begin{bmatrix} \cos(45) & -\sin(45) & 0 & 3 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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- $$T^{-1} = \begin{bmatrix} \cos(45) & \sin(45) & 0 & -2.12 \\ -\sin(45) & \cos(45) & 0 & 2.12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*note for Inverse Transformation, the last column represents the coordinates of the Old Frame WRT the New Frame.

*for the non-inverse (T), then the last column represents the coordinates of the New Frame WRT the Old Frame.

5. Now we can do T^{-1} * (1,1,1,w):

$$\begin{bmatrix} \cos(45) & \sin(45) & 0 & -2.12 \\ -\sin(45) & \cos(45) & 0 & 2.12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.41 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Solution: (1,1,1) in old frame = (1.41, 0, 1) in new frame.

(d) Find the inverse matrix for the transformation:

We already calculated this in the previous step:

$$T^{-1} = \begin{bmatrix} \cos(45) & \sin(45) & 0 & -2.12 \\ -\sin(45) & \cos(45) & 0 & 2.12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$