$\cos(-\theta) = \cos(\theta)$

$\sin(-\theta) = -\sin(\theta)$

Demetrius Johnson

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CIS-381

THE UNIVERSITY OF MICHIGAN-DEARBORN
SCHOOL OF ENGINEERING
INDUSTRIAL AND MANUFACTURING SYSTEMS ENGINEERING DEPARTMENT

IMSE/CIS 381: INDUSTRIAL ROBOTICS

ASSIGNMENT #2

1. Consider a TRL:R manipulator with the following setting (Figure 1)

Length of base link L_1	=20.0 in.
Length of extension link L	=30.0 in.
Length of the wrist, L_4	= 4.0 in.
Base angle, θ	= 15 degrees
Elevation angle ϕ	= -20 degrees
Pitch angle, ψ	= -15 degrees

a) Determine the coordinates of the point that can be reached by the end -of-arm

$X = \cos\theta(L\cos\phi + L_4\cos\psi) = \cos(15°)(30in \cdot \cos(-20°) + 4in \cdot \cos(-15))$

$X = \cos(15°)(32.05\,in.) = \boxed{30.96in.}$

position of J4 joint

$Y = \sin\theta(L\cos\phi + L_4\cos\psi)$

$Y = \sin(15°)(32.05\,in.) = \boxed{8.30in.}$

$Z = L_1 + \cancel{L}\sin\phi + L_4\sin\psi$

$Z = 20in + 30in \cdot \sin(-20°) + 4in \cdot \sin(-15)$

$Z = \boxed{8.7in}$

Solution:

$\boxed{P(30.96in, 8.30in, 8.7in)}$

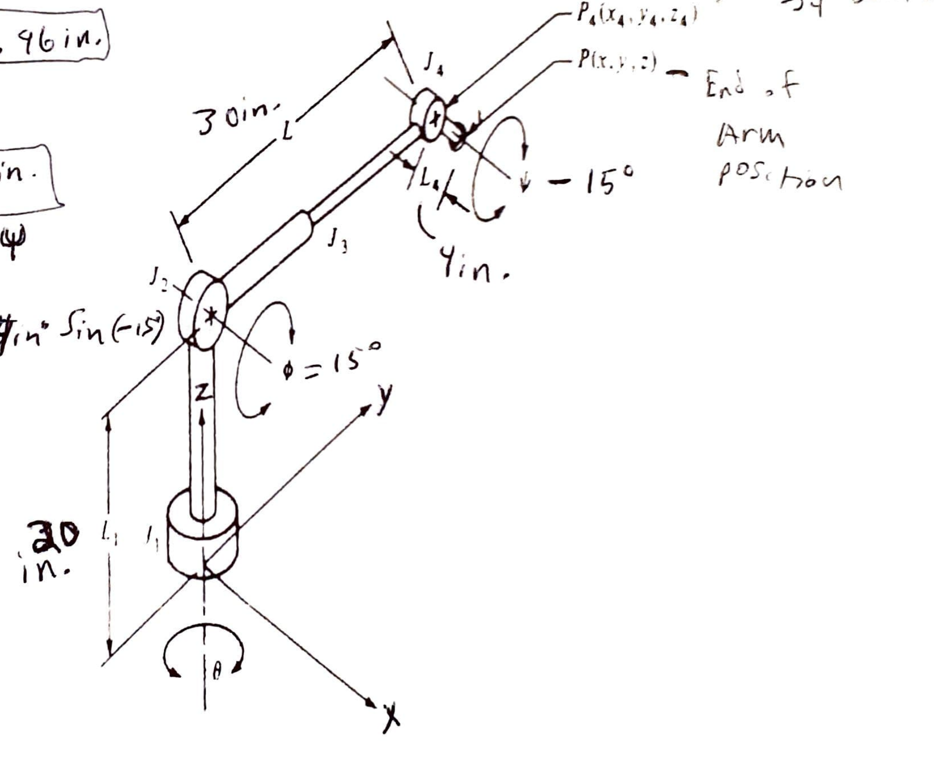


Figure 1 A 4 d.o.f. Robot in 3D.

b) Suppose the robot is commanded to move to a new position Pj ($\theta = 30$, $\phi = -10$, $\psi = 15$, L= 25), the maximum speed of any rotational joint is 10°/s and the maximum speed for the linear joint is 2.0 in/s. If the robot controller has joint interpolation routine, determine the time and speeds of each joint to move the robot to the new position.

① calculate how much each joint moves; the difference between New position and old position.

	New	Old	Difference New − old
L_1	20 in.	20 in.	0 in.
L	25 in.	30 in.	− 5 in.
L_4	4 in.	4 in	0 in.
θ	30°	15°	15°
ϕ	−10°	−20°	10°
ψ	15°	−15°	30°

- MAX speed for R joints = 10°/s
- MAX speed for L joint = 2 in/s

② Now, divide each joint difference (change) by its respective MAX speed.

- Note, Links with No change do not require this step and are therefore Not used for the calculations when we solve for interpolation.

- L_1 and L_4 $\Delta = 0$; speed = 0

- $\Delta L \Rightarrow \frac{5\text{ in}}{(2\text{ in/s})} = 2.5$ seconds

- $\Delta\theta \Rightarrow \frac{15°}{(10°/s)} = 1.5$ seconds

- $\Delta\phi \Rightarrow \frac{10°}{(10°/s)} = 1.0$ seconds

- $\Delta\psi \Rightarrow \frac{30°}{10°/s} = 3.0$ seconds

$\boxed{\text{Slowest is 3.0 seconds}}$

- Thus All other joints will coordinate with ψ to move at a velocity so as to also take 3.0s.

(3) Solve velocities based on slowest time (3.0 s) for Joint interpolation.:

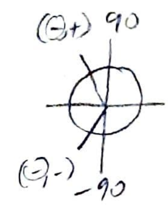
- L_1 and L_4 Velocity $= \frac{0 \text{ in}}{s}$ since they remain stationary.

- $\psi = 3.0s$ is slowest; so its velocity will be at $\boxed{10°/s}$

- $L_v \Rightarrow \frac{5 \text{ in}}{x\left(\frac{\text{in}}{\text{sec}}\right)} = 3.0 \text{ sec}; \quad x = \frac{5}{3} \frac{\text{in}}{\text{sec}} = \boxed{1.67 \frac{\text{in}}{s}}$

- $\theta_v \Rightarrow \frac{15°}{x °/s} = 3.0 \text{ sec}; \quad x = \frac{15°}{3.0 \text{ sec}} = \boxed{5°/\text{sec}}$

- $\phi_v \Rightarrow \frac{10°}{x °/s} = 3.0 \text{ sec}; \quad x = \frac{10°}{3 \text{ sec}} = \boxed{3.33°/\text{sec}}$



2. It is desired to determine the values to which angles θ_1 and θ_2 must be set in order to achieve a certain point in space for the manipulator shown in Figure 2. The length of Joint 1, L_1 = 10 in., the length of joint 2, L_2 = 12 in. The point P_w which the robot must achieve is defined by the coordinates x = 12.0 and y = 6.0. Using the reverse transformation methods, determine the angles θ_1 and θ_2 required to achieve the point in the configuration in Fig. 2.

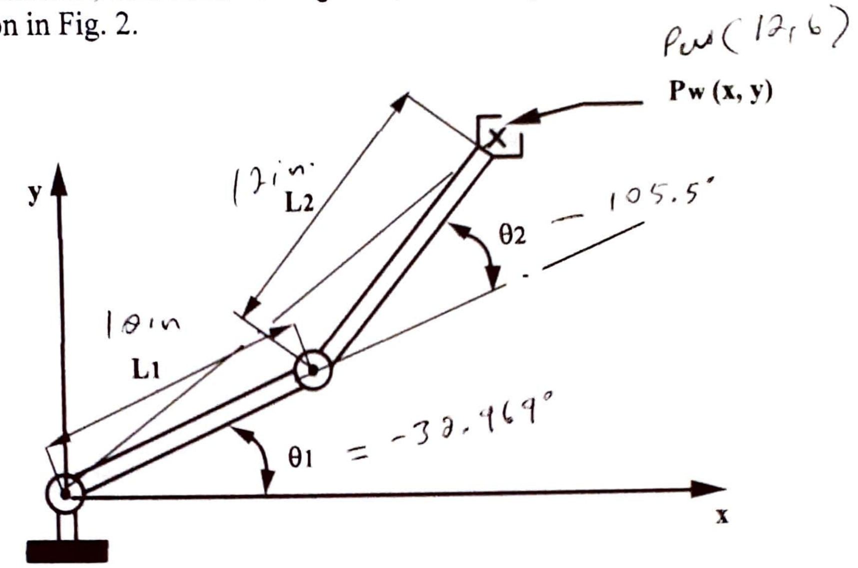


Figure 2. A 2 d.o.f. robot in 2D

$$\cos\theta_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2} = \frac{12^2+6^2-10^2-12^2}{2(10)(12)} = \frac{-64}{240}$$

$$\theta_2 = \arccos\left(\frac{-64}{240}\right) = \boxed{\pm 105.466°}$$

$\boxed{\theta_2 = 105.466°}$ (choose the positive angle)

$$\tan\theta_1 = \frac{[y(L_1 + L_2\cos\theta_2) - xL_2\sin\theta_2]}{[x(L_1 + L_2\cos\theta_2) + yL_2\sin\theta_2]}$$

$$\tan\theta_1 = \frac{[6(10 + 12\cos(105.466°)) - 12(12\sin(105.466°))]}{[12(10+12\cos(105.466°)) + 6(12\sin(105.466°))]}$$

$$\cancel{\tan\theta_1 = \frac{[6(10+12\cdot\frac{-64}{240}) - 12(12\cdot\frac{-64}{240})]}{[12(10+12\cdot\frac{-64}{240}) + 6(12\cdot\frac{-64}{240})]}} \quad \cancel{\frac{40.8+38.9}{21.6+14.2}}$$

$$\cancel{\tan\theta_1 = \frac{79.2}{62.4}} \quad \cancel{\theta = \arctan\left(\frac{79.2}{62.4}\right) \rightarrow \theta_1 =}$$

$$\tan\theta_1 = \frac{[6(6.8) - 12(11.565)]}{[12(6.8) + 6(11.565)]}$$

$$\tan\theta_1 = \frac{40.8 - 138.72}{81.6 + 69.36} = \frac{-97.92}{150.96}$$

$$\theta_1 = \arctan\left(\frac{-97.92}{150.96}\right)$$

$$\boxed{\theta_1 = -32.969^\circ}$$

3. A Cartesian coordinate robot with LLL configuration is to move from position A of coordinates (x,y,z) = (25, 10, 10) to point B of coordinates (x, y, z) = (5, 30, 20). All coordinates are given in inches. The maximum velocities for the three joints are 8 in./s, 5 in./s, and 10 in./s, respectively.

a. What is the time required to move from point A to B, if the robot controller has slew motion routine?

L Joints	(B) New	(A) Old	(distance) New − Old	MAX Join Velocity	Total time $\frac{\text{distance}}{\text{velocity}}$
X	5in	25in.	−20in.	$\frac{8in}{s}$	$\frac{20}{8} = \boxed{2.5s}$
Y	30in.	10in.	20in.	$\frac{5in}{s}$	$\frac{20}{5} = \boxed{4s}$
Z	20in.	10in.	10in.	$\frac{10in}{s}$	$\frac{10}{10} = \boxed{1s}$

Solution:

• Slew Motion: each joint moves at its max velocity over its change in position range; Joint Y takes the most time @ $\boxed{4.0s}$ So to go from A to B, it requires $\boxed{4.0s}$ of time.

b. What are motors' speeds to move from point A to B, if the robot controller has joint interpolation routine?

• Joint interpolation: All joints start and stop simultaneously based on the joint that takes most time to complete its change.

• Joint Y takes 4.0s which is greatest time; Solve other joint speeds based on this:

• $Y \Rightarrow \boxed{\frac{5in}{s}}$ for Δ 20in. $\Rightarrow$ 4s.

• $X \Rightarrow \frac{20in}{Xs} = \boxed{\frac{5in}{s}} \Rightarrow X = 4s$

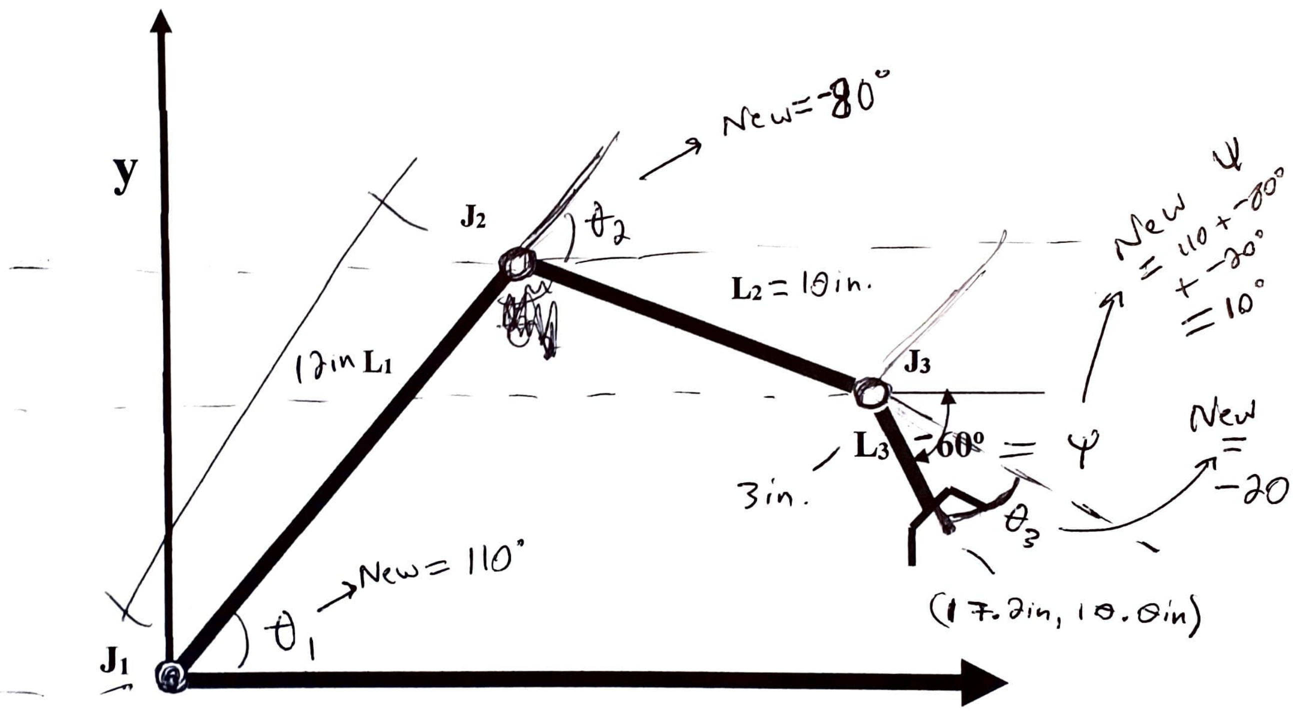
• $Z \Rightarrow \frac{10in}{4s} \Rightarrow \boxed{\frac{2.5in}{s}} \Rightarrow 4s$

Solution: $\boxed{X = \frac{5in}{s}}$ $\boxed{Y = \frac{5in}{s}}$, $\boxed{Z = \frac{2.5in}{s}}$ $\rightarrow$ will take All joints 4s to complete their position change

4. The manipulator shown in Figure 3 is at current position P_w ($x=17.2$ in, $y=10.00$ in., $\Psi= -60^\circ$). It was commanded to move to a new position Pj ($\Theta_1=110^\circ$, $\Theta_2 = -80^\circ$, $\Theta_3= -20^\circ$). The length of link 1, $L_1= 12$ in., the length of link 2, $L_2=10$ in., the length of link 3, $L_3=3$ in. The three joints are driven with motors which have a maximum joint speeds of 30 degrees/sec.

a. Determine the time to carry out this movement if the robot has joint interpolation capability.

b. Determine the coordinates (x,y) of the tip of link 3 at the end of the move.



MAX speed (all joints = $\frac{30^\circ}{s}$

New Pos

$\theta_1 = 110^\circ$ $\quad x = ?$

$\theta_2 = -80^\circ$ $\quad y = ?$

$\theta_3 = -20^\circ$

$\psi = 110 + -80 + -20 = 10^\circ$

Figure 3 Three d.o.f. robot in 2 D

Old Pos:

$\theta_1 = ?$ $\quad x = 17.2$, $y = 10$

$\theta_2 = ?$

$\theta_3 = ?$

$\psi = -60^\circ$

$x_3 = ?$, $y_3 = ?$ J_3 position

* We need to solve for these 3 angles in order to find time to carry out the movement if the robot uses joint interpolation

* Solve for θ_1 and θ_2 first, treating it as a 2 DOF problem; and solve for J_3 position:

• Solve for J_3:

$x_3 = x - L_3\cos\psi = 17.2\text{in} - 3\text{in}(\cos(-60°)) = 17.2 - 3\left(\frac{1}{2}\right)$

$\boxed{x_3 = 15.7\text{in}}$ → original position of J_3

$y_3 = y - L_3\sin\psi = 10\text{in} - 3\text{in}(\sin(-60°)) = 10\text{in} - 3(-0.866)\text{in}$

$\boxed{y_3 = 12.6\text{in}}$ → origiNal pos of J_3

• Now solve for original θ_1 and θ_2: using (x_3, y_3)

$$\cos\theta_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2} = \frac{15.7^2+12.6^2-12^2-10^2}{2(12)(10)} = \frac{161.25}{240}$$

$$\cos\theta_2 = \frac{161.25}{240};\ \theta_2 = \arccos\left(\frac{161.25}{240}\right) = \arccos(0.672) = \pm 47.79°$$

Choosing θ: $\boxed{\theta_2 = -47.79°}$

since $\psi = -60°$ based on diagram

$$\tan\theta_1 = \frac{[y(L_1+L_2\cos\theta_2) - xL_2\sin\theta_2]}{[x(L_1+L_2\cos\theta_2) + yL_2\sin\theta_2]}$$

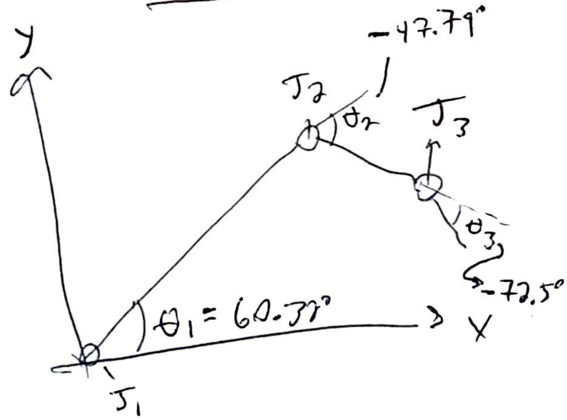
$$\tan\theta_1 = \frac{[12.6(12+10\cos(-47.79)) - 15.7(10)(\sin(-47.79°))}{[15.7(12+10\cos(-47.79)) + 12.6(10)(\sin(-47.79°))}$$

$$\tan\theta_1 = \frac{12.6(18.72) - 15.7(10)(-0.74)}{15.7(18.72) + 12.6(10)(-0.74)} = \frac{352.1}{200.7}$$

$$\tan\theta_1 = \frac{352.1}{200.7};\ \theta_1 = \arctan\left(\frac{352.1}{200.7}\right)$$

$\boxed{\theta_1 = 60.32°}$

Solved original J angles



• Now solve θ_3: $\psi = \theta_1 + \theta_2 + \theta_3$

• $\psi_{original} = -60°$

• $-60° = 60.32° + -47.79° + \theta_3$

$\boxed{\theta_3 = -72.53°}$

~~Newn~~

Now Solve Joint changes:

Joint	New	old	New − old (Δ)
J_1 (θ_1)	110°	60.32°	49.68°
J_2 (θ_2)	−80°	−47.79°	32.21°
J_3 (θ_3)	−20°	−72.53°	52.53°

← greatest displacement

- All ~~speeds~~ Joints have MAX speed at 30°/s; thus whichever joint has the most displacement will take the most amount of time to reach its new position.
- J_3 has the most displacement; thus $\frac{52.53°}{\left(\frac{30°}{s}\right)} = \boxed{1.75s}$

(a) Solution: time to carry out movement using Joint interpolation = $\boxed{1.75s}$

(b) Find (X,Y) of Link 3 tip:

$L_1 = 12 in.$; $L_2 = 10 in.$; $L_3 = 3 in.$

$\psi_{new} = 10°$; $\theta_1 = 110°$; $\theta_2 = -80°$; $\theta_3 = -20°$;

$X = L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) + L_3 \cos(\psi)$

$X = 12(\cos 110°) + 10(\cos 30) + 3(\cos 10) = 12(-0.342) + 10(0.866) + 3(0.985)$

$X = -4.104 + \cancel{8.598}\ 8.66 + 2.955 = \cancel{11.444} \rightarrow = 7.5 in.$

$Y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1+\theta_2) + L_3 \sin(\psi)$

$Y = 12\sin(110) + 10\sin 30 + 3\sin 10 = 11.28 + 5 + 0.521$

$Y = 16.8 in.$

Solution (7.5in, 16.8in)

X Y

*I wrote this one out to make writing the matrix look easier ___Demetrius Johnson

5. Consider the following transformation of a coordinate frame (CF):
"Translate 3 along X, then Rotate 45^{o} about Z,"

(a) Find the transformation matrix for the above transformation.

- Translation: (x,y,z) = (3, 0, 0)
- Rotation[about Z-axis] = 45deg.

1. First, we are translating along X.
2. Second, we are rotating about Z 45 deg.

- Note: Every transformation is a transformation of the ORIGIN of the frame with respect to the current position and orientation of the frame after each subsequent transformation.

3. Use:

Translation followed by rotation of a frame can be achieved by:

T = **Trans** · **Rot** (note the order is reversed from the previous slide)

  - This is from the slides.

4. For the Trans(x,y,z), use:

Homogeneous Transformation

- Translation by (a, b, c):

$$\text{Trans}(a,b,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$V' = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \\ z+c \\ 1 \end{bmatrix}$$

  - C. Lee
  - Trans(3, 0, 0) =

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \text{ (w = dummy coord.)} \end{bmatrix}$$

*I wrote this one out to make writing the matrix look easier ___Demetrius Johnson

5. For rotation, use:

$$\mathrm{Rot}(z,\theta)=\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

-
- Rot(z, 45deg.)=

$$\begin{bmatrix} \cos(45) & -\sin(45) & 0 & 0 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1_{\text{(w = dummy coord.)}} \end{bmatrix}$$

6. Now, for executing a series of homogenous transformations of a Coordinate Frame (CF), we use this formula in order to find the total Transformation:
   - T = Trans. * Rot., which is:

TRANS. ROT.

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{X} \begin{bmatrix} \cos(45) & -\sin(45) & 0 & 0 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \cos(45) & -\sin(45) & 0 & 3 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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7.

(b) Identify the position of the origin of the new frame after the transformation with respect to the original coordinate frame:

- Last column of T represents the origin of the new frame with respect to the old frame:
- Thus solution is: Pnew = (3, 0, 0).

(c) Find the old frame coordinate of (1,1,1) in the new frame:

- USE: [coordinates of a fixed point wrt old CF] = **T** · [coord. of the same point wrt new CF].
- Coord. wrt Old Frame = (1,1,1)
- Coord. wrt New Frame = (?,?,?)

1. (1,1,1) = **T** · [coord. of the same point wrt new CF]
2. [coord. of the same point wrt new CF] = (1,1,1,w) / T
3. [coord. of the same point wrt new CF] = T^{-1} * (1,1,1,w)
4. Use:
   - Inverse Transformation (which undoes the transformation)

     Given the inverse transformation is given,

$$\mathbf{T} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{T}^{-1} = \begin{bmatrix} n_x & n_y & n_z & -\mathbf{p}\cdot\mathbf{n} \\ o_x & o_y & o_z & -\mathbf{p}\cdot\mathbf{o} \\ a_x & a_y & a_z & -\mathbf{p}\cdot\mathbf{a} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\mathbf{p}\cdot\mathbf{n} = p_x n_x + p_y n_y + p_z n_z$, $\mathbf{p}\cdot\mathbf{o} = p_x o_x + p_y o_y + p_z o_z$,

C. Lee and $\mathbf{p}\cdot\mathbf{a} = p_x a_x + p_y a_y + p_z a_z$.

In order to solve for T^{-1}.

$$T = \begin{bmatrix} \cos(45) & -\sin(45) & 0 & 3 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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*I wrote this one out to make writing the matrix look easier ___Demetrius Johnson

- $$T^{-1} = \begin{bmatrix} \cos(45) & \sin(45) & 0 & -2.12 \\ -\sin(45) & \cos(45) & 0 & 2.12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*note for Inverse Transformation, the last column represents the coordinates of the Old Frame WRT the New Frame.

*for the non-inverse (T), then the last column represents the coordinates of the New Frame WRT the Old Frame.

5. Now we can do T^{-1} * (1,1,1,w):

$$\begin{bmatrix} \cos(45) & \sin(45) & 0 & -2.12 \\ -\sin(45) & \cos(45) & 0 & 2.12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathrm{X} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.41 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Solution: (1,1,1) in old frame = (1.41, 0, 1) in new frame.

(d) Find the inverse matrix for the transformation:

We already calculated this in the previous step:

$$T^{-1} = \begin{bmatrix} \cos(45) & \sin(45) & 0 & -2.12 \\ -\sin(45) & \cos(45) & 0 & 2.12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$