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UM-Dearborn CIS-479_RETAKE

Program 2 Report_RETAKE: Hidden Markov Model – Robot Localization

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SENSE [0,0,1,1] → highest confidence in location, since bottom right corner is only location with obstacles to the E and S, and this is the second sense in a row with these *unique* measurements .. 25

Transitional Probability

```
//moving transition probabilities:
//moving transition probabilities:
const float forward_probability = (float)0.8;   //move forward in desired direction = 80%
const float left_probability = (float)0.1;     //drift left in undesired direction = 10%
const float right_probability = (float)0.1;   //drift right in undesired direction = 10%
```

Evidence Conditional Probability

```
//obstacle sensor transition probabilities:
//-->They are complimentary --> should add up to 1.
//--> P(sense_obstacle|there is an obstacle)=0.8 + P(no_sense_obstacle|there is an
obstacle)=0.2 = 1
const float sensing_OBJ_correctly_probability = (float)0.80; //correctly sense obstacle
square as an obstacle = 80%
const float sensing_OBJ_incorrectly_probability = (float)0.20; //false negative(claiming
no obstacle) - incorrectly sense square as NOT an obstacle (even though it is an
obstacle) = 20%

//open-square (non-obstacle) sensor transition probabilities:
//-->They are complimentary --> should add up to 1.
//--> P(sense_obstacle|there is NOT an obstacle)=0.15 + P(no_sense_obstacle|there is NOT
an obstacle)=0.85 = 1
const float sensing_no_OBJ_correctly_probability = (float)0.85; //correctly sense open
square as NOT an obstacle = 85%
const float sensing_no_OBJ_incorrectly_probability = (float)0.15; //false
positive(claiming an obstacle) - incorrectly sense open square as an obstacle (even
though there is not an obstacle) = 15%
```

Filtering (from sensing) (from updateProb_sensing_filtering fx)

```
void updateProb_sensing_filtering(table& table_struct, Location*
sensor_evidence_bit_WNES) {


    //Filtering: P(St|Z1=z1, …, Zt=zt) ∝ P(Zt=zt|St) P(St|Z1=z1, …, Zt-1=zt-1)

    /* note that Z_t represents an evidence variable --> evidence from the neighbor
sensor
        Zt is composed of 4 random variables for four directions:
        Zt = (ZW,t,  ZN,t,  ZE,t, ZS,t)

        For each state St, conditionally independent:
        P(Zt|St) = P(ZW,t|St) P(ZN,t|St) P(ZE,t|St) P(ZS,t|St)


        note that:

        P(Zt=zt|St)==  likelihood
        P(St|Z1=z1, …, Zt-1=zt-1) ==  prior
        P(St|Z1=z1, …, Zt=zt) == posterior

    */

    float sum_of_proportions = 0; //sum of proportions == SUM for all States (S_t) of [
P(Zt=zt|St) P(St|Z1=z1, …, Zt-1=zt-1)  ].
```

```
                                    //add all proportions to get total probability of
evidence Z_t given S_t,
                                    //then we can use this sum to get  P(St|Z1=z1, …,
Zt=zt) == [  P(Zt=zt|St) P(St|Z1=z1, …, Zt-1=zt-1) / sum_of_proportions  ].
                                    //This is the same thing as saying: [1 of the
proportions / sum_of_propotions]...
                                    //--> and we can do this for all states to find the
probabiility the robot is at any given location given evidence(s) Z_t.

    Location neighbors_WNES[4]; //use this to get the true neighbor based om the true map
the robot has access to to compare it with the sensor evidence

    for (int row = 0; row < table_struct.numRows; row++) {
        for (int col = 0; col < table_struct.numCols; col++) {

            if (is_obstacleLocation(row, col)) //no calculation necessary for obstacle
case --> robot knows it cannot be there
                continue;

            //check neighboring squares for obstacles so we can determine which sensor
error transistion probability to use at each possible state S_t
            getNeighbors_WNES(row, col, neighbors_WNES);

            //calculate likelihood --> P(Z_t | S_t) --> obstacle or open square
probability at a given state S_t = s_t, based on evidence from sensor:
            float sensorProb_WNES[4];
            for (int i = 0; i < 4; i++) {

                //case: correctly sense an obstacle ==80%
                if (sensor_evidence_bit_WNES[i] == OBSTACLE && neighbors_WNES[i] ==
OBSTACLE)
                    sensorProb_WNES[i] = sensing_OBJ_correctly_probability;

                //case: incorrectly sense an obstacle as open square (false negative)
==20%
                else if (sensor_evidence_bit_WNES[i] == OPEN_SQUARE && neighbors_WNES[i]
== OBSTACLE)
                    sensorProb_WNES[i] = sensing_OBJ_incorrectly_probability;

                //case: correctly sense an open sqaure ==85%
                else if (sensor_evidence_bit_WNES[i] == OPEN_SQUARE && neighbors_WNES[i]
== OPEN_SQUARE)
                    sensorProb_WNES[i] = sensing_no_OBJ_correctly_probability;

                //case: incorrectly sense an open sqaure as an obstacle (false positive)
==15%
                else if (sensor_evidence_bit_WNES[i] == OBSTACLE && neighbors_WNES[i] ==
OPEN_SQUARE)
                    sensorProb_WNES[i] = sensing_no_OBJ_incorrectly_probability;
            }

            //multiply all conditionall indepdent likelihoods to get total likelihood of
a state S_t given given sensor evidence Z_t
            float state_totalLikelihood = 1;
            for (int i = 0; i < 4; i++)
                state_totalLikelihood *= sensorProb_WNES[i];

            //Now, multiply total likelihood by the prior and add that to the sum
```

```
            sum_of_proportions += (state_totalLikelihood *
table_struct.tablePos_locationProb_prior[row][col]);
        }
    }
```

Here, I demonstrate how to get the sum of all proportions of probabilities given an evidence:

```
    //now, sum of proportions has been acquired, we now can normalize and find the
posterior probabilitl for all states:
    // P(St|Z1=z1, …, Zt=zt) == [  P(Zt=zt|St) P(St|Z1=z1, …, Zt-1=zt-1) /
sum_of_proportions  ]

    for (int row = 0; row < table_struct.numRows; row++) {
        for (int col = 0; col < table_struct.numCols; col++) {

            if (is_obstacleLocation(row, col)) //no calculation necessary for obstacle
case --> robot knows it cannot be there
                continue;

            //check neighboring squares for obstacles so we can determine which sensor
error transistion probability to use at each possible state S_t
            getNeighbors_WNES(row, col, neighbors_WNES);

            //calculate likelihood --> P(Z_t | S_t) --> obstacle or open square
probability at a given state S_t = s_t, based on evidence from sensor:
            float sensorProb_WNES[4];
            for (int i = 0; i < 4; i++) {

                //case: correctly sense an obstacle ==85%
                if (sensor_evidence_bit_WNES[i] == OBSTACLE && neighbors_WNES[i] ==
OBSTACLE)
                    sensorProb_WNES[i] = sensing_OBJ_correctly_probability;

                //case: incorrectly sense an obstacle as open square (false negative)
==15%
                else if (sensor_evidence_bit_WNES[i] == OPEN_SQUARE && neighbors_WNES[i]
== OBSTACLE)
                    sensorProb_WNES[i] = sensing_OBJ_incorrectly_probability;

                //case: correctly sense an open sqaure ==95%
                else if (sensor_evidence_bit_WNES[i] == OPEN_SQUARE && neighbors_WNES[i]
== OPEN_SQUARE)
                    sensorProb_WNES[i] = sensing_no_OBJ_correctly_probability;

                //case: incorrectly sense an open sqaure as an obstacle (false positive)
==5%
                else if (sensor_evidence_bit_WNES[i] == OBSTACLE && neighbors_WNES[i] ==
OPEN_SQUARE)
                    sensorProb_WNES[i] = sensing_no_OBJ_incorrectly_probability;
            }

            //multiply all conditionall indepdent likelihoods to get total likelihood of
a state S_t given given sensor evidence Z_t
            float state_totalLikelihood = 1;
            for (int i = 0; i < 4; i++)
                state_totalLikelihood *= sensorProb_WNES[i];
```

Here, I show a repeated process, but only this time we use our sum of proportions to get the probability for all locations:

```
            //Now, multiply total likelihood by the prior
           //Now, at this moment we have calculated one proportion; now,
           //we normalize since we already have sum of all proportions, to get posterior
for the current state at [row][col]
            table_struct.tablePos_locationProb_posterior[row][col] =
(state_totalLikelihood * table_struct.tablePos_locationProb_prior[row][col]) /
sum_of_proportions;

        }
    }
}
```

Prediction (from movement) (from updateProb_moving_prediction fx)

```
void updatProb_moving_prediction(table& table_struct, Direction_Cardinal move_direction)
{


    //Prediction: P(St+1|Z1=z1, …, Zt=zt) = ∑sP(St+1|St=s) P(St|Z1=z1, …, Zt=zt)
    //The prediction is essentially just the sum of all probabilities that you can get to
state S_t+1, given all possible prior states S_t.

    //first, reinitialize the posterior table, so that we can incrementally sweep across
it and add the probability of getting to one state from one of the other possible states
    for (int i = 0; i < table_struct.numRows; i++) {

        for (int j = 0; j < table_struct.numCols; j++) {

            if (is_obstacleLocation(i, j))
                continue;
            else
                table_struct.tablePos_locationProb_posterior[i][j] = 0;
        }
    }
```

Here, I demonstrate how I don't add all probabilities for a location at one time, rather I sweep across the entire matrix and update probabilities as a location should absorb it given do a move from one location to another:

```
     //now sweep across all locations adding any probability from neighbor states that it
can be reached from - when done, then all probability for reaching a state S_t+1 from all
other possible states S_t will be complete
    for (int row = 0; row < table_struct.numRows; row++) {
        for (int col = 0; col < table_struct.numCols; col++) {

            if (is_obstacleLocation(row, col)) //no need to check impossible cases since
robot cannot be at an obstacle location
                continue;

            switch (move_direction) {



            case Direction_Cardinal::WEST:
```

```
                prob_moving_WEST(row, col, table_struct);
                break;
            case Direction_Cardinal::NORTH:

                prob_moving_NORTH(row, col, table_struct);
                break;
            case Direction_Cardinal::EAST:

                prob_moving_EAST(row, col, table_struct);
                break;
            case Direction_Cardinal::SOUTH:

                prob_moving_SOUTH(row, col, table_struct);
                break;
            }
        }

    }
}
```

And here, I demonstrate how I create a transition function that sweeps across the matrix allowing the proper location to absorb probability for all directions as shown in the function above, below I will show you just one of the functions for the sake of space and unnecessary redundancy:

```
void prob_moving_WEST(int row, int col, table& table_struct) {


    //WEST == FORWARD-->(col - 1), LEFT-->(row + 1), RIGHT-->(row - 1):

    //forward
        //check wall bounce back case, otherwise use (col - 1)
    if (is_obstacleLocation(row, col - 1))
        //current [row][col] location absorbs the probability
        table_struct.tablePos_locationProb_posterior[row][col] += (forward_probability *
table_struct.tablePos_locationProb_prior[row][col]);
    else
        //otherwise, forward location absorbs the probability
        table_struct.tablePos_locationProb_posterior[row][col - 1] +=
(forward_probability * table_struct.tablePos_locationProb_prior[row][col]);
    //left
        //check bounce back case, otherwise use (row + 1)
    if (is_obstacleLocation(row + 1, col))
        //current [row][col] location absorbs the probability
        table_struct.tablePos_locationProb_posterior[row][col] += (left_probability *
table_struct.tablePos_locationProb_prior[row][col]);
    else
        //otherwise, left location absorbs the probability
        table_struct.tablePos_locationProb_posterior[row + 1][col] += (left_probability *
table_struct.tablePos_locationProb_prior[row][col]);
    //right
       //check bounce back case, otherwise use (row - 1)
    if (is_obstacleLocation(row - 1, col))
        table_struct.tablePos_locationProb_posterior[row][col] += (right_probability *
table_struct.tablePos_locationProb_prior[row][col]);
    else
        //otherwise, right location absorbs the probability
```

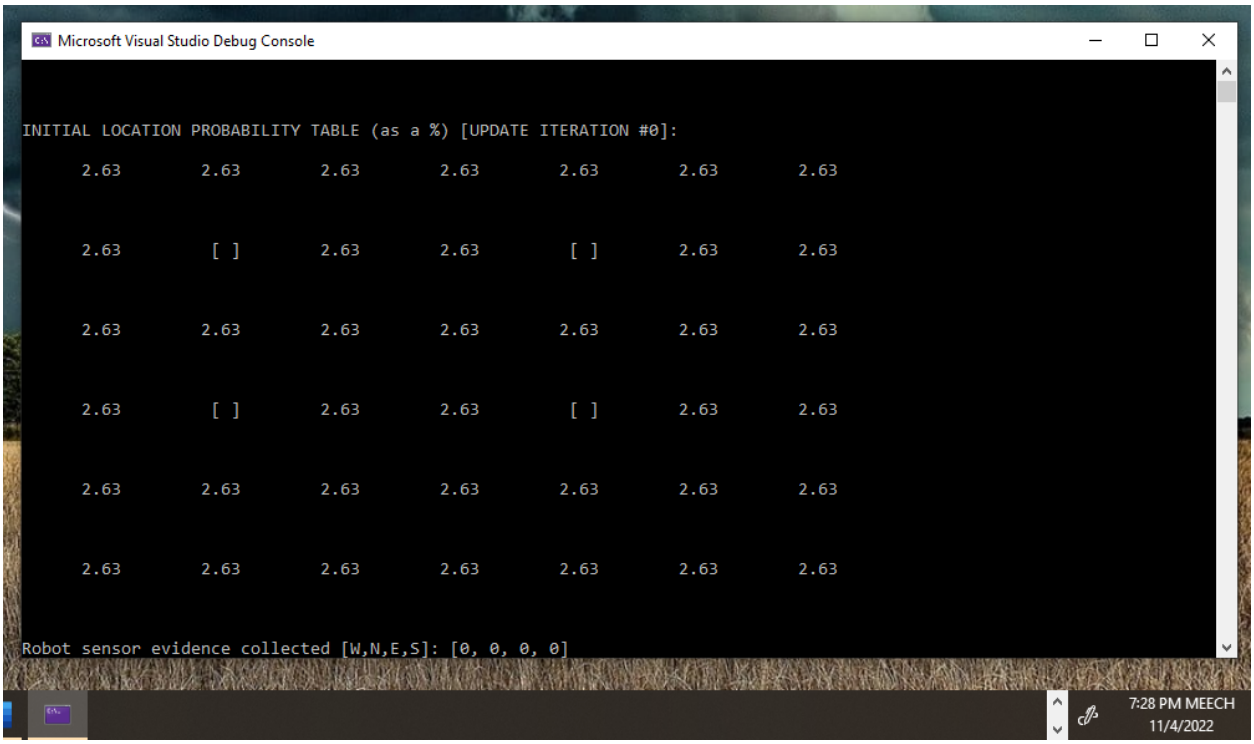
```
        table_struct.tablePos_locationProb_posterior[row - 1][col] += (right_probability
* table_struct.tablePos_locationProb_prior[row][col]);
}
```

Screenshots of solution output. Notice my last name and the time on the bottom right of each screenshot:

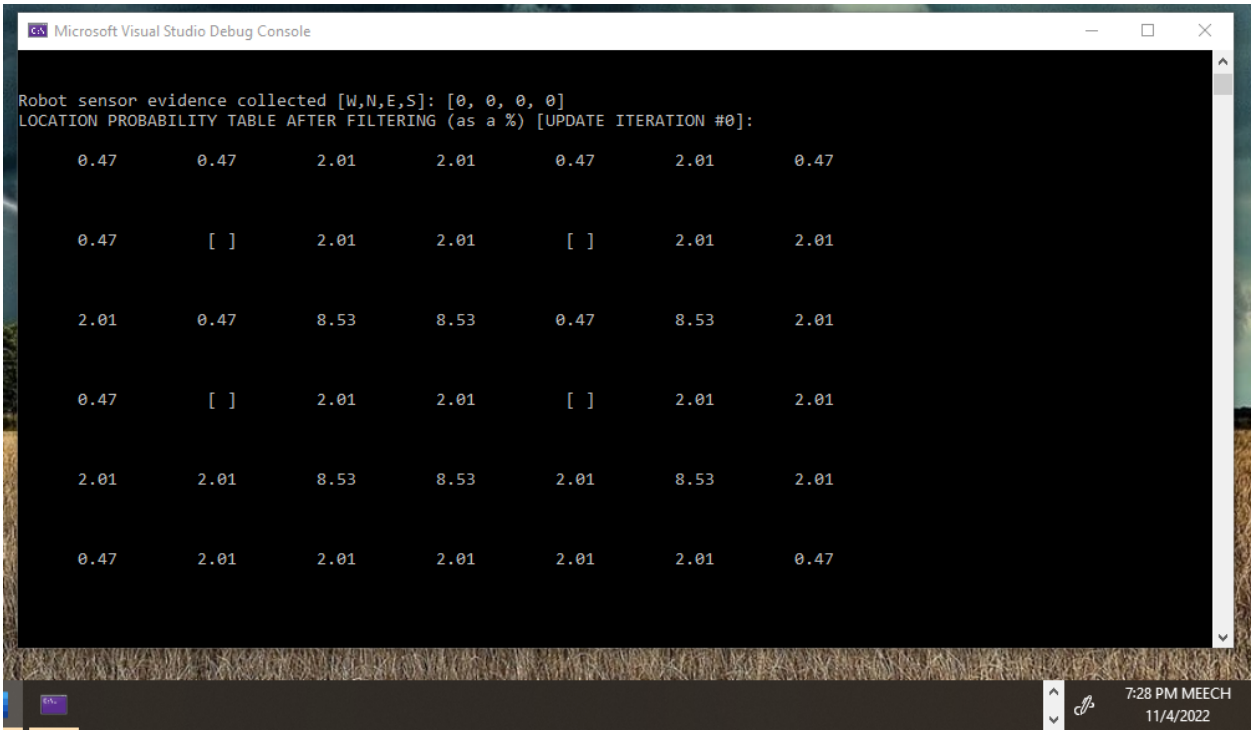
**Notice how you are never 100% certain where the robot is, and also whenever the robot moves once the values converge, it is pretty easy to tell where it is and what square it moves too (with much greater certainty). This is especially demonstrated in my bonus test cases after the standard test cases show a convergence of where the robot most likely is. Also, if we were to just keep sensing and filtering, the result would be that we would become more and more certain – even with the error of the robot adding up. Over time, however, eventually all of the error over adds up (over many, many iterations) and will cause the probabilities to converge to a uniform distribution in this stochastic setup (especially if you moved the robot to every location over and over). I also notice that if you move the robot to locations that are more unique, you can get the probabilities to converge on that location with a much higher certainty.

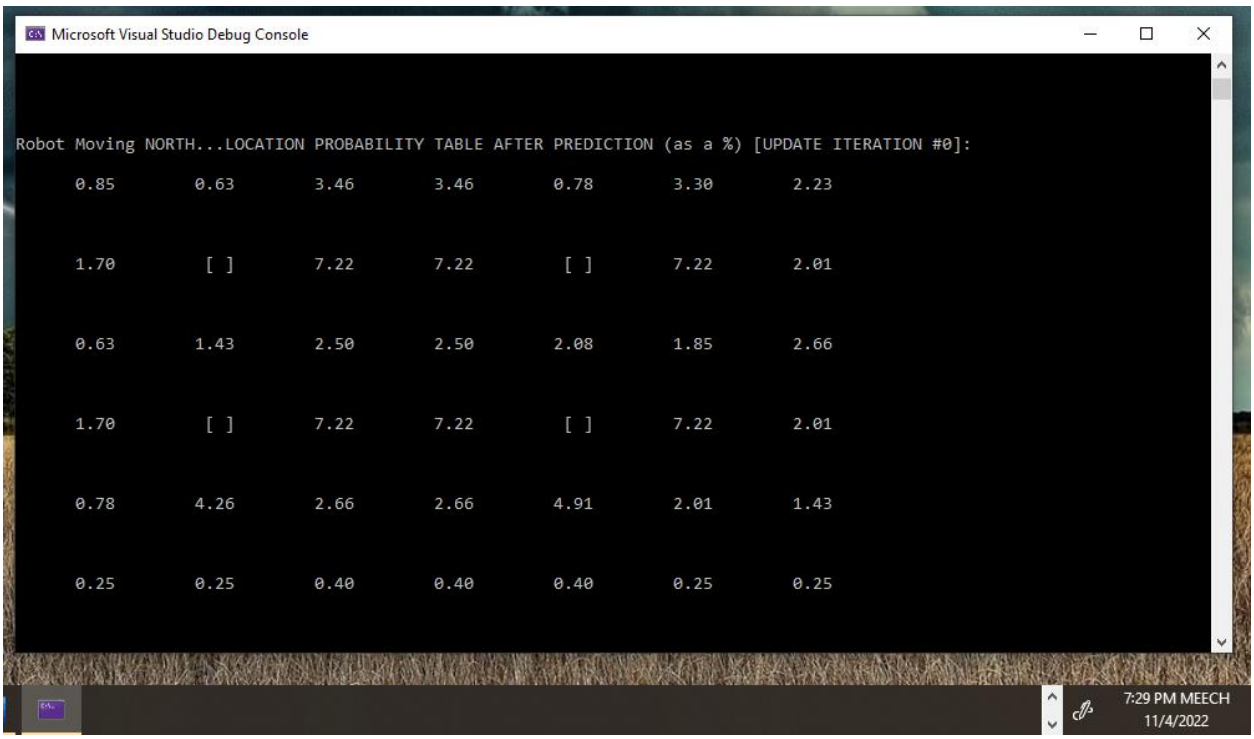
Standard Test Case Screenshots:

Initial Probabilities == uniform

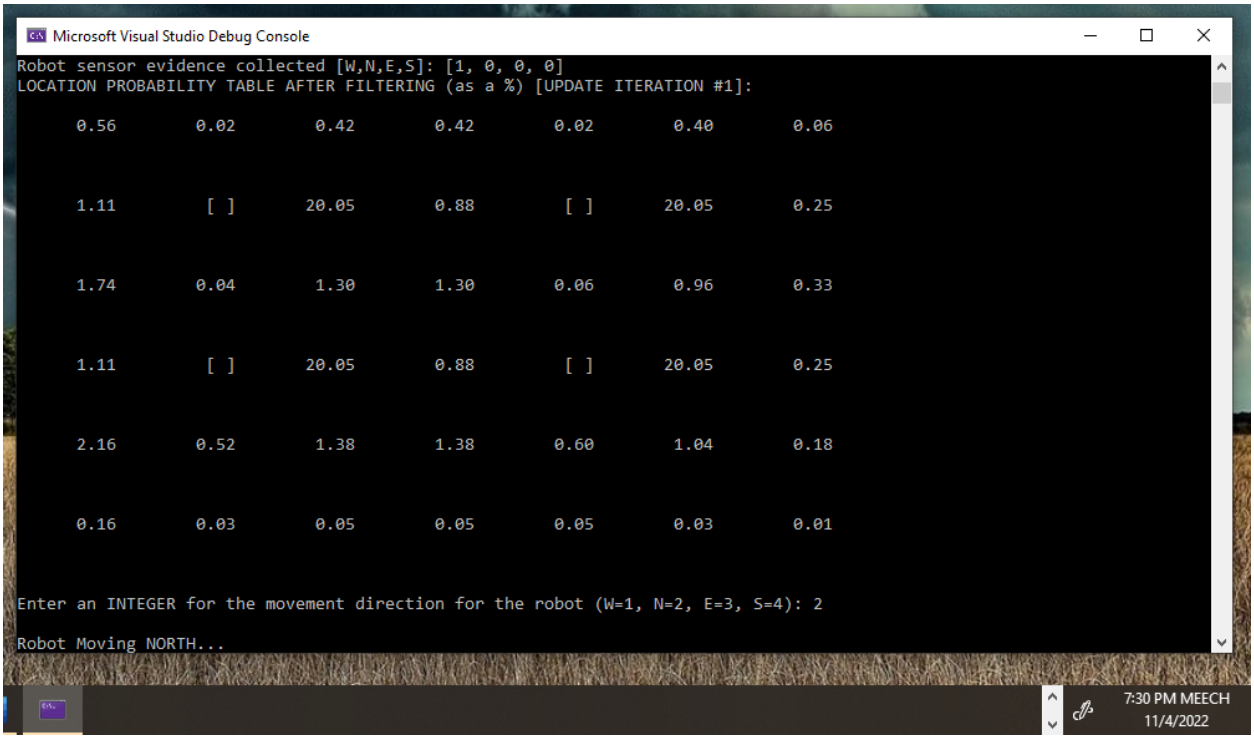


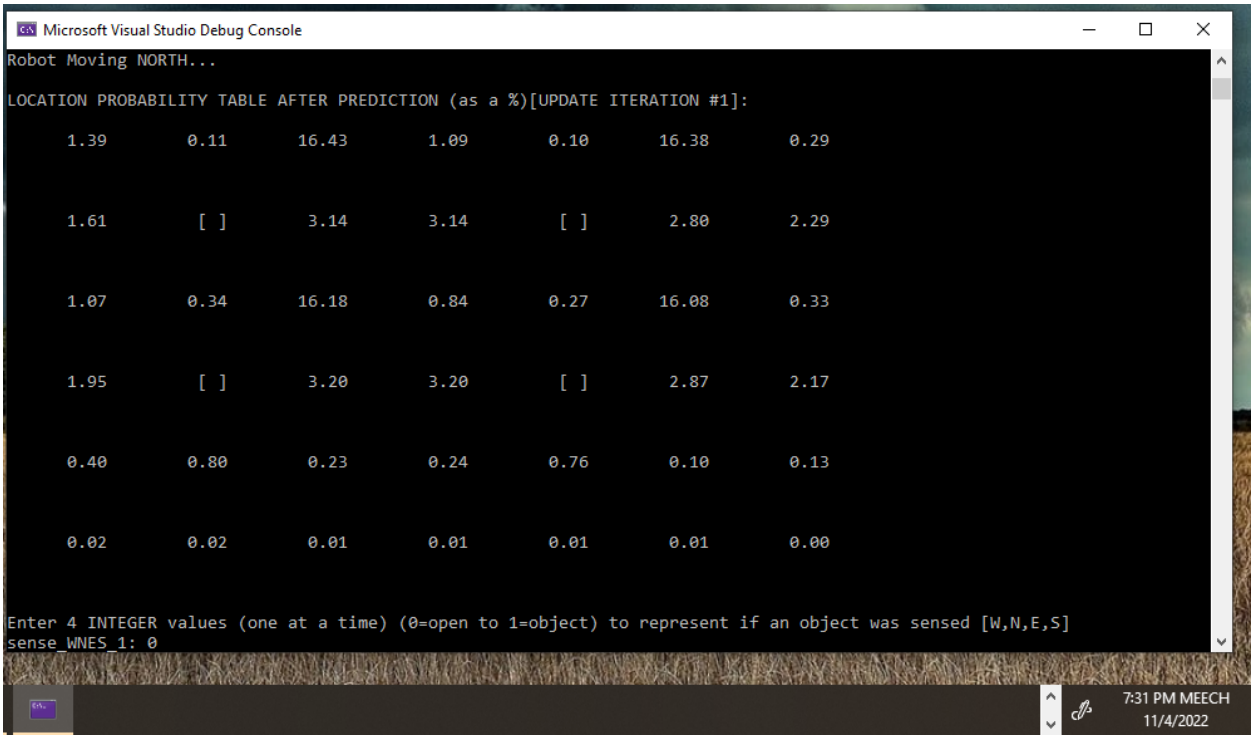
SENSE [0,0,0,0], MOVE N



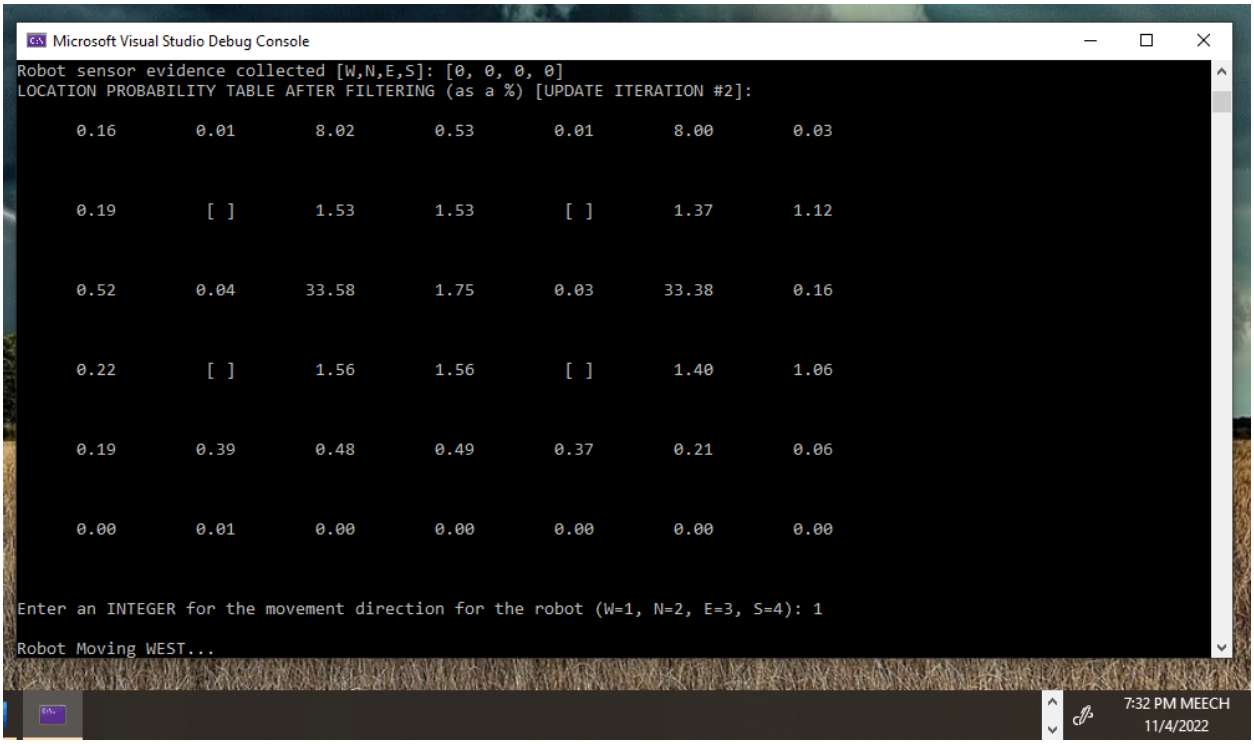


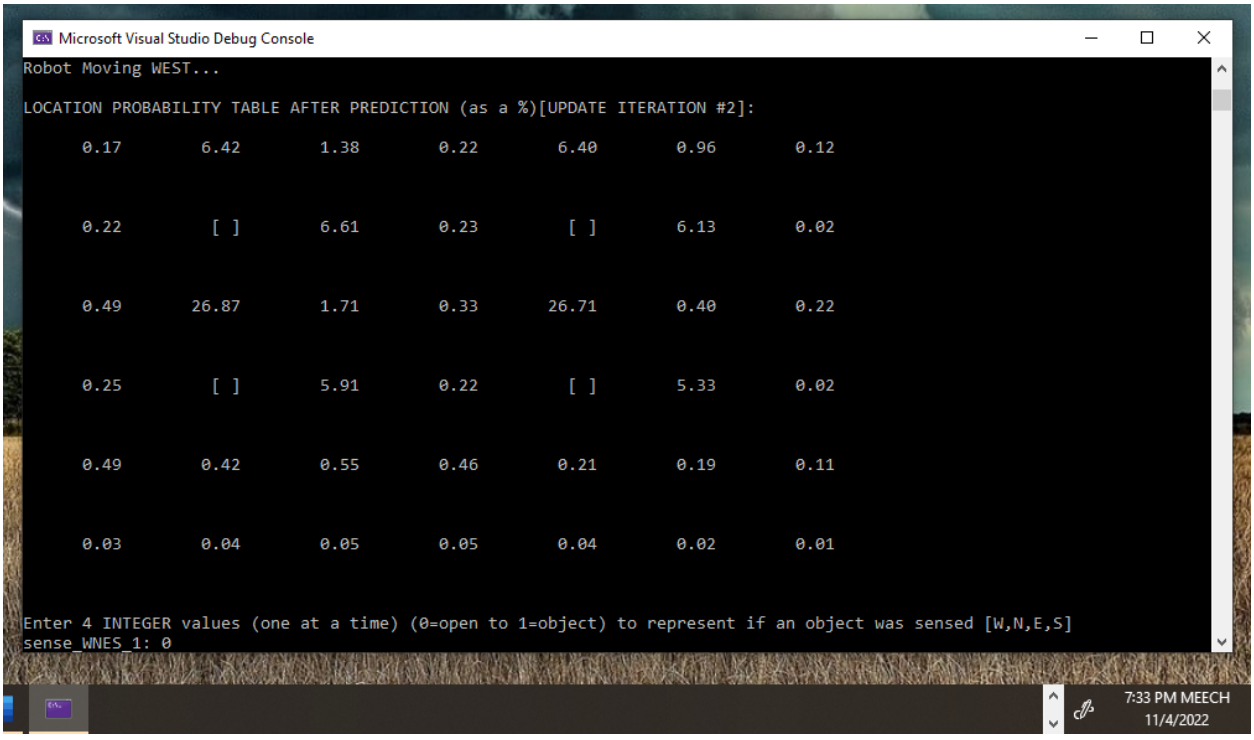
SENSE [1,0,0,0], MOVE N



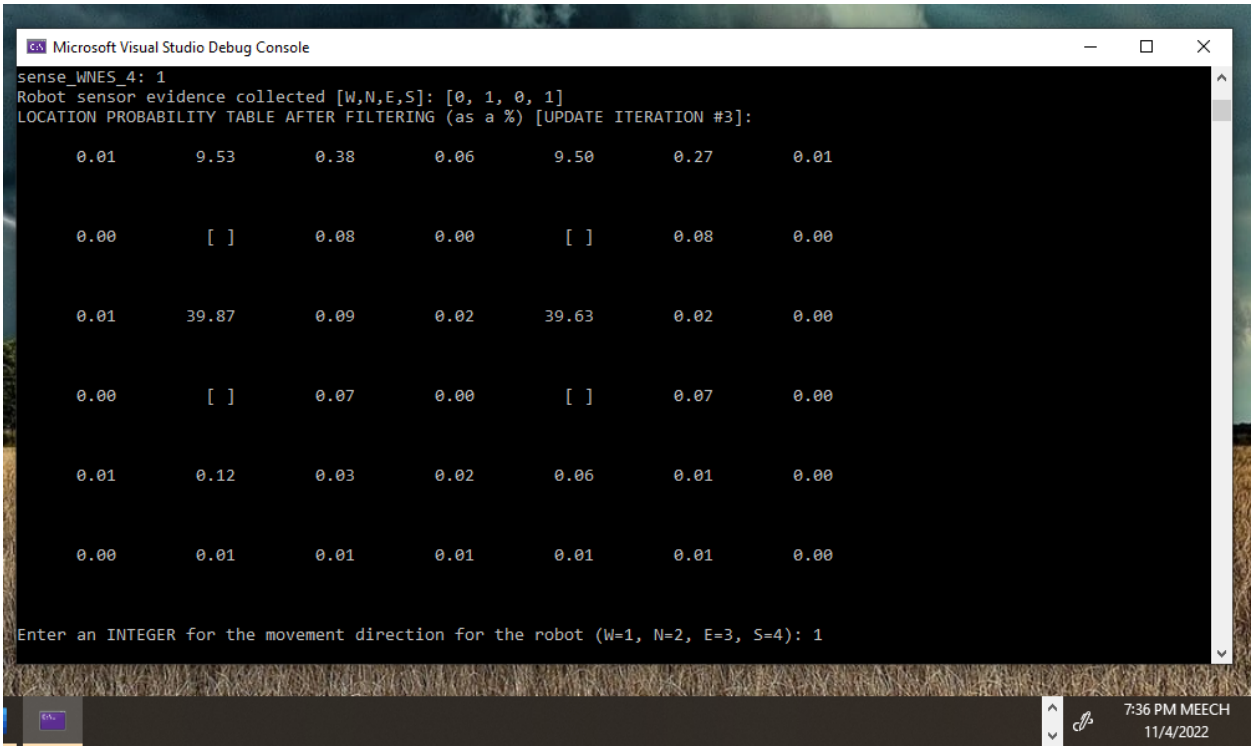


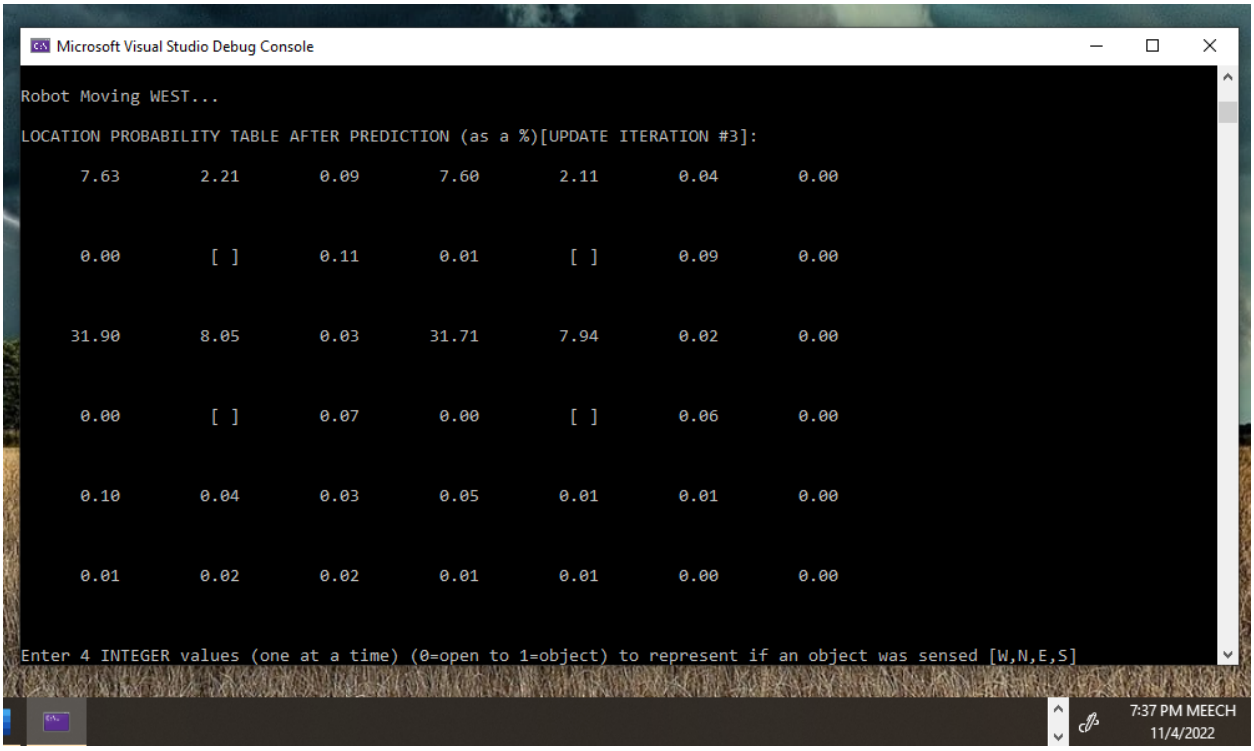
SENSE [0,0,0,0], MOVE W



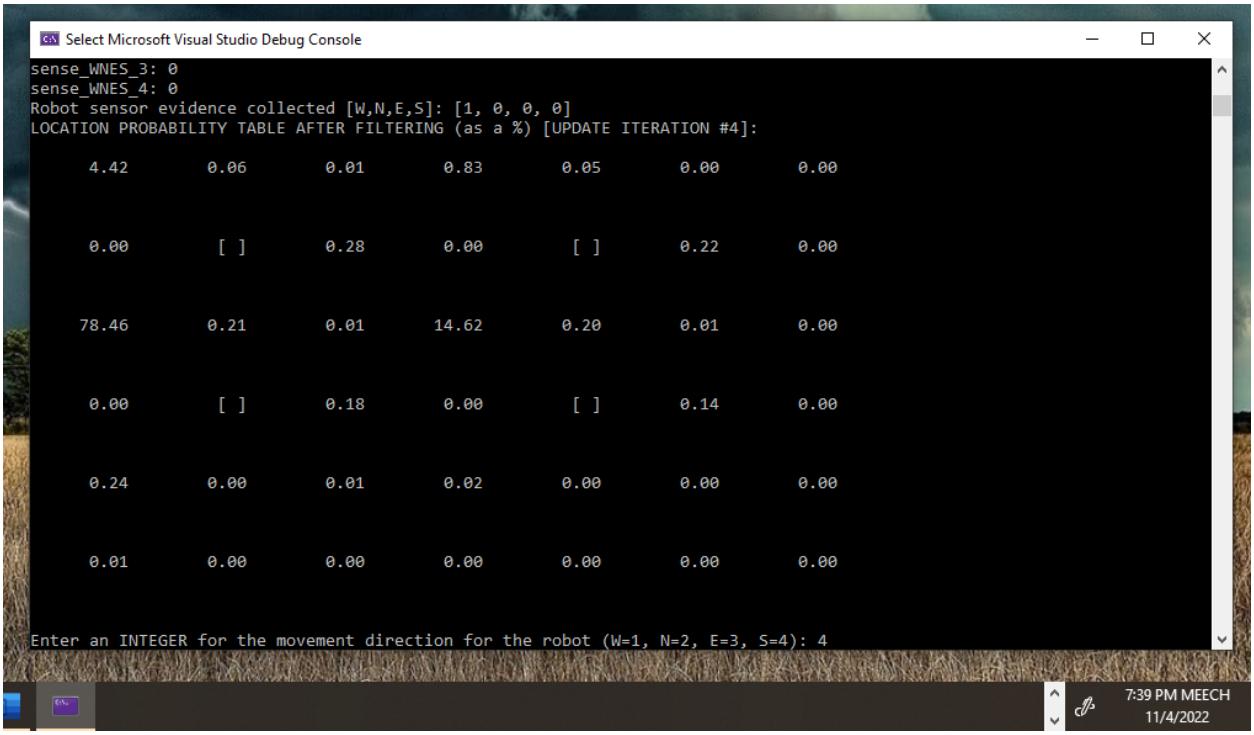


SENSE [0,1,0,1], MOVE W



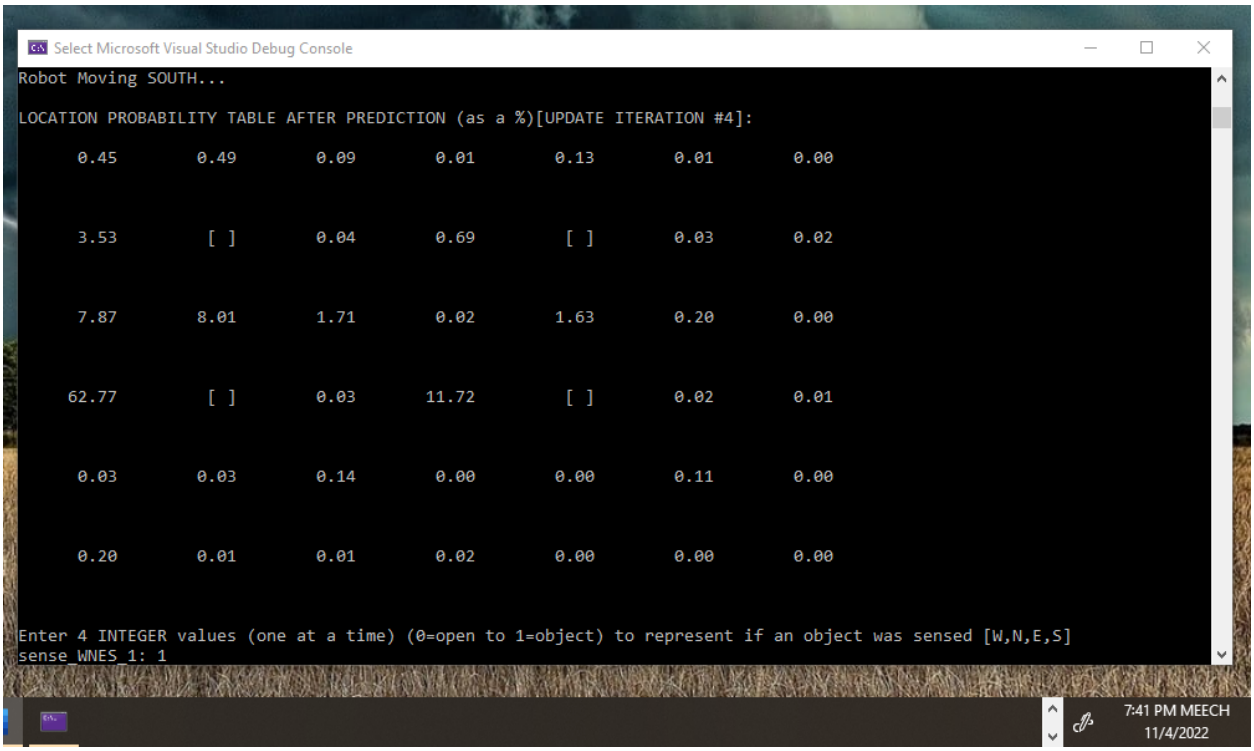


SENSE [1,0,0,0]

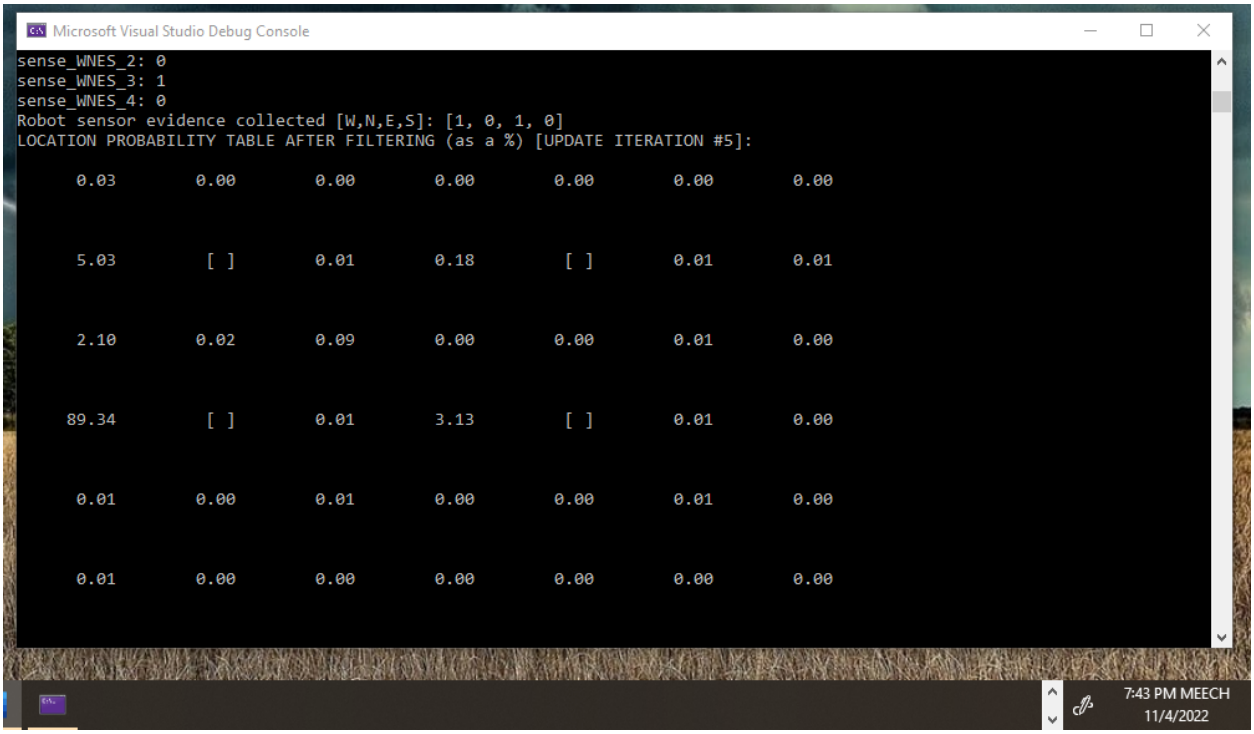


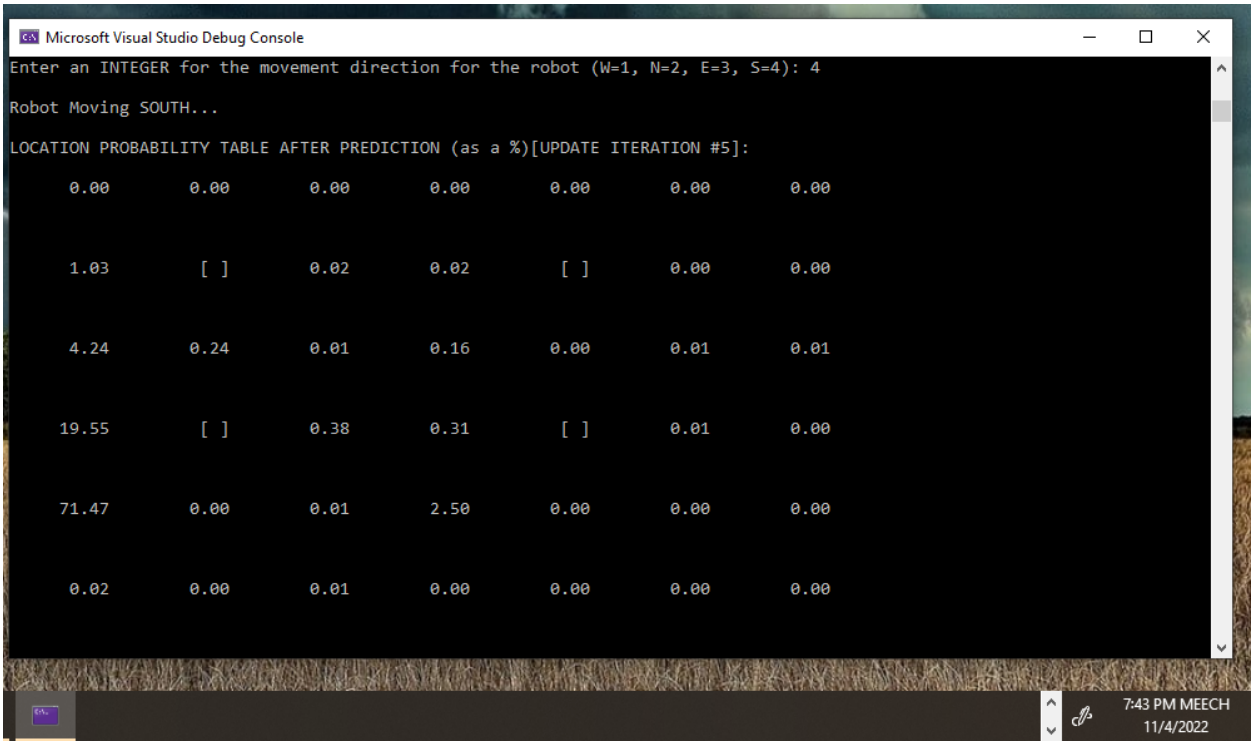
BONUS moving EAST and SOUTH screenshots:

MOVE S (PREVIOUS SENSE AS SHOWN IN LAST SCREENSHOT WAS 1,0,0,0)

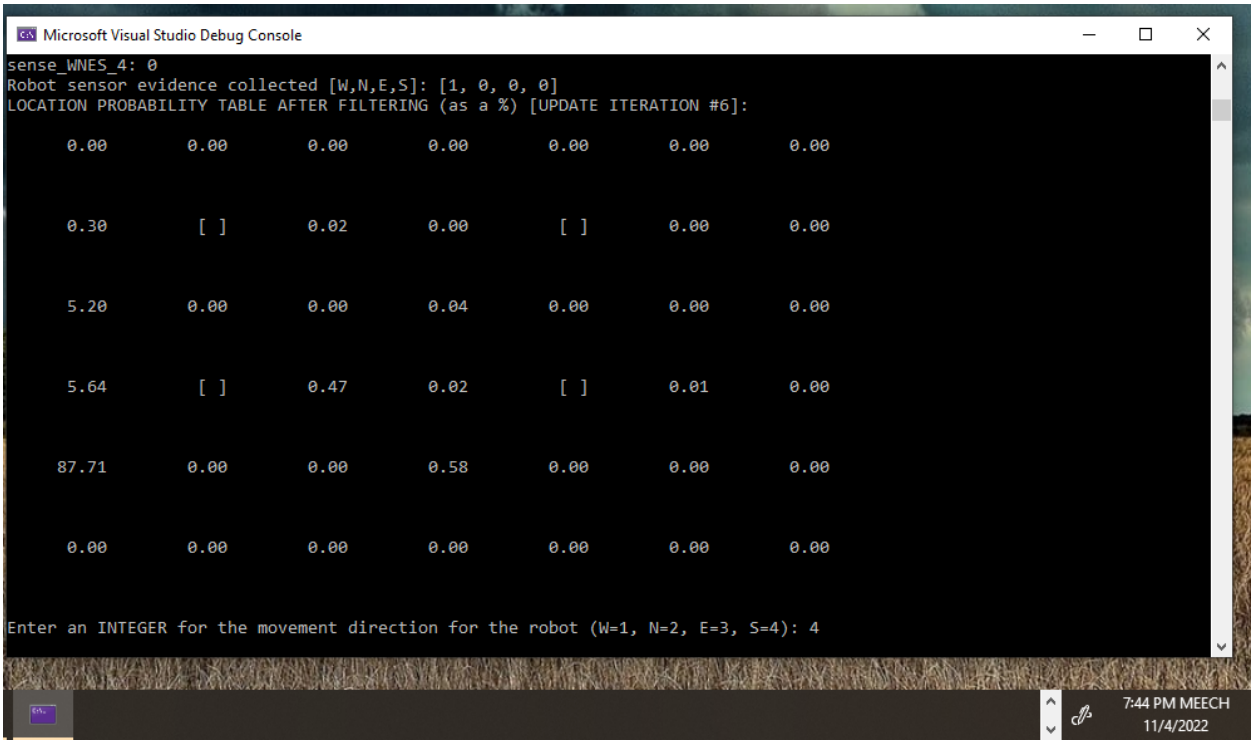


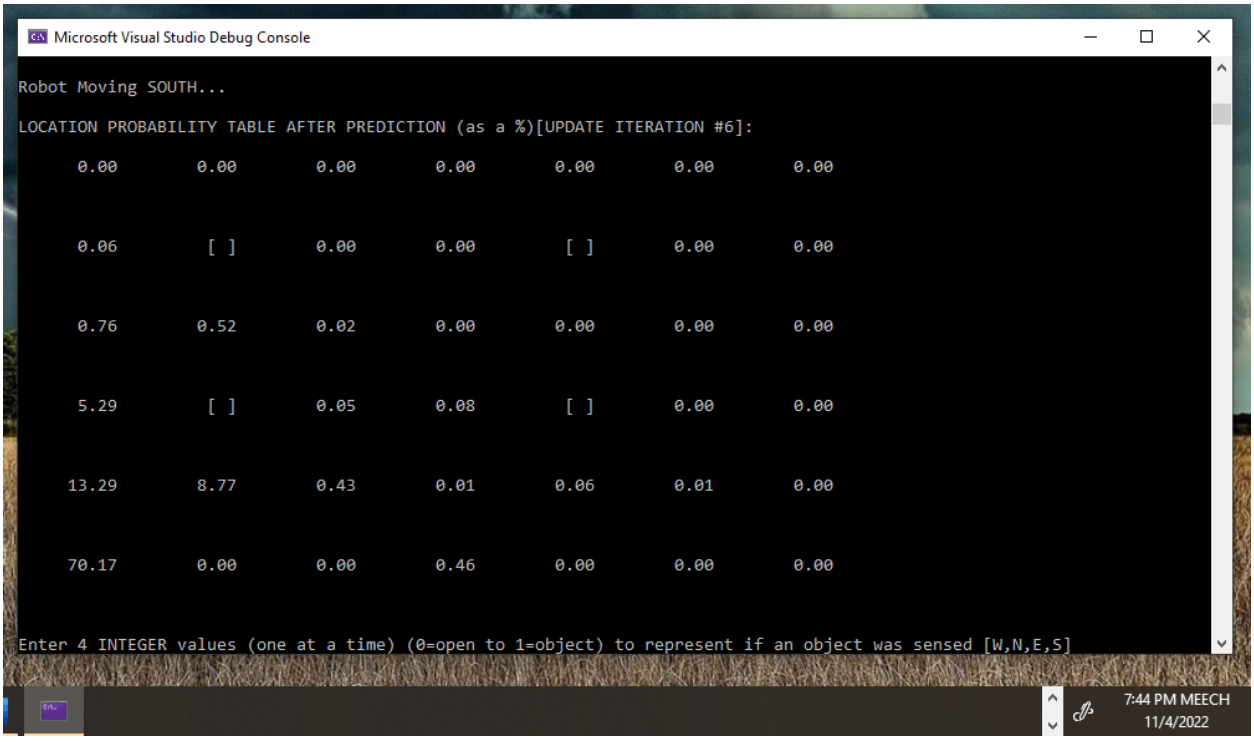
SENSE [1,0,1,0], MOVE S



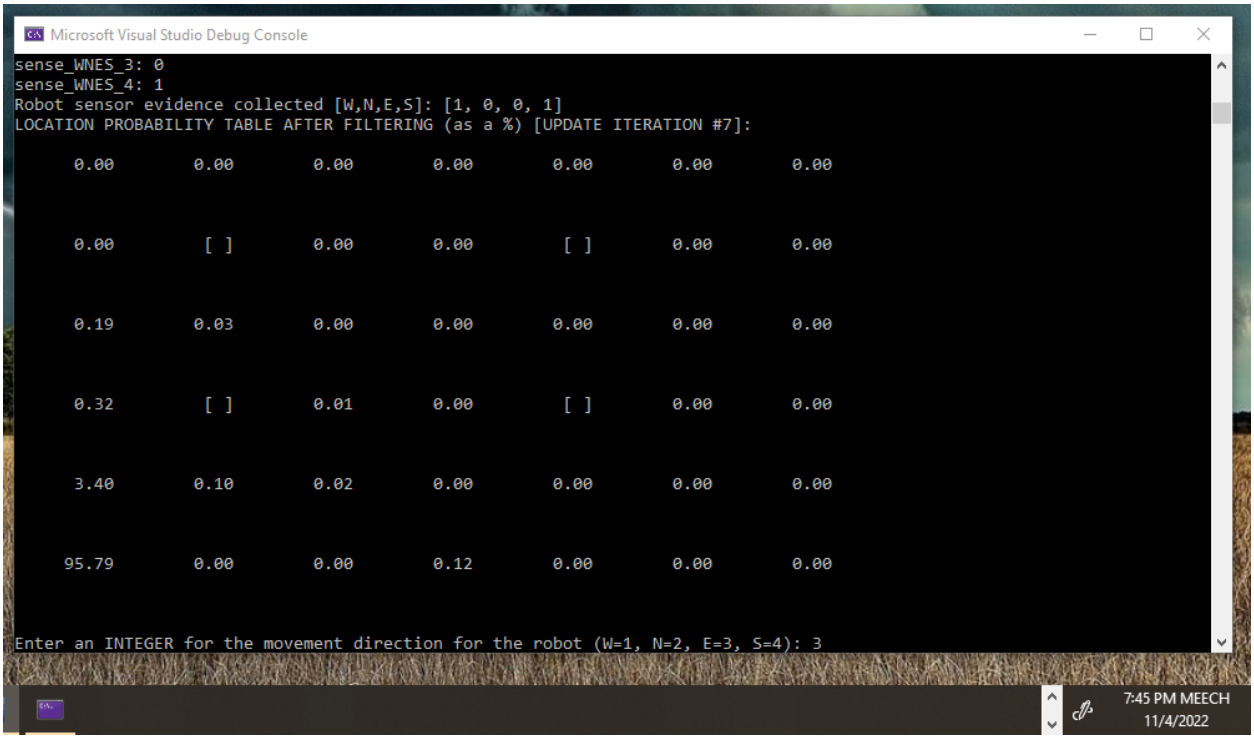


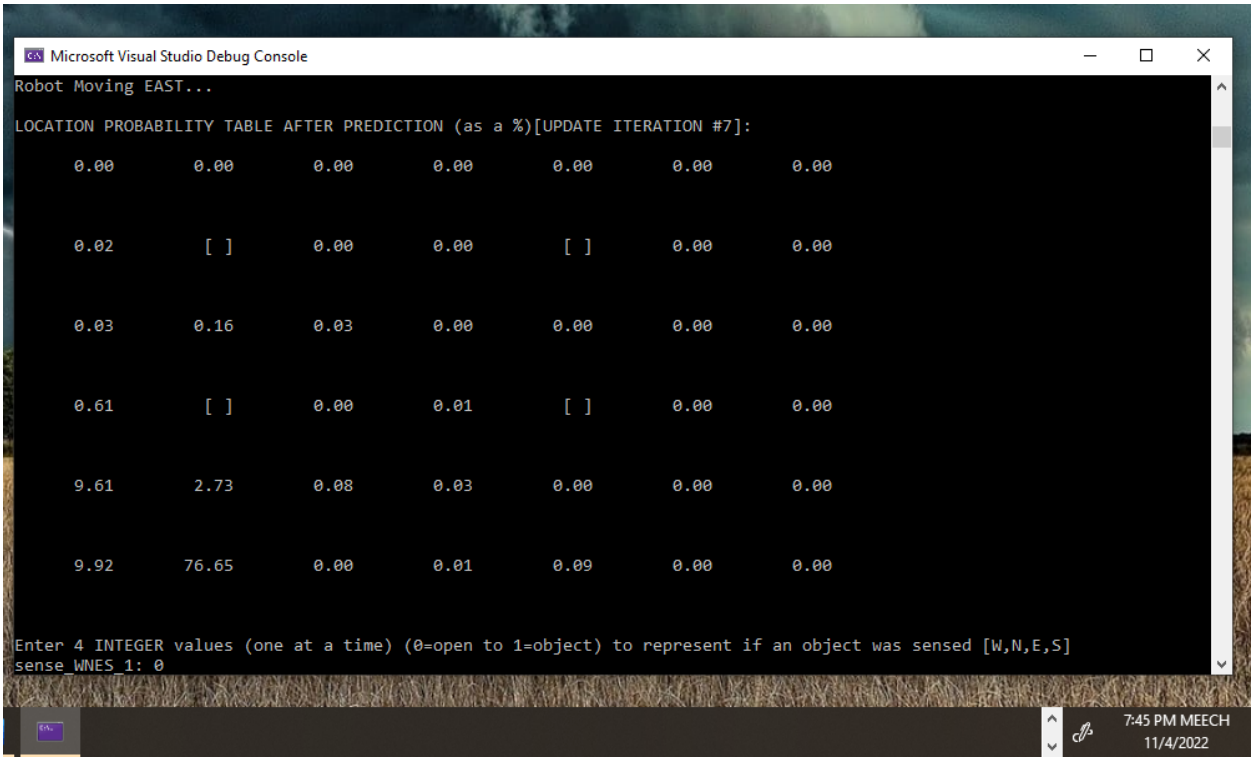
SENSE [1,0,0,0], MOVE S



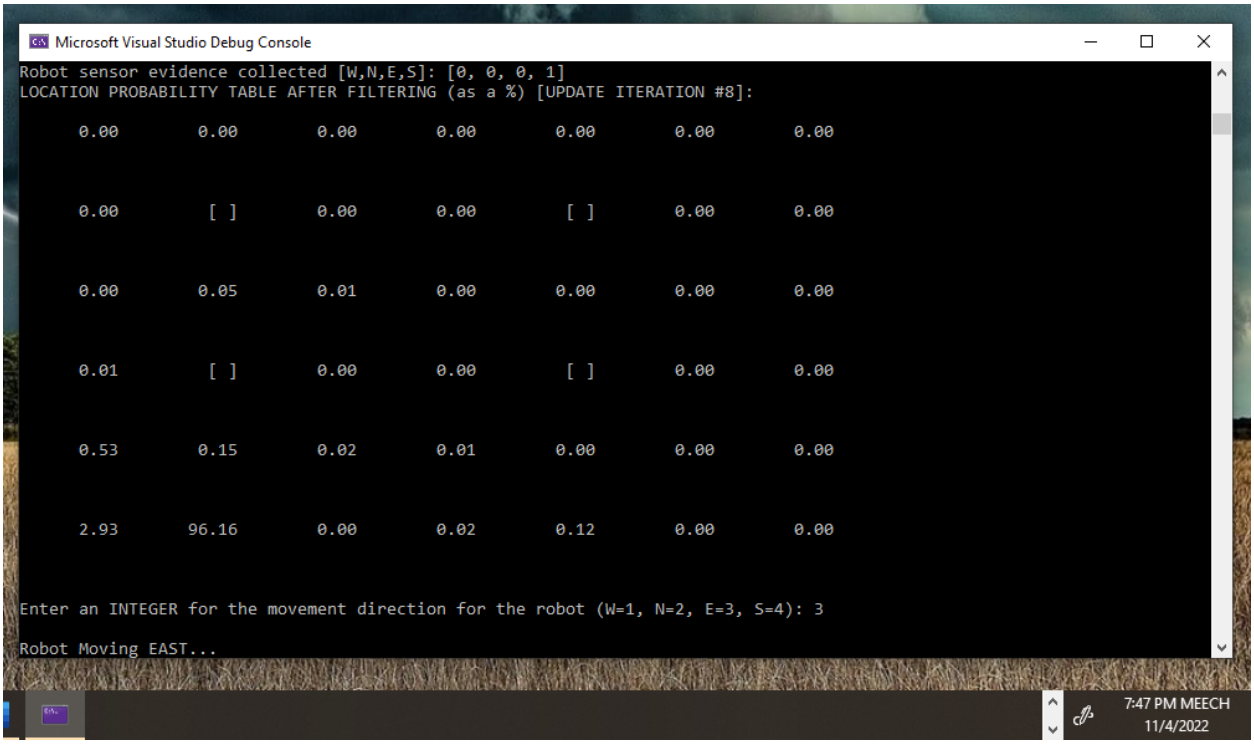


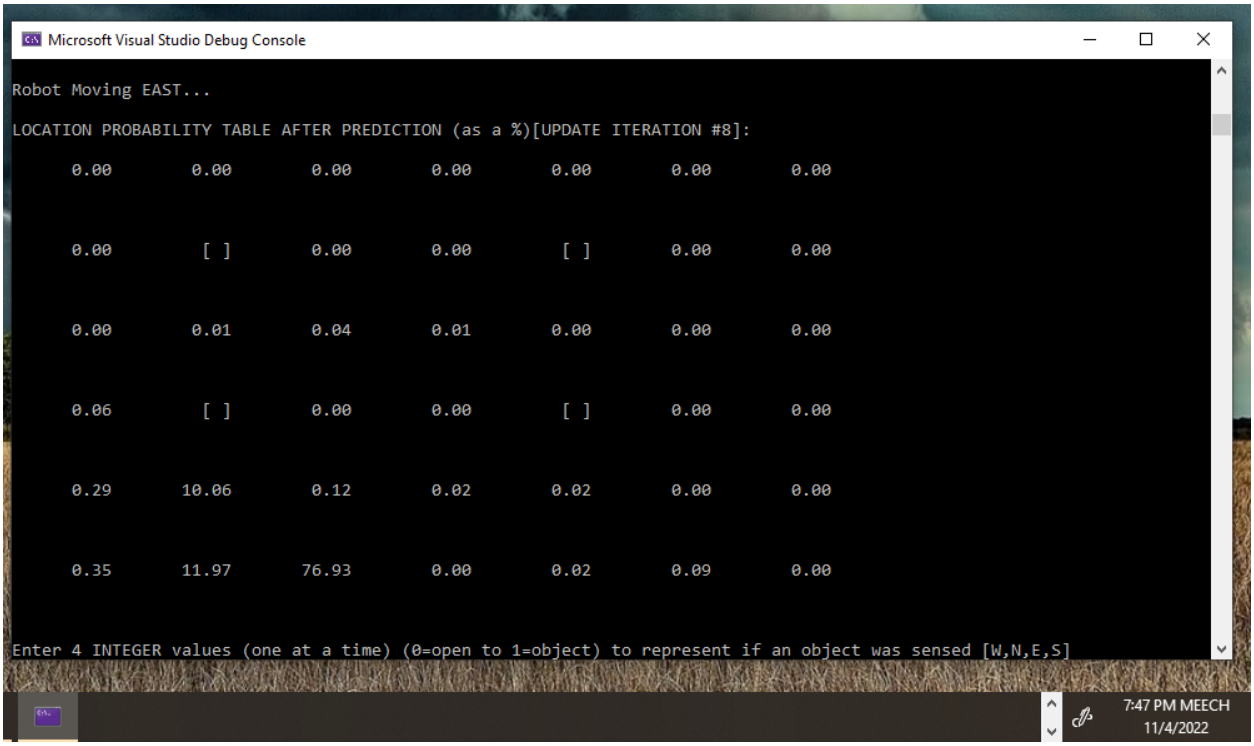
SENSE [1,0,0,1], MOVE E





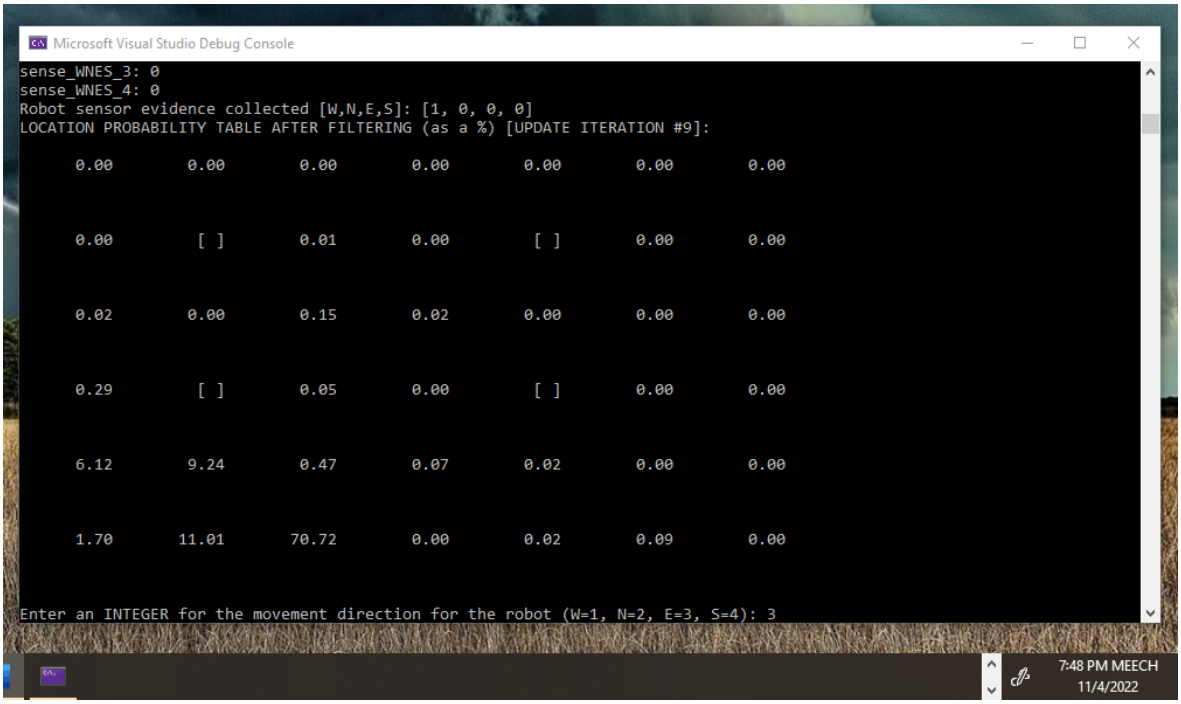
SENSE [0,0,0,1], MOVE E

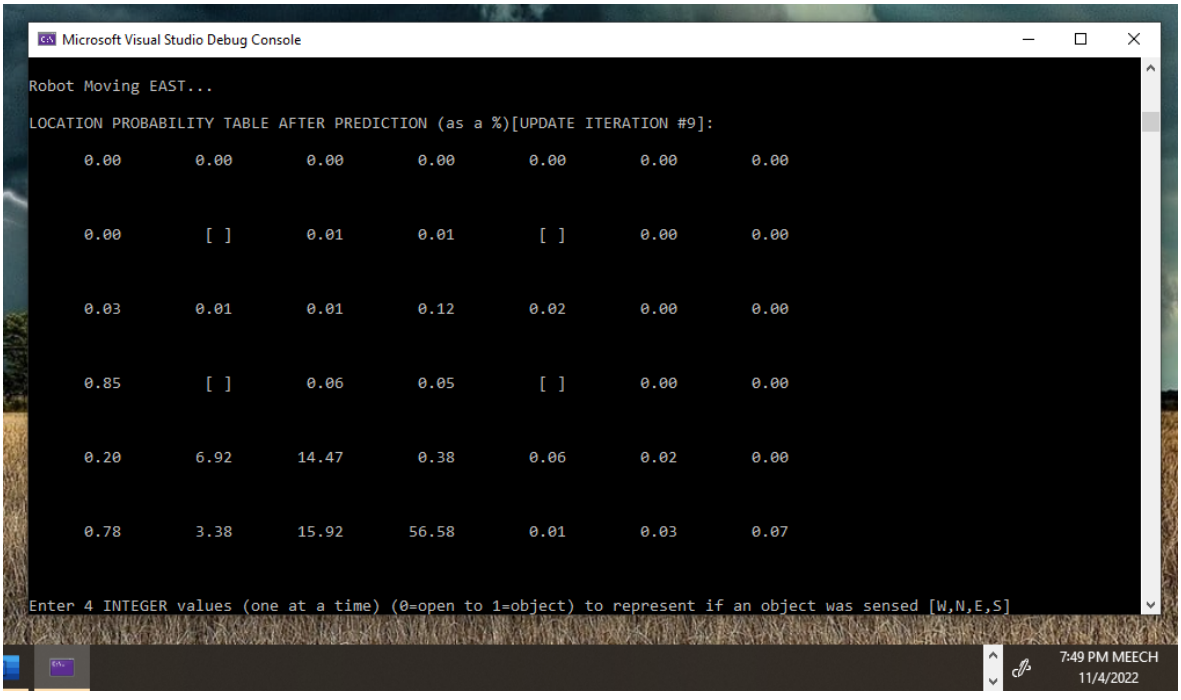




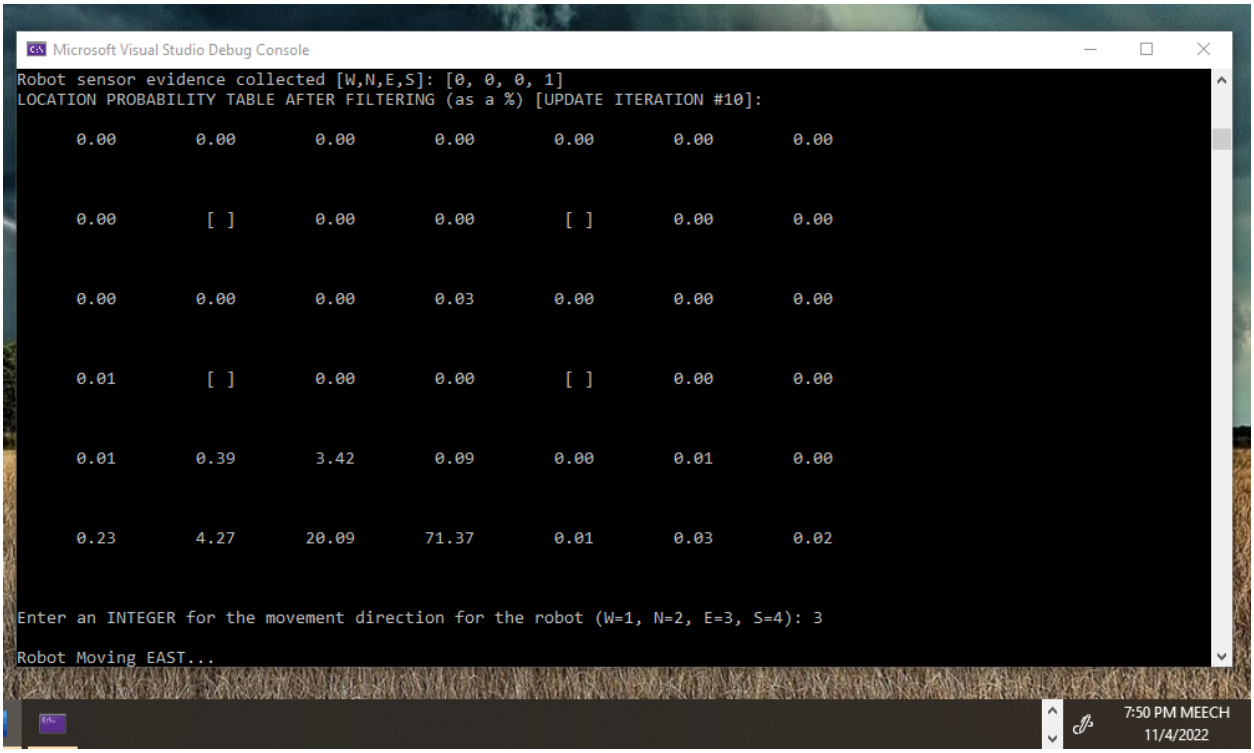
SENSE [1,0,0,0], MOVE E (***sensor error scenario***)

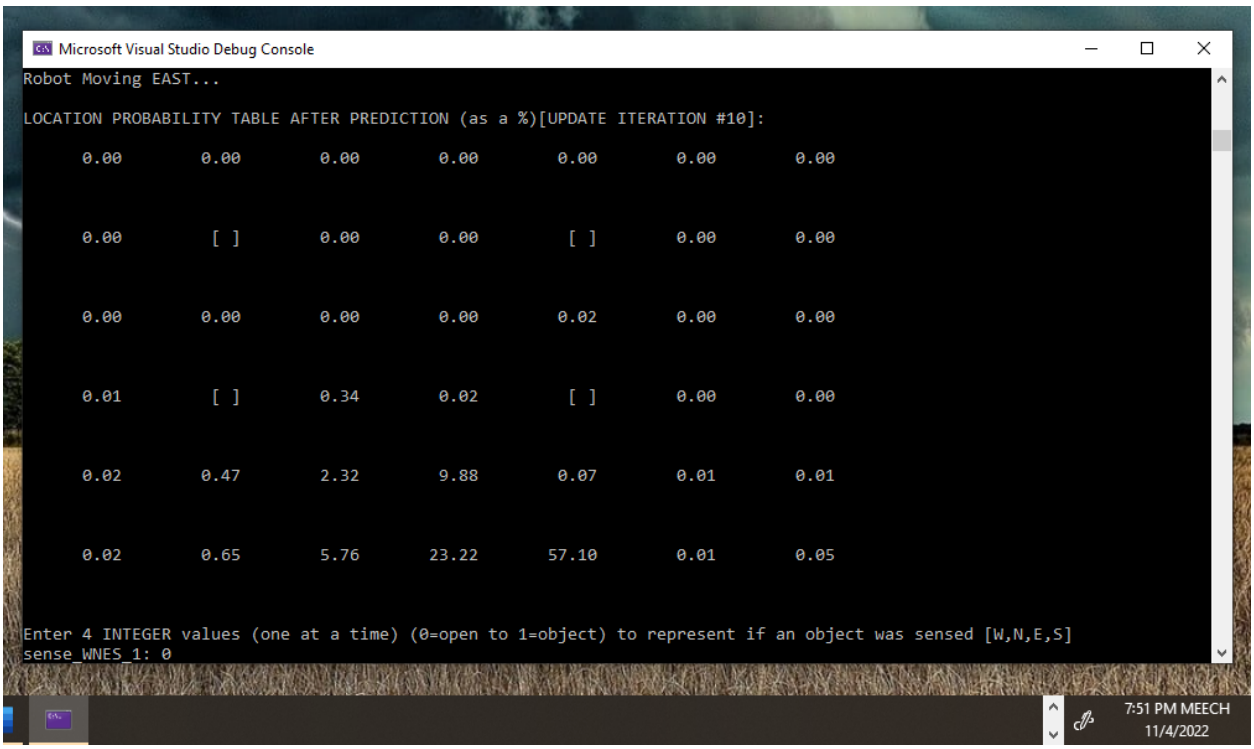
Notice, now robot is a lot more uncertain about where it is and the probability distribution is less concentrated on one location, and more distributed across 3-4 more locations (although it is still over 56% confident it is in the location as shown below). This error will not propagate in the next sense and move updates and eventually start to converge again with more certainty, particularly when we get to a corner wall location that is more unique/distinguished so that the probability distribution will start to more strongly converge as before on one location.



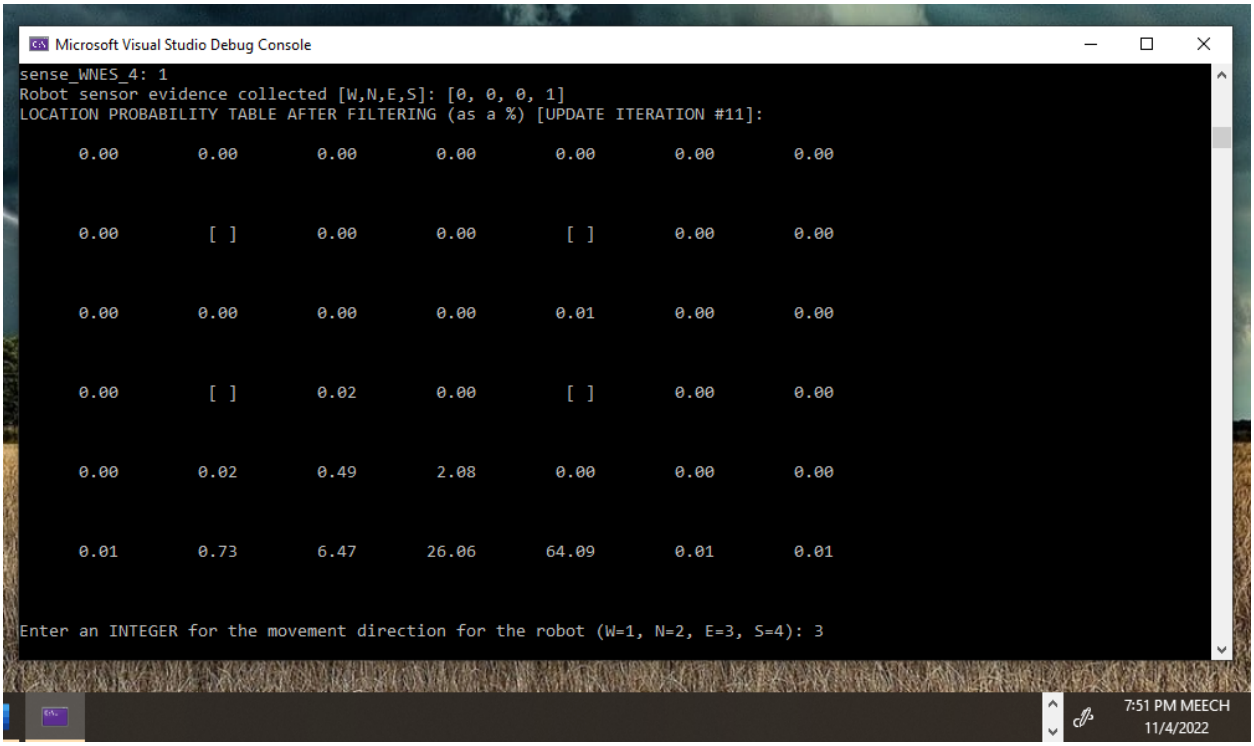


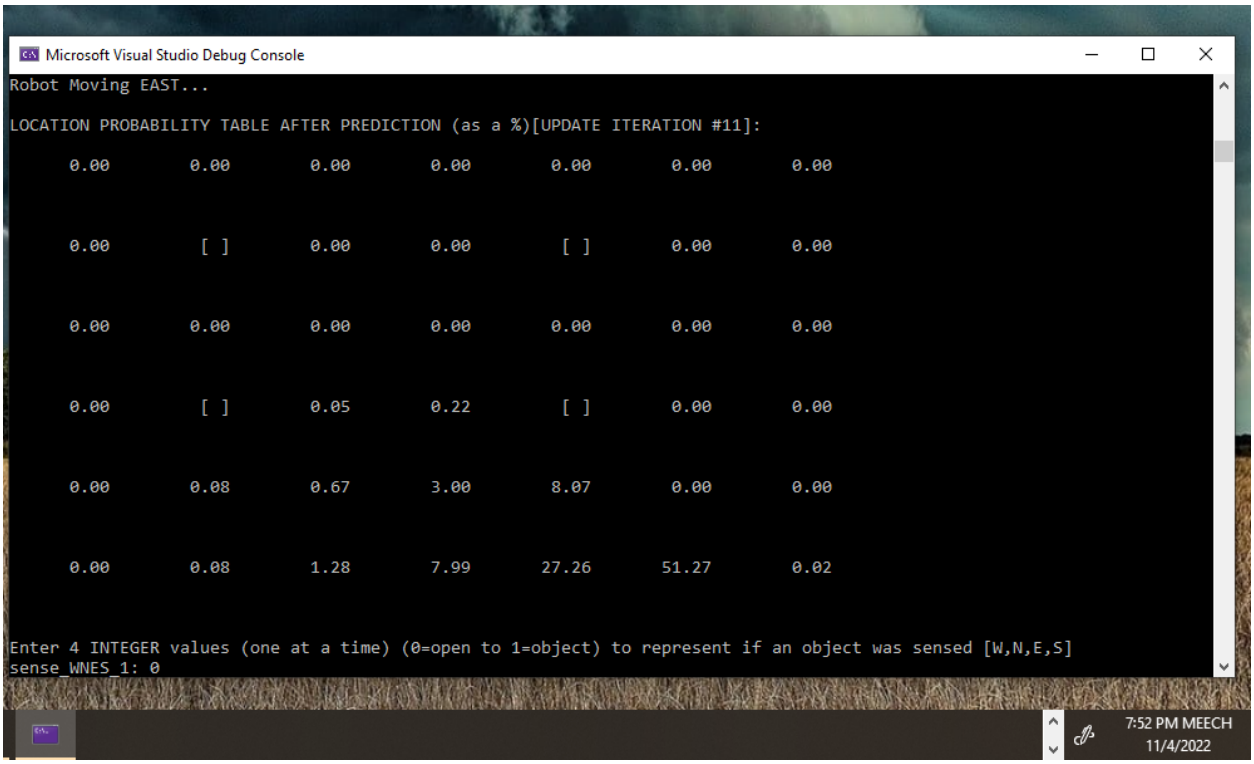
SENSE [0,0,0,1], MOVE E



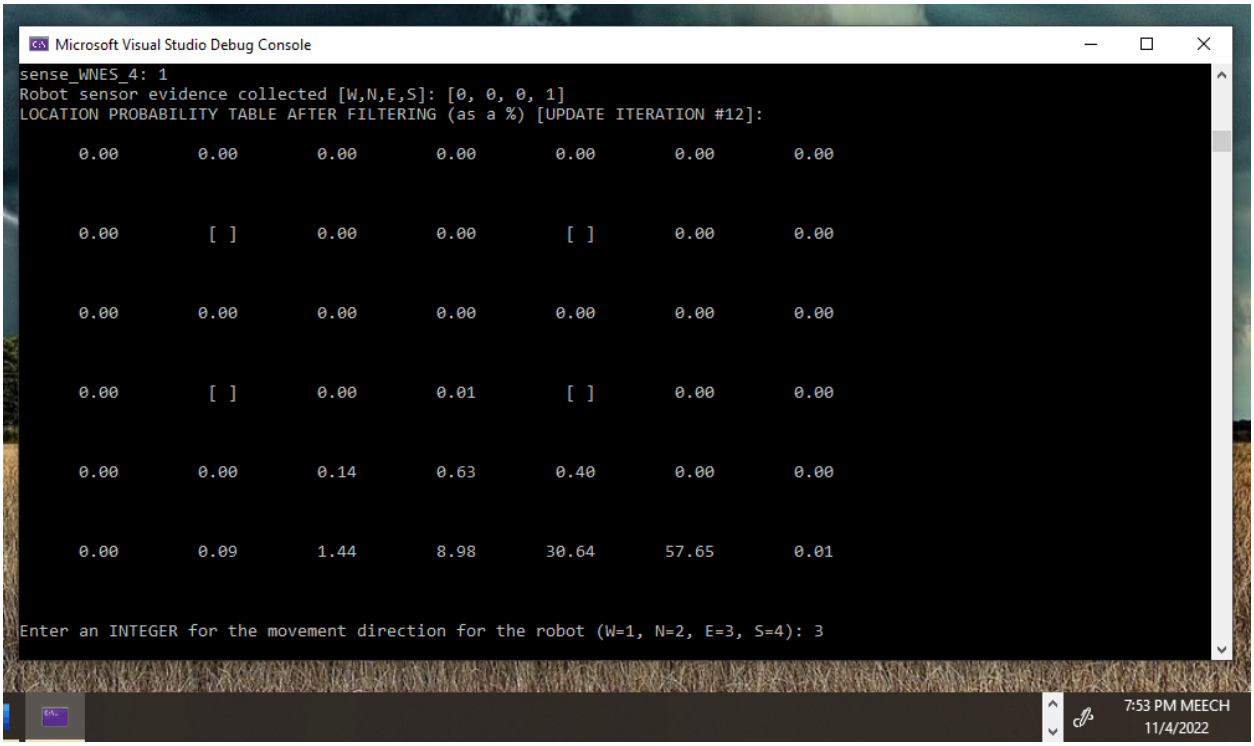


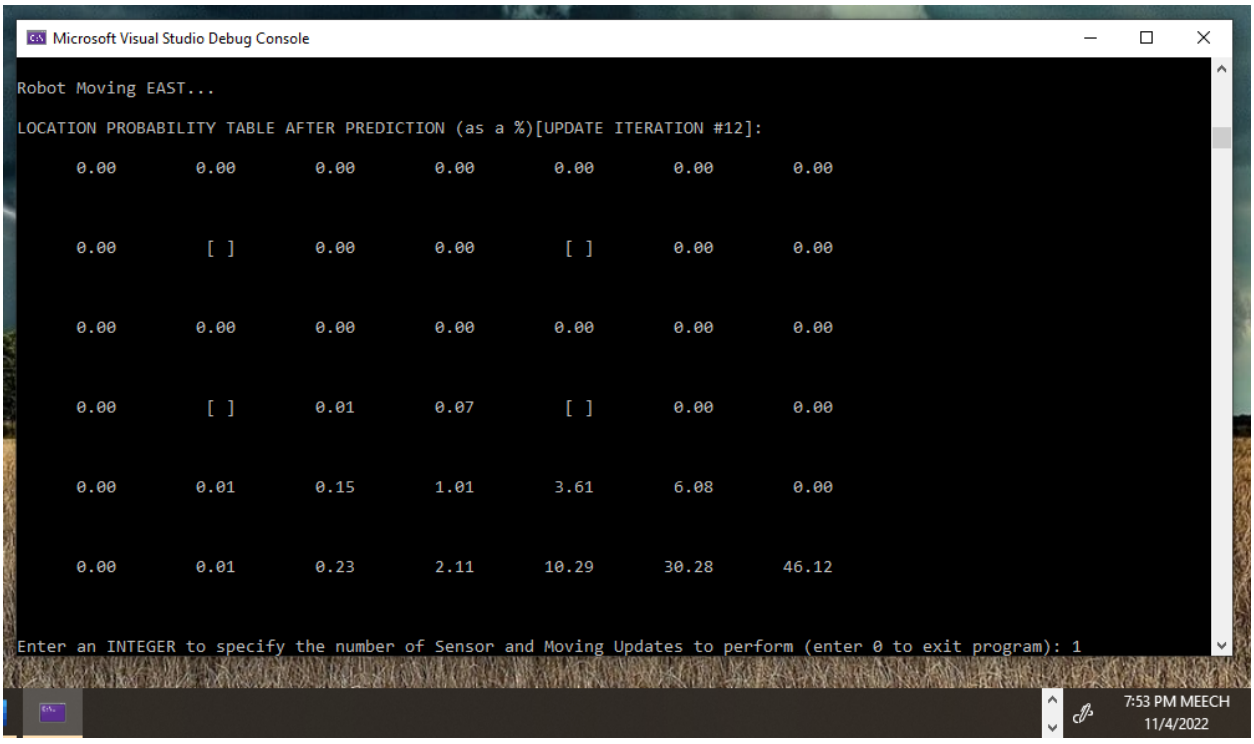
SENSE [0,0,0,1], MOVE E



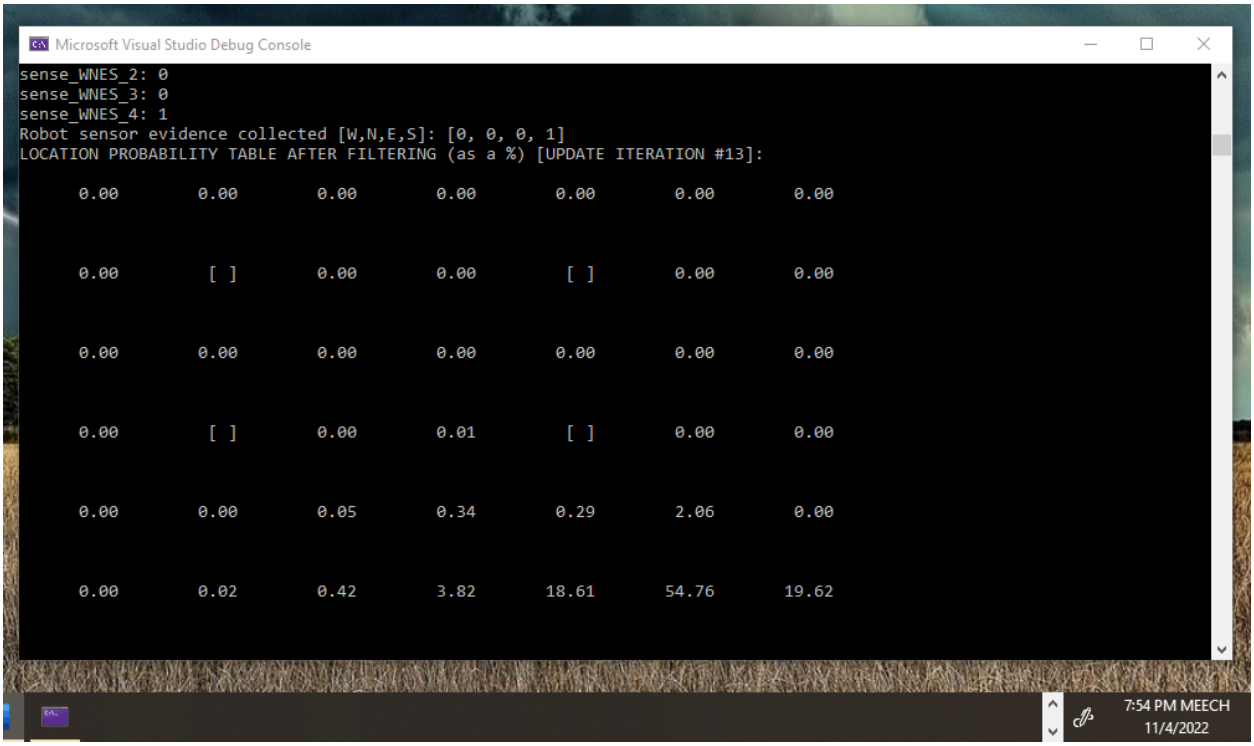


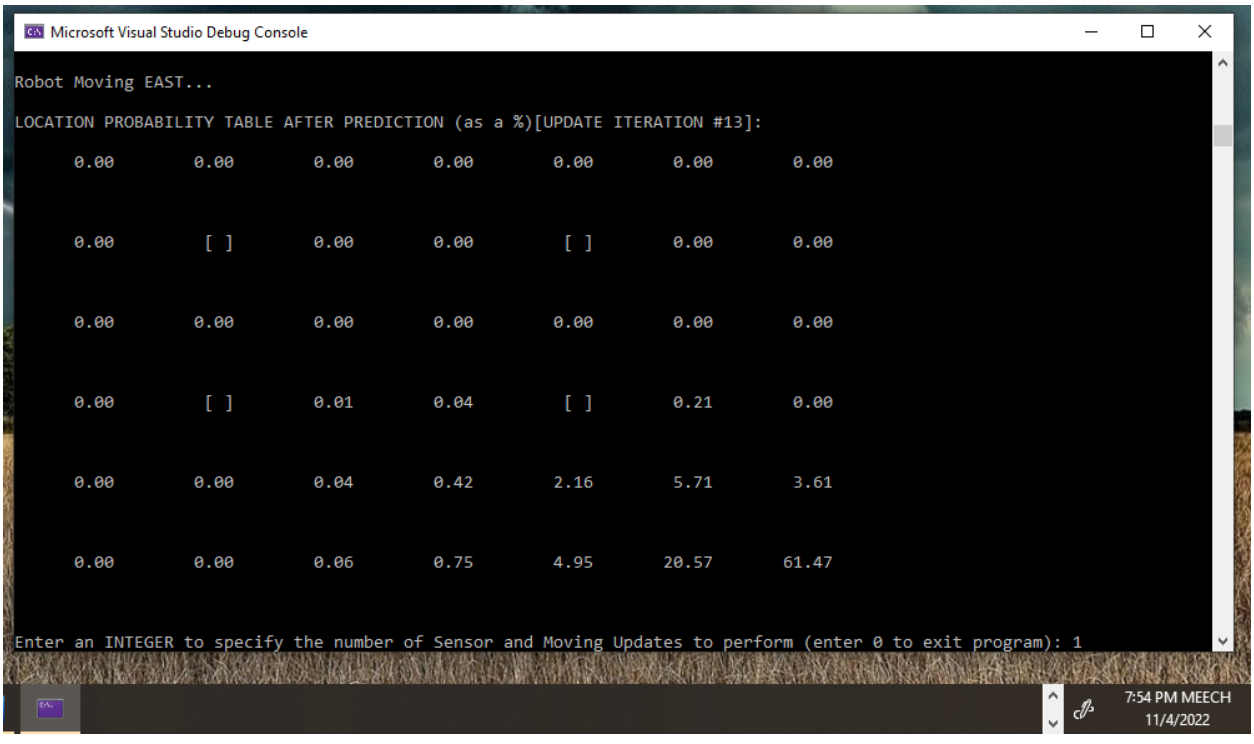
SENSE [0,0,0,1], MOVE E



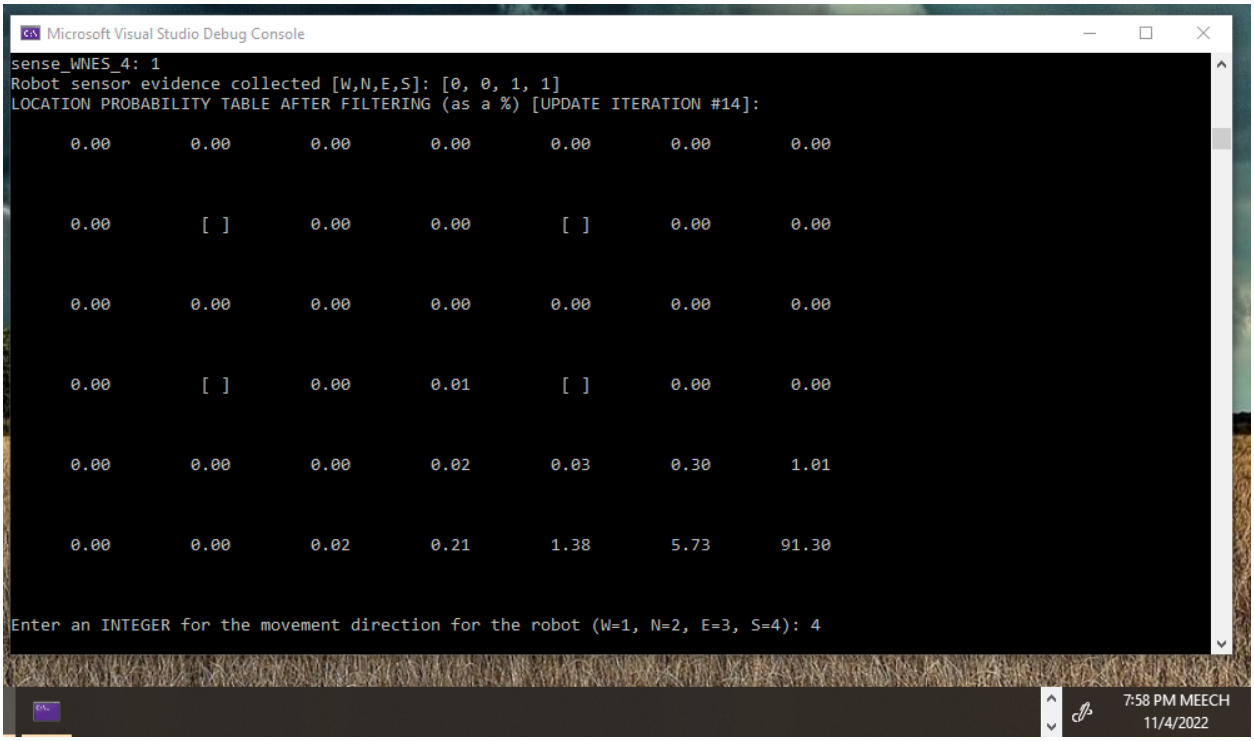


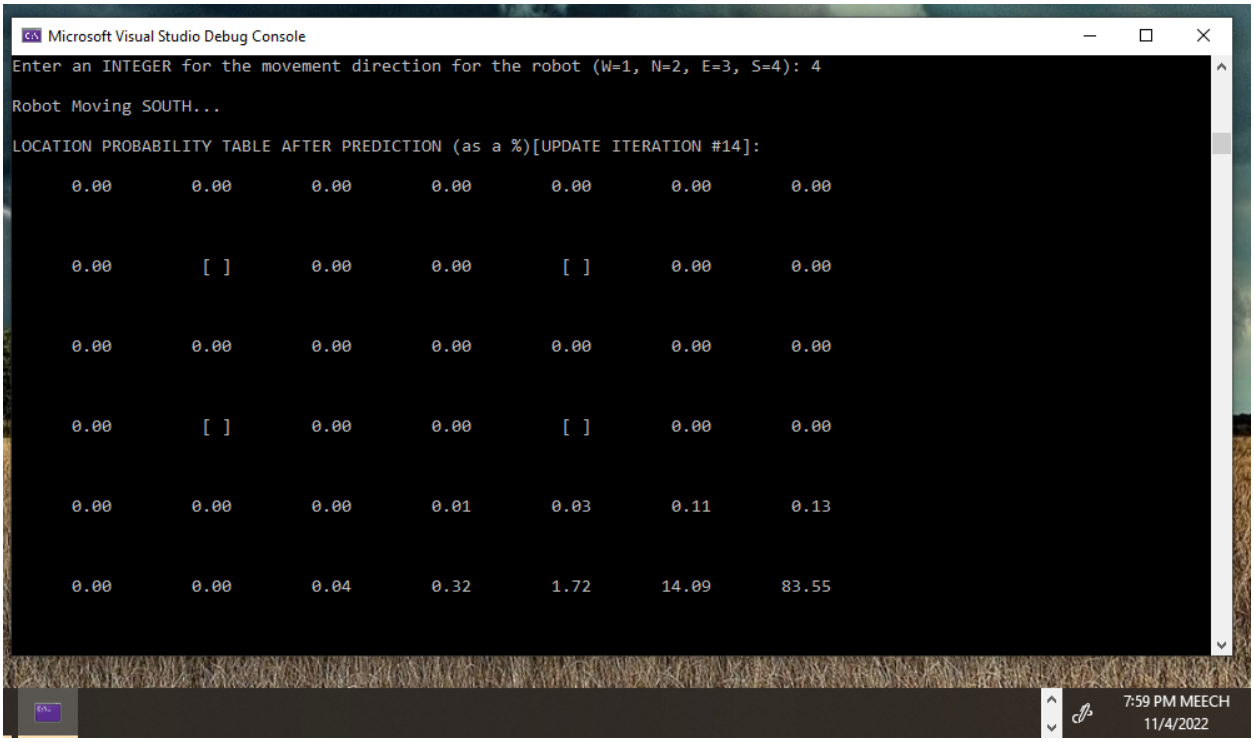
SENSE [0,0,0,1], MOVE E





SENSE [0,0,1,1], MOVE S (**scenario where probabilities converge more strongly on 1 location due to the unique corner location which has obstacles that make up the corner. Also, this is a bounce back movement scenario)





SENSE [0,0,1,1] → highest confidence in location, since bottom right corner is only location with obstacles to the E and S, and this is the second sense in a row with these *unique* measurements