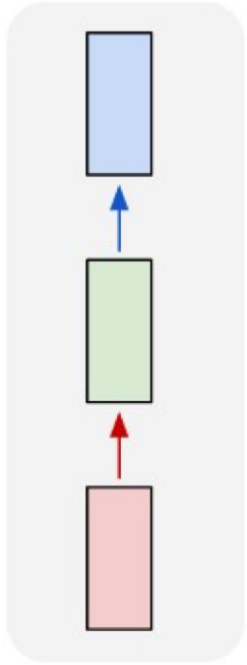


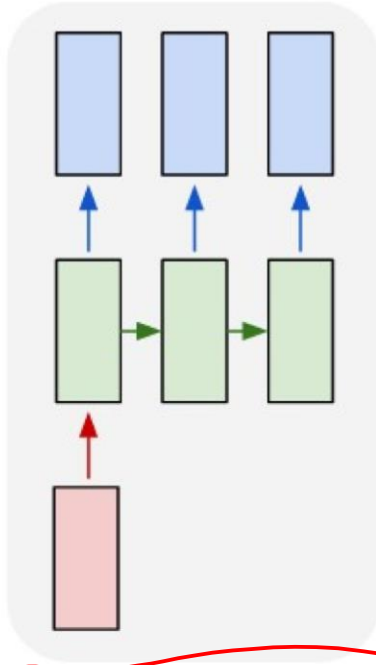
Lecture 7: RNN-II

Last Time: Recurrent Neural Networks

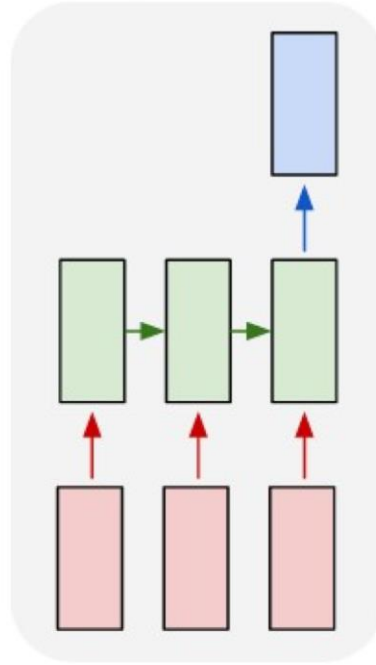
one to one



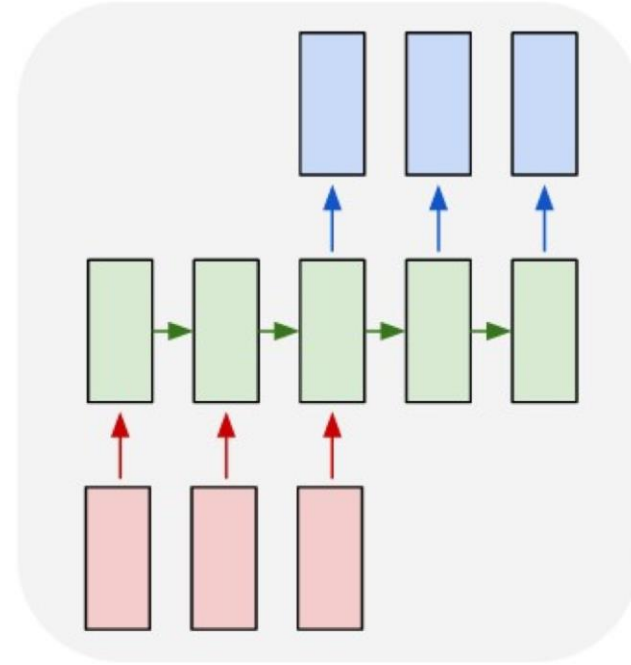
one to many



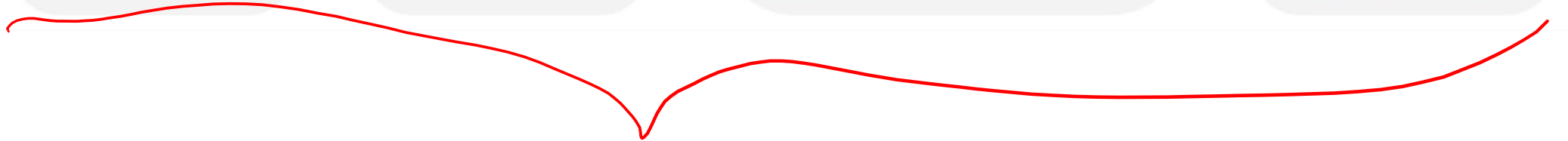
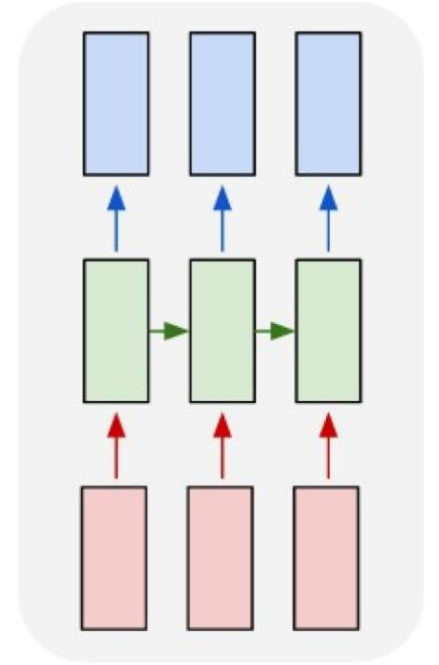
many to one



many to many



many to many

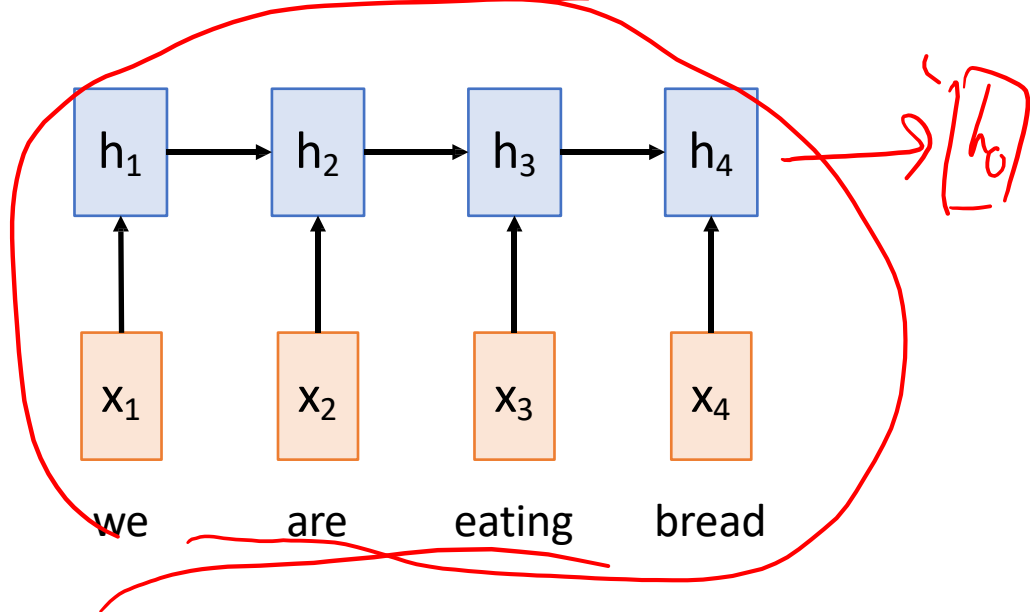


Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

Output: Sequence $y_1, \dots, y_{T'}$

Encoder: $h_t = f_w(x_t, h_{t-1})$



Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

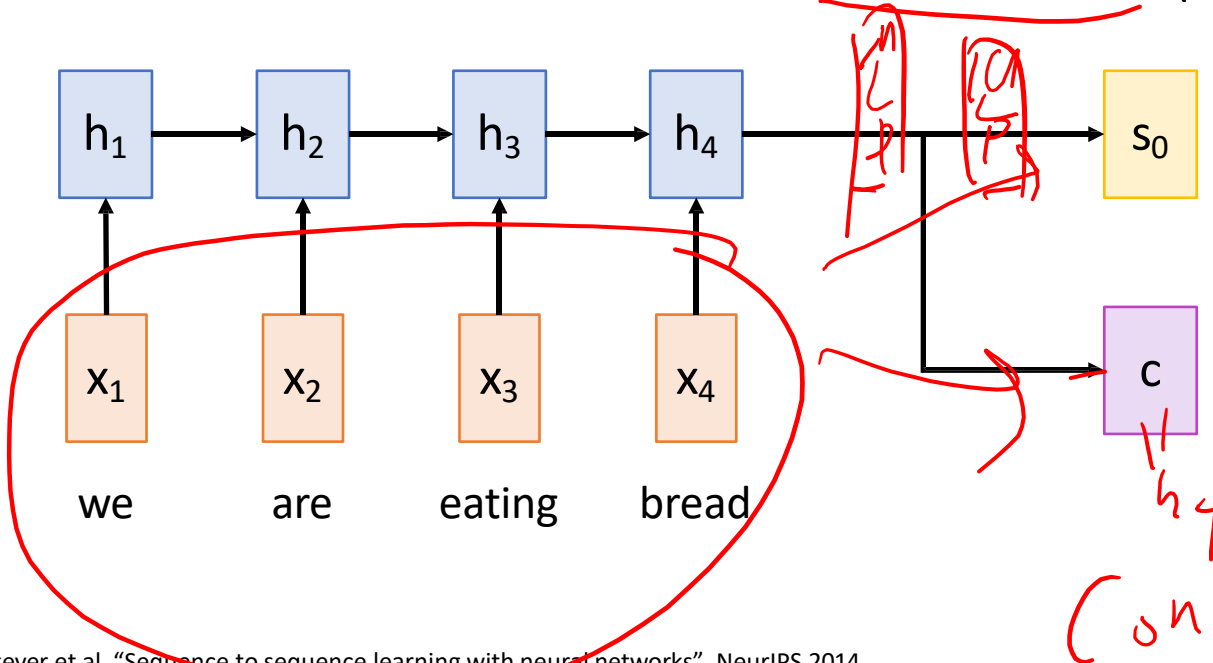
Output: Sequence $y_1, \dots, y_{T'}$

Encoder: $h_t = f_W(x_t, h_{t-1})$

From final hidden state predict:

Initial decoder state s_0

Context vector c (often $c = h_T$)



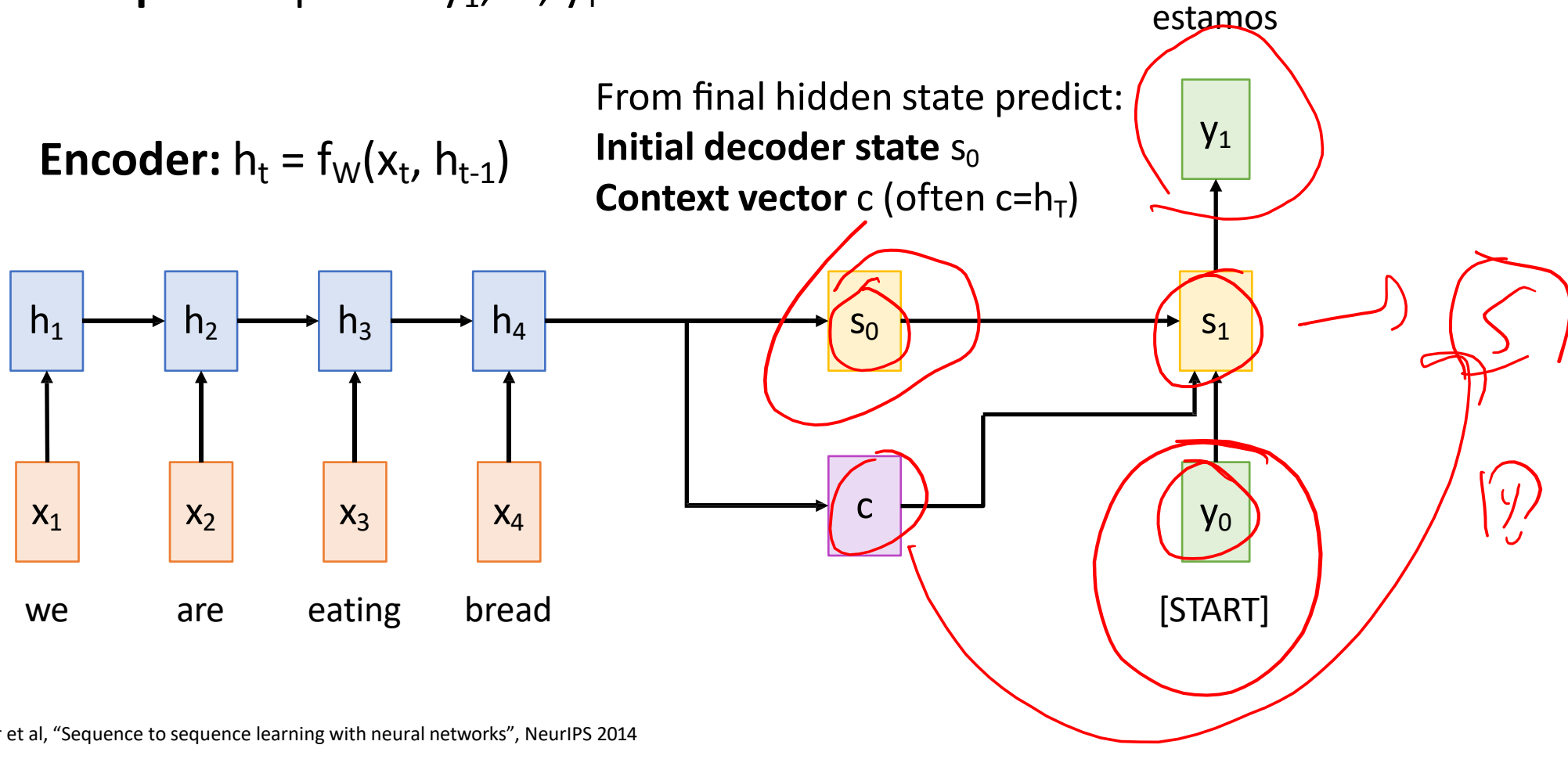
Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

Output: Sequence $y_1, \dots, y_{T'}$

Decoder: $s_t = g_U(y_{t-1}, h_{t-1}, c)$

Encoder: $h_t = f_W(x_t, h_{t-1})$



Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

Output: Sequence $y_1, \dots, y_{T'}$

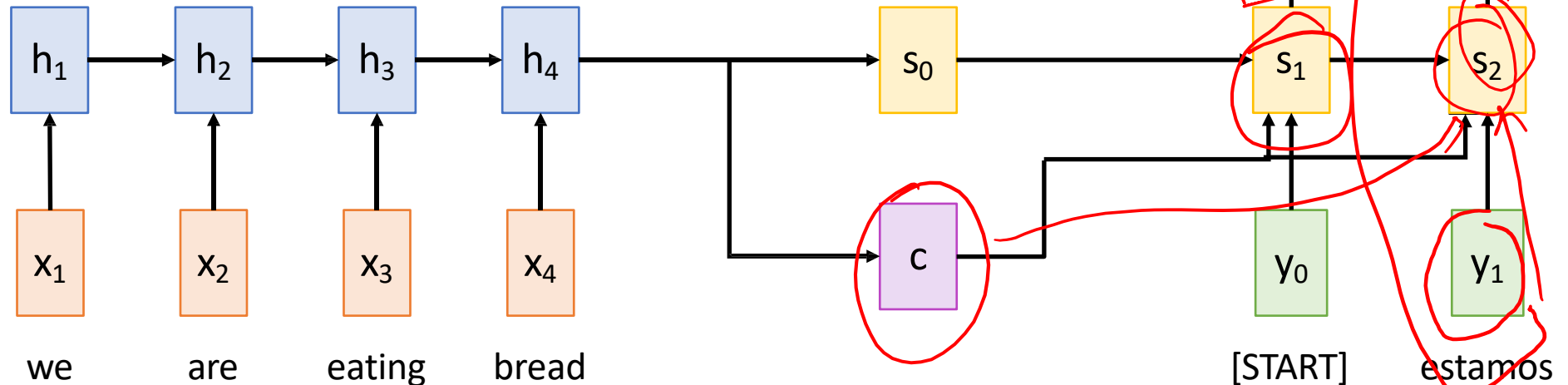
Encoder: $h_t = f_W(x_t, h_{t-1})$

From final hidden state predict:

Initial decoder state s_0

Context vector c (often $c=h_T$)

Decoder: $s_t = g_U(y_{t-1}, h_{t-1}, c)$

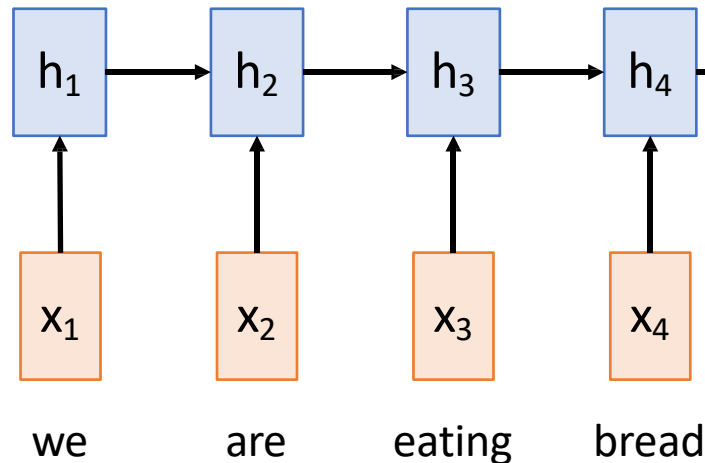


Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

Output: Sequence $y_1, \dots, y_{T'}$

Encoder: $h_t = f_W(x_t, h_{t-1})$



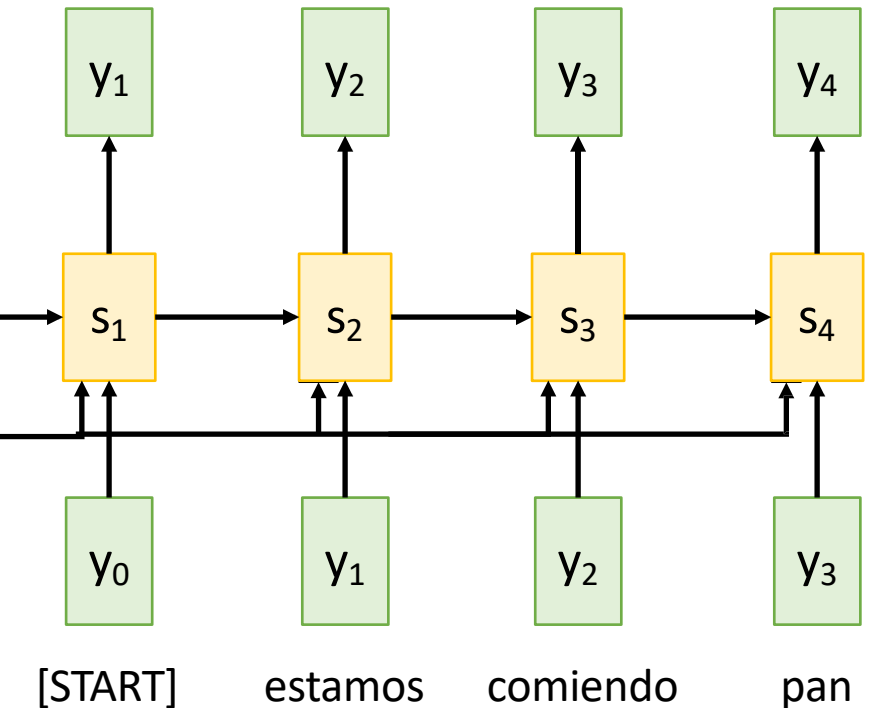
From final hidden state predict:

Initial decoder state s_0

Context vector c (often $c=h_T$)

Decoder: $s_t = g_U(y_{t-1}, h_{t-1}, c)$

~~estamos~~ ~~comiendo~~ ~~pan~~ ~~[STOP]~~



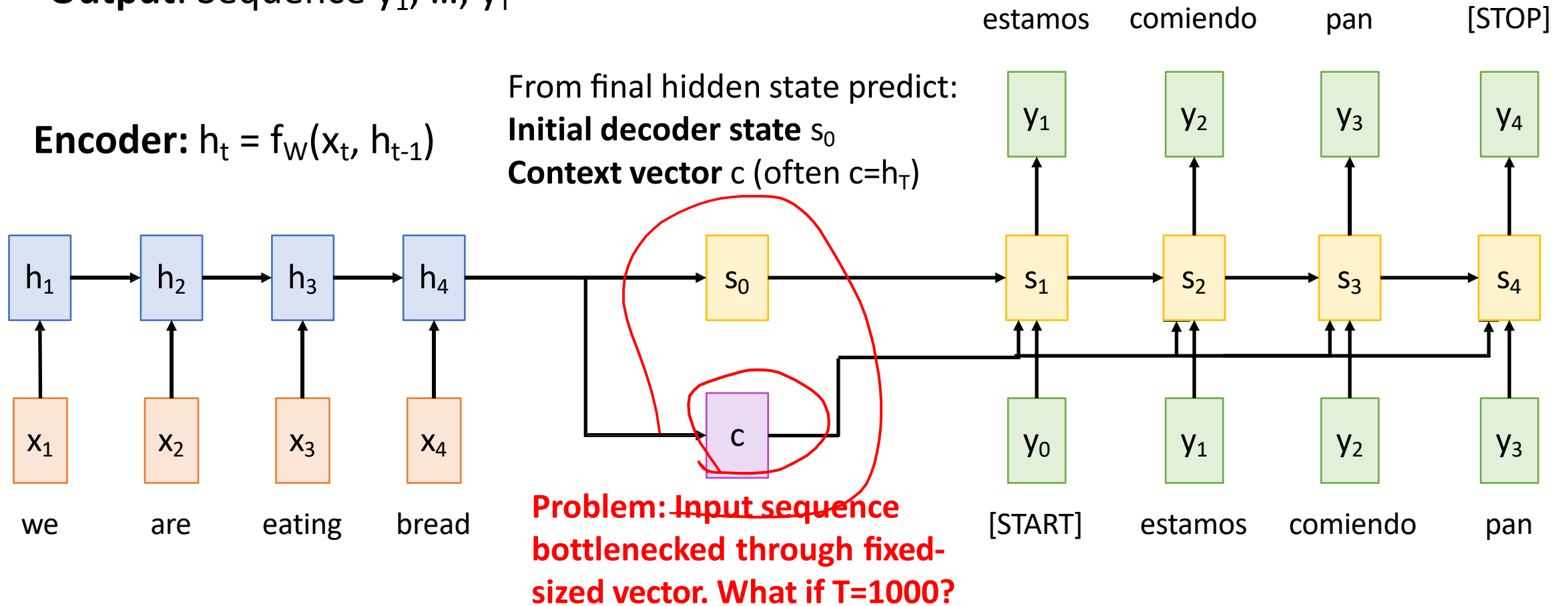
Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

Output: Sequence $y_1, \dots, y_{T'}$

Decoder: $s_t = g_U(y_{t-1}, h_{t-1}, c)$

Encoder: $h_t = f_W(x_t, h_{t-1})$



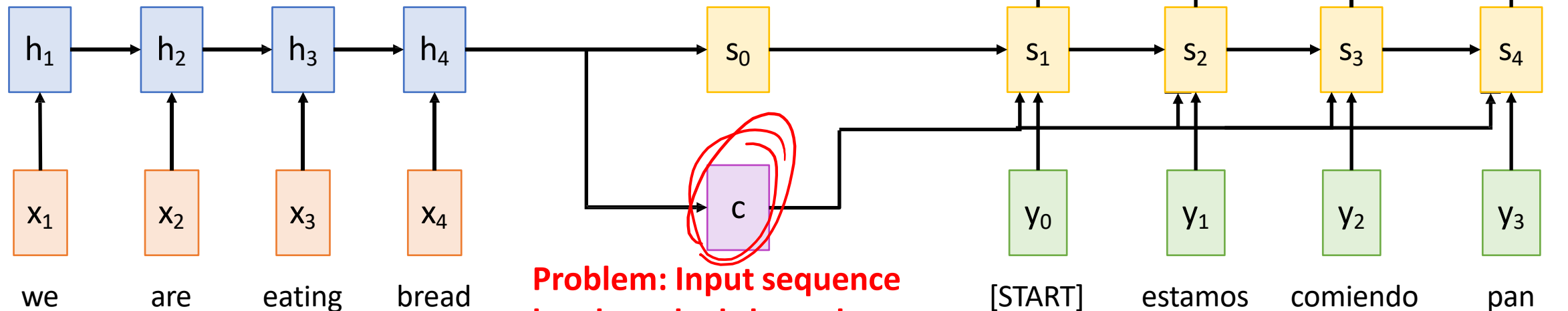
Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

Output: Sequence $y_1, \dots, y_{T'}$

Decoder: $s_t = g_U(y_{t-1}, h_{t-1}, c)$

Encoder: $h_t = f_W(x_t, h_{t-1})$



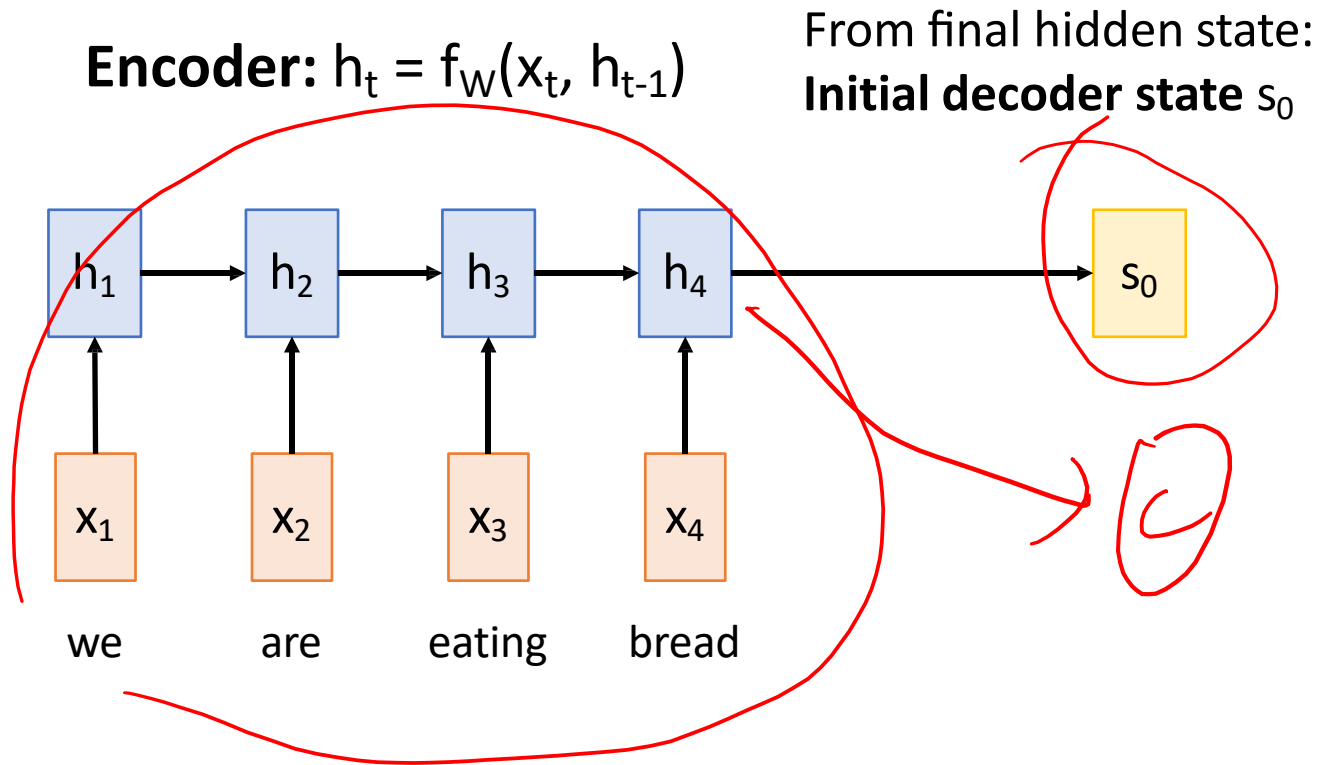
Problem: Input sequence bottlenecked through fixed-vector. What if $T=1000$?

Idea: use new context vector at each step of decoder!

Sequence-to-Sequence with RNNs and attention

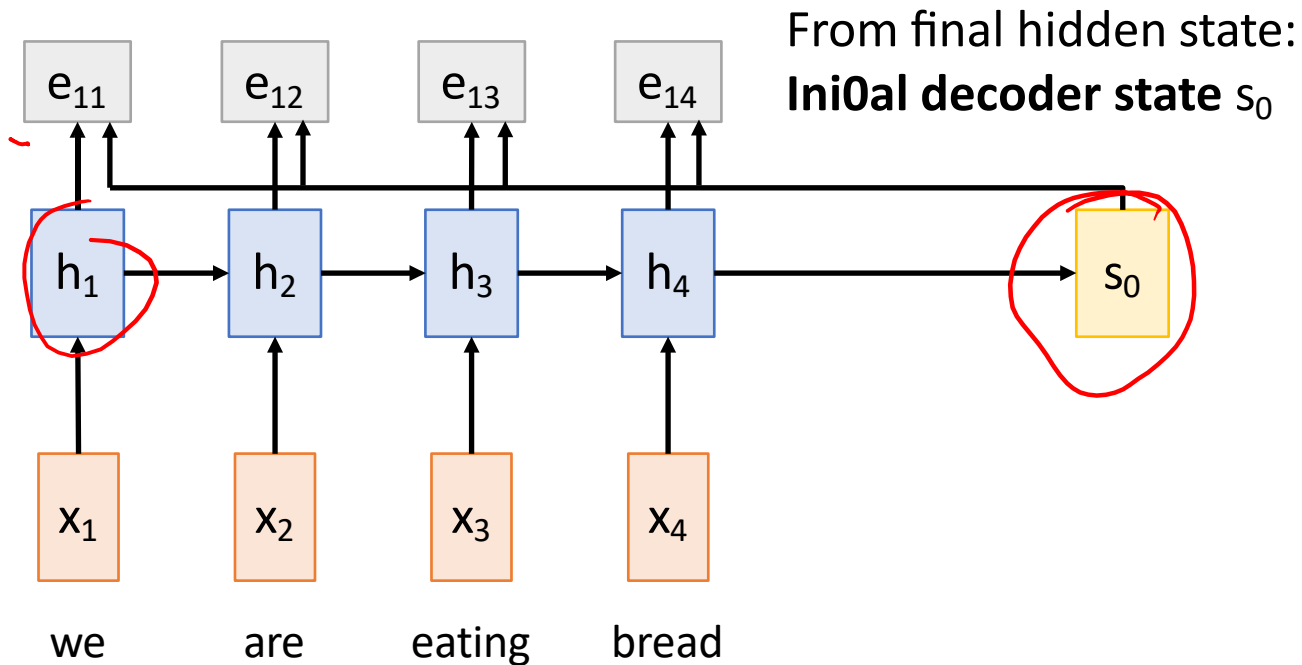
Input: Sequence x_1, \dots, x_T

Output: Sequence $y_1, \dots, y_{T'}$

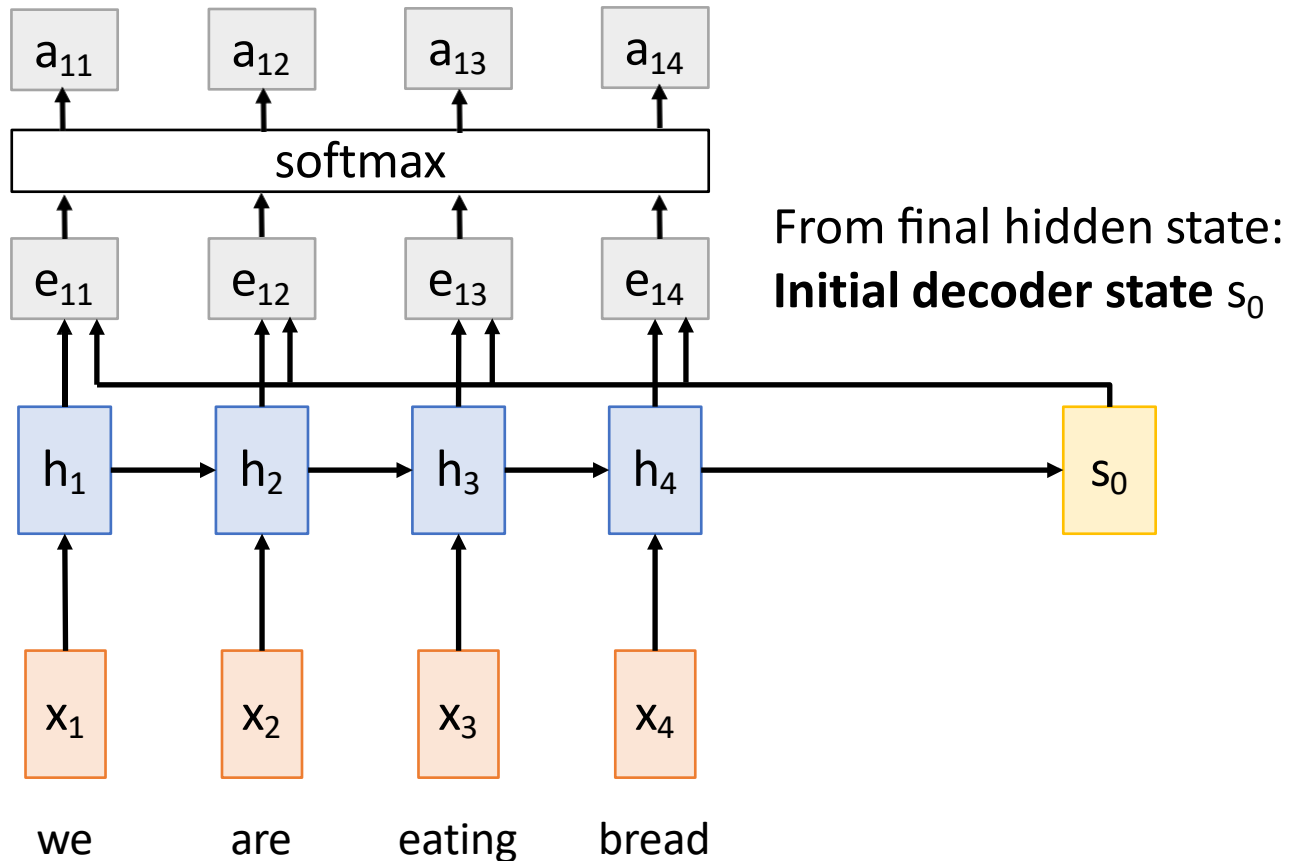


Sequence-to-Sequence with RNNs and attention

Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)



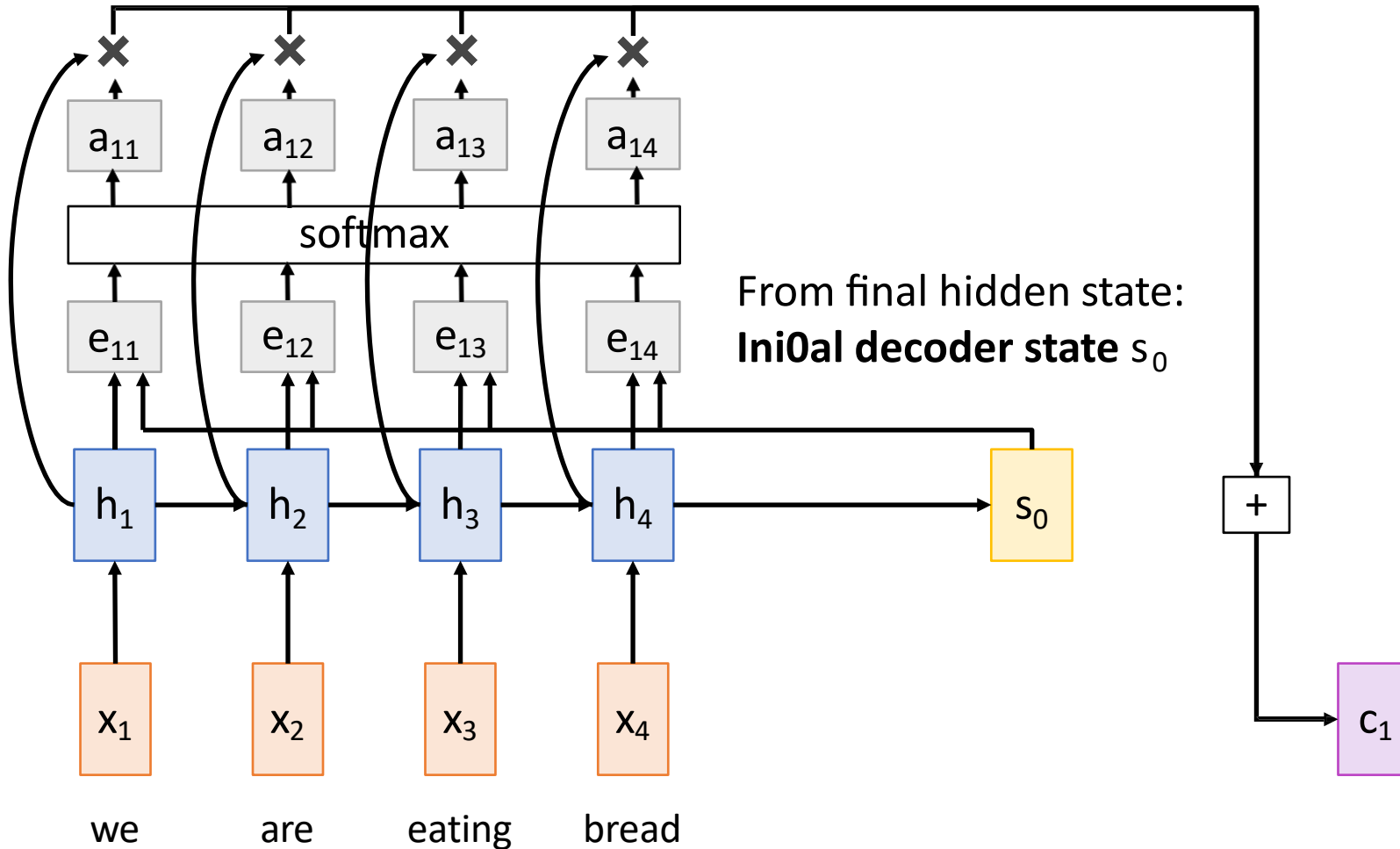
Sequence-to-Sequence with RNNs and Attention



Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)

Normalize alignment scores
to get **attention weights**
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

Sequence-to-Sequence with RNNs and Attention

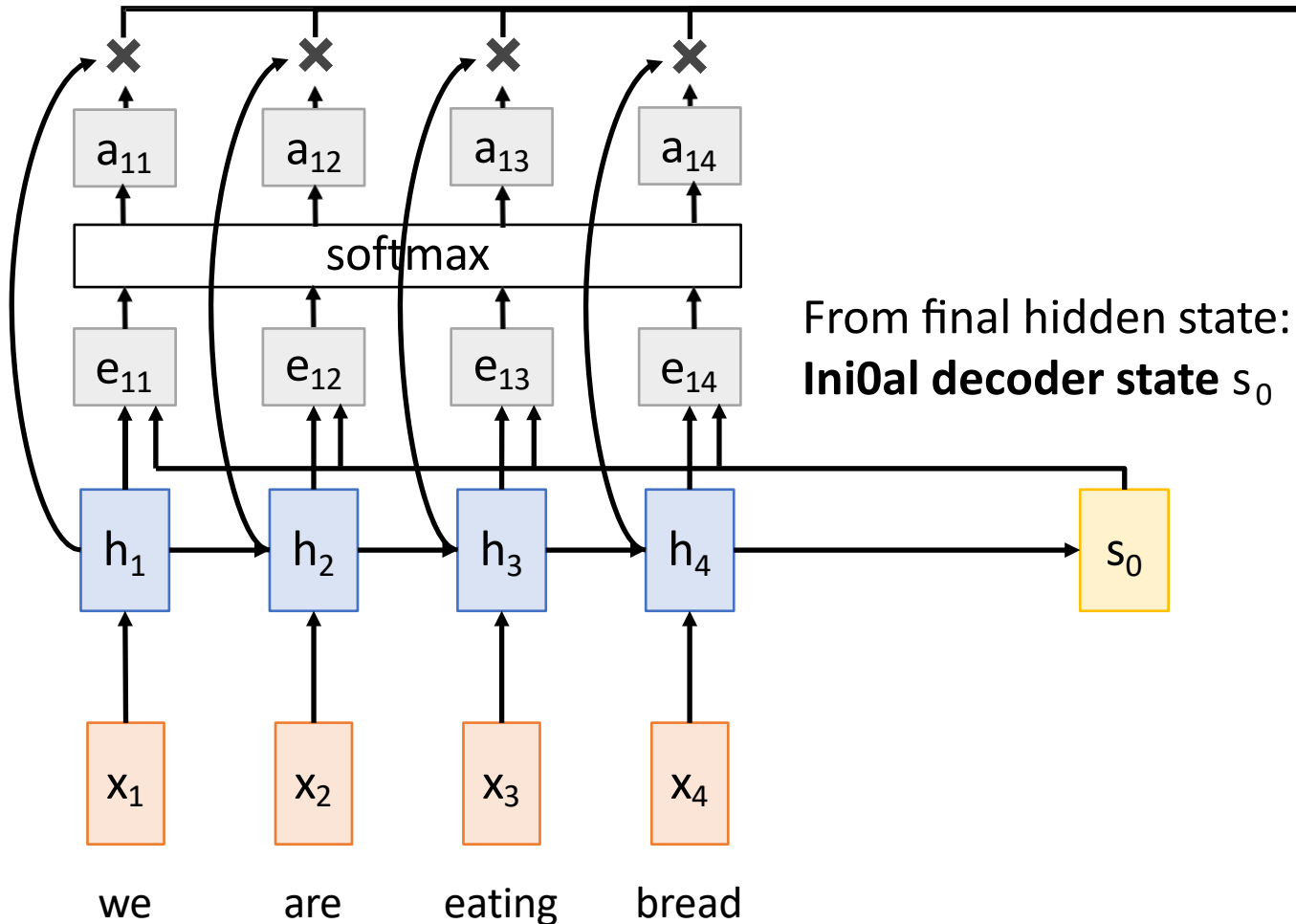


Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)

Normalize alignment scores
to get **attention weights**
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

Compute context vector as linear
combination of hidden states
 $c_t = \sum_i a_{t,i} h_i$

Sequence-to-Sequence with RNNs and Attention



Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)

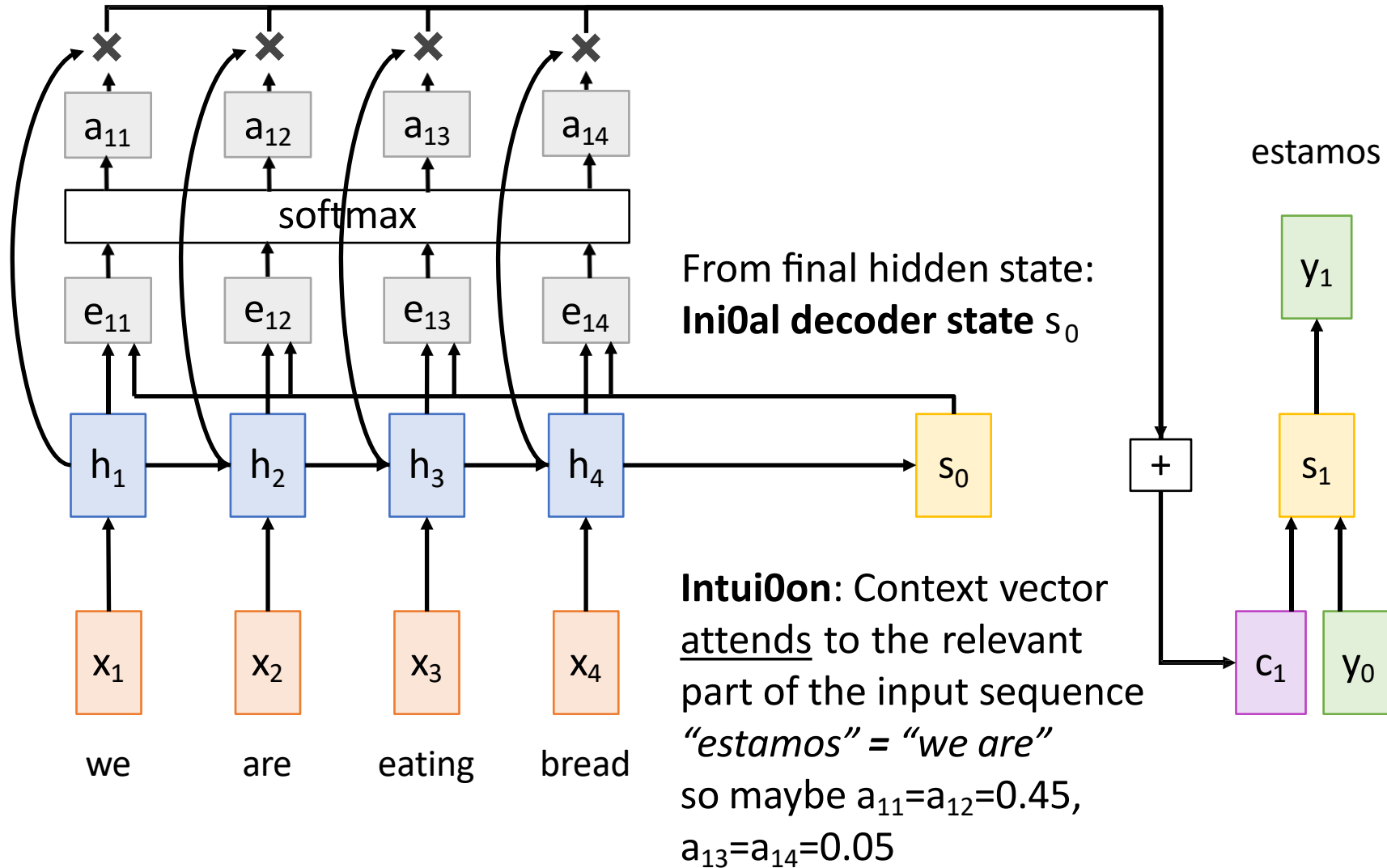
Normalize alignment scores
to get **Attention weights**
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

Compute context vector as linear
combination of hidden states
 $c_t = \sum_i a_{t,i} h_i$

Use context vector in
decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$

**This is all differentiable! Do not
supervise attention weights –
backprop through everything**

Sequence-to-Sequence with RNNs and Attention



Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)

Normalize alignment scores
to get **Attention weights**
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

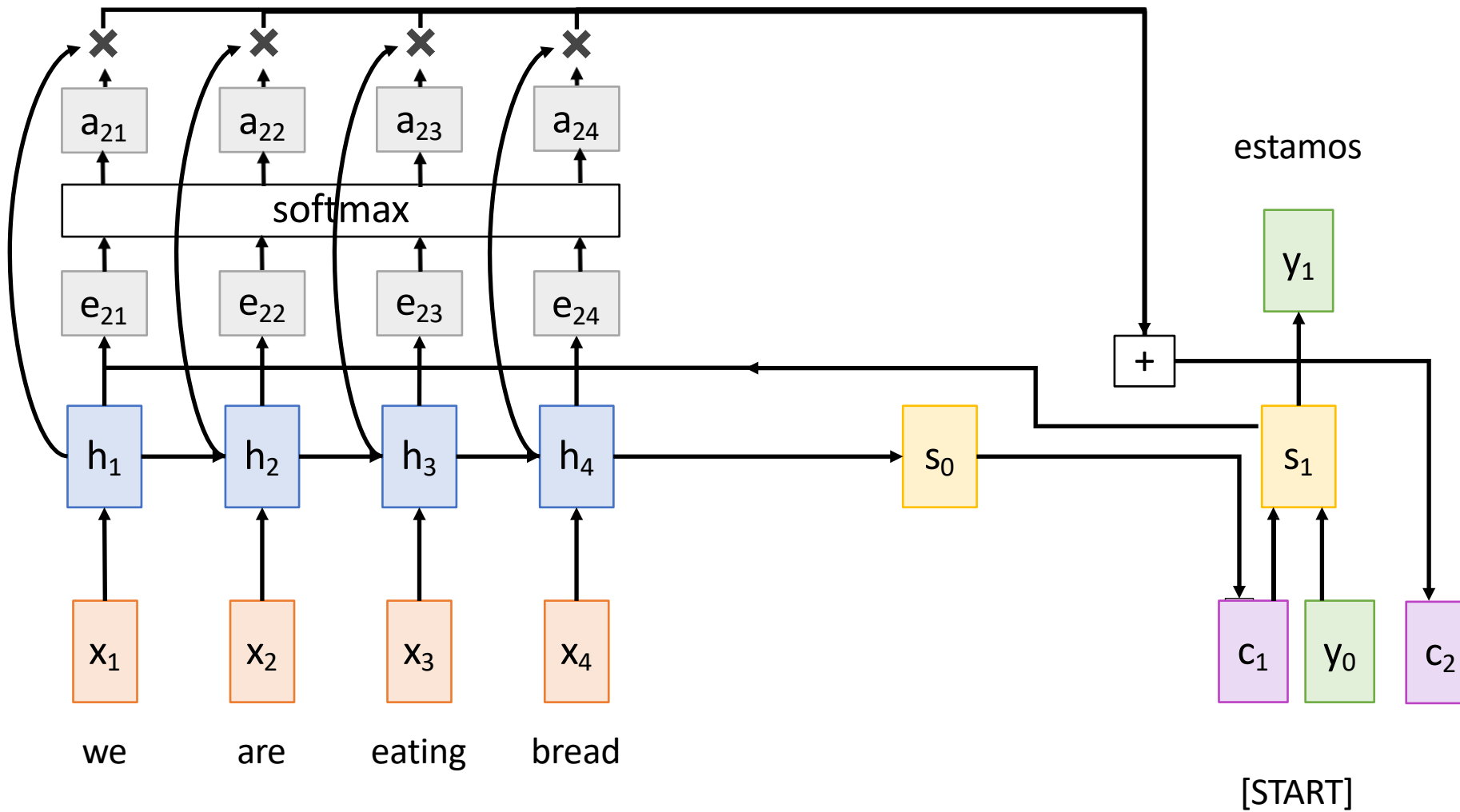
Compute context vector as linear
combination of hidden states
 $c_t = \sum_i a_{t,i} h_i$

Use context vector in
decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$

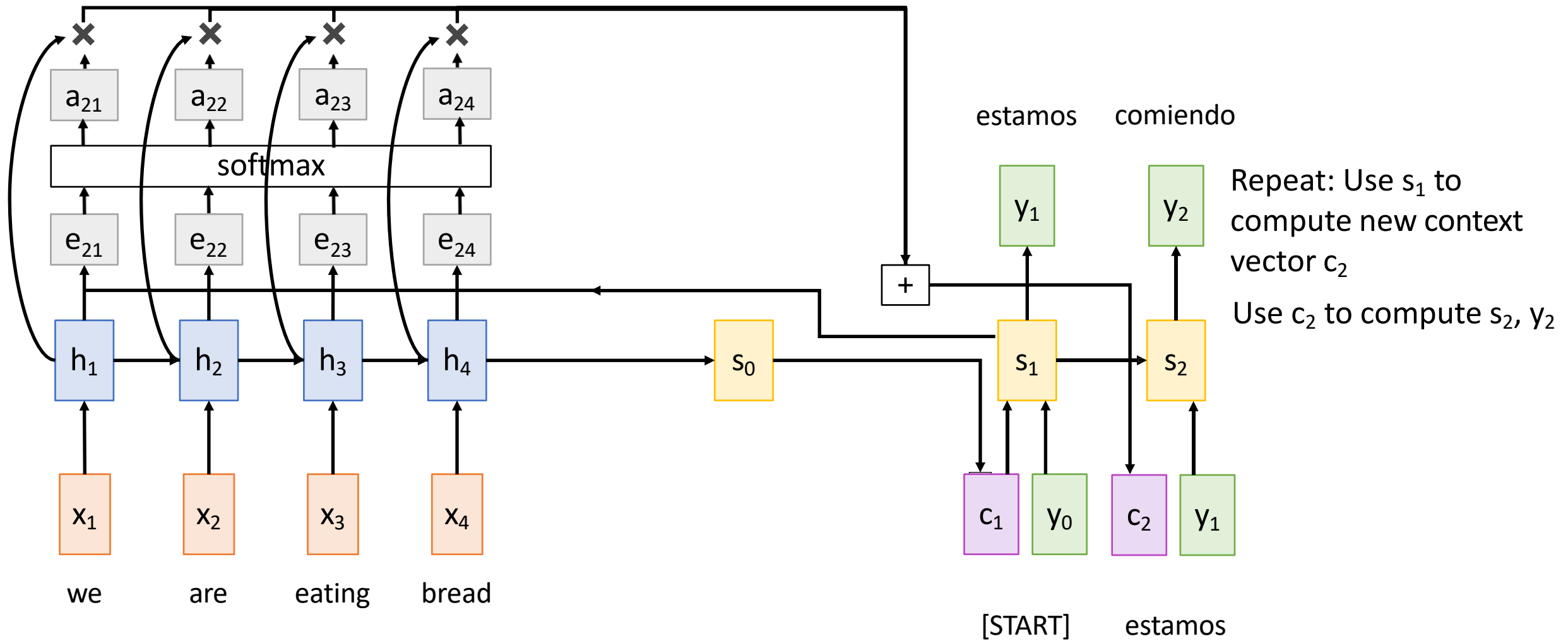
**This is all differentiable! Do not
supervise attention weights –
backprop through everything**

Sequence-to-Sequence with RNNs

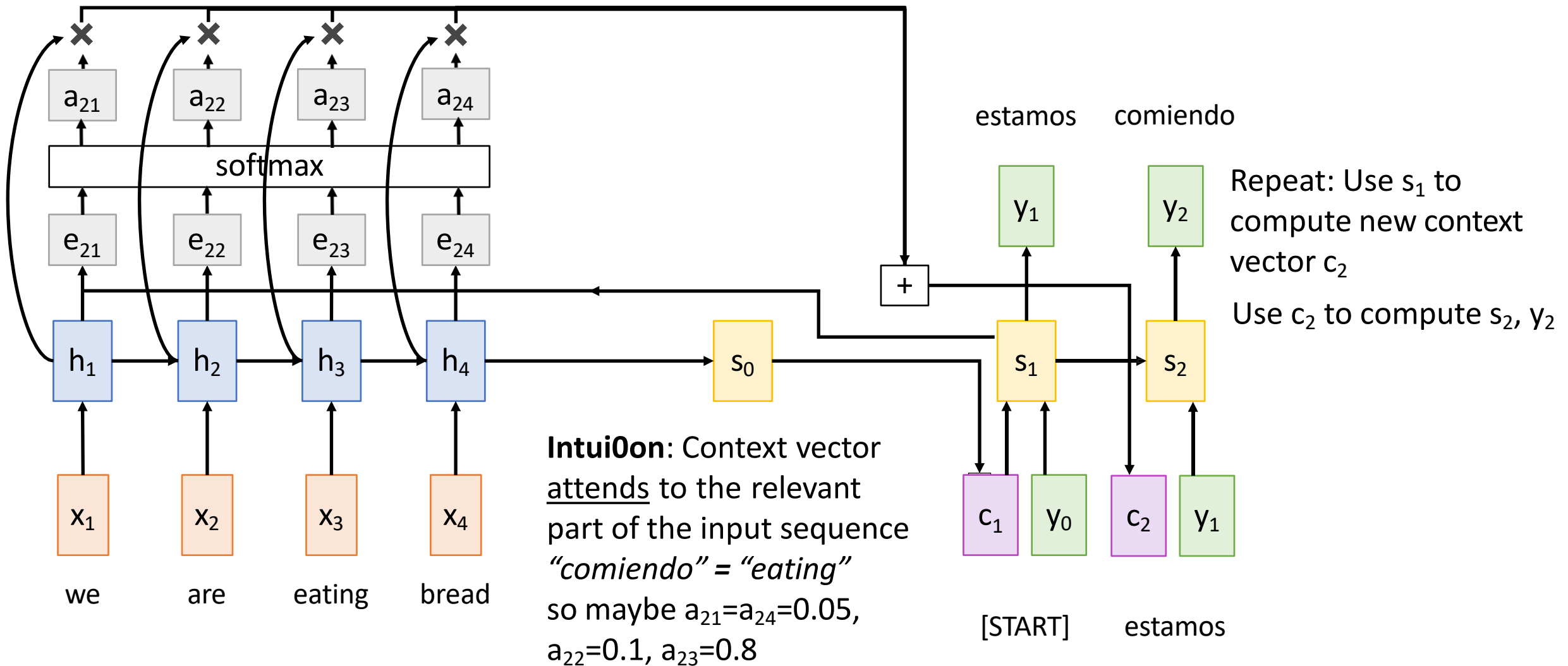
Repeat: Use s_1 to compute new context vector c_2



Sequence-to-Sequence with RNNs and Attention



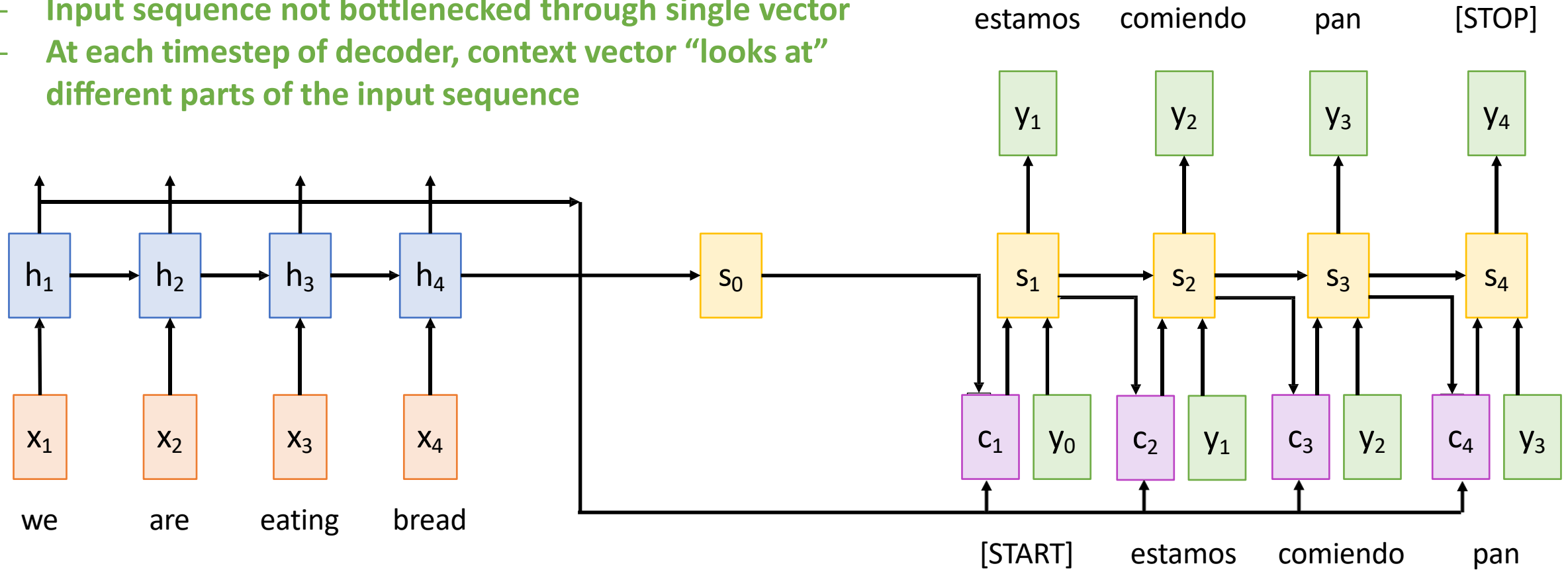
Sequence-to-Sequence with RNNs and Attention



Sequence-to-Sequence with RNNs and Attention

Use a different context vector in each timestep of decoder

- Input sequence not bottlenecked through single vector
- At each timestep of decoder, context vector “looks at” different parts of the input sequence



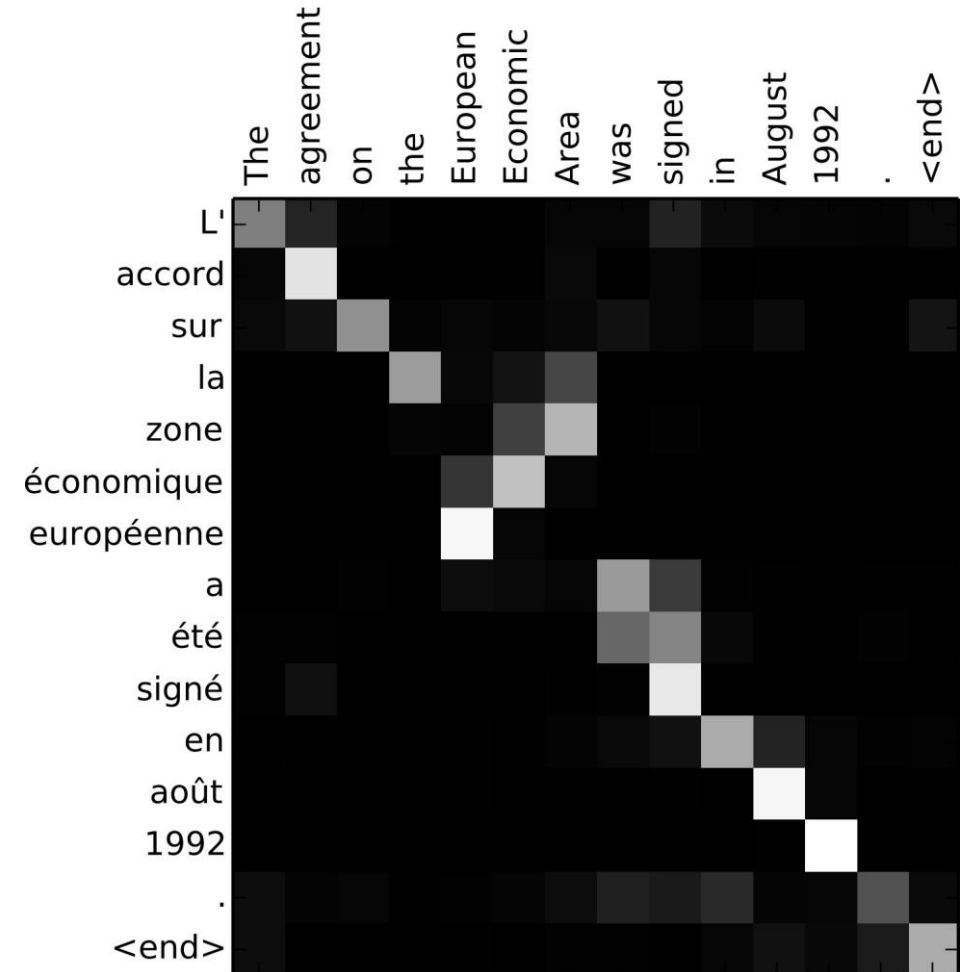
Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

Output: “L’accord sur la zone économique européenne a été signé en août 1992.”

Visualize attention weights $a_{t,i}$



Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

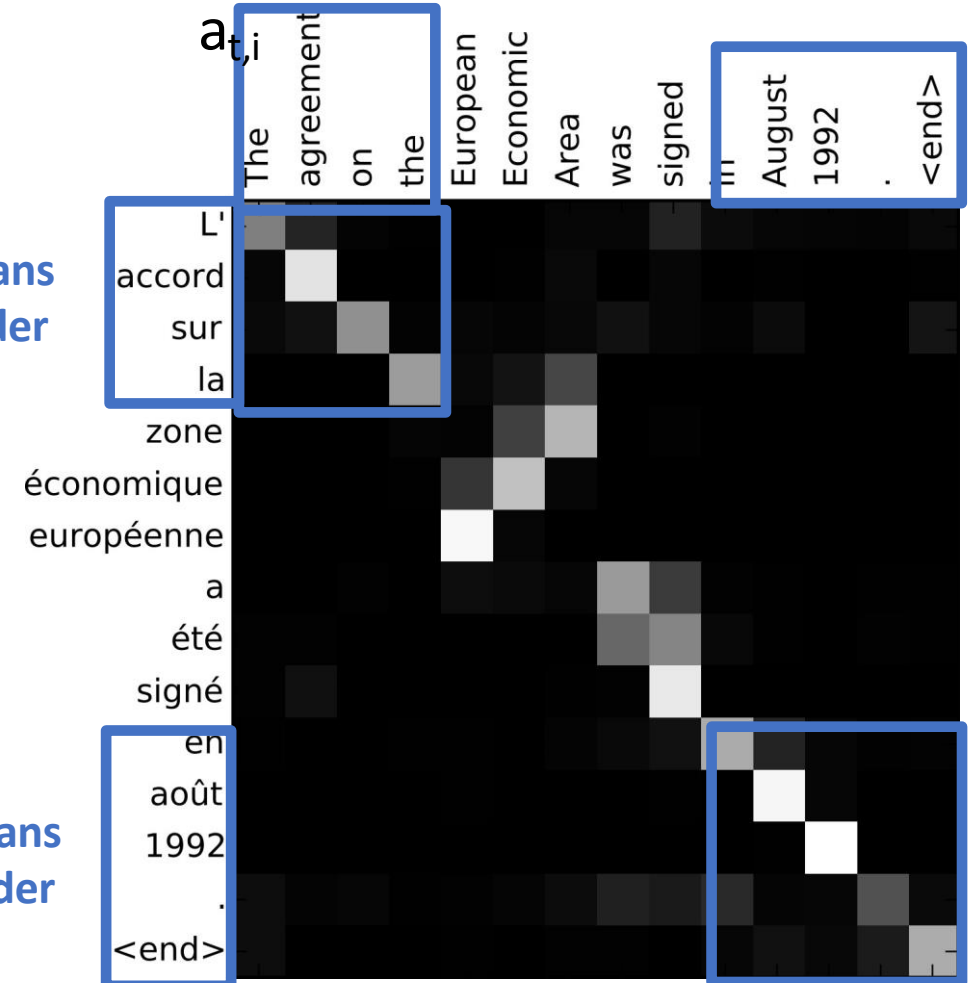
Input: “The agreement on the European Economic Area was signed in August 1992.”

Output: “L’accord sur la zone économique européenne a été signé en août 1992.”

Diagonal Attention means words correspond in order

Diagonal Attention means words correspond in order

Visualize attention weights



Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

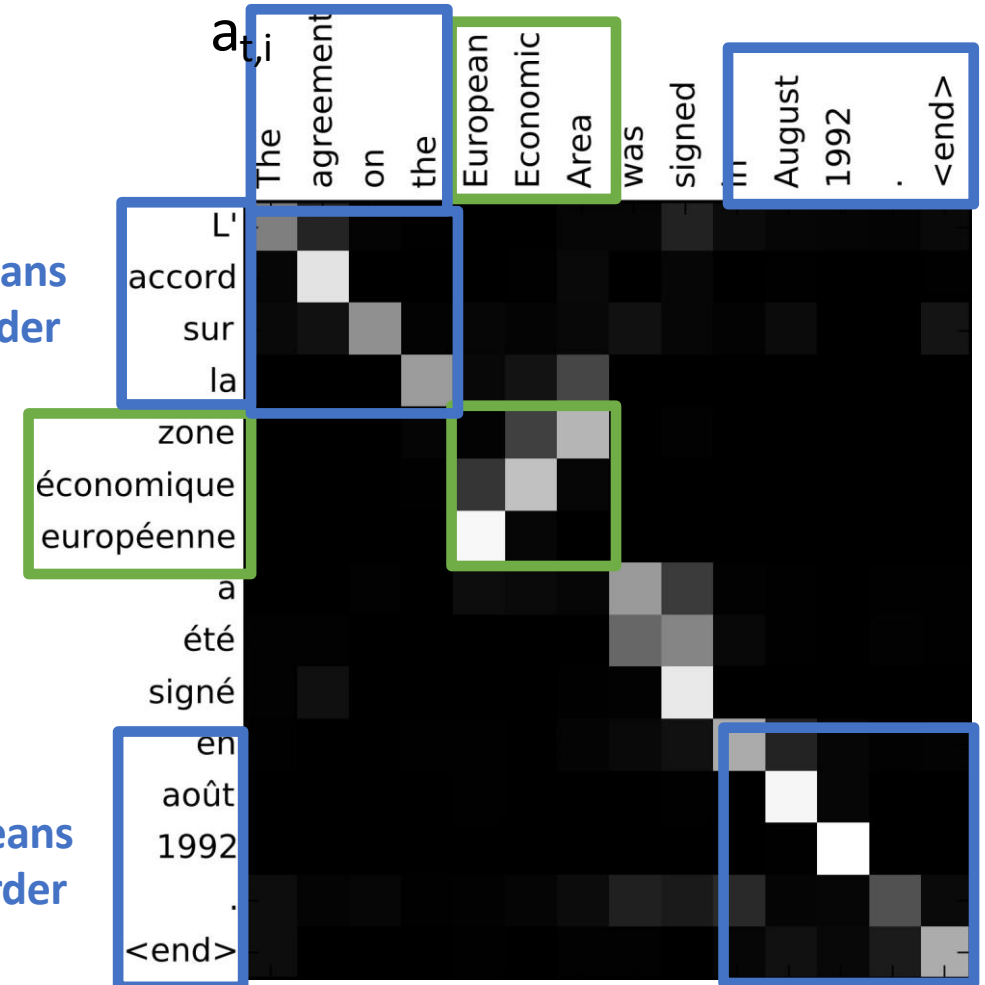
Output: “L'accord sur la zone économique européenne a été signé en août 1992.”

Diagonal Attention means words correspond in order

Attention figures out different word orders

Diagonal Attention means words correspond in order

Visualize attention weights



Sequence-to-Sequence with RNNs and Attention

The decoder doesn't use the fact that h_i form an ordered sequence – it just treats them as an unordered set $\{h_i\}$

Can use similar architecture given any set of input hidden vectors $\{h_i\}$!

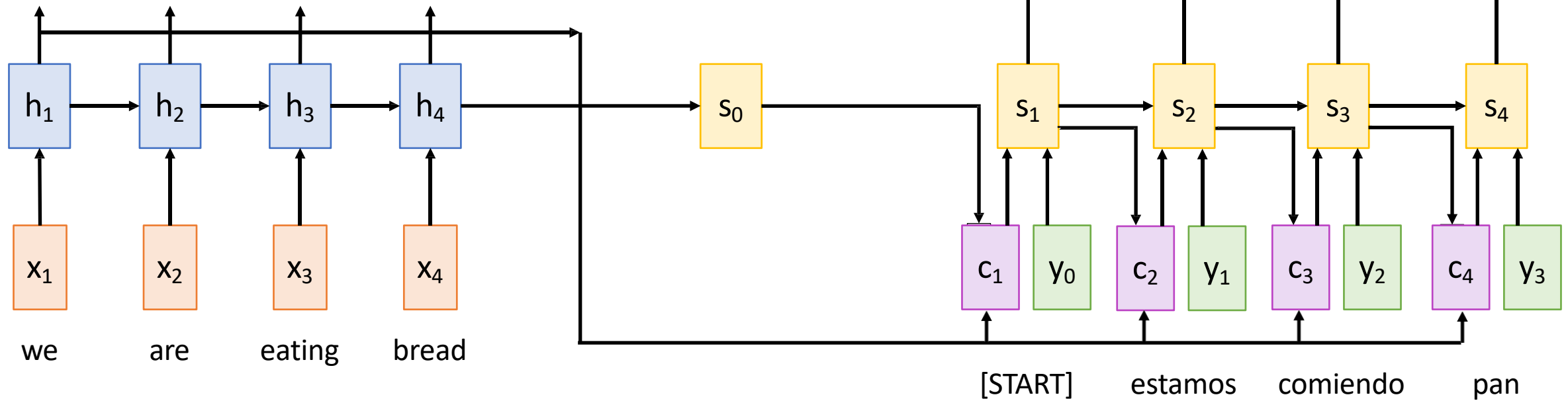
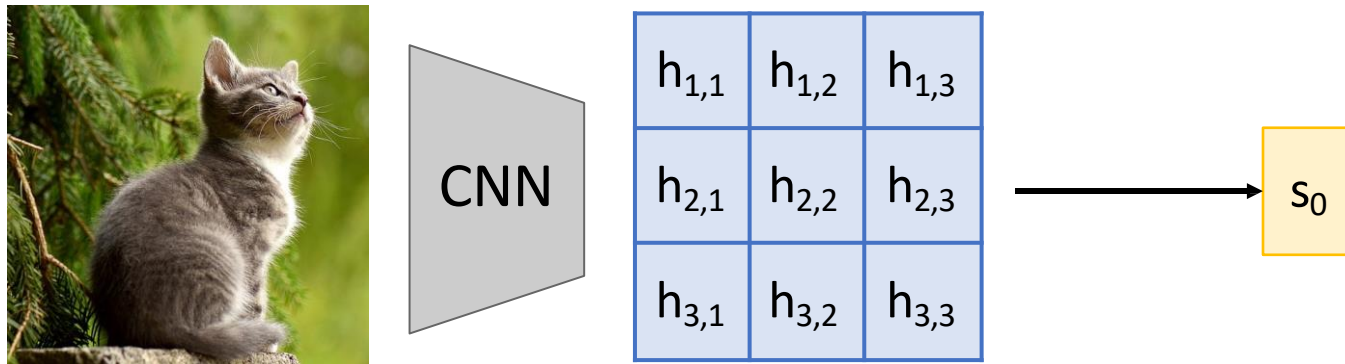


Image Captioning with RNNs and Attention



Use a CNN to compute a
grid of features for an image

Image Captioning with RNNs and Attention

$$e_{t,i,j} = f_{\text{att}}(s_{t-1}, h_{i,j})$$

Alignment scores

$e_{1,1,1}$	$e_{1,1,2}$	$e_{1,1,3}$
$e_{1,2,1}$	$e_{1,2,2}$	$e_{1,2,3}$
$e_{1,3,1}$	$e_{1,3,2}$	$e_{1,3,3}$

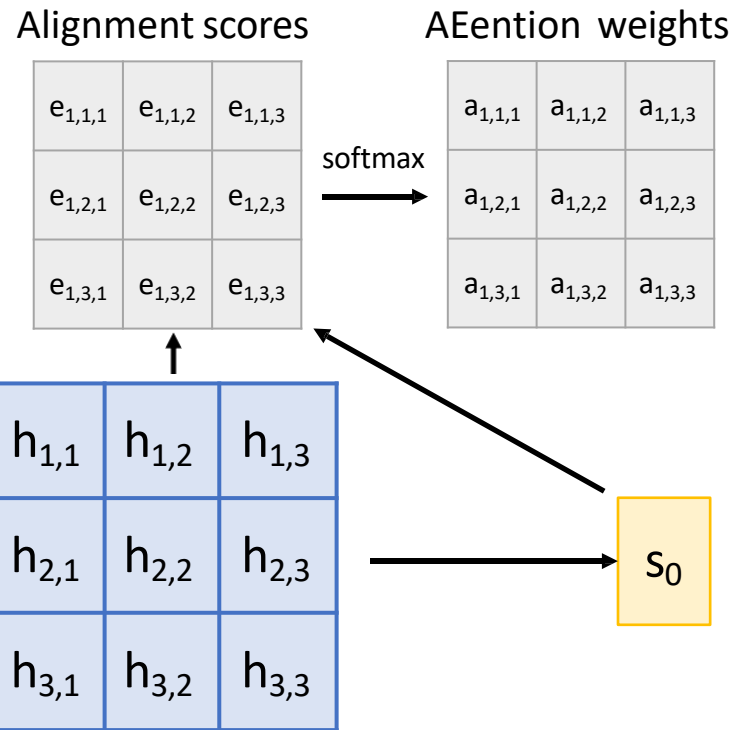
$h_{1,1}$	$h_{1,2}$	$h_{1,3}$
$h_{2,1}$	$h_{2,2}$	$h_{2,3}$
$h_{3,1}$	$h_{3,2}$	$h_{3,3}$

s_0

Use a CNN to compute a
grid of features for an image

Image Captioning with RNNs and Attention

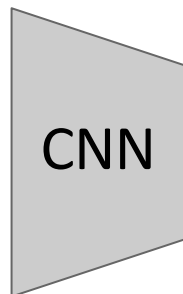
$$e_{t,i,j} = f_{\text{att}}(s_{t-1}, h_{i,j})$$
$$a_{t,:} = \text{softmax}(e_{t,:,:})$$



Use a CNN to compute a grid of features for an image

Image Captioning with RNNs and Attention

$$e_{t,i,j} = f_{\text{att}}(s_{t-1}, h_{i,j})$$
$$a_{t,:} = \text{softmax}(e_{t,:})$$
$$c_t = \sum_{i,j} a_{t,i,j} h_{i,j}$$



Use a CNN to compute a grid of features for an image

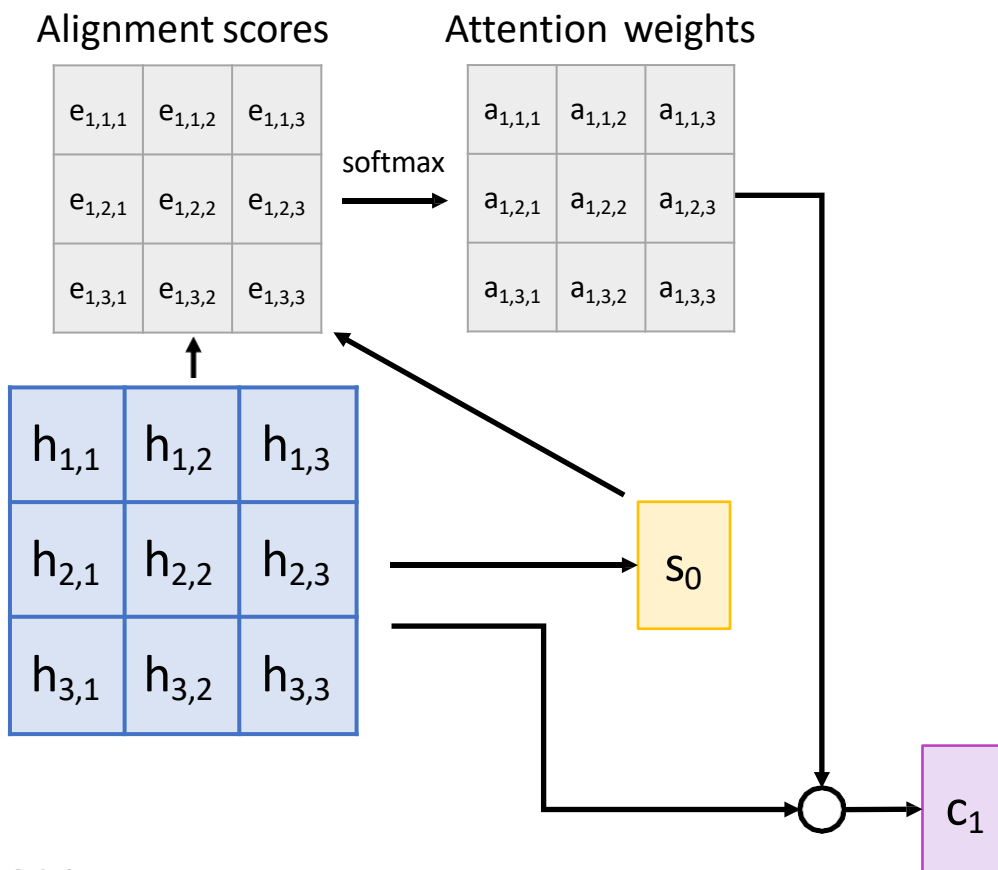


Image Captioning with RNNs and Attention

$$e_{t,i,j} = f_{\text{att}}(s_{t-1}, h_{i,j})$$
$$a_{t,:} = \text{softmax}(e_{t,:})$$
$$c_t = \sum_{i,j} a_{t,i,j} h_{i,j}$$



Use a CNN to compute a grid of features for an image

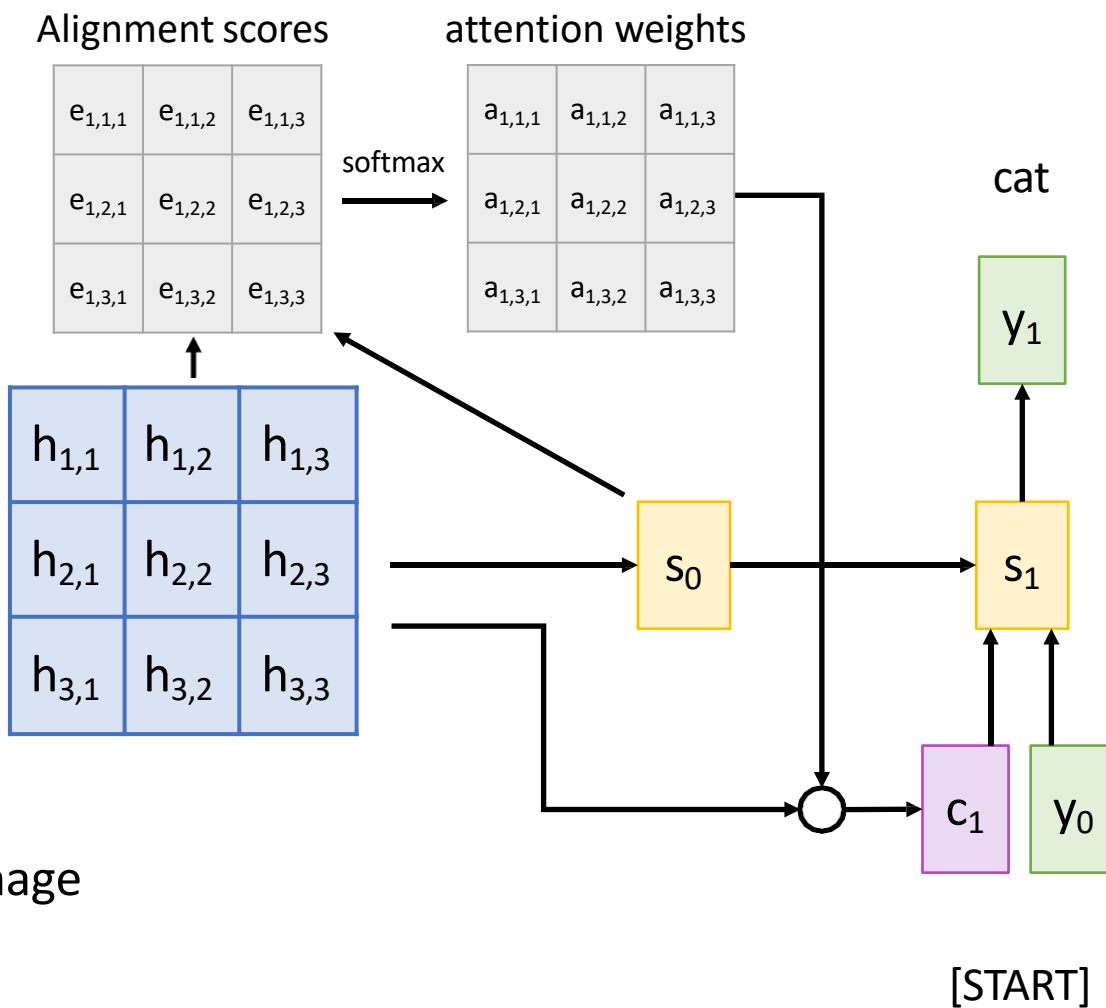


Image Captioning with RNNs and Attention

$$e_{t,i,j} = f_{\text{att}}(s_{t-1}, h_{i,j})$$
$$a_{t,:} = \text{softmax}(e_{t,:})$$
$$c_t = \sum_{i,j} a_{t,i,j} h_{i,j}$$

Alignment scores

$e_{2,1,1}$	$e_{2,1,2}$	$e_{2,1,3}$
$e_{2,2,1}$	$e_{2,2,2}$	$e_{2,2,3}$
$e_{2,3,1}$	$e_{2,3,2}$	$e_{2,3,3}$



Use a CNN to compute a grid of features for an image

$h_{1,1}$	$h_{1,2}$	$h_{1,3}$
$h_{2,1}$	$h_{2,2}$	$h_{2,3}$
$h_{3,1}$	$h_{3,2}$	$h_{3,3}$

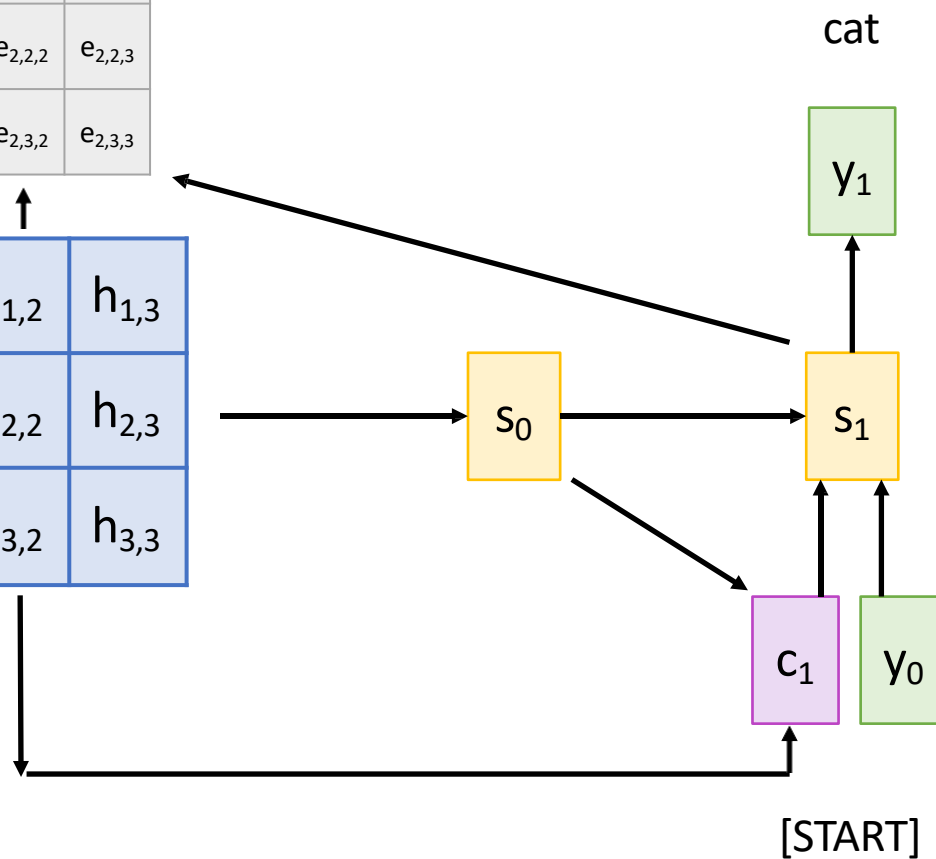
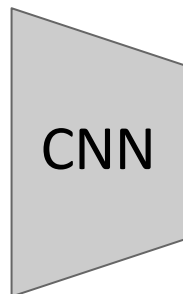


Image Captioning with RNNs and Attention

$$e_{t,i,j} = f_{\text{att}}(s_{t-1}, h_{i,j})$$

$$a_{t,:} = \text{softmax}(e_{t,:})$$

$$c_t = \sum_{i,j} a_{t,i,j} h_{i,j}$$



Use a CNN to compute a grid of features for an image

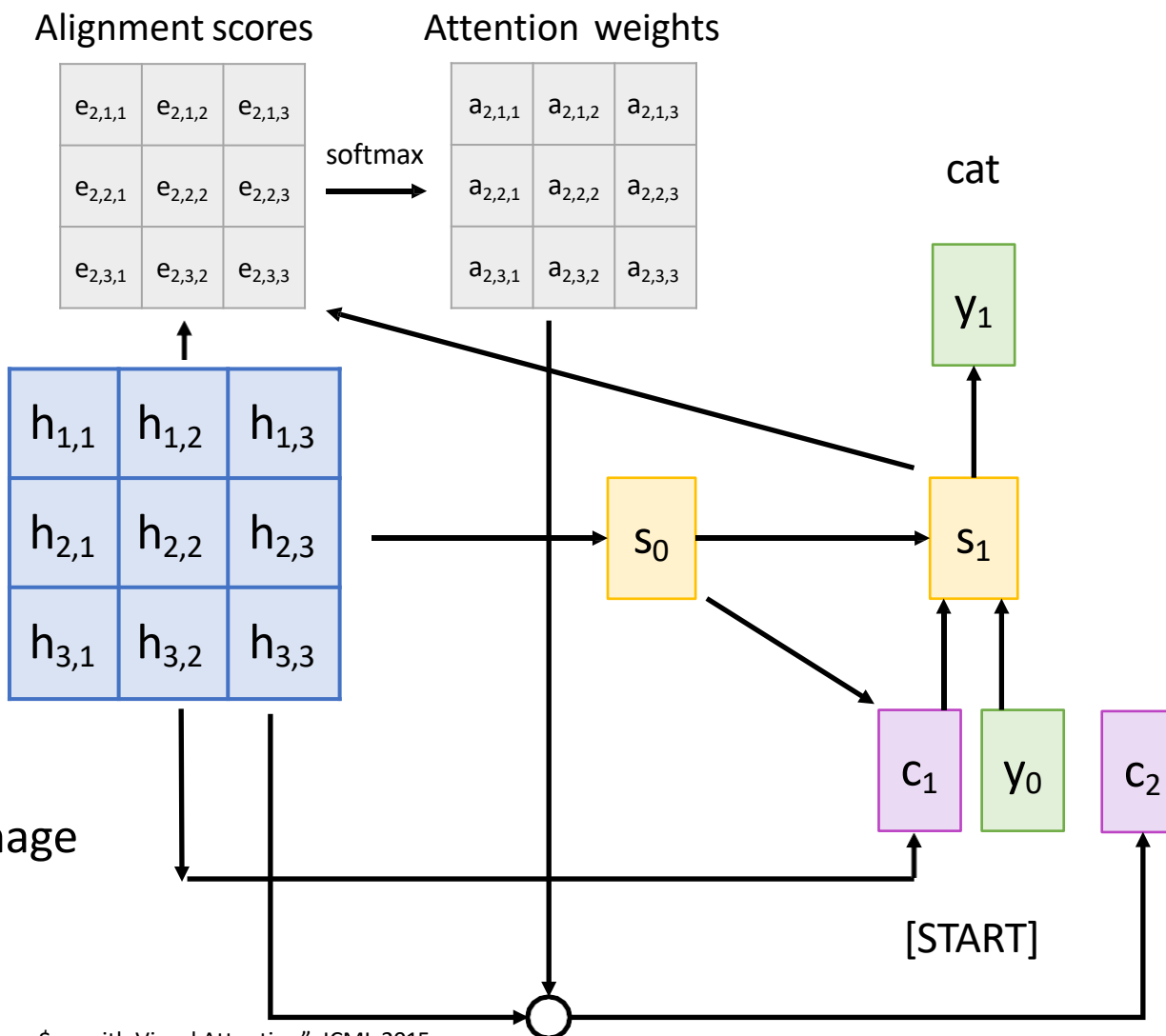


Image Captioning with RNNs and Attention

$$e_{t,i,j} = f_{\text{att}}(s_{t-1}, h_{i,j})$$

$$a_{t,:} = \text{softmax}(e_{t,:})$$

$$c_t = \sum_{i,j} a_{t,i,j} h_{i,j}$$



Use a CNN to compute a grid of features for an image

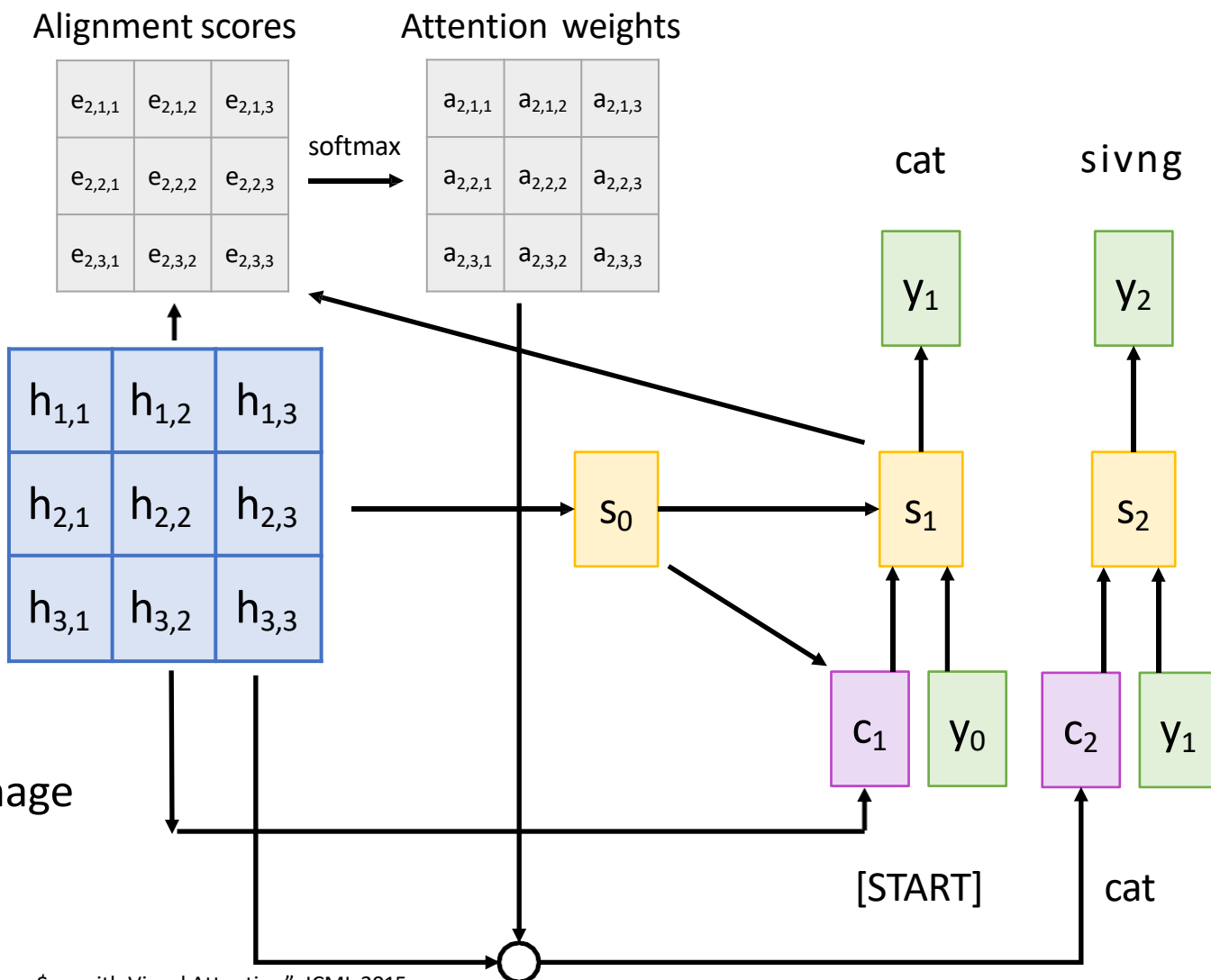
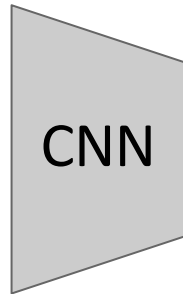


Image Captioning with RNNs and Attention

$$e_{t,i,j} = f_{\text{att}}(s_{t-1}, h_{i,j})$$
$$a_{t,:} = \text{softmax}(e_{t,:})$$
$$c_t = \sum_{i,j} a_{t,i,j} h_{i,j}$$

Each timestep of decoder
uses a different context
vector that looks at different
parts of the input image



$h_{1,1}$	$h_{1,2}$	$h_{1,3}$
$h_{2,1}$	$h_{2,2}$	$h_{2,3}$
$h_{3,1}$	$h_{3,2}$	$h_{3,3}$

Use a CNN to compute a
grid of features for an image

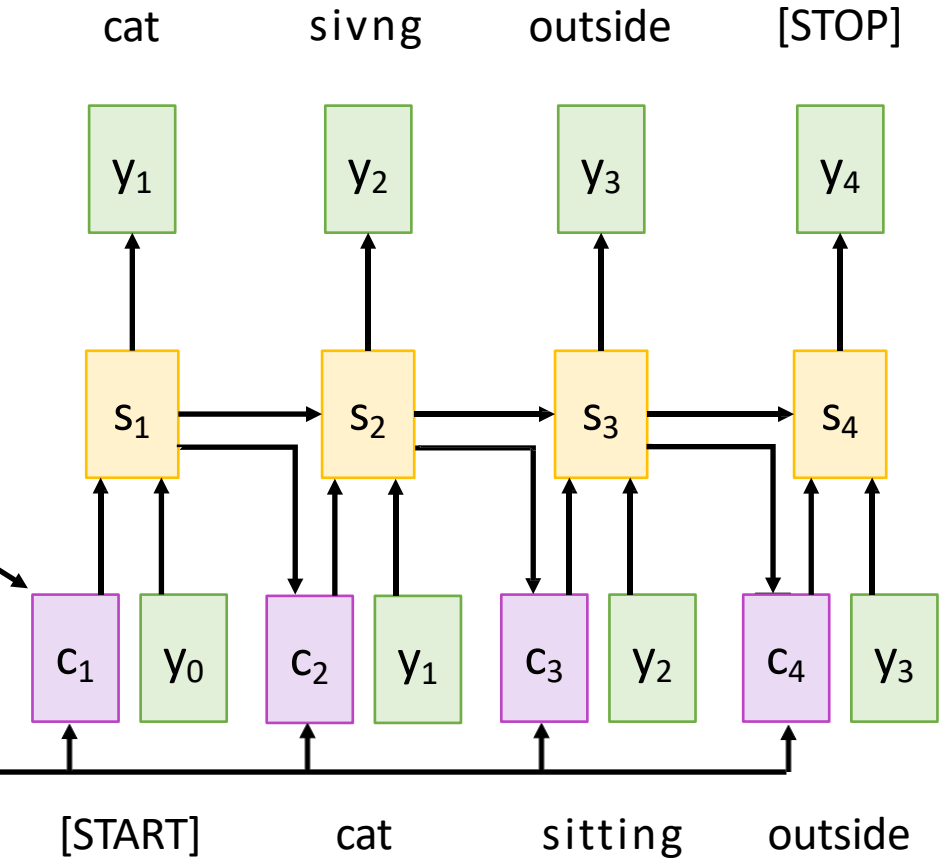


Image Captioning with RNNs and Attention

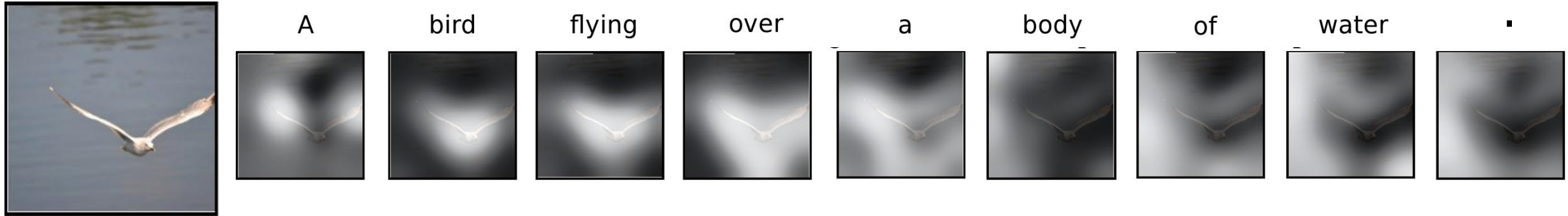


Image Captioning with RNNs and Attention



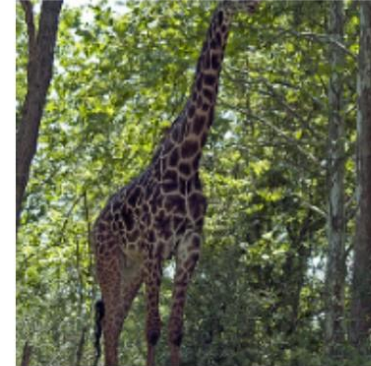
A dog is standing on a hardwood floor.



A stop sign is on a road with a mountain in the background.



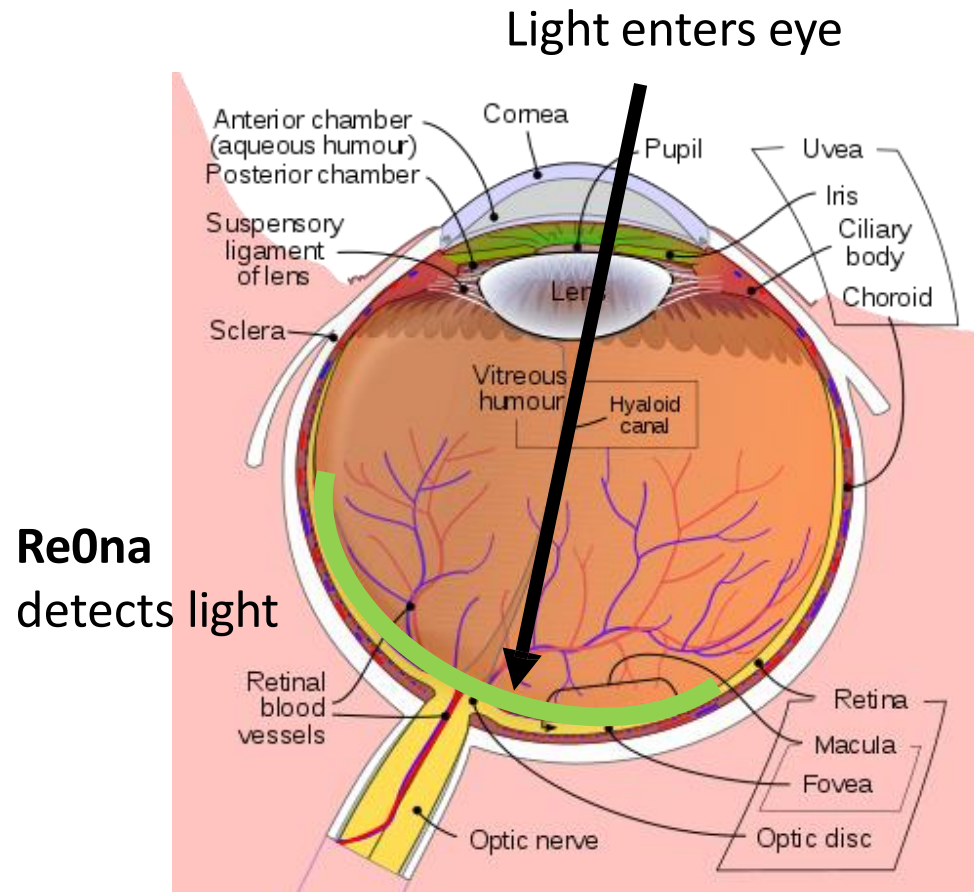
A group of people sitting on a boat in the water.



A giraffe standing in a forest with trees in the background.

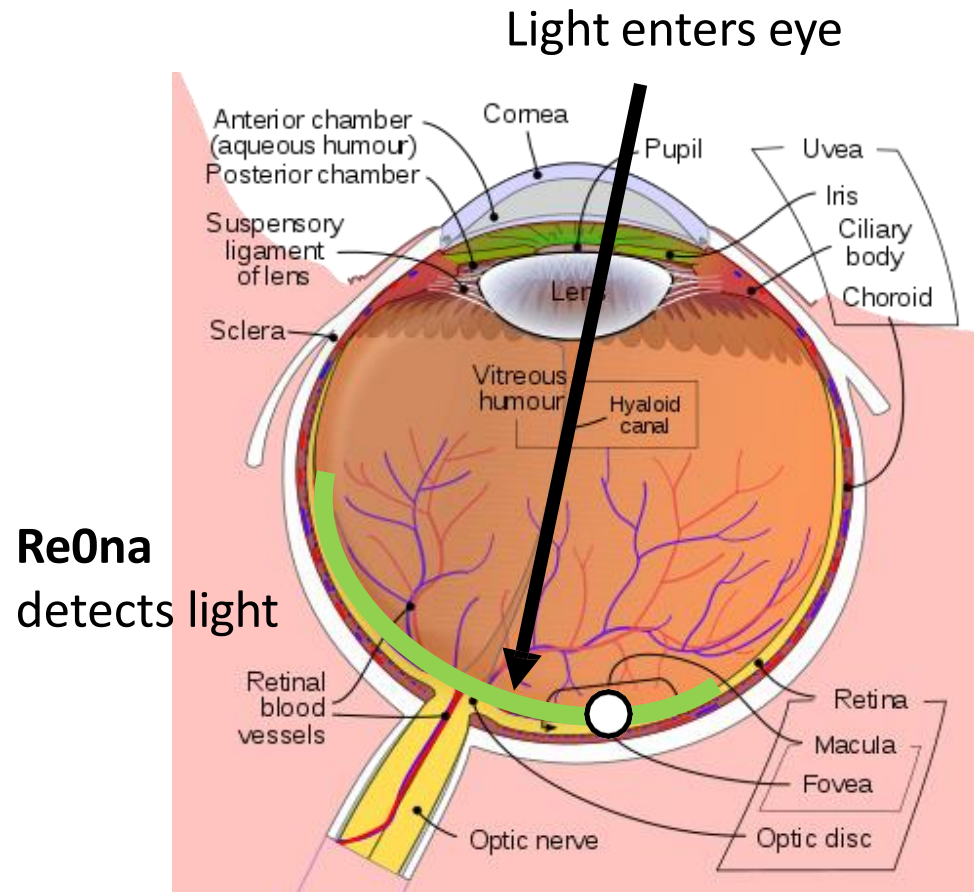
Human Vision:

Fovea

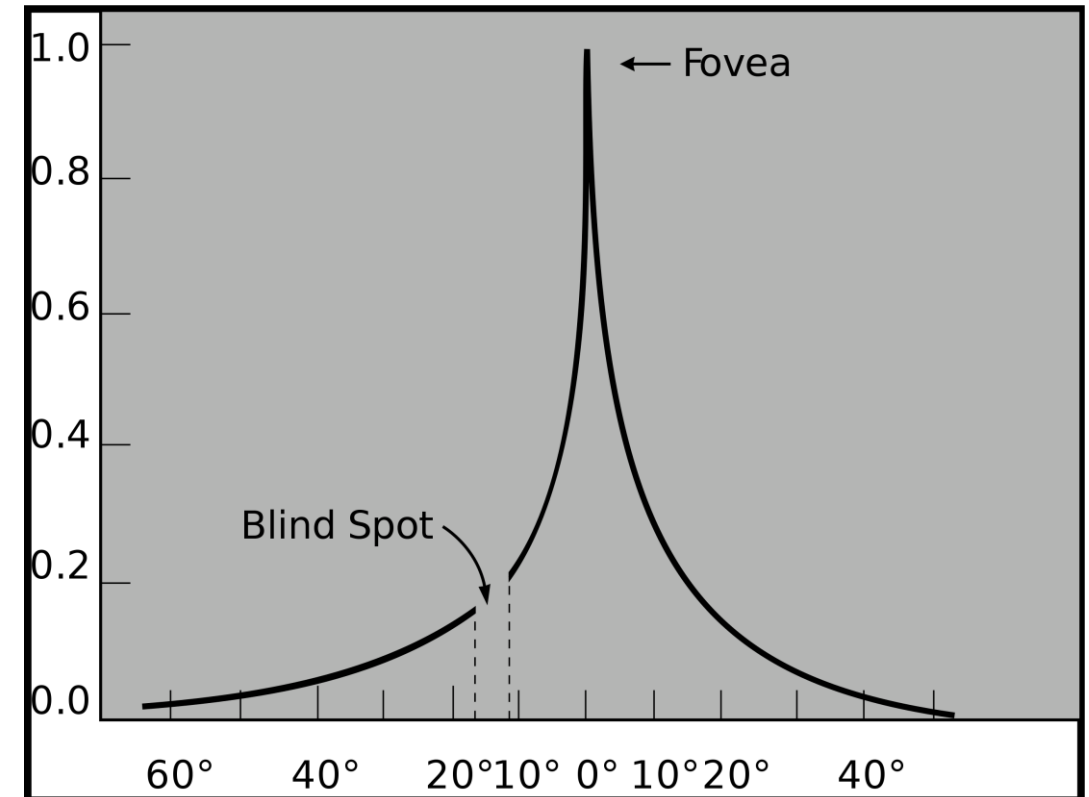


Human Vision:

Fovea



The **fovea** is a tiny region of the retina that can see with high acuity



Human Vision:

Saccades

Human eyes are constantly moving so we don't notice



The **fovea** is a tiny region of the retina that can see with high acuity

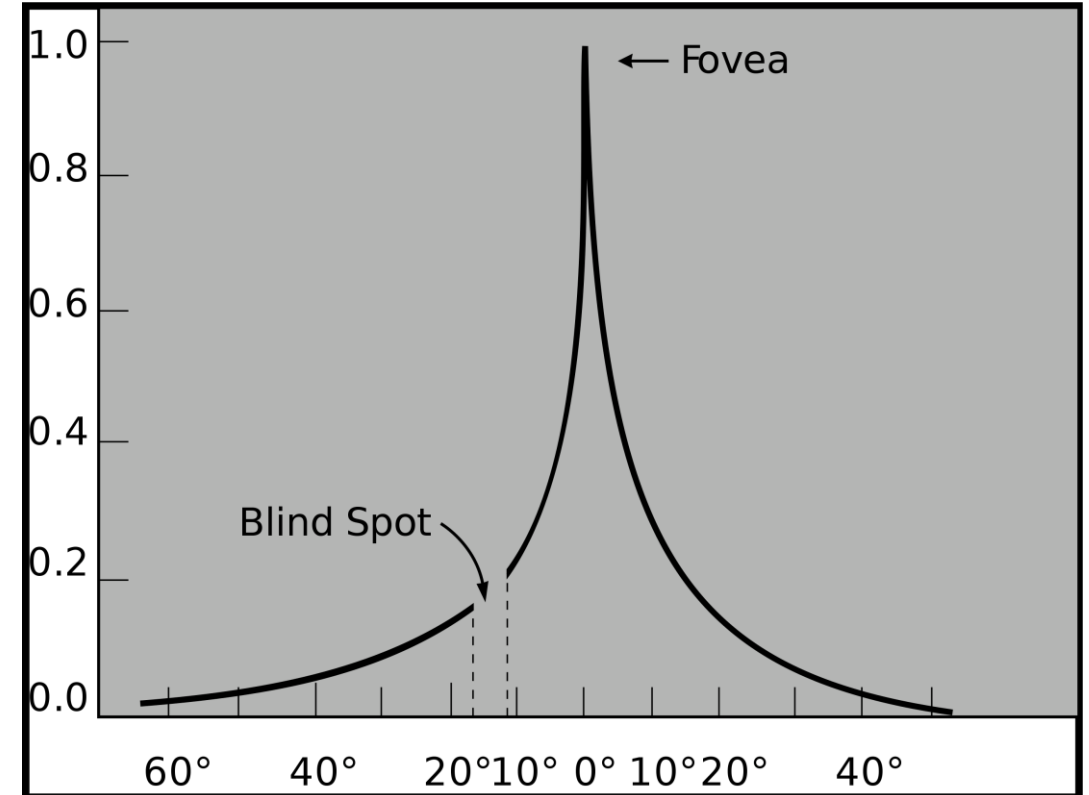
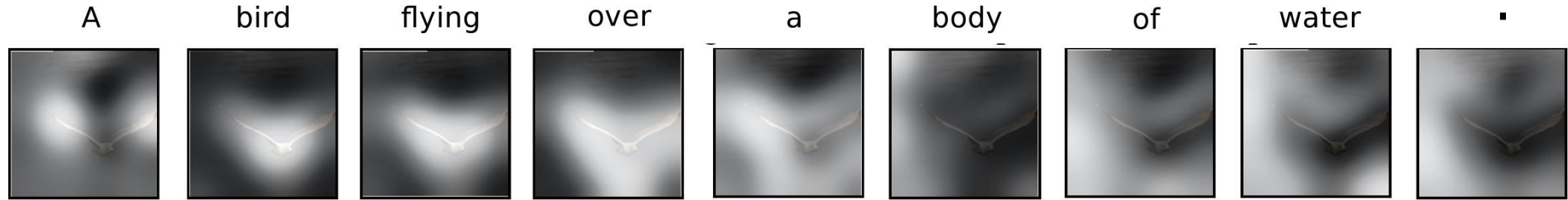


Image Captioning with RNNs and Attention



Attention weights at each timestep kind of like saccades of human eye



X, Attend, and Y

“Show, attend, and tell” (*Xu et al, ICML 2015*)

Look at image, attend to image regions, produce question

“Ask, attend, and answer” (*Xu and Saenko, ECCV 2016*)

“Show, ask, attend, and answer” (*Kazemi and Elqursh, 2017*)

Read text of question, attend to image regions, produce answer

“Listen, attend, and spell” (*Chan et al, ICASSP 2016*)

Process raw audio, attend to audio regions while producing text

“Listen, attend, and walk” (*Mei et al, AAAI 2016*)

Process text, attend to text regions, output navigation commands

“Show, attend, and interact” (*Qureshi et al, ICRA 2017*)

Process image, attend to image regions, output robot control commands

“Show, attend, and read” (*Li et al, AAAI 2019*)

Process image, attend to image regions, output text

Attention Layer

Inputs:

Query vector: \mathbf{q} (Shape: D_Q)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

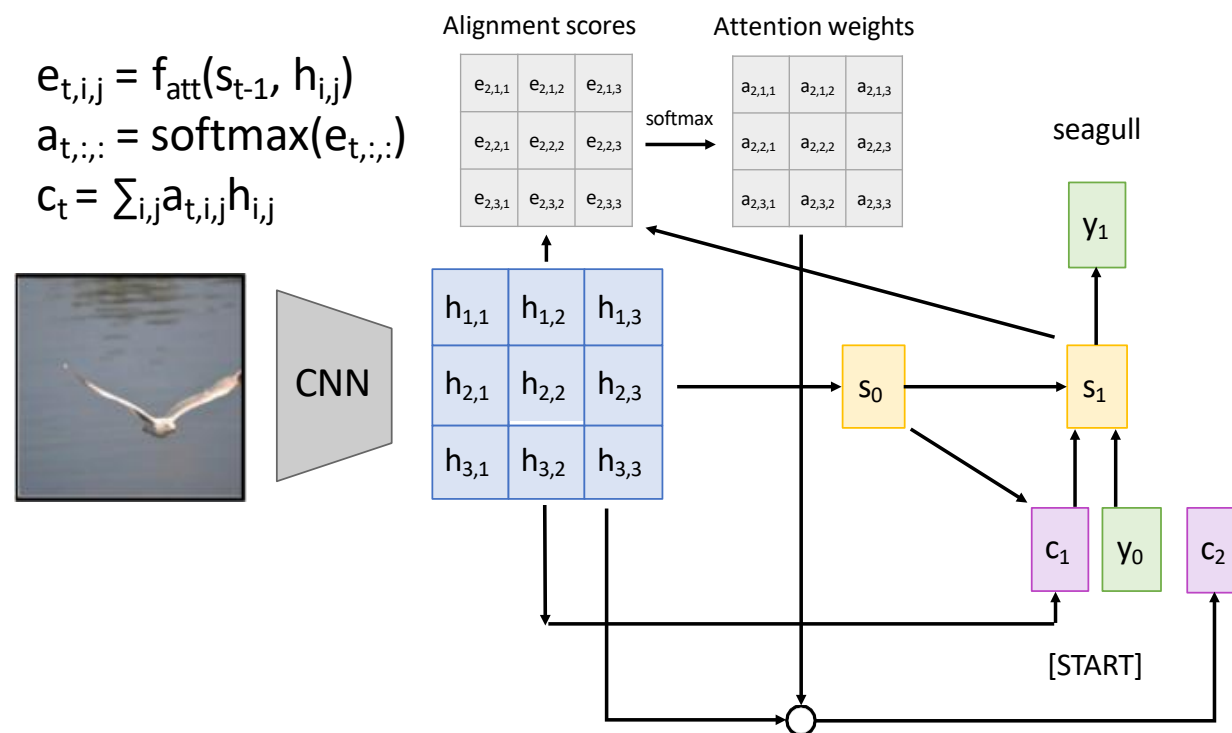
Similarity function: f_{att}

Computation:

Similarities: \mathbf{e} (Shape: N_X) $e_i = f_{\text{att}}(\mathbf{q}, \mathbf{X}_i)$

Attention weights: $\mathbf{a} = \text{softmax}(\mathbf{e})$ (Shape: N_X)

Output vector: $\mathbf{y} = \sum_i a_i \mathbf{X}_i$ (Shape: D_X)



Attention Layer

Inputs:

Query vector: \mathbf{q} (Shape: D_Q)

Input vectors: \mathbf{X} (Shape: $N_X \times D_Q$)

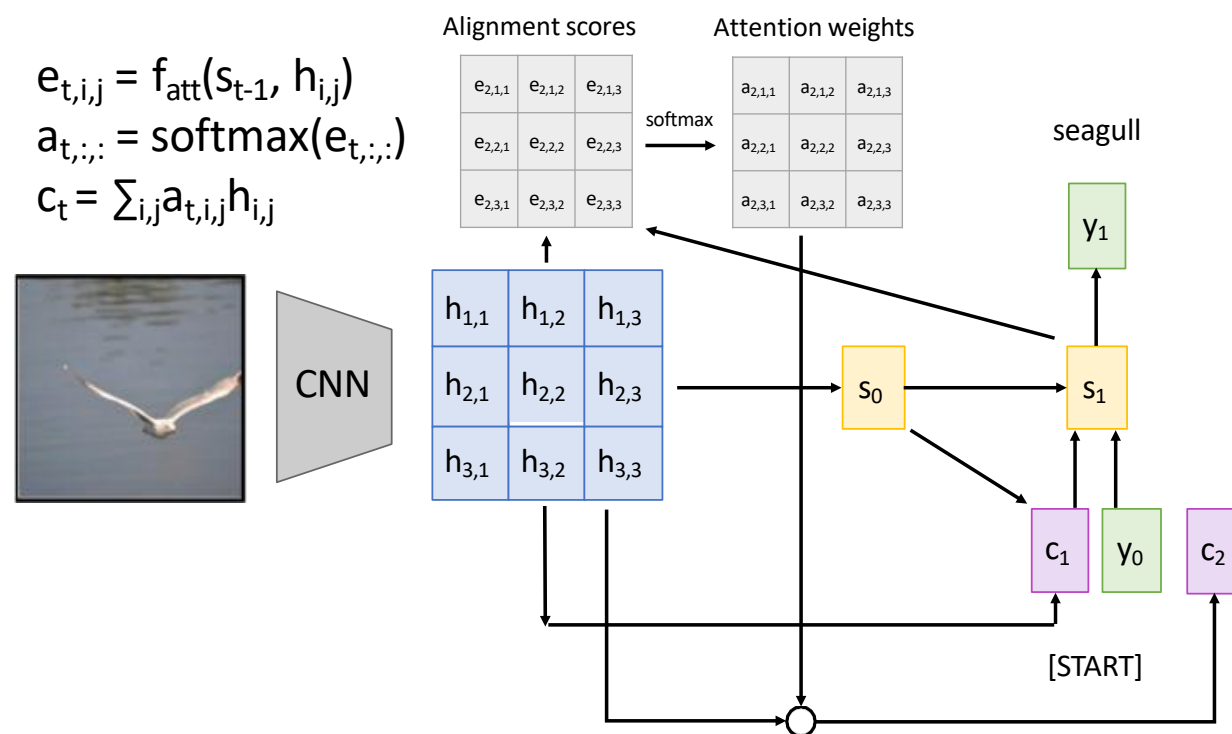
Similarity function: dot product

Computation:

Similarities: \mathbf{e} (Shape: N_X) $\mathbf{e}_i = \mathbf{q} \cdot \mathbf{X}_i$

Attention weights: $\mathbf{a} = \text{softmax}(\mathbf{e})$ (Shape: N_X)

Output vector: $\mathbf{y} = \sum_i \mathbf{a}_i \mathbf{X}_i$ (Shape: D_X)



Changes:

- Use dot product for similarity

Attention Layer

Inputs:

Query vector: \mathbf{q} (Shape: D_Q)

Input vectors: \mathbf{X} (Shape: $N_X \times D_Q$)

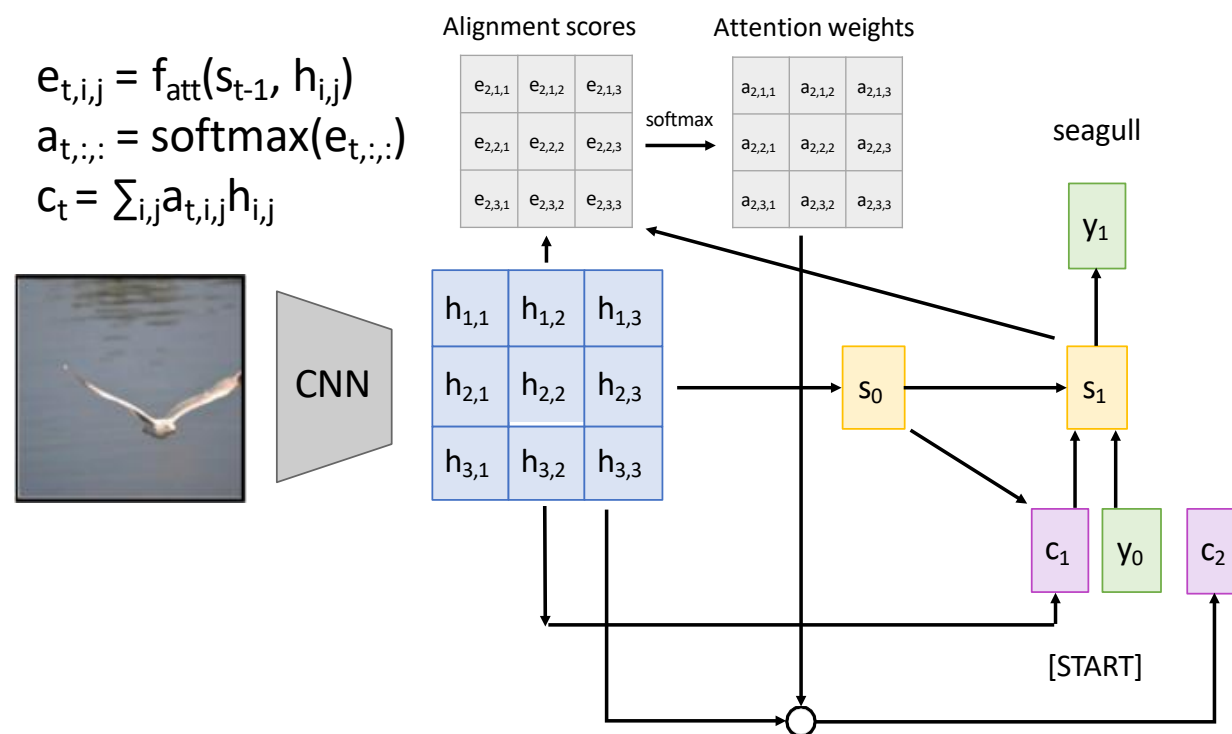
Similarity function: scaled dot product

Computation:

Similarities: e (Shape: N_X) $e_i = \mathbf{q} \cdot \mathbf{X}_i / \sqrt{D_Q}$

Attention weights: $a = \text{softmax}(e)$ (Shape: N_X)

Output vector: $\mathbf{y} = \sum_i a_i \mathbf{X}_i$ (Shape: D_X)



Changes:

- Use **scaled** dot product for similarity

Attention Layer

Inputs:

Query vector: \mathbf{q} (Shape: D_Q)

Input vectors: \mathbf{X} (Shape: $N_X \times D_Q$)

Similarity function: scaled dot product

Large similarities will cause softmax to saturate and give vanishing gradients

Recall $a \cdot b = |a| |b| \cos(\text{angle})$

Suppose that a and b are constant vectors of dimension D

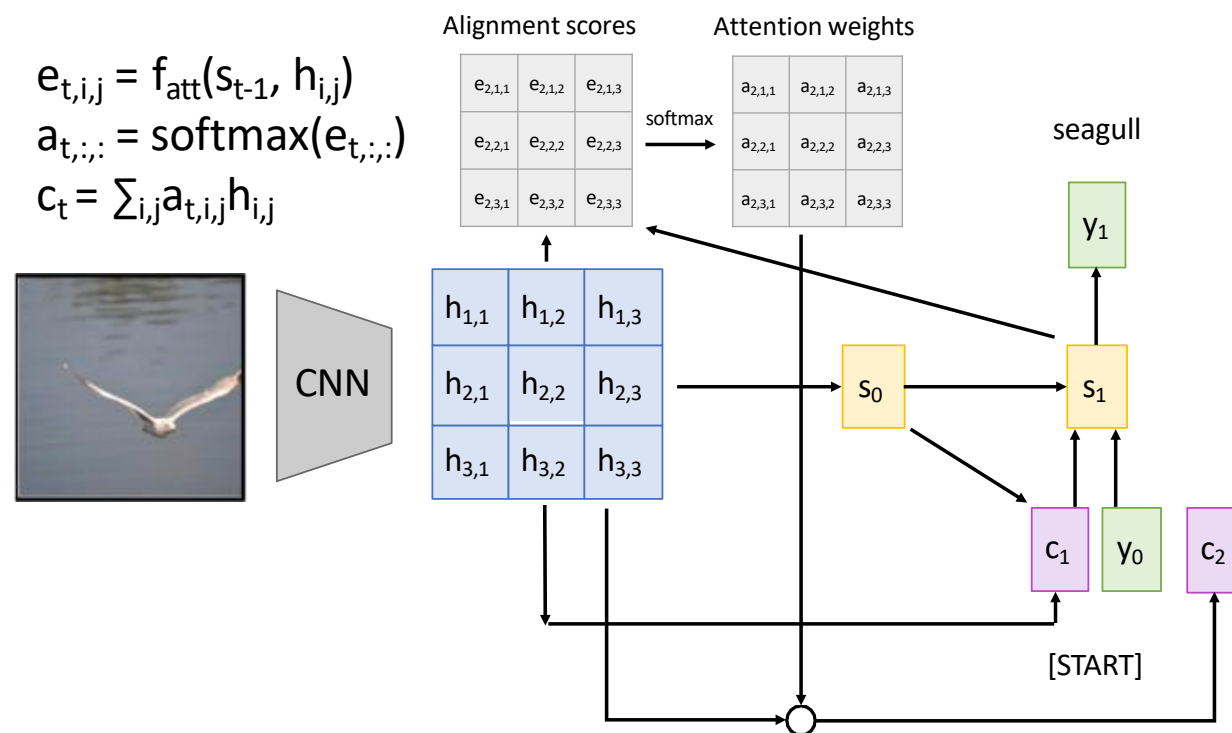
Then $|a| = (\sum_i a_i^2)^{1/2} = a \text{ sqrt}(D)$

Computation:

Similarities: e (Shape: N_X) $e_i = \mathbf{q} \cdot \mathbf{X}_i / \text{sqrt}(D_Q)$

Attention weights: $a = \text{softmax}(e)$ (Shape: N_X)

Output vector: $\mathbf{y} = \sum_i a_i \mathbf{X}_i$ (Shape: D_X)



Changes:

- Use **scaled** dot product for similarity

Attention Layer

Inputs:

Query vectors: **Q** (Shape: $N_Q \times D_Q$)

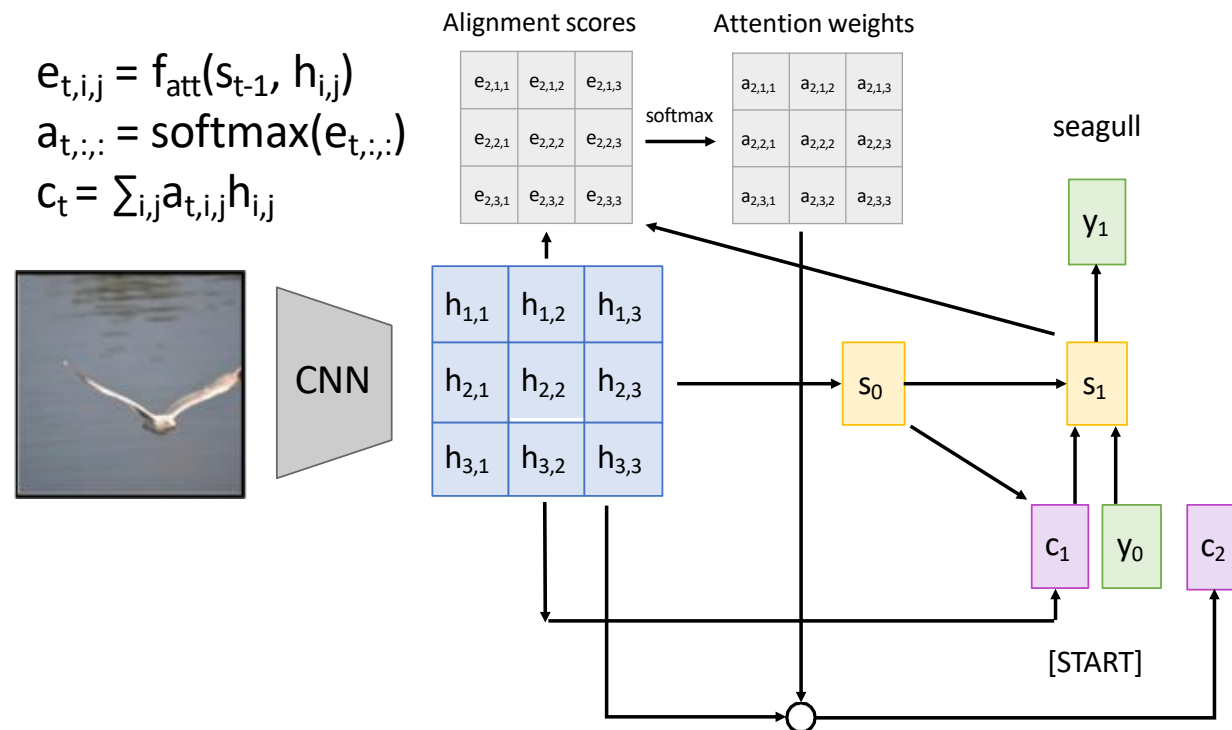
Input vectors: **X** (Shape: $N_X \times D_X$)

Computation:

Similarities: $E = QX^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = Q_i \cdot X_j / \sqrt{D_Q}$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $Y = AX$ (Shape: $N_Q \times D_X$) $Y_i = \sum_j A_{i,j} X_j$



Changes:

- Use dot product for similarity
- Multiple **query** vectors

Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

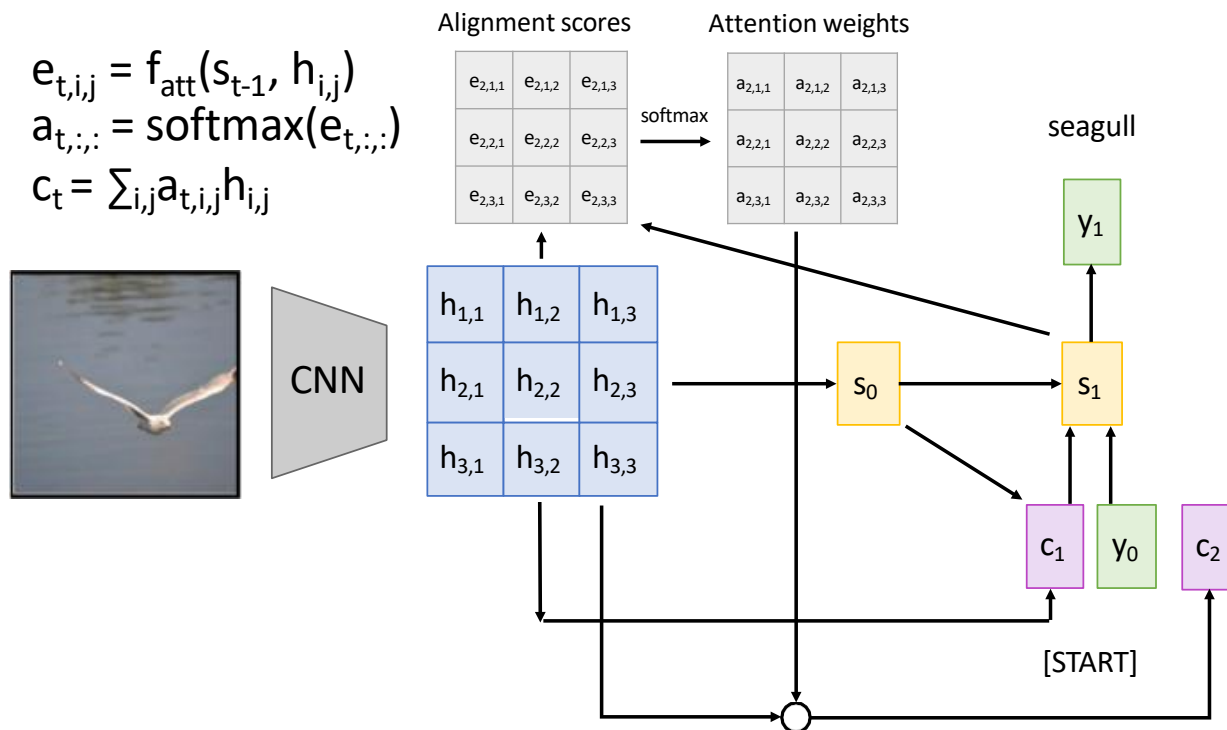
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Changes:

- Use dot product for similarity
- Multiple **query** vectors
- Separate **key** and **value**

Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \sqrt{D_Q}$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

X_1

X_2

X_3

Q_1

Q_2

Q_3

Q_4

Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

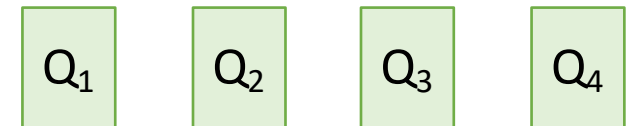
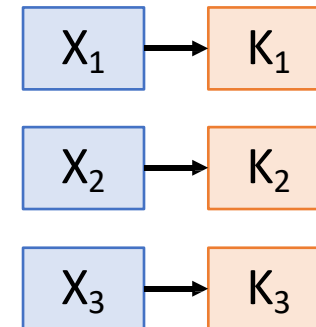
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \sqrt{D_Q}$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

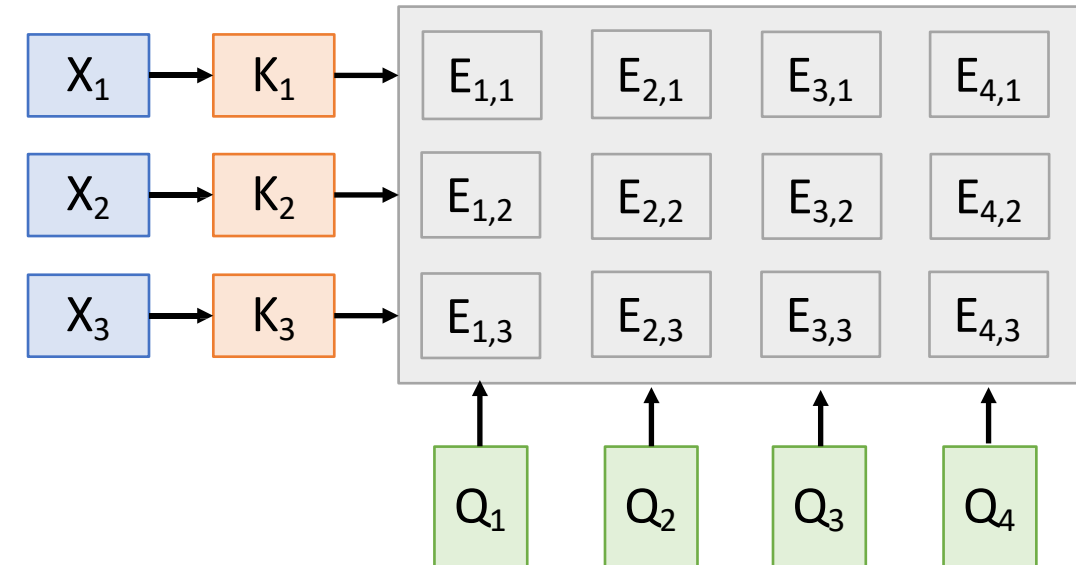
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

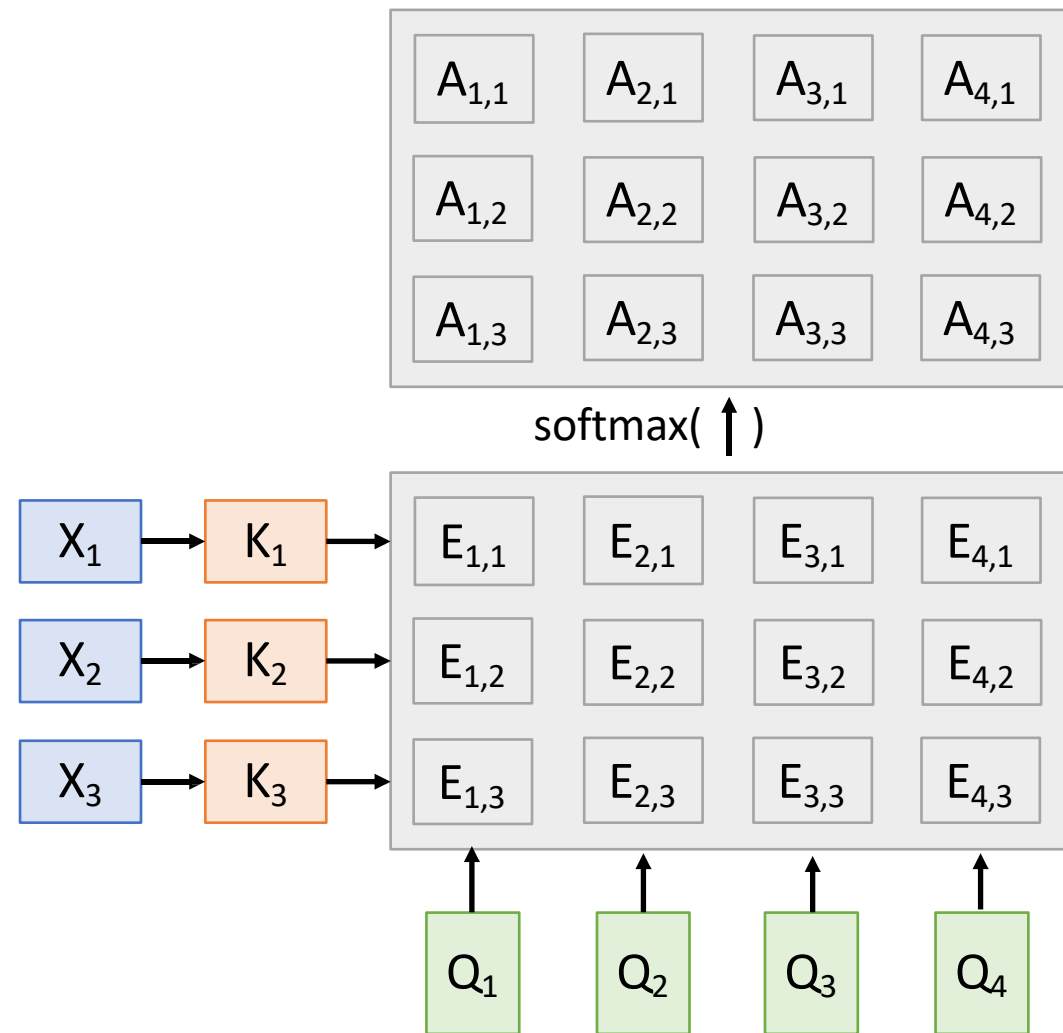
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

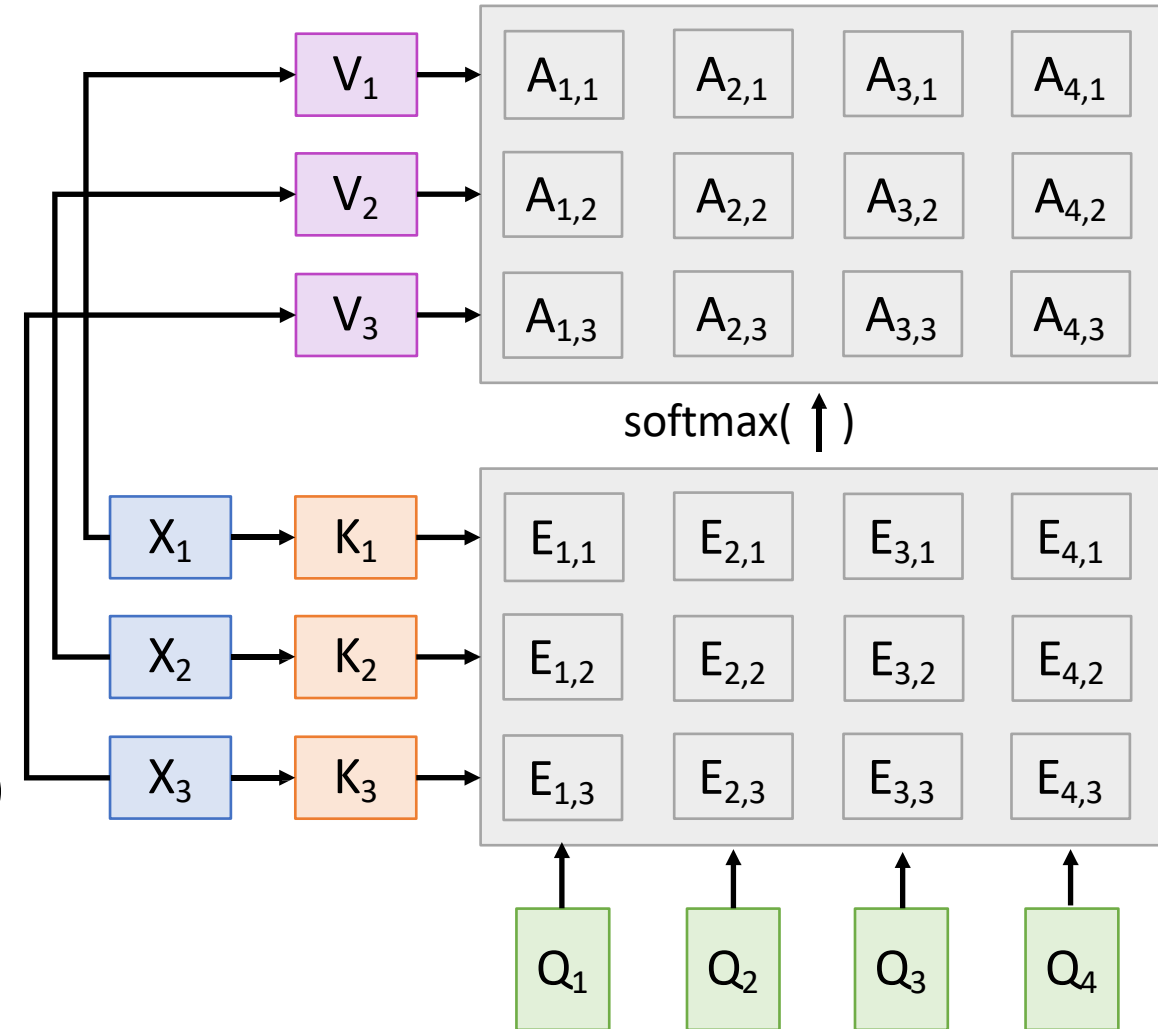
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

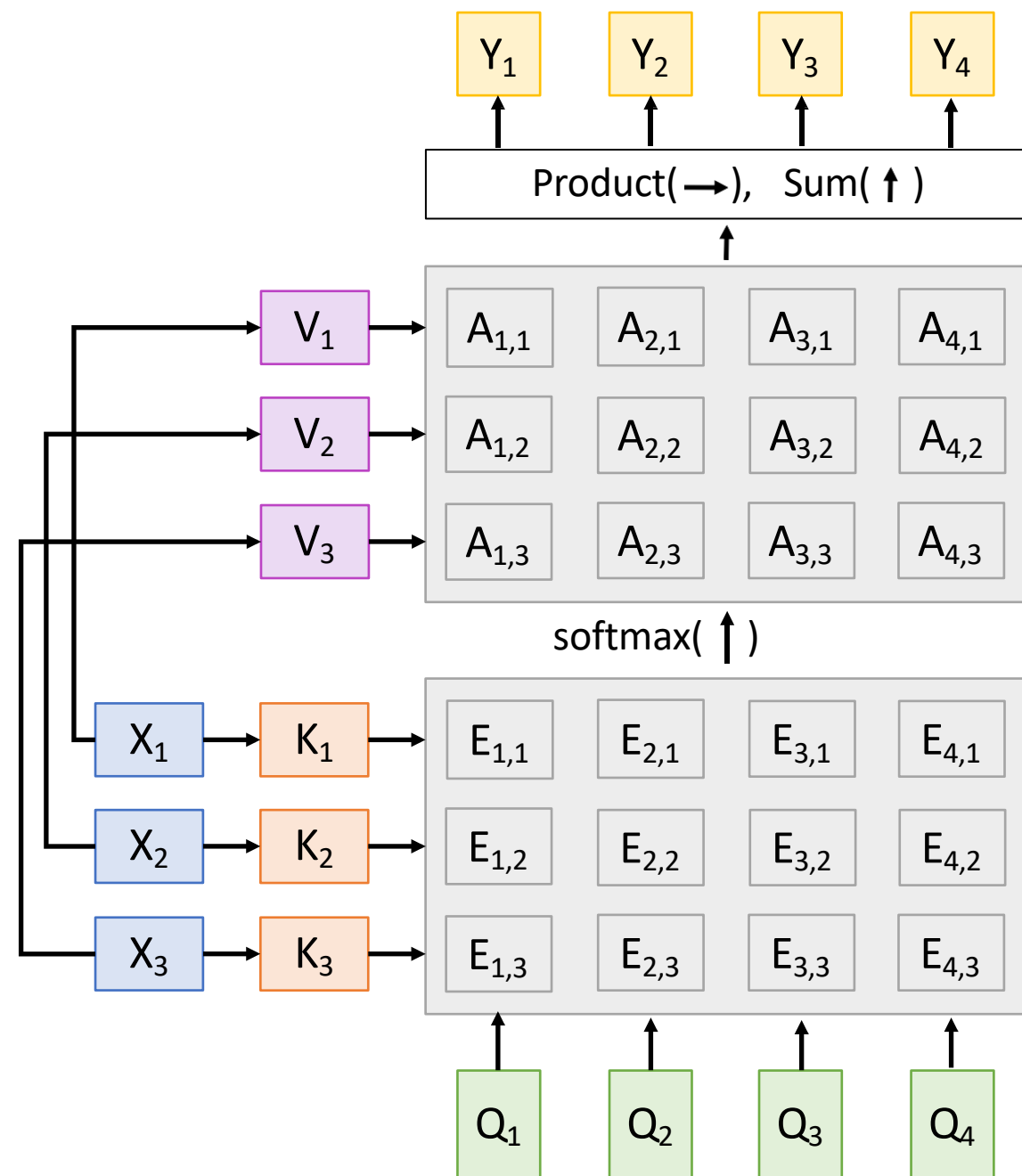
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

X_1

X_2

X_3

Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

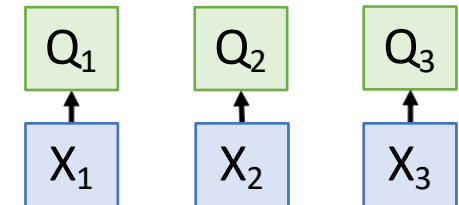
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

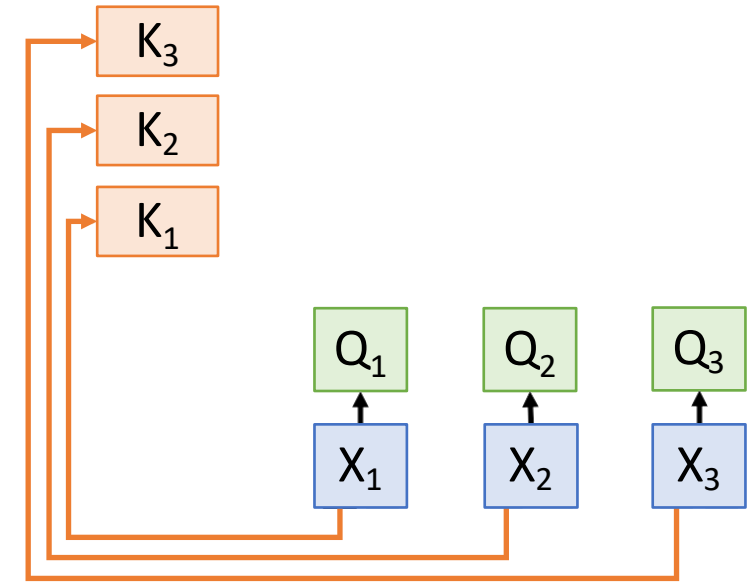
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

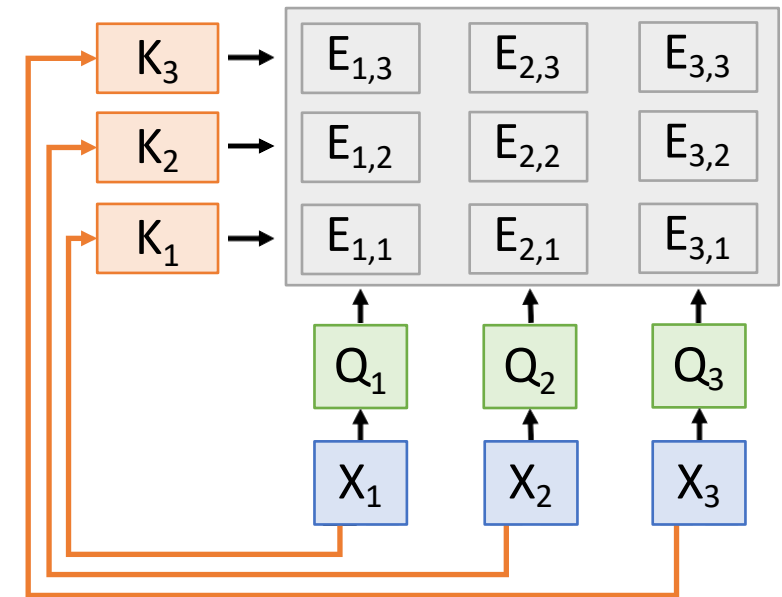
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

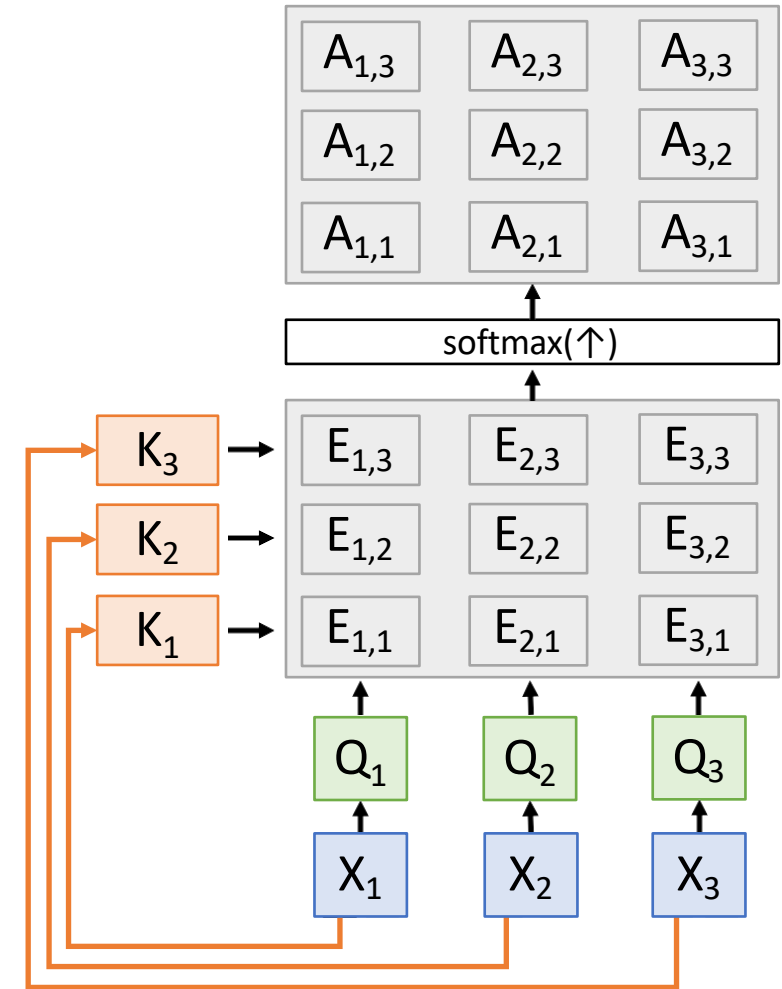
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

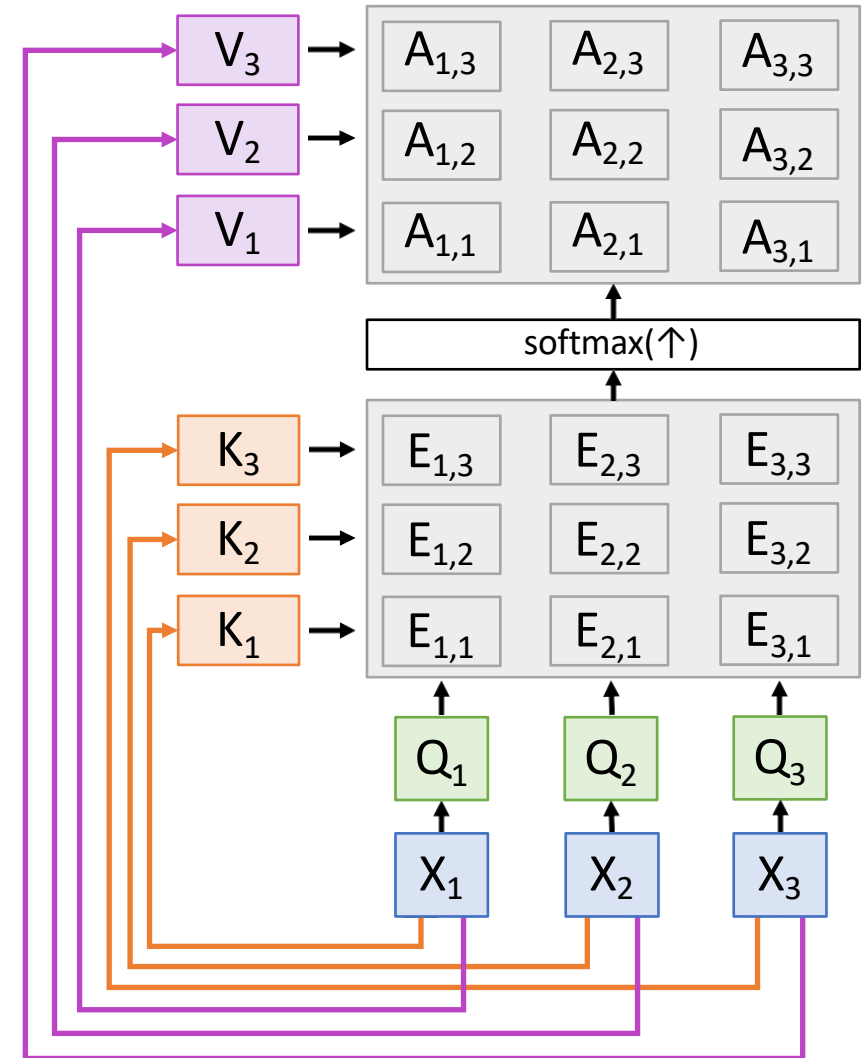
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

Inputs: One **query** per **input vector**

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

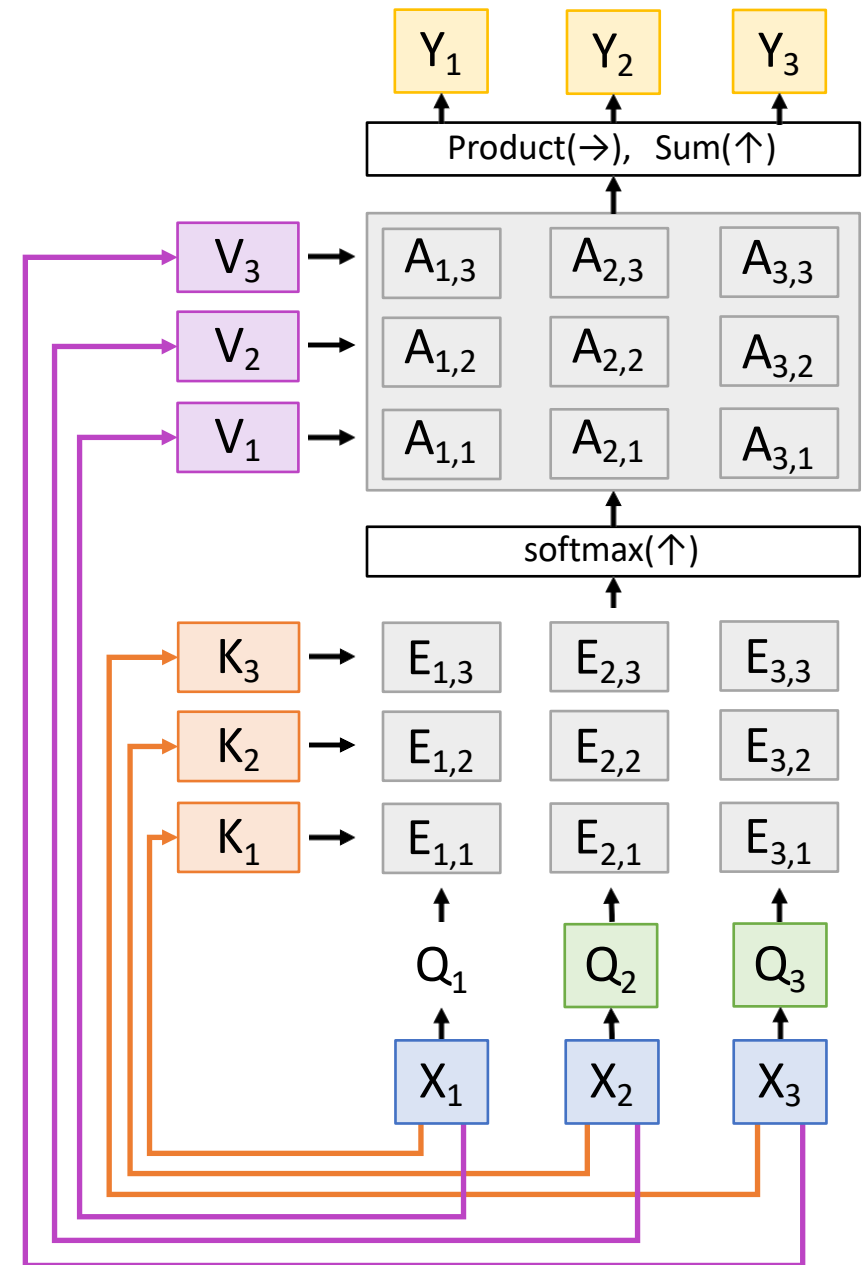
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

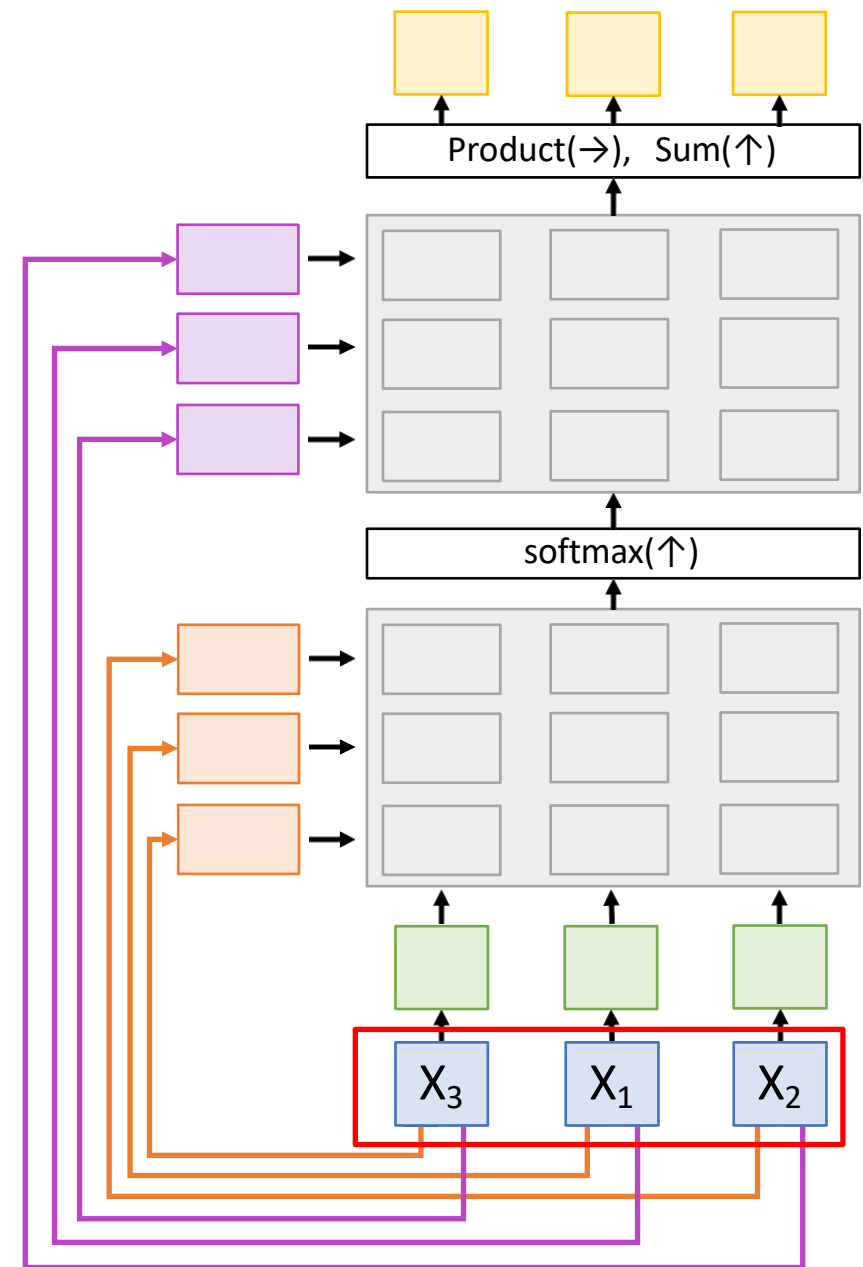
Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

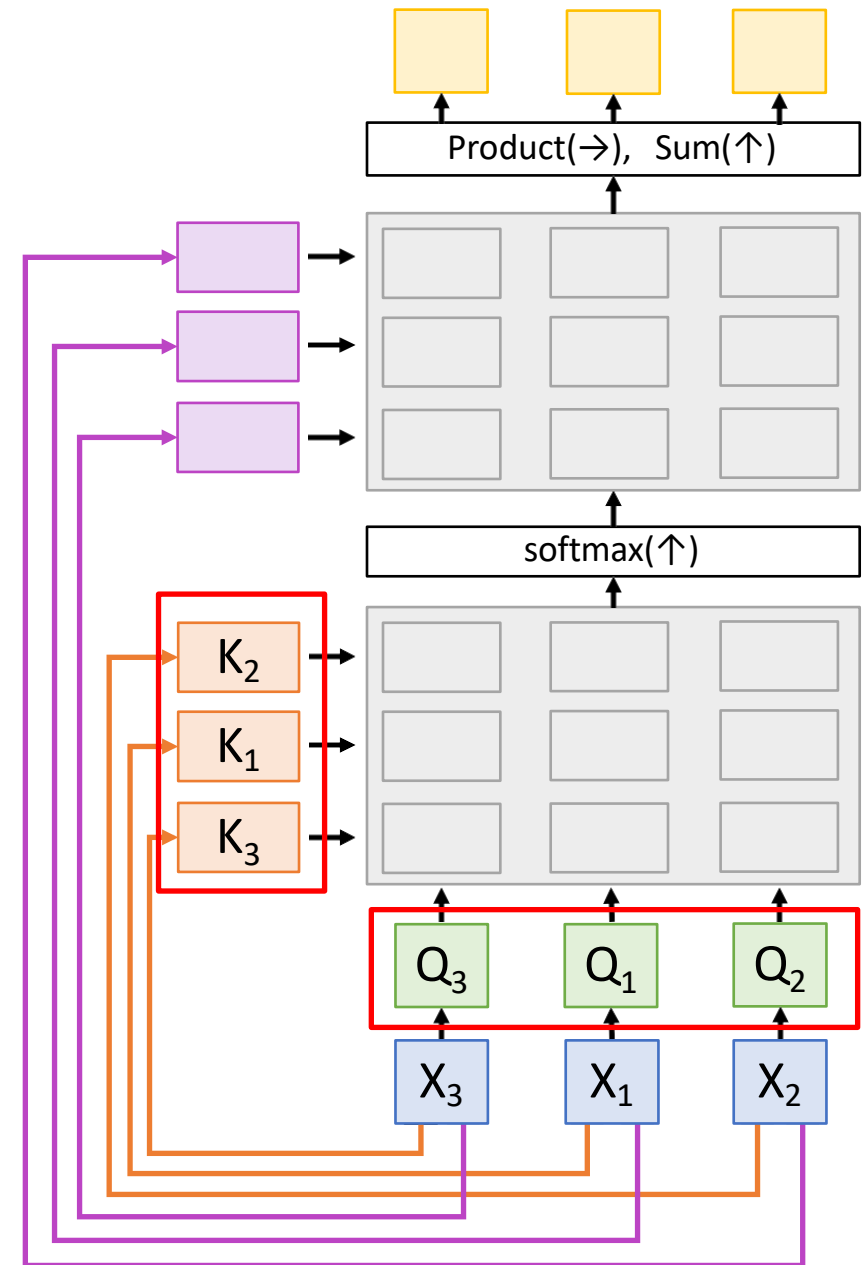
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Queries and Keys will be
the same, but permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

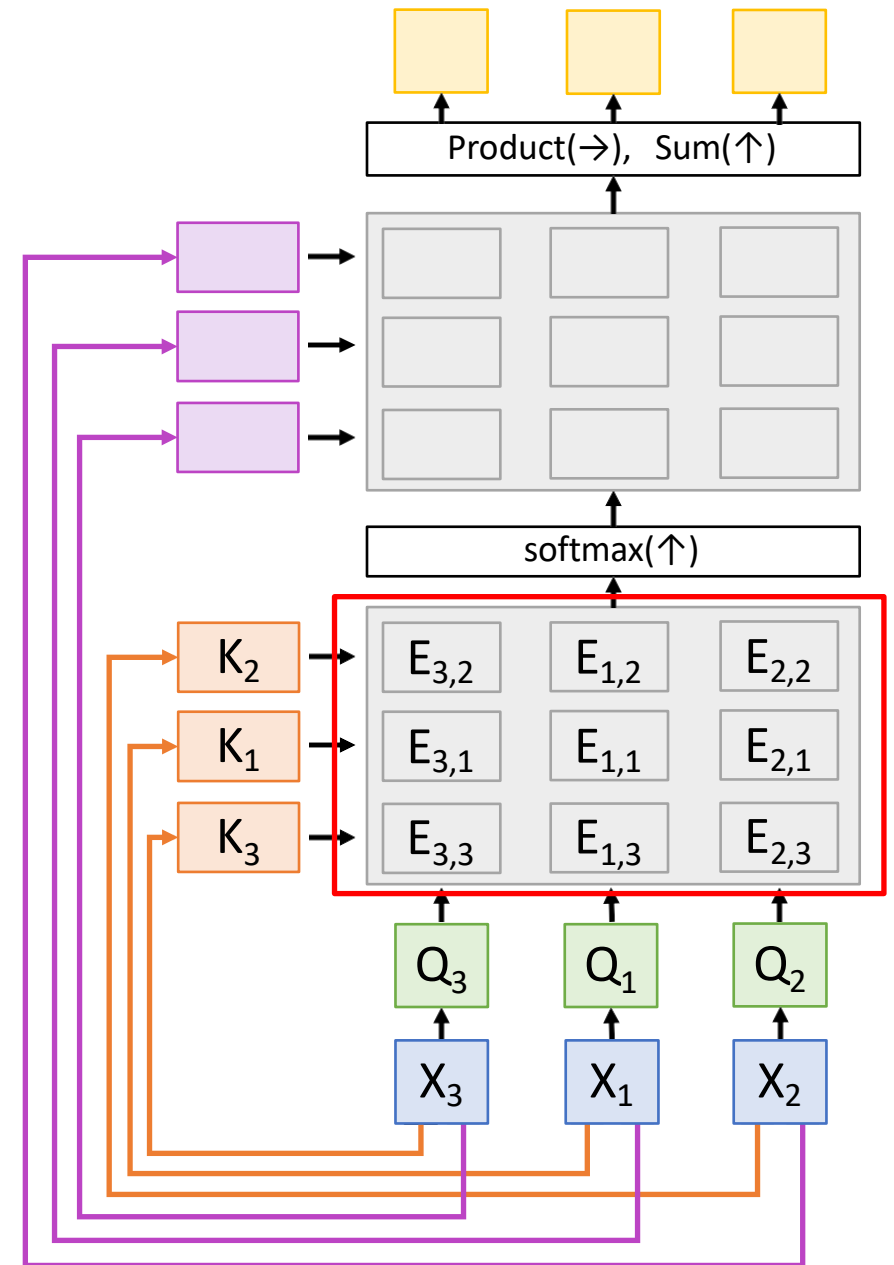
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Similarities will be the
same, but permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

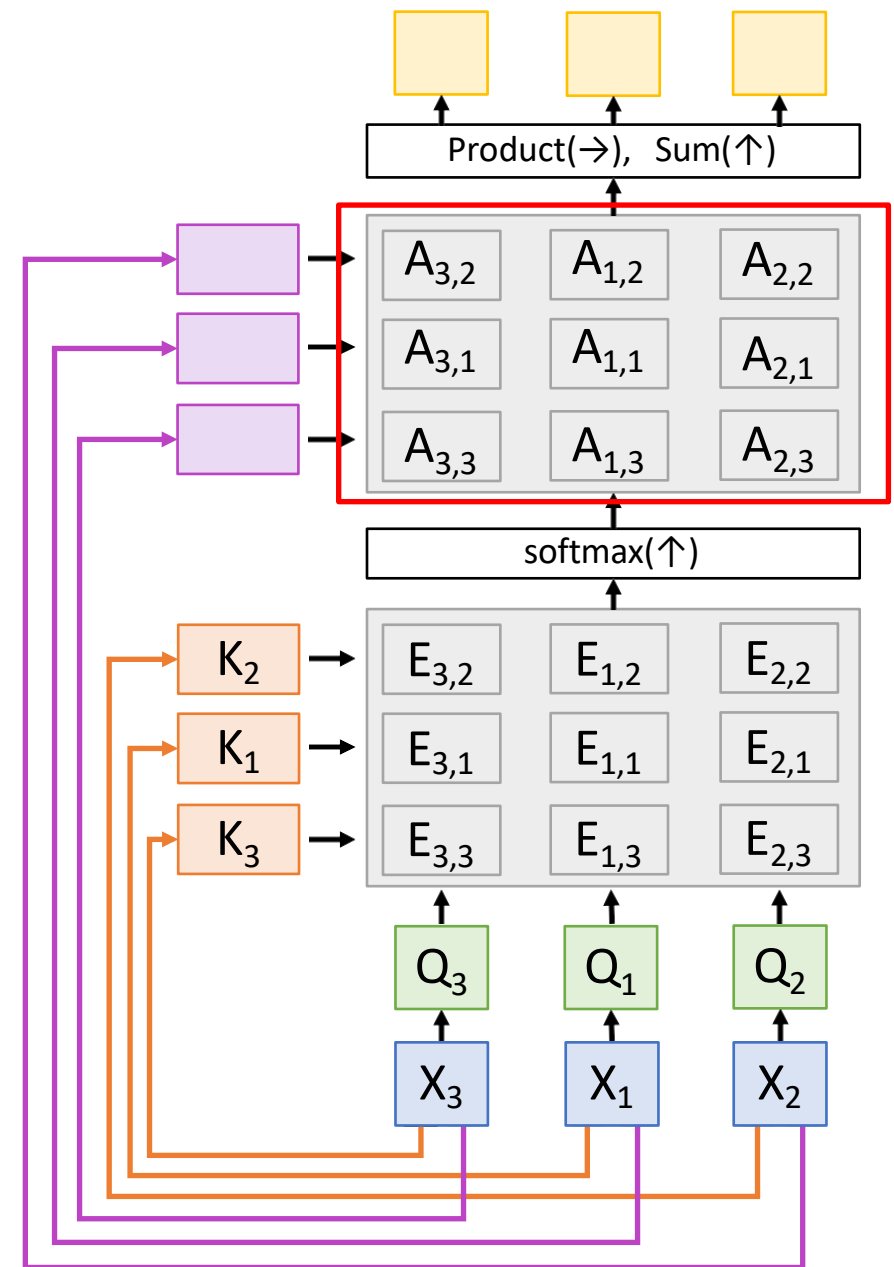
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Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Attention weights will
be the same, but
permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

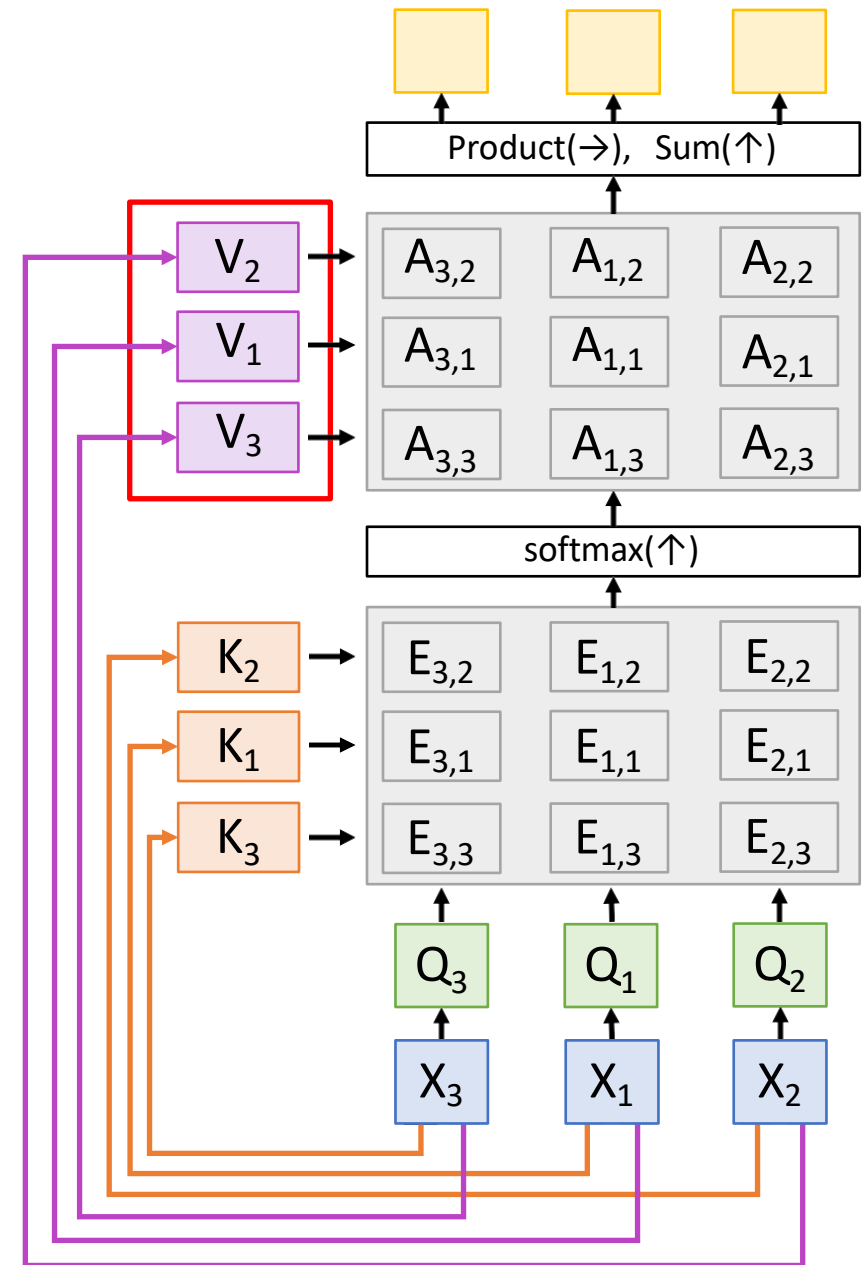
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Values will be the
same, but permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

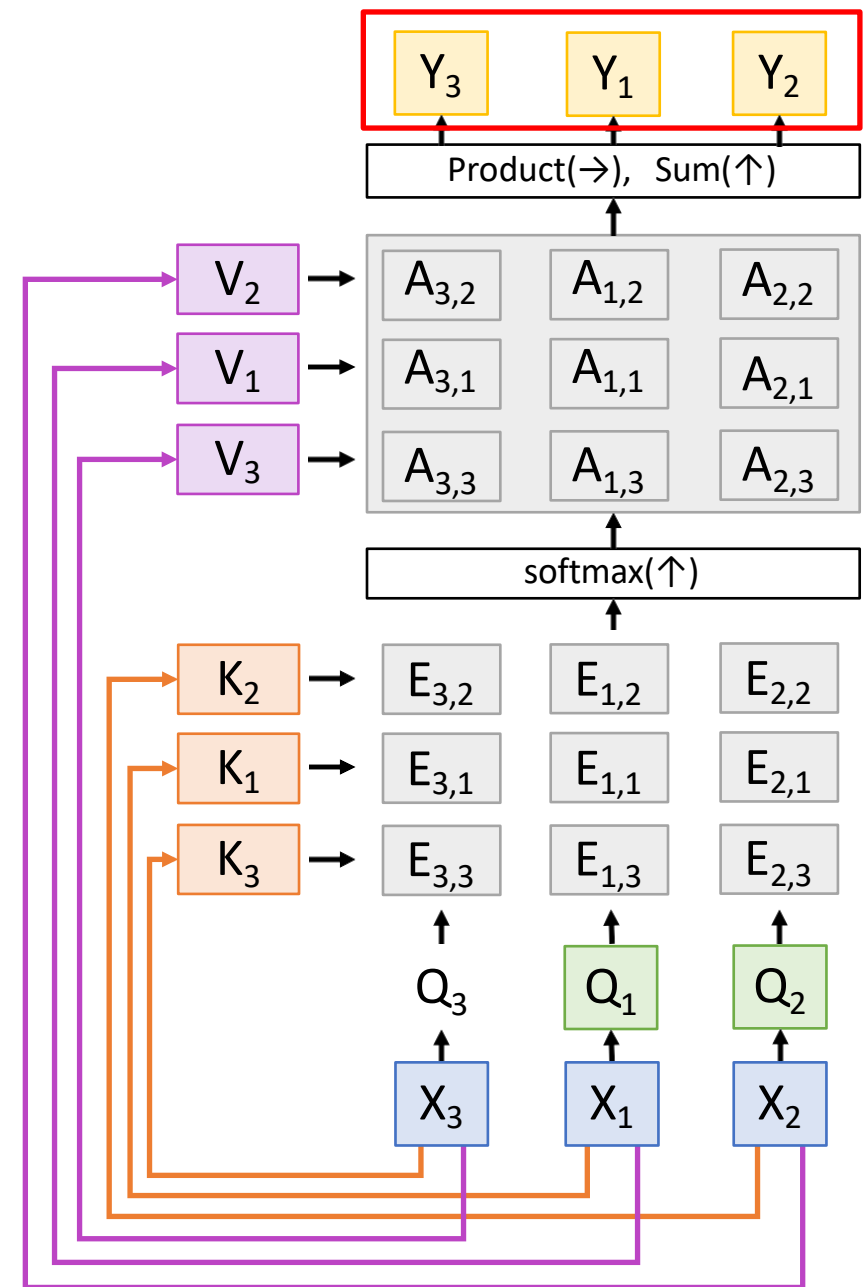
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Outputs will be the
same, but permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_x \times D_x$)

Key matrix: \mathbf{W}_K (Shape: $D_x \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_x \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_x \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_x \times N_x$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_x \times N_x$)

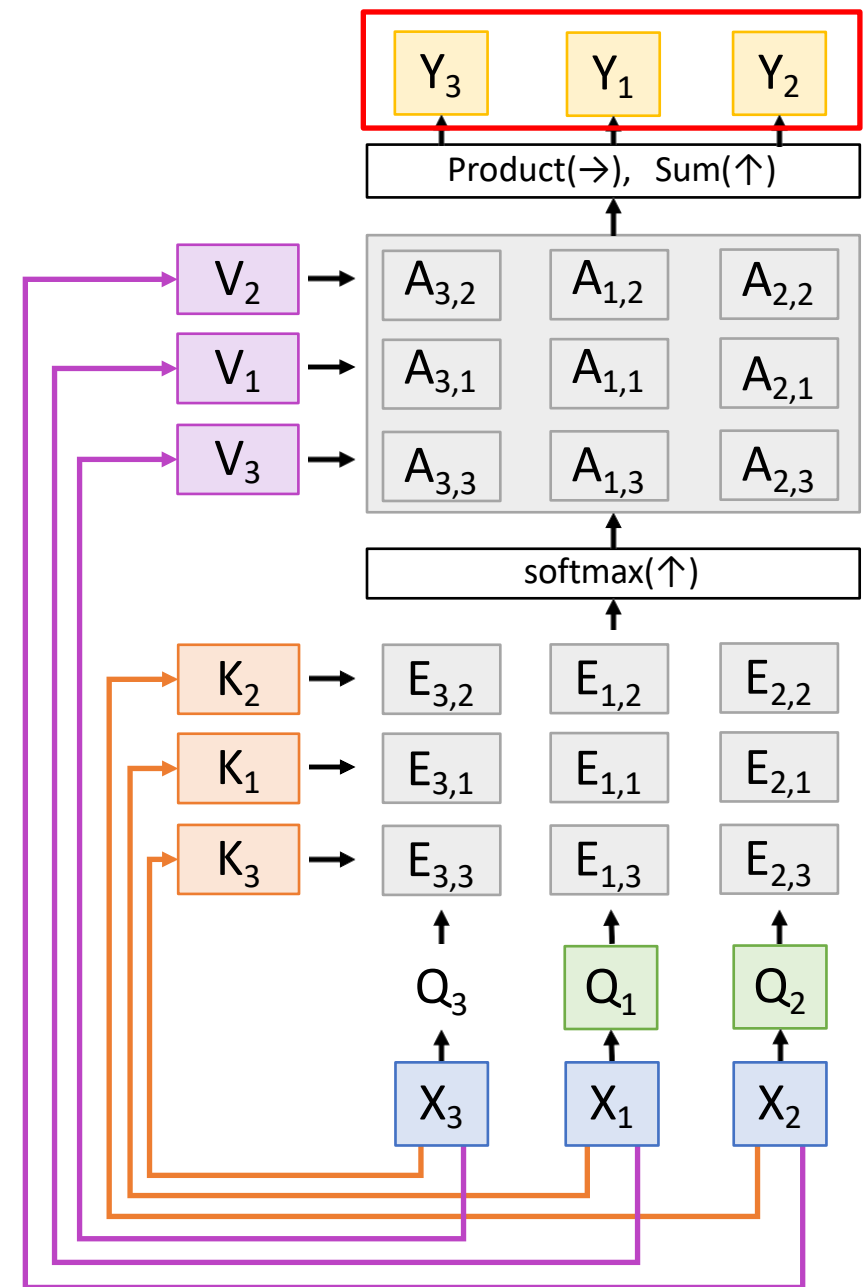
Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Outputs will be the
same, but permuted

Self-attention layer is
Permutation Equivariant
 $f(s(x)) = s(f(x))$

Self-attention layer works
on **sets** of vectors



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

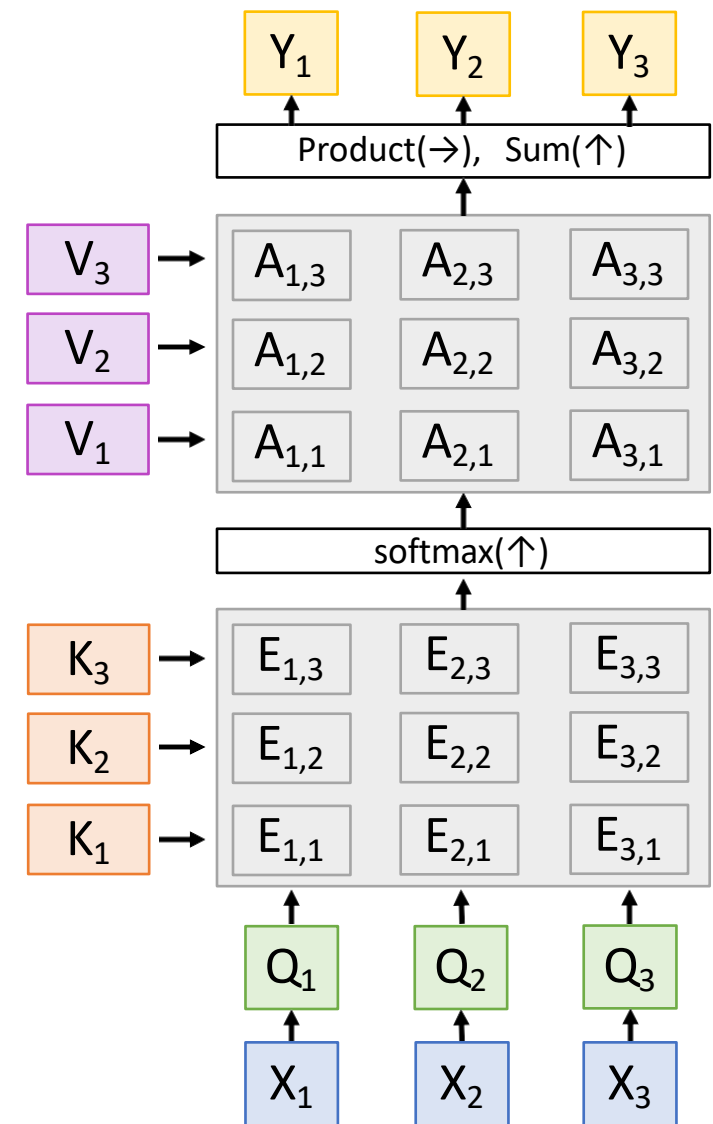
Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Self attention doesn't
"know" the order of the
vectors it is processing!



Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

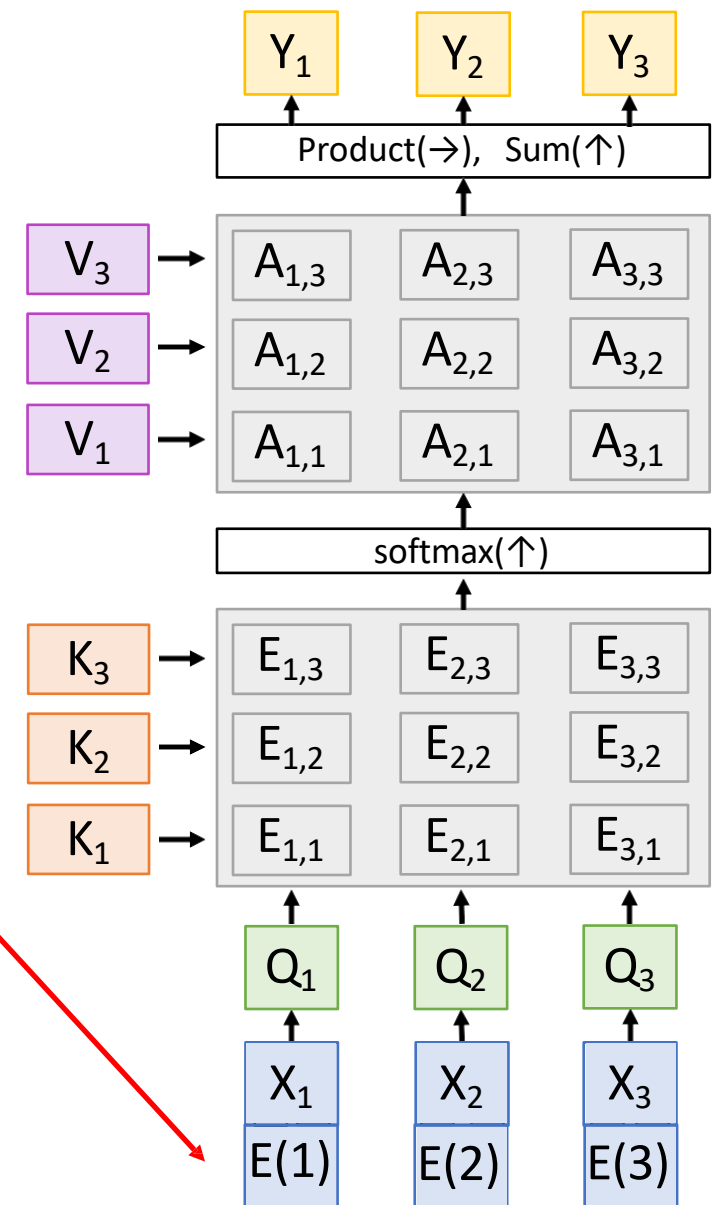
Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

Self attention doesn't
"know" the order of the
vectors it is processing!

In order to make
processing position-
aware, concatenate input
with **positional encoding**

E can be learned lookup
table, or fixed function



Masked Self-Attention Layer

Inputs: Don't let vectors "look ahead" in the sequence

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

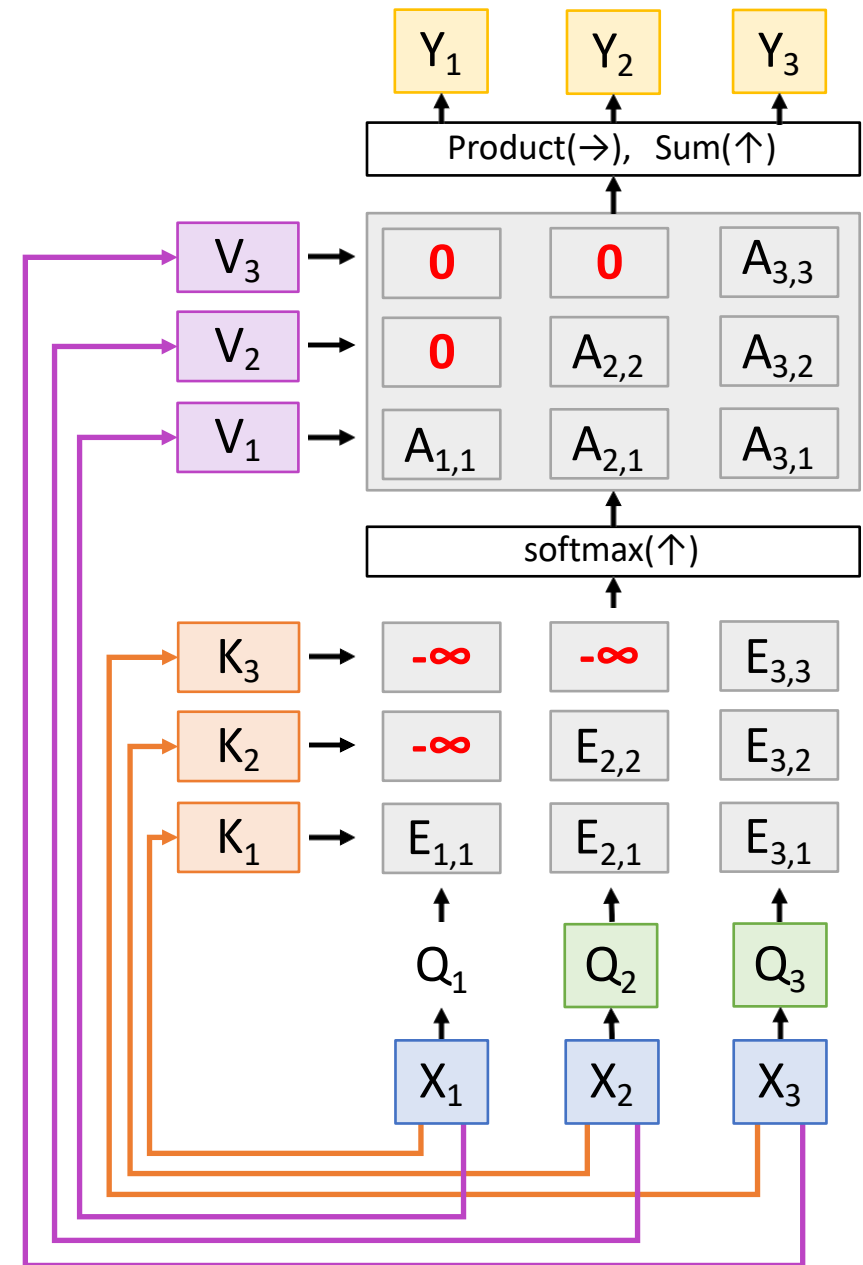
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Masked Self-Attention

Layer Don't let vectors "look ahead" in the sequence
Used for language modeling (predict next word)

Inputs:

Input vectors: \mathbf{X} (Shape: $N_x \times D_x$)

Key matrix: \mathbf{W}_K (Shape: $D_x \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_x \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

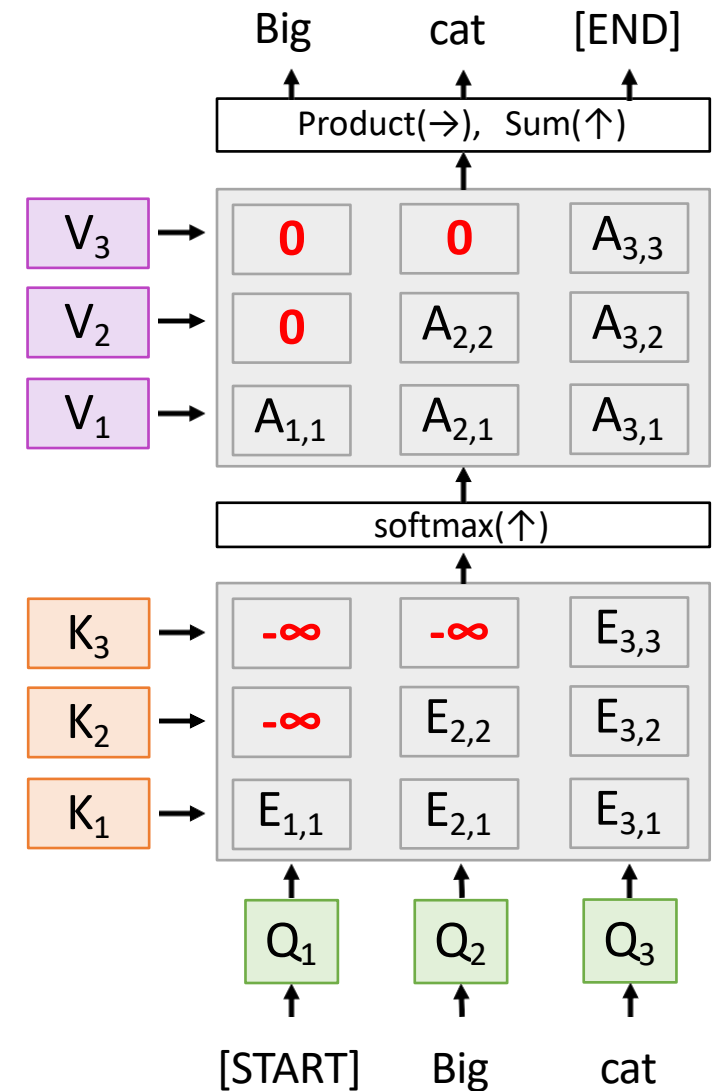
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_x \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_x \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_x \times N_x$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Multihead Self-Attention Layer

Use H independent
“attention Heads” in parallel

Key matrix: W_K (Shape: $D_X \times D_Q$)

Input vectors: X (Shape: $N_X \times D_X$)

Query matrix: W_Q (Shape: $D_X \times D_Q$)

Value matrix: W_V (Shape: $D_X \times D_V$)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $V = XW_V$ (Shape: $N_X \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_X \times N_X$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

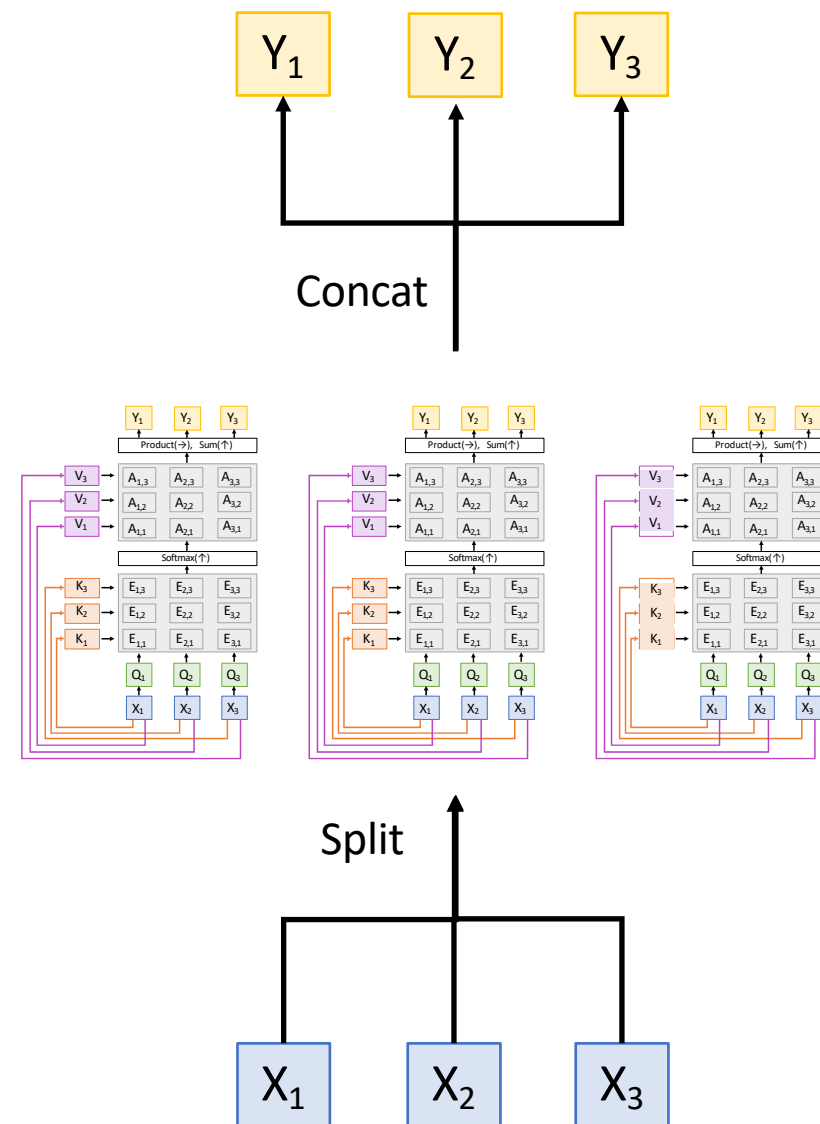
Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $Y = AV$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

Hyperparameters:

Query dimension D_Q

Number of heads H

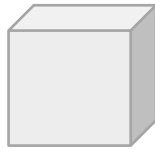


Example: CNN with Self-Attention

Input Image

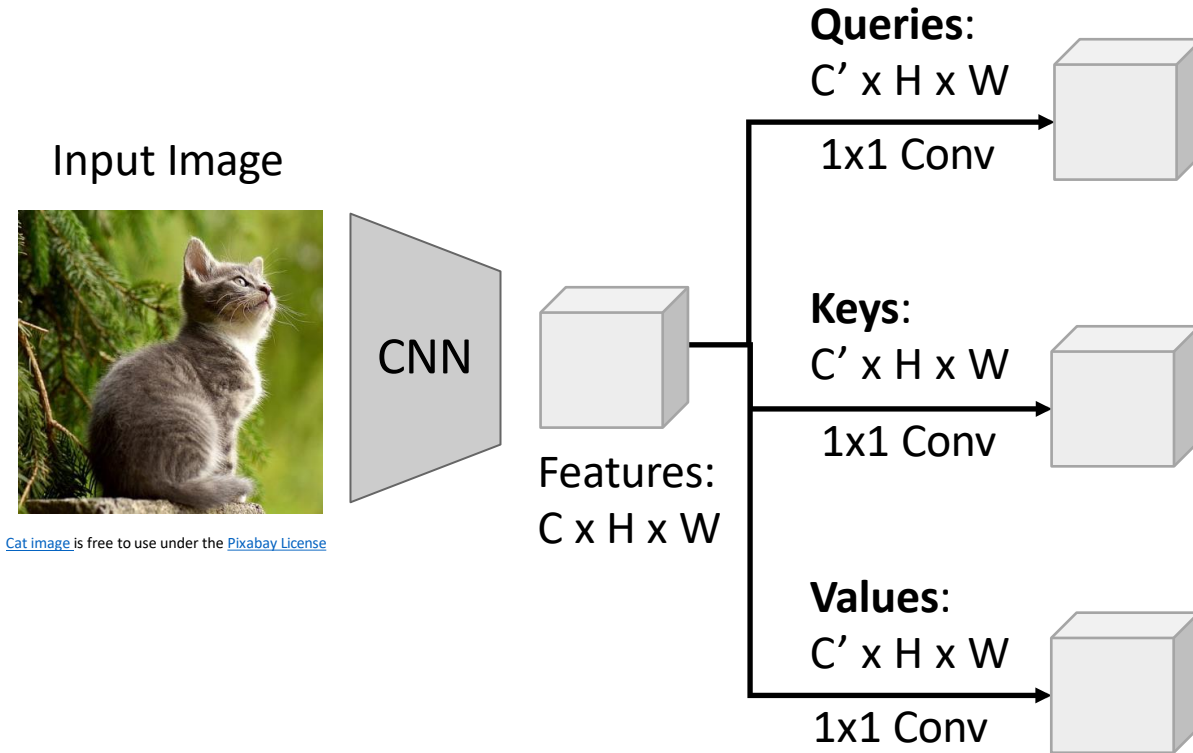


[Cat image](#) is free to use under the [Pixabay License](#)

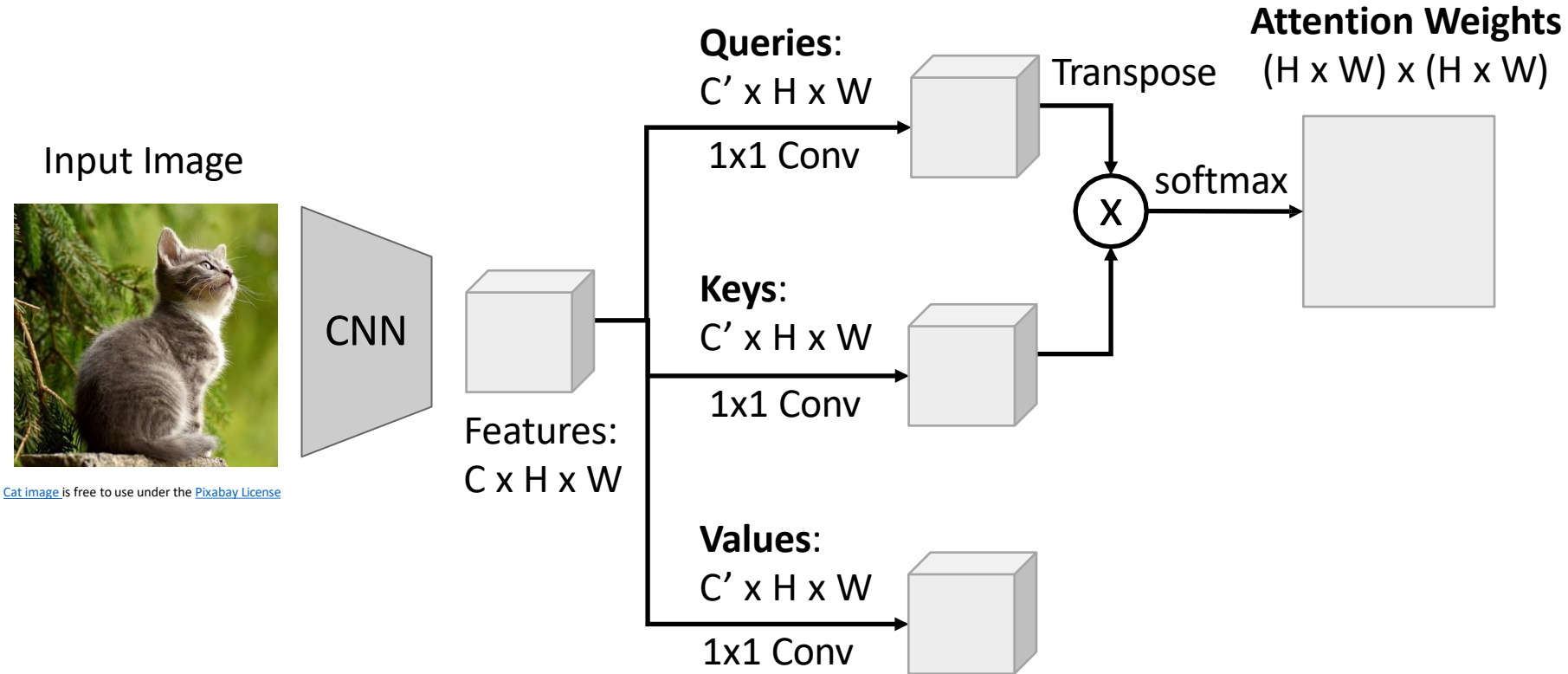


Features:
 $C \times H \times W$

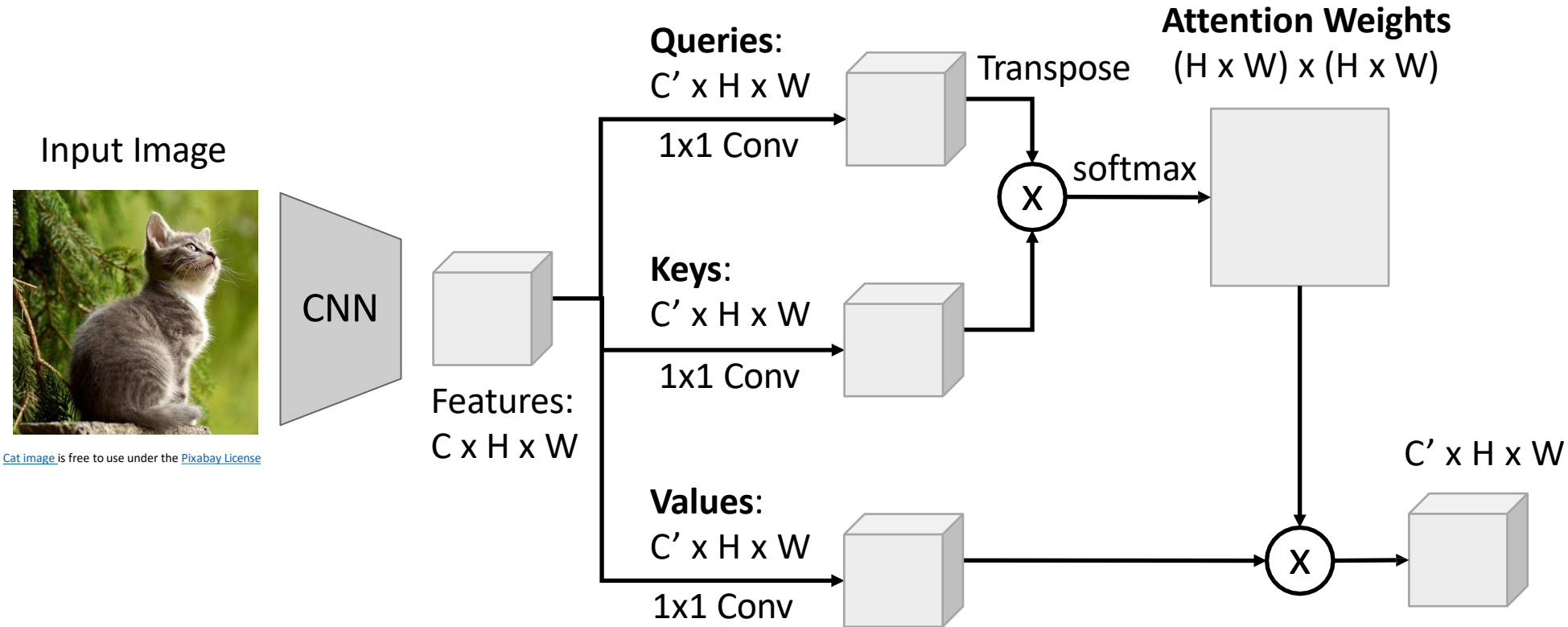
Example: CNN with Self-Attention



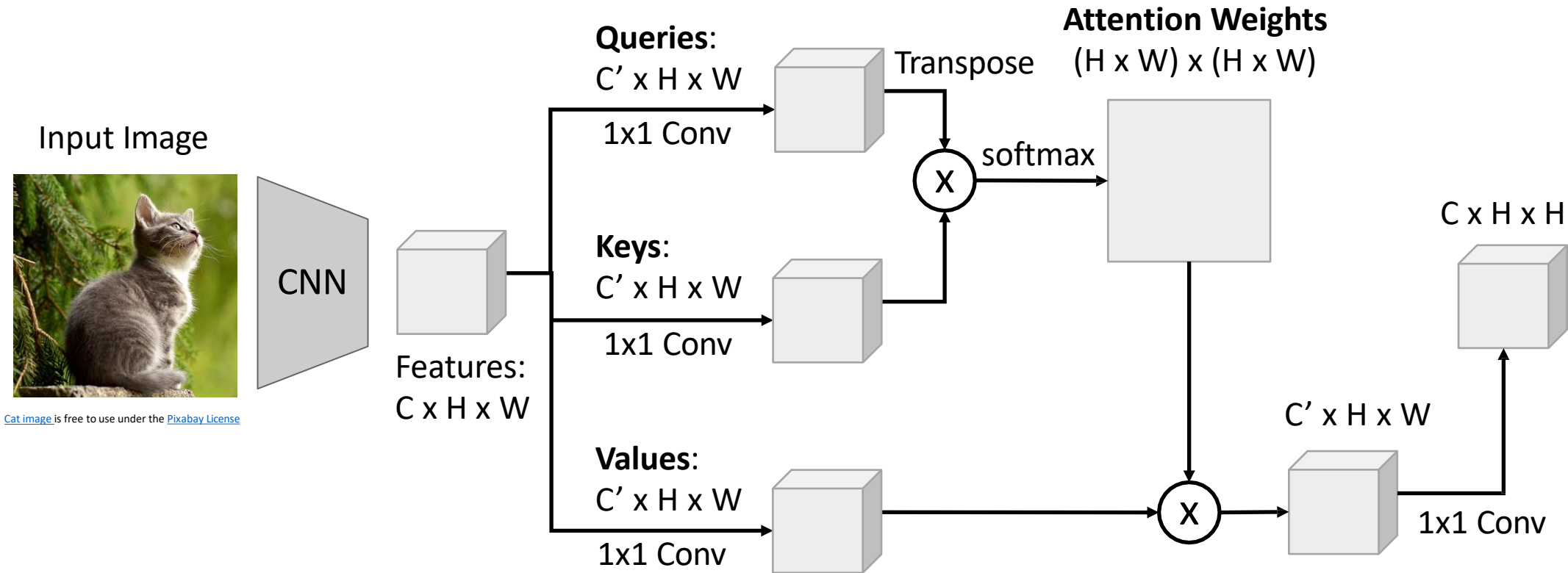
Example: CNN with Self-Attention



Example: CNN with Self-Attention



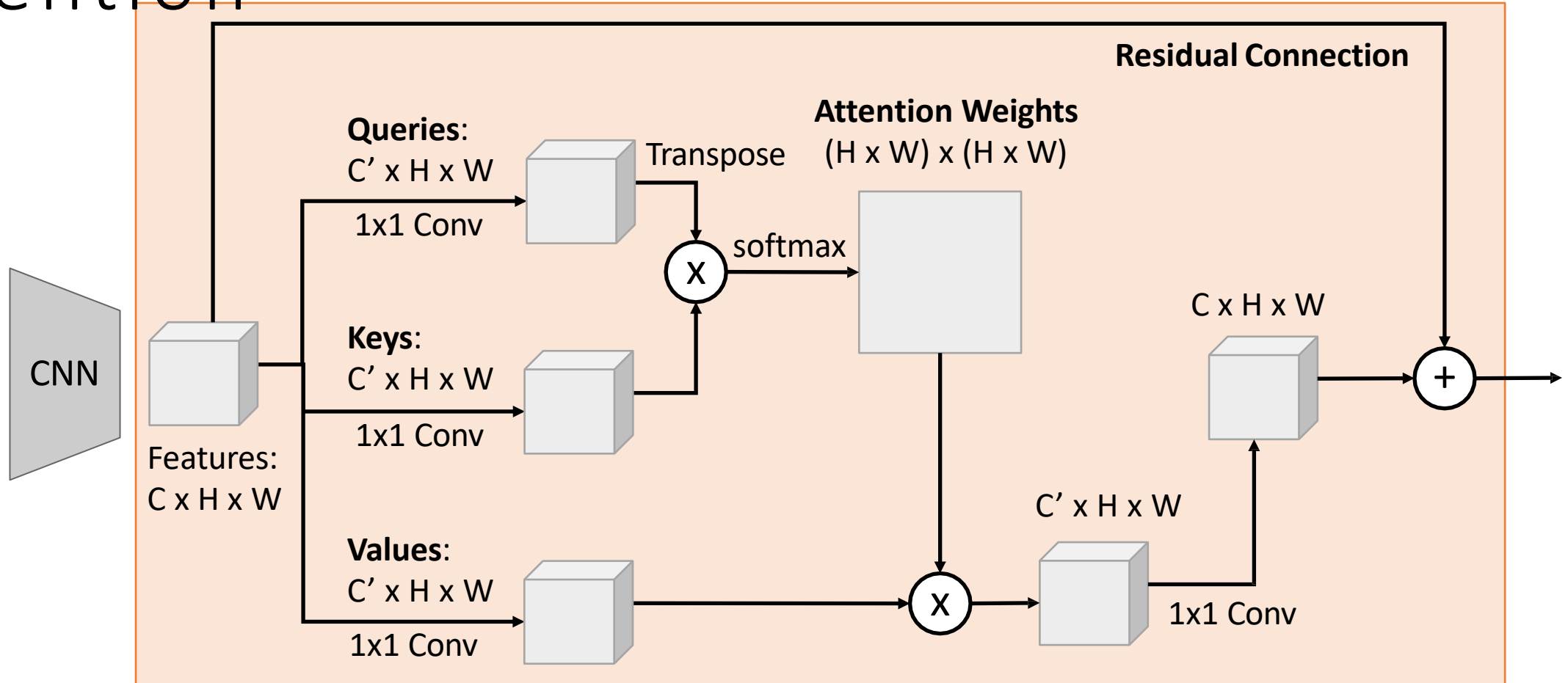
Example: CNN with Self-Attention



Example: CNN with Self-Attention



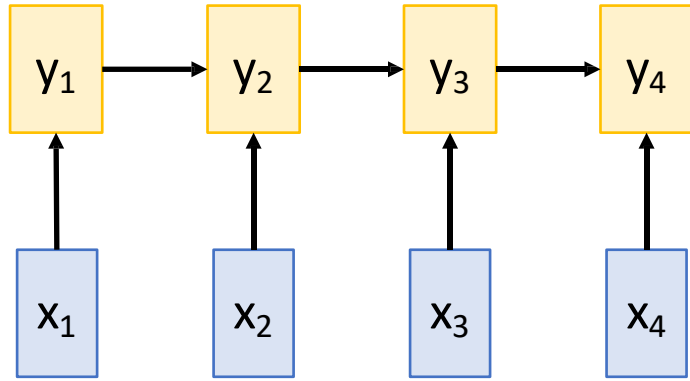
[Cat image](#) is free to use under the [Pixabay License](#)



Self-attention
Module

Three Ways of Processing Sequences

Recurrent Neural Network



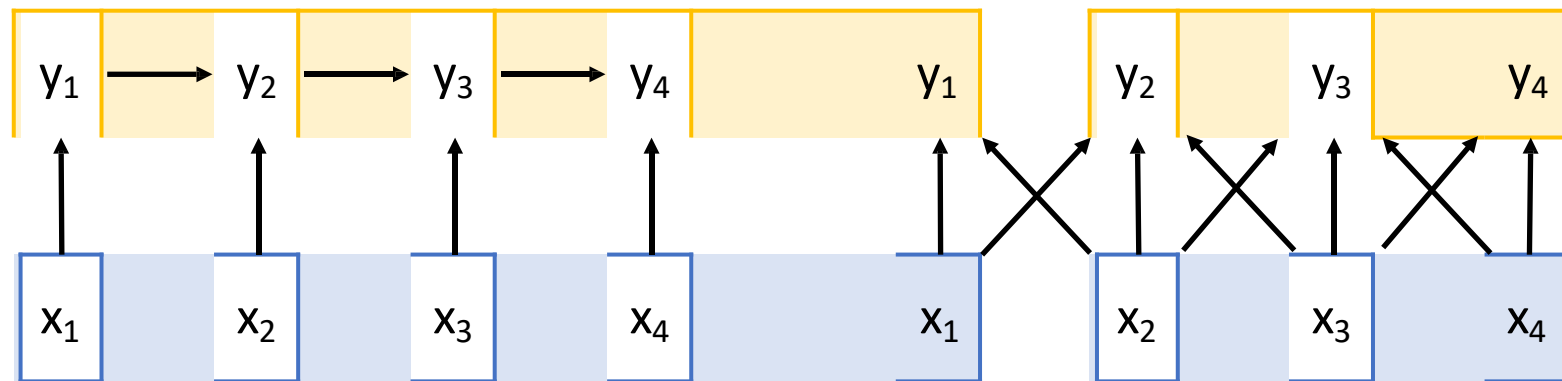
Works on **Ordered Sequences**

(+) **Good at long sequences:** After one RNN layer, h_T "sees" the whole sequence

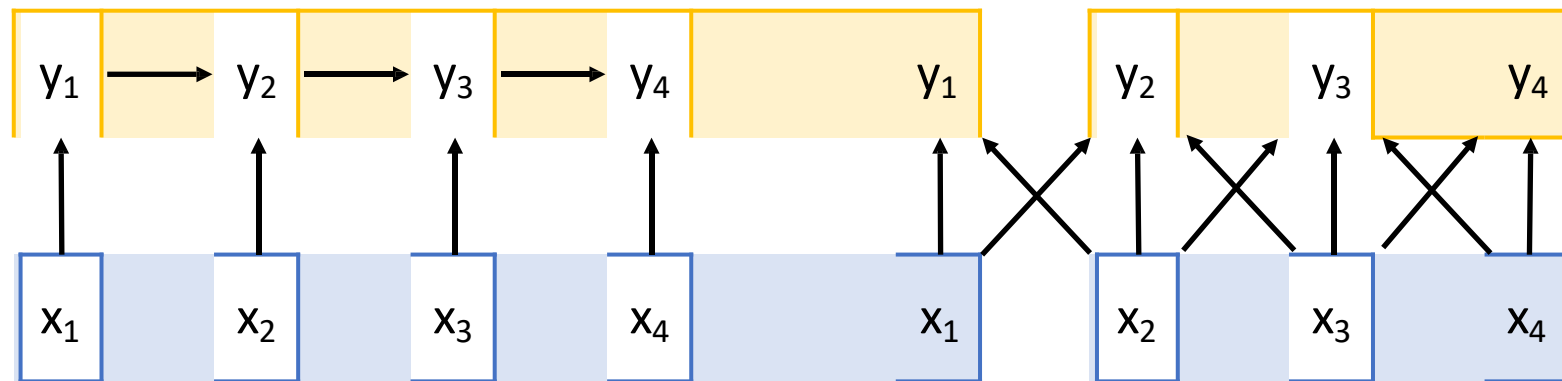
(-) **Not parallelizable:** need to compute hidden states sequentially

Three Ways of Processing Sequences

Recurrent Neural Network



1D Convolution



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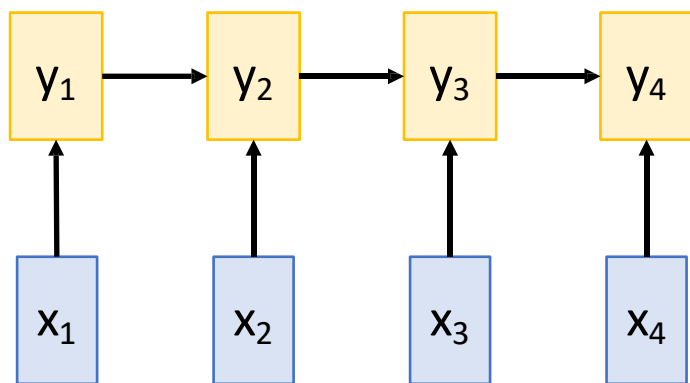
Works on **Multidimensional Grids**

(-) **Bad at long sequences:** Need to stack many conv layers for outputs to "see" the whole sequence

(+) **Highly parallel:** Each output can be computed in parallel

Three Ways of Processing Sequences

Recurrent Neural Network

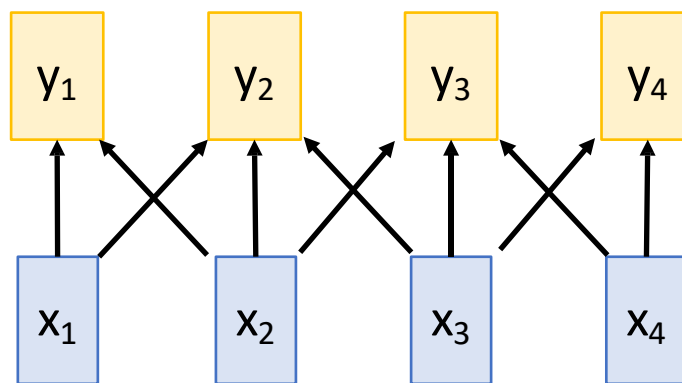


Works on **Ordered Sequences**

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1D Convolution

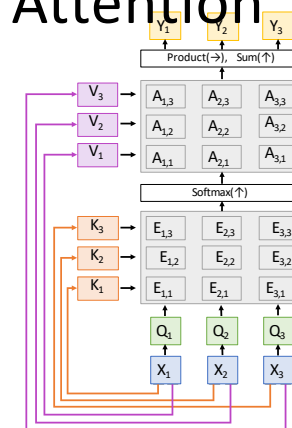


Works on **1D dimensional Grids**

(-) **Bad at long sequences:** Need to stack many conv layers for outputs to "see" the whole sequence

(+) **Highly parallel:** Each output can be computed in parallel

Self-Attention



Works on **Sets of Vectors**

(-) **Good at long sequences:** after one self-Attention layer, each output "sees" all inputs!

(+) **Highly parallel:** Each output can be computed in parallel

(-) **Very memory intensive**

Three Ways of Processing Sequences

Recurrent Neural Network

1D Convolution

Self-Attention

attention is all you need

Vaswani et al, NeurIPS 2017

Works on **Ordered Sequences**

(+) **Good at long sequences:** After one RNN layer, h_T "sees" the whole sequence

(-) **Not parallelizable:** need to compute hidden states sequentially

Works on **Multidimensional Grids**

(-) **Bad at long sequences:** Need to stack many conv layers for outputs to "see" the whole sequence

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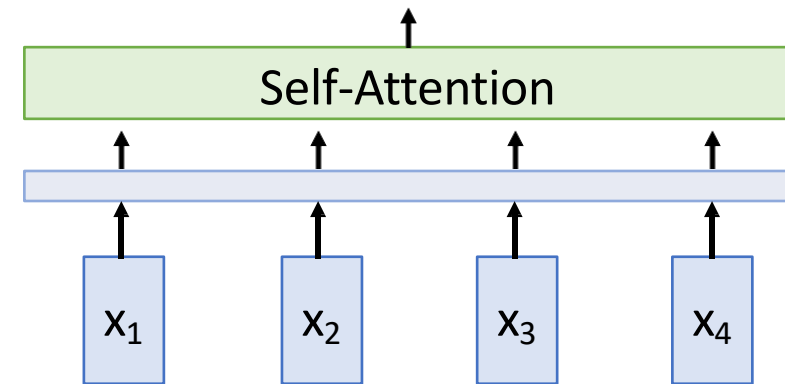
(-) **Very memory intensive**

The Transformer



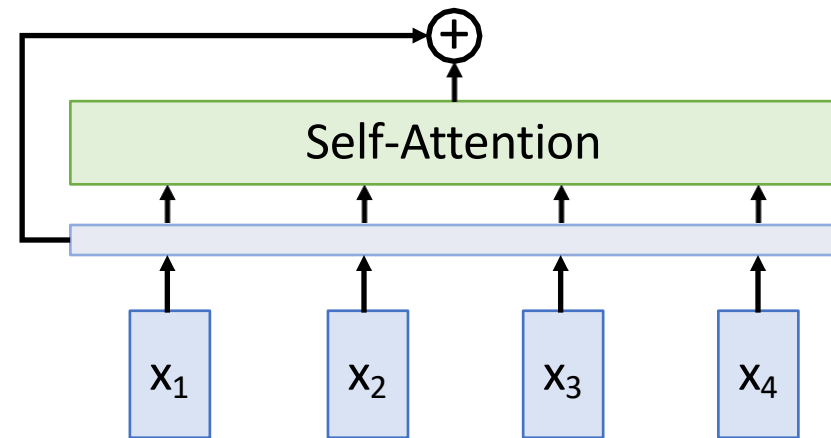
The Transformer

All vectors interact
with each other



The Transformer

Residual connection
All vectors interact
with each other



The Transformer

Recall **Layer Normalization**:

Given h_1, \dots, h_N (Shape: D)

scale: γ (Shape: D)

shift: β (Shape: D)

$\mu_i = (1/D) \sum_j h_{i,j}$ (scalar)

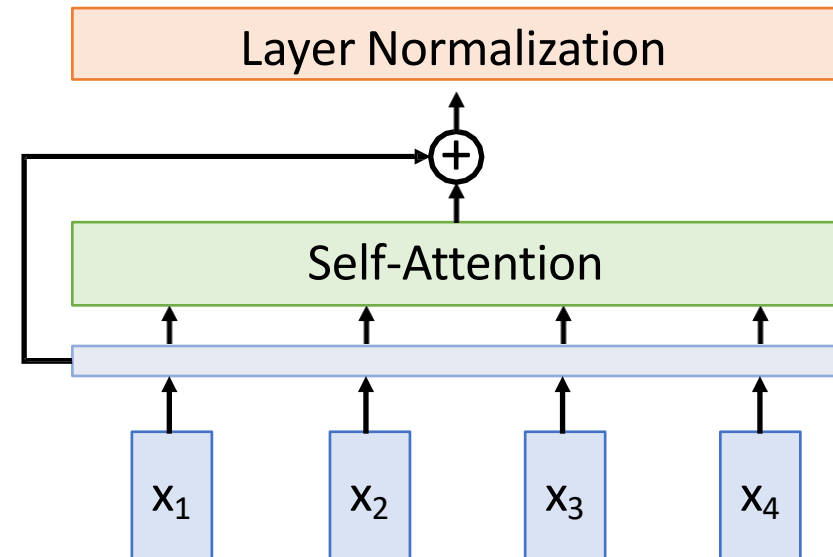
$\sigma_i = (\sum_j (h_{i,j} - \mu_i)^2)^{1/2}$ (scalar)

$z_i = (h_i - \mu_i) / \sigma_i$

$y_i = \gamma * z_i + \beta$

Ba et al, 2016

Residual connection
All vectors interact
with each other



The Transformer

Recall **Layer Normalization**:

Given h_1, \dots, h_N (Shape: D)

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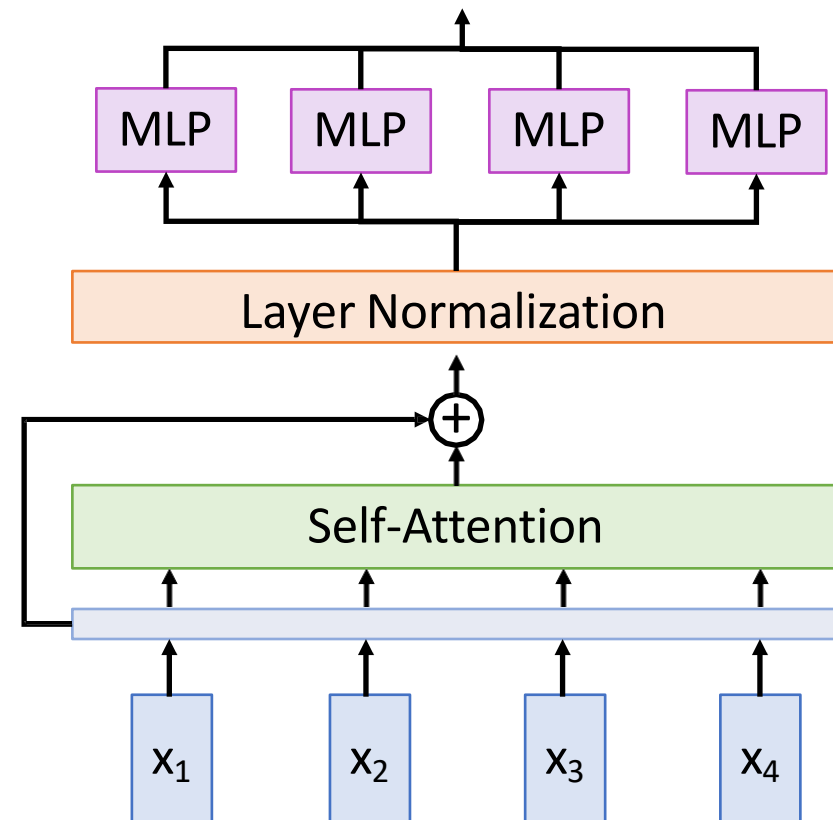
$z_i = (h_i - \mu_i) / \sigma_i$

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Ba et al, 2016

MLP independently
on each vector

Residual connection
All vectors interact
with each other



The Transformer

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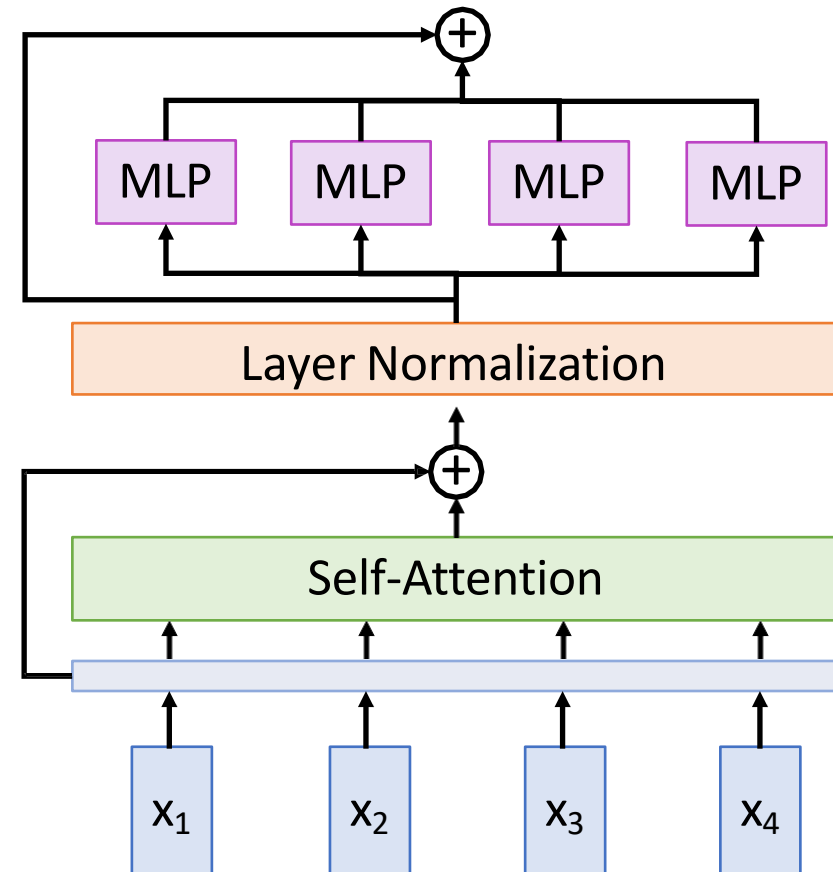
Ba et al, 2016

Residual connection

MLP independently
on each vector

Residual connection

All vectors interact
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The Transformer

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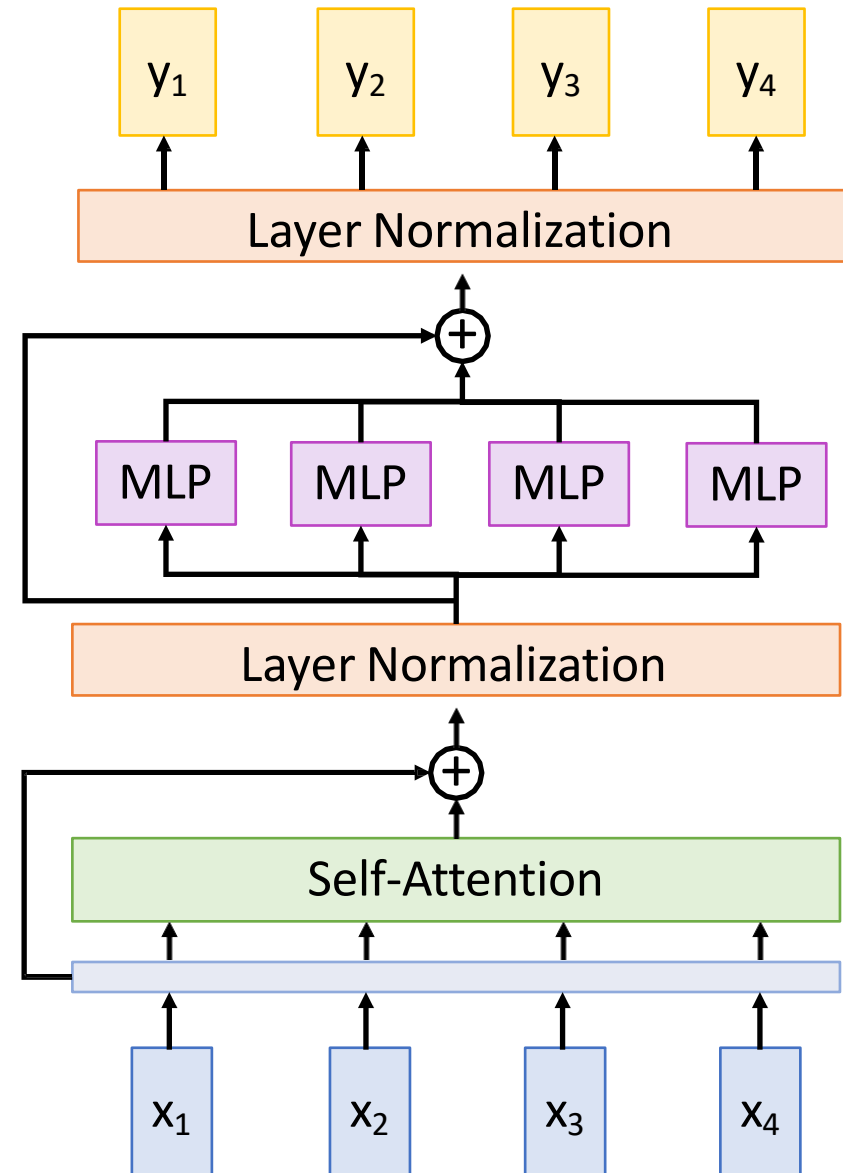
Ba et al, 2016

Residual connection

MLP independently
on each vector

Residual connection

All vectors interact
with each other



The Transformer

Transformer Block:

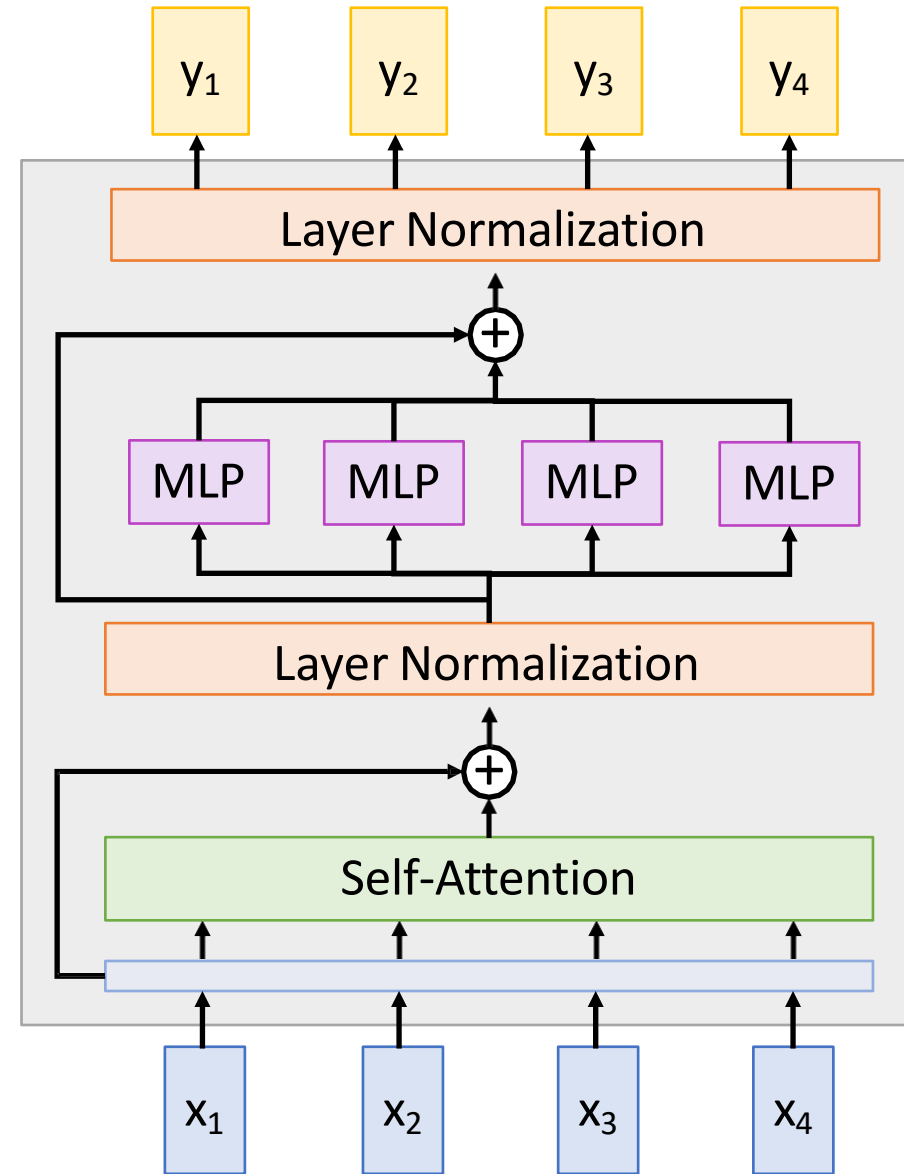
Input: Set of vectors x

Output: Set of vectors y

Self-attention is the only interaction between vectors!

Layer norm and MLP work independently per vector

Highly scalable, highly parallelizable

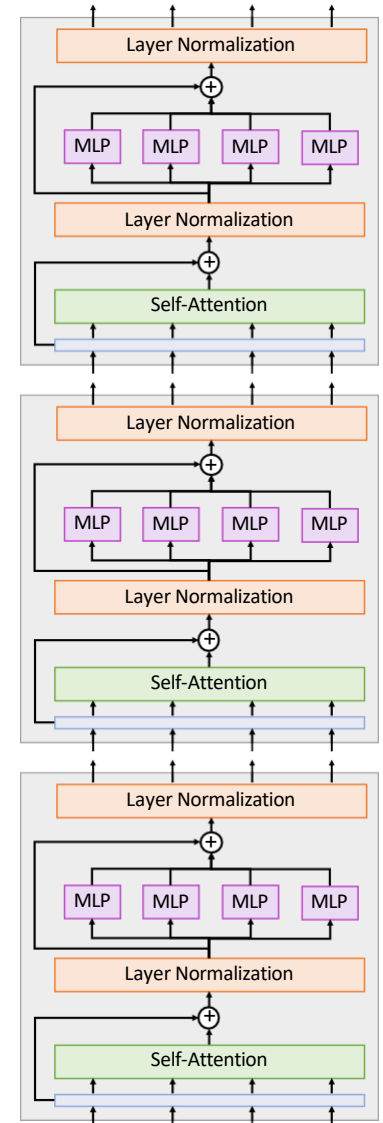


The Transformer

- **Transformer Block:**
- **Input:** Set of vectors x
- **Output:** Set of vectors y
- Self-attention is the only interaction between vectors!
- Layer norm and MLP work independently per vector
- Highly scalable, highly parallelizable

A **Transformer** is a sequence of transformer blocks

Vaswani et al:
12 blocks, $D_Q=512$, 6 heads



The Transformer: Transfer Learning

“ImageNet Moment for Natural Language Processing”

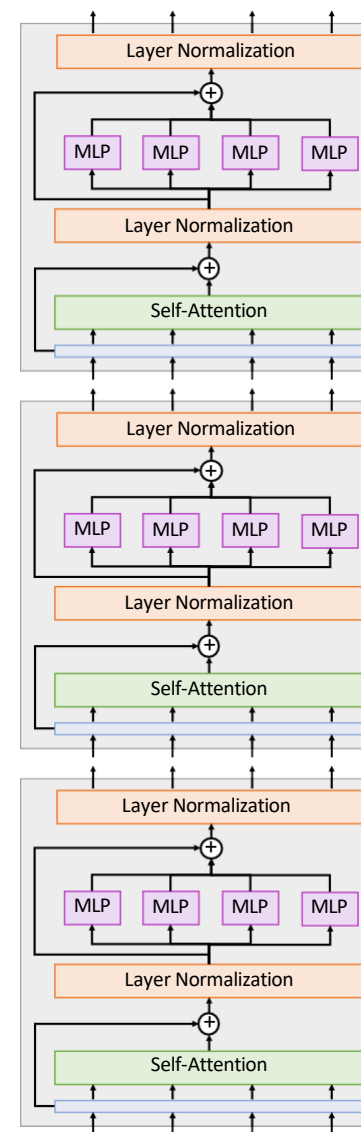
Pretraining:

Download a lot of text from the internet

Train a giant Transformer model for language modeling

Finetuning:

Fine-tune the Transformer on your own NLP task



Scaling up Transformers

Model	Layers	Width	Heads	Params	Data	Training
Transformer-Base	12	512	8	65M		8x P100 (12 hours)
Transformer-Large	12	1024	16	213M		8x P100 (3.5 days)

Scaling up Transformers

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BERT-Base	12	768	12	110M	13 GB	
BERT-Large	24	1024	16	340M	13 GB	

Scaling up Transformers

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Yang et al, XLNet: Generalized Autoregressive Pretraining for Language Understanding", 2019
Liu et al, "RoBERTa: A Robustly Optimized BERT Pretraining Approach", 2019

Scaling up Transformers

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Megatron-LM	54	1920	20	2.5B	174 GB	128x V100 GPU
Megatron-LM	64	2304	24	4.2B	174 GB	256x V100 GPU (10 days)
Megatron-LM	72	3072	32	8.3B	174 GB	512x V100 GPU (9 days)

Shoeybi et al, "Megatron-LM: Training Multi-Billion Parameter Language Models using Model Parallelism", 2019

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~\$430,000 on Amazon AWS!

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COMPLETION (Transformer-written): The scientist named the population, after their distinctive horn, Ovid's Unicorn. These four-horned, silver-white unicorns were previously unknown to science.

Now, after almost two centuries, the mystery of what sparked this odd phenomenon is finally solved.

Dr. Jorge Pérez, an evolutionary biologist from the University of La Paz, and several companions, were exploring the Andes Mountains when they found a small valley, with no other animals or humans. Pérez noticed that the valley had what appeared to be a natural fountain, surrounded by two peaks of rock and silver snow.

Pérez and the others then ventured further into the valley. "By the time we reached the top of one peak, the water looked blue, with some crystals on top," said Pérez.

Pérez and his friends were astonished to see the unicorn herd. These creatures could be seen from the air without having to move too much to see them – they were so close they could touch their horns.

While examining these bizarre creatures the scientists discovered that the creatures also spoke some fairly regular English. Pérez stated, "We can see, for example, that they have a common 'language,' something like a dialect or dialectic."

Dr. Pérez believes that the unicorns may have originated in Argentina, where the animals were believed to be descendants of a lost race of people who lived there before the arrival of humans in those parts of South America.

While their origins are still unclear, some believe that perhaps the creatures were created when a human and a unicorn met each other in a time before human civilization. According to Pérez, "In South America, such incidents seem to be quite common."

However, Pérez also pointed out that it is likely that the only way of knowing for sure if unicorns are indeed the descendants of a lost alien race is through DNA. "But they seem to be able to communicate in English quite well, which I believe is a sign of evolution, or at least a change in social organization," said the scientist.

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- Try it yourself: <https://talktotransformer.com>

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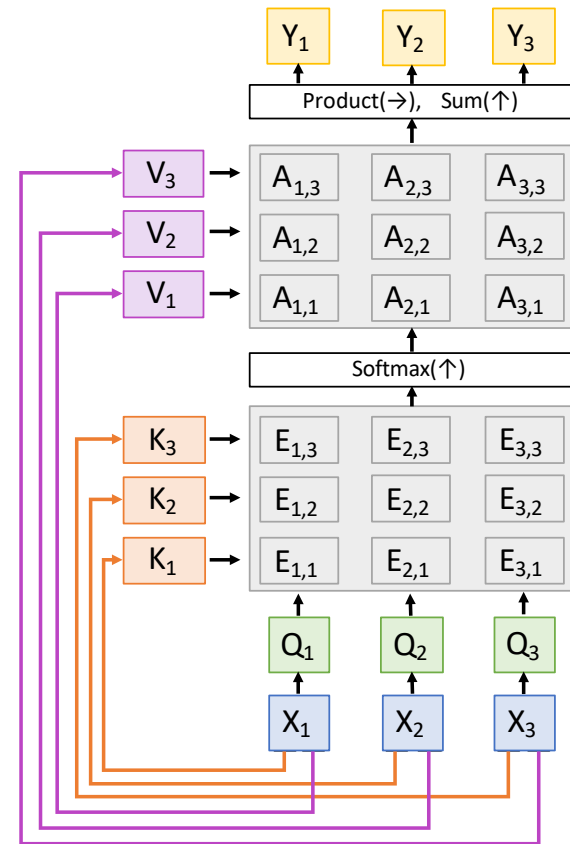
Summary

Adding **Attention** to RNN models lets them look at different parts of the input at each timestep



A dog is standing on a hardwood floor.

Generalized **Self-Attention** is new, powerful neural network primitive



Transformers are a new neural network model that only uses attention

