**Problem 1.**

**9.** (4 points) Solve the system by finding the reduced row-echelon form of the augmented matrix.

$$\begin{cases} x+5y-4z &= 3 \\ 3x+16y-15z &= 5 \\ 3x+12y-3z &= 21 \end{cases}$$

reduced row-echelon form:

$$\left[\begin{array}{ccc|c} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{array}\right]$$

How many solutions are there to this system?

- A. None
- B. Exactly 1
- C. Exactly 2
- D. Exactly 3
- E. Infinitely many
- F. None of the above

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If there is one solution, give its coordinates in the answer spaces below.

If there are infinitely many solutions, enter $z$ in the answer blank for $z$, enter a formula for $y$ in terms of $z$ in the answer blank for $y$ and enter a formula for $x$ in terms of $z$ in the answer blank for $x$.

If there are no solutions, leave the answer blanks for $x$, $y$ and $z$ empty.

$x =$ ________________

$y =$ ________________

$z =$ ________________

*Answer(s) submitted:*

- `1`
- `0`
- `11`
- `23`
- `0`
- `1`
- `-3`
- `-4`
- `0`
- `0`
- `0`
- `0`
- `E`
- `-11z+23`
- `3z-4`
- `z`

(correct)

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**Problem 2.**

**2.** (4 points)

Determine if the subset of $\mathbb{R}^2$ consisting of vectors of the form $\begin{bmatrix} a \\ b \end{bmatrix}$, where $a+b=1$ is a subspace.

Select true or false for each statement.

$\boxed{?}$ 1. This set is a subspace

$\boxed{?}$ 2. This set is closed under vector addition

$\boxed{?}$ 3. The set contains the zero vector

$\boxed{?}$ 4. This set is closed under scalar multiplications

*Answer(s) submitted:*

- `False`
- `True`
- `False`
- `True`

(score 0.5)

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**Problem 3.**

**1.** (3 points) Let

$$A = \begin{bmatrix} 4 & -2 & -2 \\ -4 & 2 & 2 \\ -8 & 4 & 4 \end{bmatrix}.$$

A basis for the null space of $A$ is

$$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}.$$

*Answer(s) submitted:*

- `(1/2)`

(correct)

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**Problem 4.**

**3.** (3 points) Given that

$$\begin{bmatrix} -1 & 5 & -2 & 4 \\ -2 & -2 & -2 & 4 \\ 1 & 2 & -4 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -\frac{63}{29} \\ 0 & 1 & 0 & \frac{9}{29} \\ 0 & 0 & 1 & -\frac{4}{29} \end{bmatrix},$$

write $\begin{bmatrix} 4 \\ 4 \\ -1 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -4 \end{bmatrix}$.

$\begin{bmatrix} 4 \\ 4 \\ -1 \end{bmatrix} =$ ____ $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} +$ ____ $\begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix} +$ ____ $\begin{bmatrix} -2 \\ -2 \\ -4 \end{bmatrix}$.

*Answer(s) submitted:*

- `-((63)/(29))`

(correct)

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**Problem 5.**

**13.** (3 points)

Find $k$ such that the following matrix $M$ is singular.

$$M = \begin{bmatrix} -3 & -4 & 4 \\ -6 & -11 & 10 \\ 1+k & -10 & 8 \end{bmatrix}$$

$k =$ ____

*Answer(s) submitted:*

- `-4`

(correct)

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**Problem 6.**

**10.** (3 points)

Given $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 2$, find the following determinants.

$\det \begin{bmatrix} g & h & i \\ a & b & c \\ d & e & f \end{bmatrix} =$ ___

$\det \begin{bmatrix} a & b & c \\ -6d+a & -6e+b & -6f+c \\ g & h & i \end{bmatrix} =$ ___

$\det \begin{bmatrix} -6d+a & -6e+b & -6f+c \\ d & e & f \\ g & h & i \end{bmatrix} =$ ___

*Answer(s) submitted:*

- `2`
- `-12`
- `2`

(correct)

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**Problem 7.**

**12.** (4 points)

$A$ and $B$ are $n \times n$ matrices.

Check the true statements below:

- A. An elementary row operation on $A$ does not change the determinant.
- B. $(detA)(detB) = detAB$.
- C. The determinant of $A$ is the product of the diagonal entries in $A$.
- D. If $\lambda + 5$ is a factor of the characteristic polynomial of $A$, then 5 is an eigenvalue of $A$.

*Answer(s) submitted:*

- `B`

(correct)

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**Problem 8.**

**11.** (4 points)

Use determinants to determine whether each of the following sets of vectors is linearly dependent or independent.

$\boxed{?}$1. $\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \end{bmatrix}$,

$\boxed{?}$2. $\begin{bmatrix} -4 \\ 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 12 \\ -13 \\ 7 \end{bmatrix}, \begin{bmatrix} -24 \\ 28 \\ -10 \end{bmatrix}$,

$\boxed{?}$3. $\begin{bmatrix} -5 \\ -15 \end{bmatrix}, \begin{bmatrix} -2 \\ -6 \end{bmatrix}$,

$\boxed{?}$4. $\begin{bmatrix} -1 \\ -4 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ -16 \\ -8 \end{bmatrix}, \begin{bmatrix} 5 \\ 20 \\ 10 \end{bmatrix}$,

*Answer(s) submitted:*

- `Linearly Independent`
- `Linearly Independent`
- `Linearly Dependent`
- `Linearly Dependent`

(correct)

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**Problem 9.**

**6.** (3 points) Let $A = \begin{bmatrix} -27 & -10 \\ 60 & 23 \end{bmatrix}$

Find a matrix $S$, a diagonal matrix $D$ and $S^{-1}$ such that $A = SDS^{-1}$.

$S = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, D = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, S^{-1} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

*Answer(s) submitted:*

-
-
-

(incorrect)

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**Problem 10.**

**5.** (3 points) Given that $\vec{v}_1 = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are eigenvectors of the matrix

$$A = \begin{bmatrix} -9 & 6 \\ -4 & 1 \end{bmatrix}$$

determine the corresponding eigenvalues.

$\lambda_1 =$ ___.
$\lambda_2 =$ ___.

*Answer(s) submitted:*

- `-5`
- `-3`

(correct)

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**Problem 11.**
**7.** (2 points)
Suppose that $A$ is a $6 \times 5$ matrix which has a null space of dimension 2.

The rank of $A$ is $\text{rank}(A) =$ ___

*Answer(s) submitted:*

- `3`

(correct)

---

**Problem 12.**

**14.** (4 points) (a) Find the coordinate vector of $x = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$ with respect to the ordered basis

$$E = \left\{ \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

of $\mathbb{R}^3$:

$$[x]_E = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

(b) Let $F_1$ be the ordered basis of $\mathbb{R}^2$ given by

$$F_1 = \left\{ \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \end{bmatrix} \right\}$$

and let $F_2$ be the ordered basis given by

$$F_2 = \left\{ \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$$

Find the transition matrix $P_{F_2 \leftarrow F_1}$ such that $[x]_{F_2} = P_{F_2 \leftarrow F_1} [x]_{F_1}$ for all $x$ in $\mathbb{R}^2$:

$$P = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$$

*Answer(s) submitted:*

- `5`
- `-22`
- `-108`
- `-12`
- `-1`
- `-19`
- `-4`

(correct)

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**Problem 13.**
**8.** (3 points) Find a basis for the column space of

$$A = \begin{bmatrix} -2 & 3 & 0 & 4 \\ 2 & -3 & 2 & -1 \\ -4 & 6 & 0 & 8 \end{bmatrix}.$$

Basis = $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

*Answer(s) submitted:*

- `-2`

(correct)

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**Problem 14.**
**4.** (4 points)
Let $A = \begin{bmatrix} 2 & -2 & 1 \\ 0 & 3 & 1 \\ -3 & 0 & 3 \end{bmatrix}$.

(a) Find the determinant of $A$.
$\det(A) =$ ____,

(b) Find the matrix of cofactors of $A$.

$C = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

(c) Find the adjoint of $A$.

$\text{adj}(A) = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

(d) Find the inverse of $A$.

$A^{-1} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

*Answer(s) submitted:*

- `33`
- `9`
- `9`
- `(9/(33))`

(correct)