Final Exam Part B, Math 227-Version T

Please answer the following problems in details and attach your solution as a single pdf file to the canvas link : Final Exam Part B Work Upload

Please write your answers in order according to this test

Problem 1:

Consider the vectors $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$ in R^3.

Consider the vector space of all polynomials of degree less than or equal to 2, P_2. Let $\beta = \{1, t, t^2\}$ be the standard basis for P_2. Find three vectors (i.e. polynomials) p_1, p_2 and p_3 in P_2, such that $[p_1]_\beta = v_1$, $[p_2]_\beta = v_2$ and $[p_3]_\beta = v_3$.

Problem 2:

the following matrix $A = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ and the linear operator $T_A : R^3 \to R^3$ that is defined by the matrix

Find the rank(A) and the nullity of T_A and determine whether that transformation is one-to-one or not with a clear explanation

Problem 3:

Diagonalize the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 2 \end{bmatrix}$ given that the eigenvalues of A are $\lambda = -1, 5$.

Problem 4:

Find the matrix that defines a rotation of the $xy-plane$ about the origin through an angle of $\frac{\pi}{3}$ radians (in the counterclockwise direction), and find the image of the point $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ under that transformation.

Problem5:

Consider the bases $B = \{(1,3),(3,0)\}$ and $B' = \{(2,1),(0,2)\}$ of $\mathbf{R}^2$. If u is a vector in $\mathbf{R}^2$ such that $u_B = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ (the coordinate vector of u relative to the basis B), find $u_{B'}$ (the coordinate vector of u relative to the basis B').