Appendix

Hyperchaotic System Inflow Simulations

The hyperchaotic system governing inflow dynamics for the simulation was

$$(x, y, z, w) \coloneqq \begin{cases} \frac{dx}{dt} = a(y - x) - ew, \\ \frac{dy}{dt} = xz - hy, \\ \frac{dz}{dt} = b - xy - cz, \\ \frac{dw}{dt} = ky - dw \end{cases} \tag{16}$$

where "a, b, c, d, e, h are positive parameters of system" [15] and with the specific attractor a = 5, b = 20, c = 1, d = 0.1, e = 20.6, h = 1, and k = 0.1. As stated by the paper, the largest Lyapunov exponent of the attractor above is 0.24. As a result, the Lyapunov time of the system is $1/0.24 \approx 4.167$. Because each system time-step is 0.01, the Lyapunov time expressed in time-steps is $4.167/0.01 \approx 417$, which amounts to $417/60 \approx 6.95$ minutes, far greater than any reasonable maximum traffic light activation time. As a result, the cumulative urgency formula is not expected to significantly diverge from the true chaotic distribution as the Lyapunov time is far greater than any expected result.

The system was solved by using the Runge-Kutta family's Euler method with a step size of 0.01 over $[0, 2000]$ by treating the system as the derivative of a vector-valued function.

Euler's method allows for discrete approximations of differential equations. It states that for function y and its derivative y',

$$\mathbf{h}_{n+1} = \mathbf{h}_n + \mathbf{h}'(t_n)\Delta_n \tag{17}$$

This may be extended to vector-valued functions, such as the previously-described hyperchaotic system [15], where

$$\mathbf{h} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}, \qquad \mathbf{h}' = \begin{bmatrix} a(y - x) - ew \\ xz - hy \\ b - xy - cz \\ ky - dw \end{bmatrix} \tag{18}$$

and $\Delta_n = 0.01$.

The system was normalized by dividing the system by the L1 norm of its integral over $[0, 2000]$, then scaling it up by a factor of 800. This effectively modeled chaotic vehicle inflow so that the total inflow over the 2,000 timesteps was 800 vehicles. The integral of the solution was calculated by summing all of the data points calculated through Euler.

Because the step size was 0.01 and the stiffness ratio (ignoring the eigenvalue of 0) was $-7.56/0.23 \approx -32.8695$, which is not significantly less than -1, it is safe to say that the system is non-stiff for the estimation technique used.

LSTM Prediction Model

The LSTM prediction model trained on 10,000 data points. The data was generated by choosing a random real vector $v \in \mathrm{R}^4$ so that v is a vector randomly sampled from the 4-dimensional hypercube whose boundaries are defined by the fourfold Cartesian product $\{\{0, 1\} \times \{0, 1\} \times \{0, 1\} \times \{0, 1\}\}$. The LSTM's training data was split 80-20 as traditionally split when training and testing data for machine learning models. The first 8,000 data points were used only training while the last 2,000 points were used for simulation. Every data point in the last 2,000 points was trained pointwise on the LSTM as a single-sample batch.

The LSTM itself consisted of 16 units (determined empirically through tests over different ranges of units to be the optimal number of units to facilitate low errors while preventing overfitting for the specific attractor used) and was trained over 100 epochs with a batch size of 10 samples using the Adam optimizer with a mean squared error loss function.

Traffic Intersection Simulation

Few details were given on the ns2 map used in the ITLC paper [1]; as a result, some assumptions were made about the simulation. The turning speed was set at a constant 10 meters per second, chosen from the thorough analysis done in *MODELING SPEED PROFILES OF TURNING VEHICLES AT SIGNALIZED INTERSECTIONS* [14]. Furthermore, the outflow rate (q) was calculated using a numerical Greenshield model [3] to be

$$q = kv_f \left(1 - \frac{k}{k_j}\right) \tag{19}$$

where k is the density, v_f is the free-flow speed, and k_j is the jam density. The capacity of each street was set to 1,000 vehicles. Each inroad was initialized to a load of 200 vehicles with the total vehicle inflow over the 2,000-second testing interval being 800, with a resulting 4,000 vehicles total and an average of 1,000 vehicles per inroad, only accomplished if all 4,000 vehicles flow out over the 2,000-second interval. The capacity for each inroad was set to 1,000 with a jam density of 1.