Tutorial No. - 04.

Ques-1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Here, $a = 3$, $b = 2$

$f(n) = n^2$

So, $n^{\log_b a} = n^{\log_2 3}$

Since, $n^{\log_2 3} < n^2$

So, according to Master's Theorem -

$T(n) = O(n^2)$.

Ques-2. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

Here, $a = 4$, $b = 2$ and $f(n) = n^2$

So, $n^{\log_b a} = n^{\log_2 4} = n^{\log_2 (2^2)}$

$= n^{2\log_2 2}$

$= n^2$

Since, $n^{\log_b a} = f(n)$

According to Master's Theorem,

$T(n) = O(n^2 \log n)$

Ques-3. $T(n) = T\left(\frac{n}{2}\right) + 2^n$

Here, $a = 1$, $b = 2$ and $f(n) = 2^n$

So, $n^{\log_b a} = n^{\log_2 1} = n^{\log_2 2^0}$

$\Rightarrow n^0 = 1$

Since, $1 < f(n)$

According to Master's Theorem,

$T(n) = O(2^n)$

Ques-4. $T(n) = 2^n T\left(\frac{n}{2}\right) + n^2$

Master's Theorem is not applicable since 'a' is a function

Ques-5. $T(n) = 16T\left(\frac{n}{4}\right) + n$

Here, $a = 16$, $b = 4$ and $f(n) = n$

So, $n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^{2\log_4 4}$

$= n^2$

Since, $n^2 > n$

Therefore, according to Master's theorem,

$T(n) = O(n^2)$

Ques-6. $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

Here, $a = 2$, $b = 2$ and $f(n) = n\log n$.

So, $n^{\log_b a} = n^{\log_2 2} = n$.

Since, $n^{\log_b a} < f(n)$

According to Master's Theorem,

$T(n) = O(n\log n)$.

Ques-7. $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$.

Here, $a = 2$, $b = 2$ and $f(n) = n/\log n$.

So, $n^{\log_b a} = n^{\log_2 2} = n$.

Since, $n^{\log_b a} > f(n)$
According to Master's Theorem,
$T(n) = O(n)$.

Ques-8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Here, $a = 2$, $b = 4$ and $f(n) = n^{0.51}$
So, $n^{\log_b a} = n^{\log_4 2} = n^{0.5}$
Since, $n^{\log_b a} < f(n)$
According to Master's Theorem,
$T(n) = O(n^{0.51})$

Ques-9. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$
Master's Theorem is not applicable since $a < 1$.

Ques-10. $T(n) = 16T\left(\frac{n}{4}\right) + n!$

Here, $a = 16$, $b = 4$ and $f(n) = n!$

So, $n^{\log_b a} = n^{\log_4 16} = n^{\log_4 (4^2)} = n^2$

Since, $n^{\log_b a} < n!$
$\therefore$ According to Master's Theorem,

$T(n) = O(n!)$

Ques - 11. $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

Here, $a = 4$, $b = 2$ and $f(n) = \log n$.

So, $n^{\log_b a} = n^{\log_2 4} = n^2$

Since, $n^{\log_b a} > f(n)$

According to Master's Theorem,

$T(n) = O(n^2)$

Ques - 12. $T(n) = \text{sqrt}(n)\, T\left(\frac{n}{2}\right) + \log n$

Since, $a \neq$ constant.

$\therefore$ Master's Theorem is not applicable.

Ques - 13. $T(n) = 3T\left(\frac{n}{2}\right) + n$

Here, $a = 3$, $b = 2$ and $f(n) = n$

So, $n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

Since, $n^{\log_b a} > f(n)$

$\therefore$ According to Master's Theorem,

$T(n) = O(n^{1.58})$

Ques - 14. $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

Here, $a = 3$, $b = 3$ and $f(n) = \sqrt{n}$

So, $n^{\log_b a} = n^{\log_3 3} = n$

Since, $n^{\log_b a} > f(n)$

$\therefore$ According to Master's Theorem,
$$T(n) = O(n).$$

Ques-15. $T(n) = 4T\left(\frac{n}{2}\right) + cn$

Here, $a = 4$, $b = 2$ and $f(n) = n$.

So, $n^{\log_b a} = n^{\log_2 4}$
$= n^2$

Since, $n^{\log_b a} > f(n)$

$\therefore$ According to Master's Theorem,
$$T(n) = O(n^2)$$

Ques-16. $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

Here, $a = 3$, $b = 4$ and $f(n) = n\log n$.

So, $n^{\log_b a} = n^{\log_4 3} = n^{0.187}$

Since, $n^{\log_b a} < f(n)$

$\therefore$ According to Master's Theorem,
$$T(n) = O(n\log n)$$

Ques-17. $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

Here, $a = 3$, $b = 3$ and $f(n) = \frac{n}{2}$

So, $n^{\log_b a} = n^{\log_3 3} = n$

Since, $n^{\log_b a} > f(n)$

$\therefore$ According to Master's Theorem,
$$T(n) = O(n).$$

Ques-18. $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$.

Here, $a = 6$, $b = 3$ and $f(n) = n^2 \log n$.

So, $n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

Since, $n^{\log_b a} < n^2 \log n$.

$\therefore$ According to Master's Theorem,

$T(n) = O(n^2 \log n)$.

Ques-19. $T(n) = 4T\left(\frac{n}{2}\right) + n/\log n$.

Here, $a = 4$, $b = 2$ and $f(n) = n/\log n$.

So, $n^{\log_b a} = n^{\log_2 4} = n^2$

Since, $n^{\log_b a} > f(n)$

$\therefore$ According to Master's Theorem,

$T(n) = O(n^2)$.

Ques-20. $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

Here, $f(n)$ is not an increasing function. Therefore, Master's Theorem is not applicable.

Ques-21. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

Here, $a = 7$, $b = 3$ and $f(n) = n^2$

So, $n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

Since, $n^{\log_b a} < f(n)$

$\therefore$ According to Master's Theorem,

$T(n) = O(n^2)$

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Ques-22. $T(n) = T\left(\frac{n}{2}\right) + n(2-\cos n)$

Here, Master's Theorem is not applicable due to violation of regularity condition.