# Merge Sort - 1

## Lecture-31

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# Already discussed sorting algorithms

## Revisiting their time complexity!

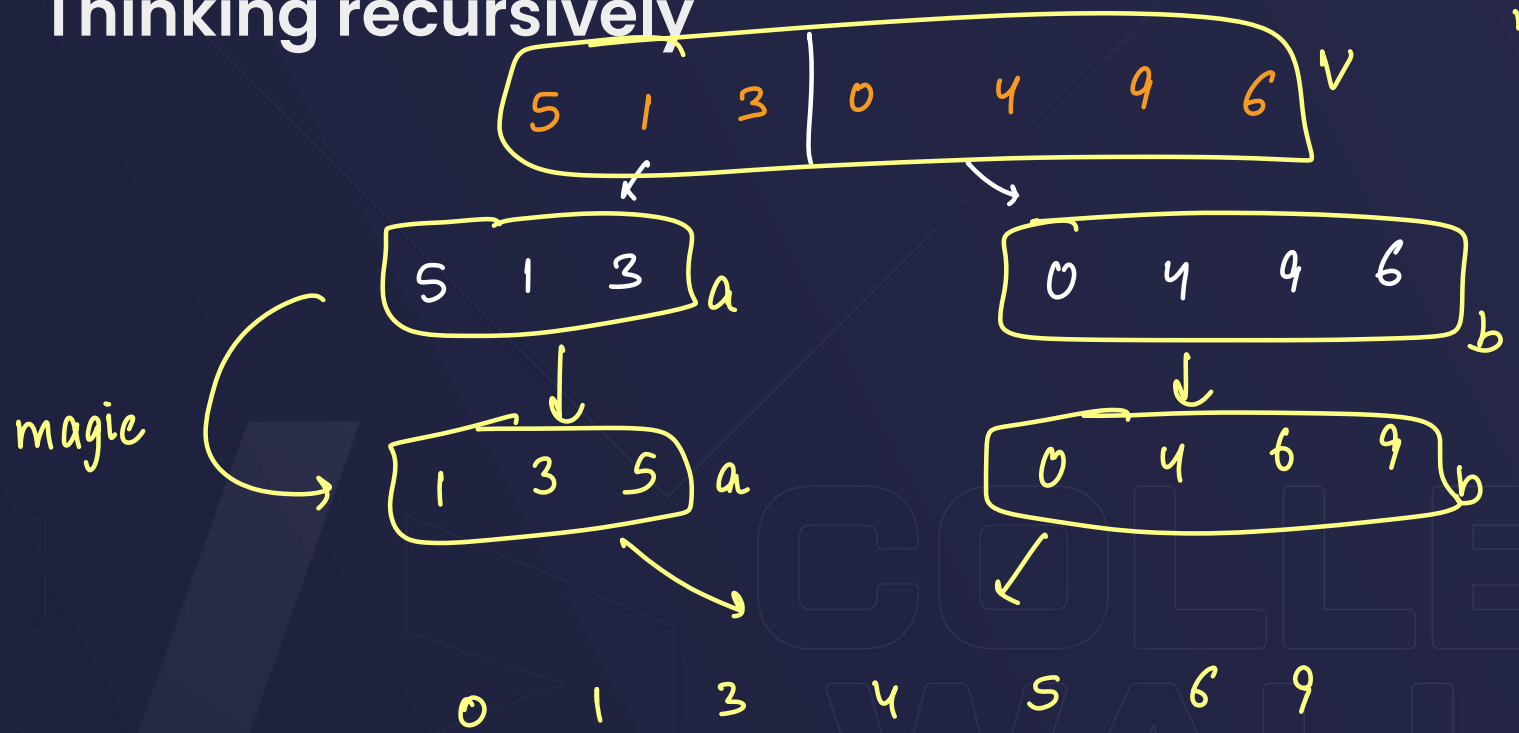
| | |
|---|---|
| Bubble Sort | $O(n^2)$ |
| Selection Sort | $O(n^2)$ |
| Insertion Sort | $O(n^2)$ |
| Merge Sort | $O(n \cdot \log n)$ |
| Quick Sort | $O(n \cdot \log n)$ |

# What if we had two sorted arrays?

## Building the intuition

# Introducing divide and conquer

## Thinking recursively

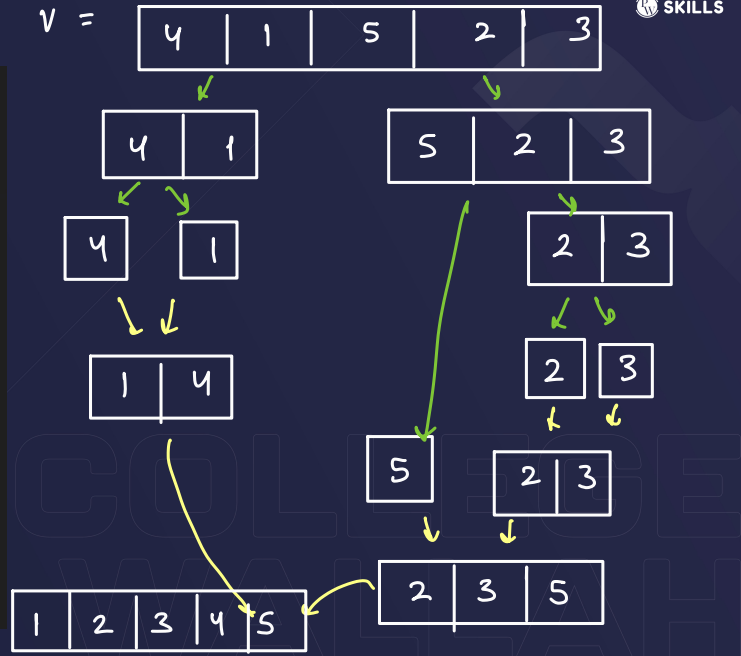


# Merge sort algorithm

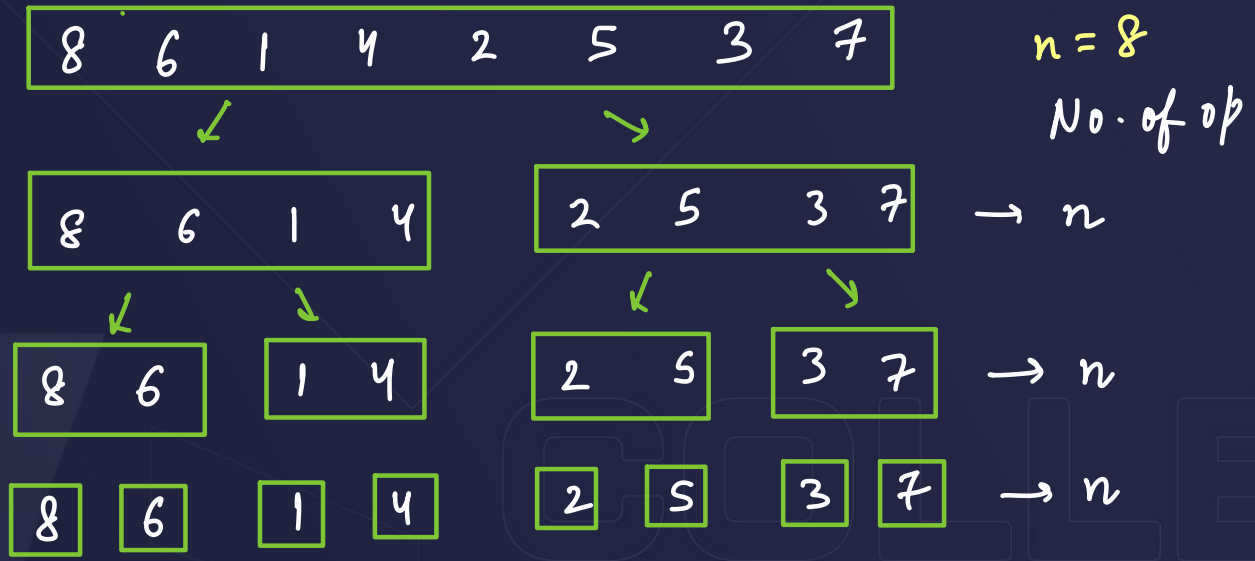
Dividing the array into two equal parts and then merging them.

# Code for Merge Sort

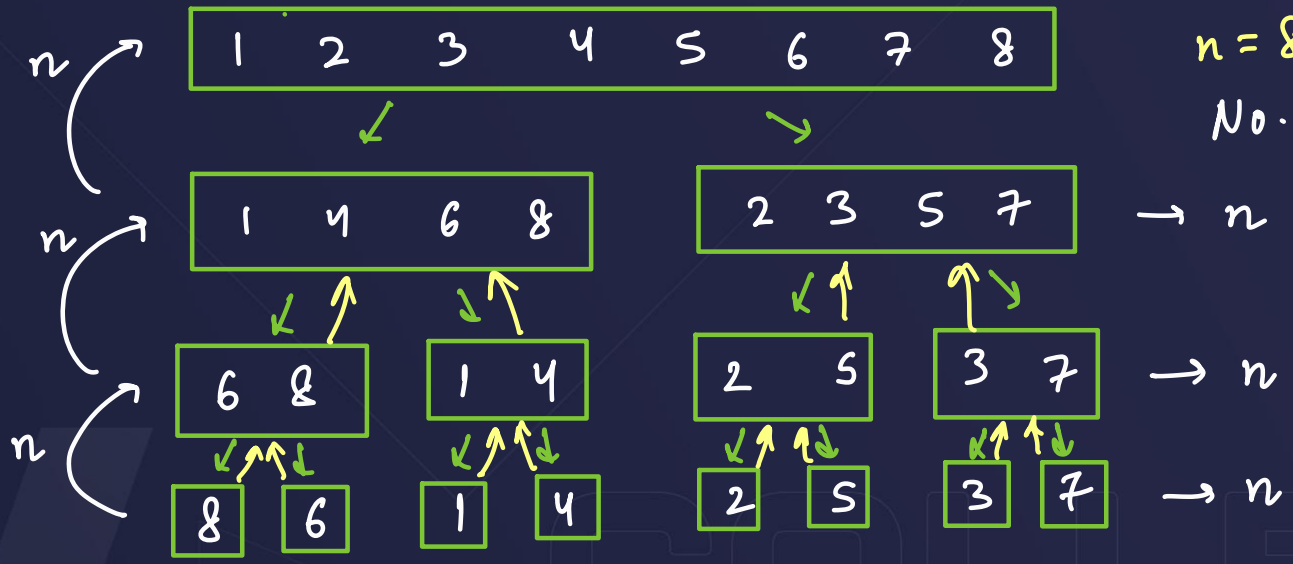
```
void mergeSort(vector<int>& v){
    int n = v.size();
    if(n==1) return;
    int n1 = n/2, n2 = n - n/2;
    vector<int> a(n1), b(n2);
    // copy pasting
    for(int i=0;i<n1;i++)
        a[i] = v[i];
    for(int i=0;i<n2;i++)
        b[i] = v[i+n1];
    // magic aka recursion
    mergeSort(a);
    mergeSort(b);
    // merge
    merge(a,b,v);
}
```



# Does it improve the time complexity?



# Does it improve the time complexity?




Total no. of operations = $Kn + Kn = K \cdot (2n)$

T.C. = $O(Kn) \rightarrow O(n \cdot \log n)$

# Does it improve the time complexity?

$n, \frac{n}{2}, \frac{n}{4}, \frac{n}{8} \ldots\ldots$

$\rightarrow \quad n, \frac{n}{2^1}, \frac{n}{2^2}, \frac{n}{2^3} \ldots \frac{n}{2^x}$

$x$ terms

$\Rightarrow \quad \frac{n}{2^x} = 1 \Rightarrow n = 2^x$

$\Rightarrow 2^x = n$

$\Rightarrow \boxed{x = \log_2 n}$

$n = 128$

$\downarrow$

$B.S. = \frac{n(n-1)}{2} = 64 \cdot 127$

$= 8128$ ops

$M.S \rightarrow 2n \log_2 n$

$\Rightarrow 2 \times 128 \times \log_2 128$

$= 256 \times 7$

$= 1792$ operations

# Does it improve the time complexity?

$n = 1024$ → Yess!!

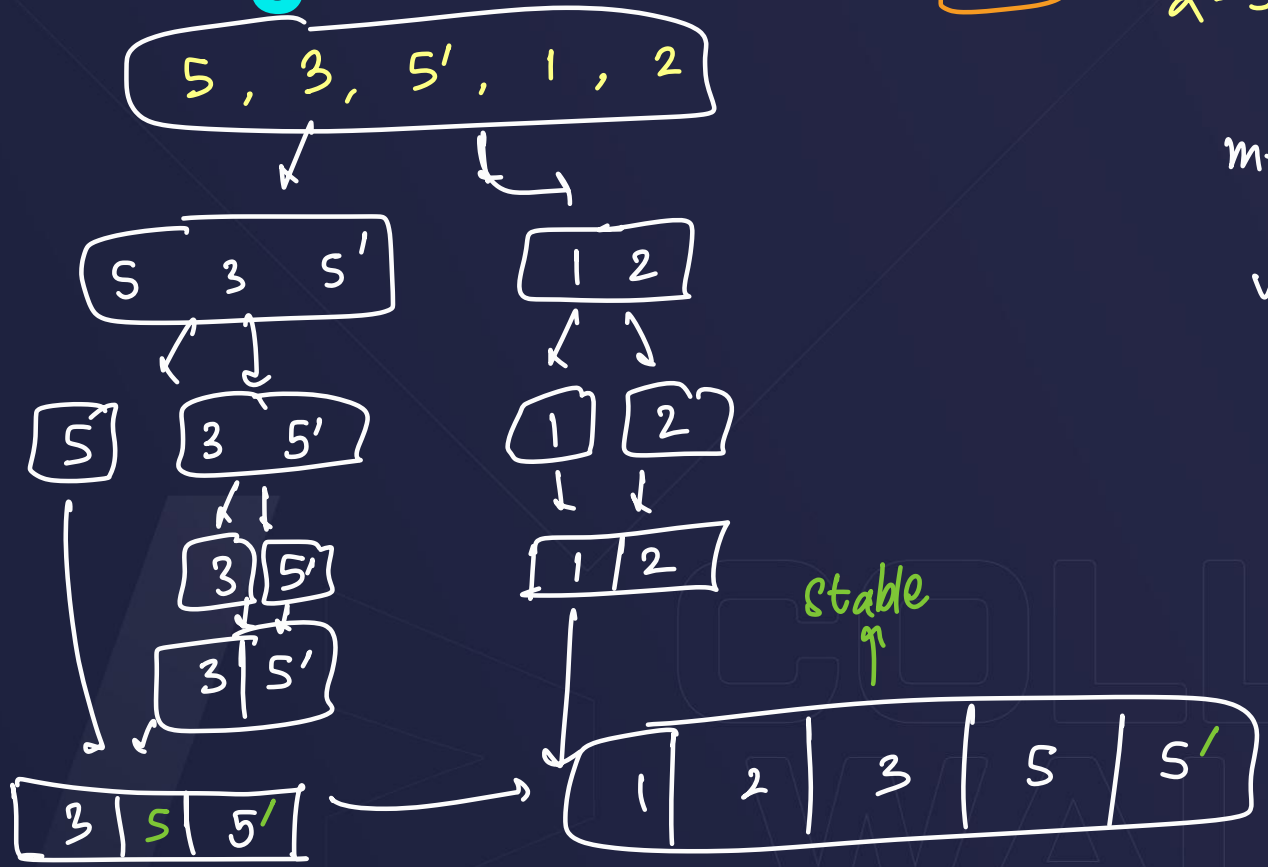
$B.S. = \frac{n(n-1)}{2} = \frac{1024}{2} \cdot 1023 = 5,23,776$ ops

$M.S. = 2 \times n \times \log n = 2 \times 1024 \times \log_2 1024$

$= 2048 \times 10$

$= 20480$ ops

# Is merge sort stable? yes

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# Applications of Merge Sort

1) It is used in sorting linked list

2) It is used in count inversion problem

3) External sorting

# Drawbacks of Merge Sort

- Time Complexity → O(n log n)
- Space Complexity → O(n log n) → improved
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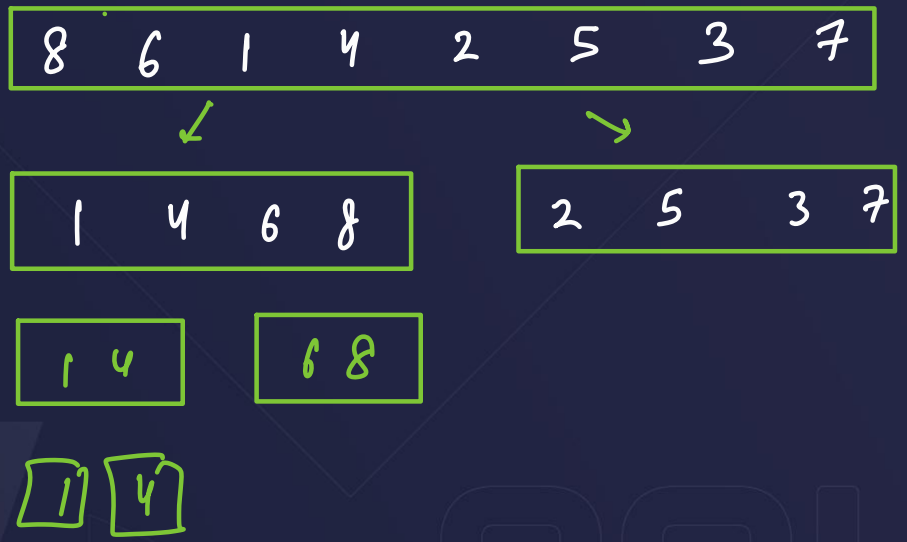
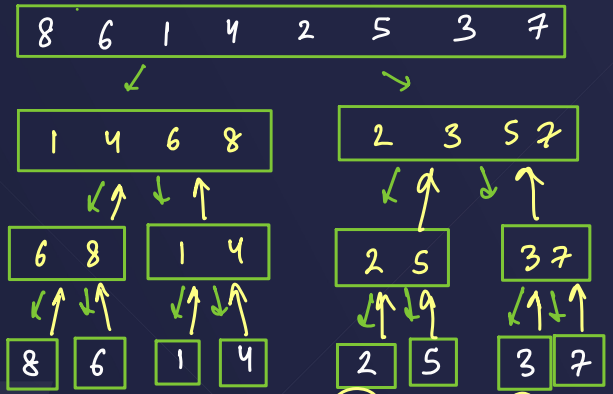
  - O(n)

# Drawbacks of Merge Sort




Extra Space Used: $n + \frac{n}{2} + \frac{n}{4} \rightarrow < 2n \rightarrow O(n)$




Extra Space Used : $n + \frac{n}{2} + \frac{n}{4} + \frac{n}{4} + \frac{n}{2} + \frac{n}{4} + \frac{n}{4} \rightarrow 3n$

# Practice

→ Homework

'Implement' merge sort algorithm to sort an array of elements in decreasing order.

# Count Inversions

Two elements of an array a, a[i] and a[j] form an inversion if a[i] > a[j] and i < j. Given an array of integers. Find the Inversion Count in the array.

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| a → | 5 | 1 | 8 | 2 | 3 |

(5, 1)
(5, 2)
(5, 3)
(8, 2)
(8, 3)

Total no. of inversions = 5

a[0] > a[1]

0 < 1

# M-I: Brute Force :

total no. of ops = $\frac{n(n+1)}{2}$

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| a → | 5 | 1 | 8 | 2 | 3 |

```
for(int i=0; i<n-1; i++){
    for(int j=i+1; j<n; j++){
        if(a[i] > a[j]) count++;
    }
}
```

T.C. = $O(n^2)$

S.C. = $O(1)$

# What if...?

We have an array made up of two subarrays, both sorted.

What can be said about the inversions including a certain element?

{ 3 5 6 7 }
  i

{ 2, 2, 3, 4 }
     j

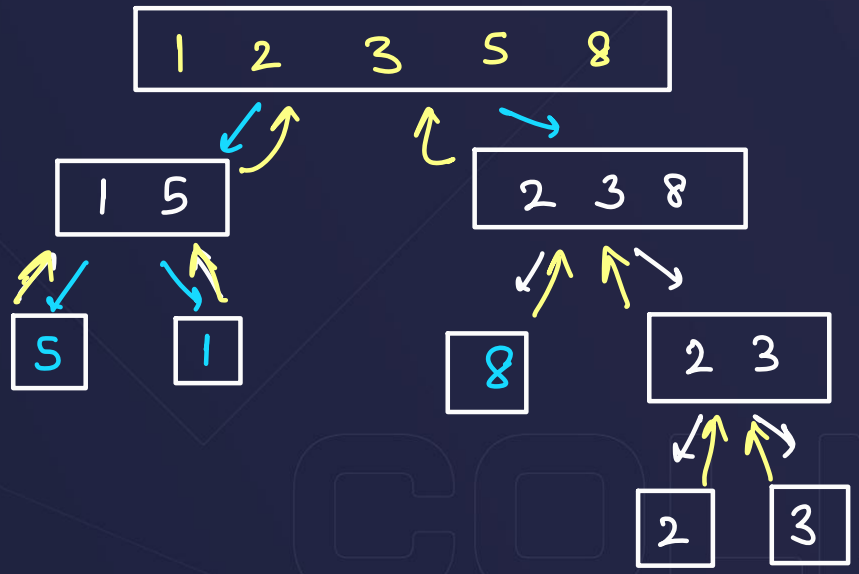
Count = ∅ ∪ ∅ ∪ ∅ inversions

# Recalling Merge Sort with this respect

## The efficient solution

Algorithm

count = 0 1 2 3 4 5



# Coding and dry run

# Time complexity

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Merge Sort

↓

T.C. = $O(n \log n)$

S.C. = $O(n)$

# Next Lecture

Next sorting algorithm: Quick Sort & Quick Select