Seive of Eratosthenes

Prime Numbers & Thodi si Math

Lecture- 37

Raghav Garg

What is a prime number ?

↓

2 factors only → 1 & the number itself

2, 3, 5, 7, 11, 13

→ only even prime

4 → 1, 2, 4

9 → 1, 3, 9

Checking if a number is Prime or Not

If your number is n

→ 2 to n-1

60 → 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Prime → 2 factors

Composite → even number of factors except perfect squares

* if 'i' is a factor of 'n' then 'n/i' is also a factor

9 → 1, 3, 3, 9

2, 2, 5, 10

16 → 1, 2, 4, 8, 16

Checking if a number is Prime or Not

$n = 49 \rightarrow$ 2 to 48 ∝ | 2 to 7 | T.C. = $O(\sqrt{n})$ instead of $O(n)$

60 = 1, 2, 3, 4, 5, 6, | 10, 12, 15, 20, 30, 60

↓ sqrt(60)

$sqrt(49) < sqrt(60) < sqrt(64)$

$7 < sqrt(60) < 8$

$n = 41$
$i = 2$ to 40 ∝
$i = 2$ to $sqrt(41)/6$

If any number has factors except 1 & n, then for sure half of the factors will lie before $\leq sqrt(n)$

Ques: Prime in Diagonal [Leetcode - 2614]

↓

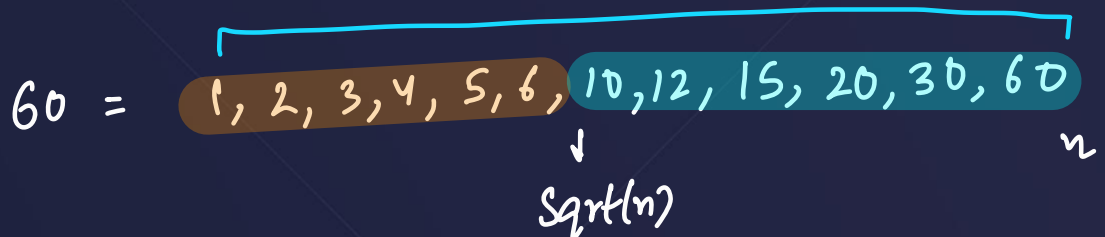
Very Simple

↓

T.C. = $O(n\sqrt{n})$ or $O(n^{3/2})$

Finding factors of a number

if 'n' is divisible by 'i' , it means 'i' is a factor of n



Ques: Perfect Number [Leetcode - 507]

Ques: Smallest Value After Replacing With Sum of Prime Factors

[Leetcode - ~~294~~ 2507]

→ n → sum of prime factors

⇒ 60 → $2^2 \cdot 3^1 \cdot 5^1$ → 2+2+3+5

⇒ 12 → $2^2 \cdot 3$ → 2+2+3

⇒ 7

---

4 ⇒ 2^2 → 2+2

4

$n = 46$

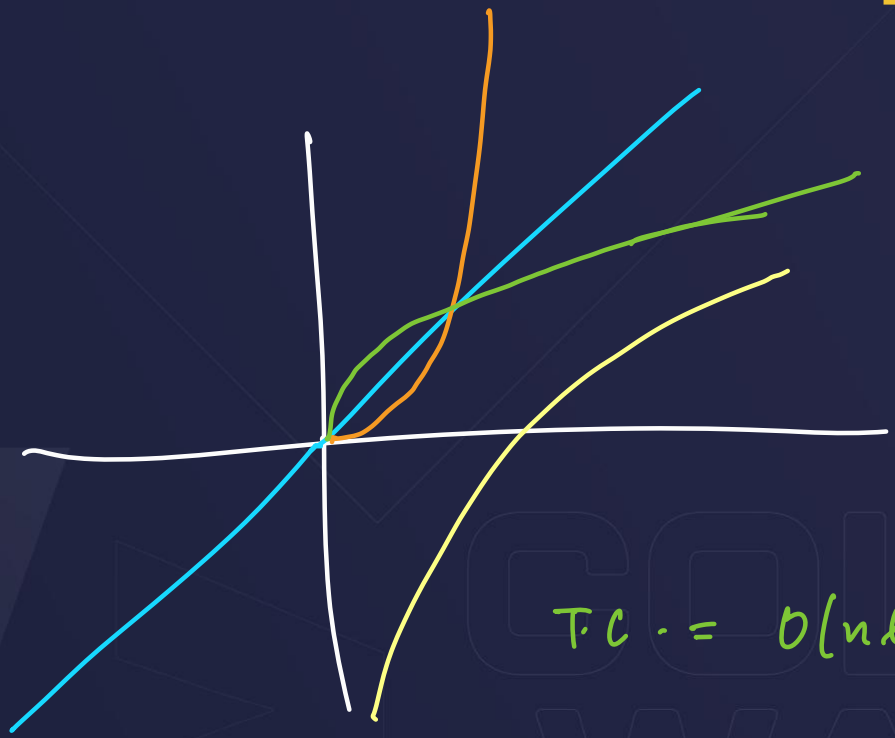
1, 2, 23, 46

$8 \times 9 = 72$

$= 2^3 \cdot 3^2$

$n = 1$ to $n = 8$

Ques: Smallest Value After Replacing With Sum of Prime Factors [Leetcode - 204]



$x = a \cdot b \cdot c \cdot d \cdot e$

$\downarrow$

$a+b+c+d+e$

T.C. = $O(n \log n)$

Ques: Count Primes

[Leetcode - 204]

Sieve of Eratosthenesis → Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

n = 100

S.C. = O(n)

T.C. =

```
for(int i=2; i<= sqrt(n); i++){
    if(arr[i]==1)
```

$$\frac{n}{2} + \frac{n}{3} + \cdots \frac{n}{\text{sqrt}(n)}$$

$$\Rightarrow n\left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{\text{sqrt}(n)}\right]$$

K terms; Sqrt(n)

$$\Rightarrow O(n) \rightarrow O(n\sqrt{n})$$

Time Complexity of Sieve

↳ $\sim O(n)$

↳ $O(n * \log(\log n))$

$$\rightarrow \text{no. of ops} = n\left[\frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{\text{sqrt}(n)}\right]$$

$$\approx n \cdot \log_2 \log_2 (n^{1/2}) \rightarrow \log_2 n^{1/2} = \frac{1}{2}\log_2 n$$

$$= n \cdot \log_2 \left(\frac{\log_2 n}{2}\right)$$

$$= n \cdot [\log_2 \log_2 n - \log_2 2]$$

$$= n \cdot \boxed{\log_2 (\log_2 n)}$$

Time Complexity of Sieve

$n \rightarrow 2^{32}$

$\sqrt{n} = 2^{16}$

$\log_2 \sqrt{n} = \log_2 2^{16} = 16 \log_2 2 = 16$

$\log_2 [\log_2 \sqrt{n}] = \log_2 [16] = \log_2 2^4 = 4 \rightarrow$ max value

$O(n \cdot \log(\log n)) \approx O(n)$

Time Complexity of Sieve

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots \quad \frac{1}{n} \Rightarrow \infty$$

$$\Rightarrow \quad \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \frac{1}{13} + \frac{1}{17} + \frac{1}{19} + \cdots \quad \frac{1}{n} \approx \log(\log n)$$

Ques: Distinct prime factors of Product of Array [Leetcode - 2521]

arr = {2, 4, 3, 10, 6}

sieve =

0	1	2	3	4	5	6	7	8	9	10
0	0	1	1	0	1	0	1	0	0	0

primes = {2, 3, 5, 7}

taken = {0, 0, 0, 0}

Ques: 2 Keys Keyboard

[Leetcode - 650]

A A A A A A A A A A A A

Copy All

Paste

A A A A

n = 8	n = 12
C	C
P	P
C	C
P	P
C	C
P	P
	P

n = prime number

n = 7 → 7

C P P P P P P

A A A A A A A

Ques: 2 Keys Keyboard [Leetcode - 650]

n = 20

Hint : • Think in reverse order

• Current no. uska highest factor dhoondo

20 ← 10 → 20/10 = 2 ops count = 2

10 ← 5 → 10/5 → 2 → count = 4

5 ← 1 → 5/1 → 5 → count = 9

AAAAA AAAAA AAA AAAAAA

CPPPPCPCP

Ques: 2 Keys Keyboard [Leetcode - 650]

n= 90 ← 45 90/45 = 2

45 ← 15 45/15 = 3

15 ← 5 15/5 = 3

5 ← 1 5/1 = 5

49 → 1, 7, 49

```
for(int i=2; i<=sqrt(n); i++){
```

Ques: 2 Keys Keyboard [Leetcode - 650]

130
↓ /2
65
↓ /5
13

1, 5, 13, 65

T.C. = $O(\sqrt{n} \cdot \log n)$

```
while (n>1){ → O(log2 n)
    if (isPrime(n)){ → O(√n)
        count += n ;
        break ;
    }
    int hf → O(√n)
    n = hf
```

Ques: Ugly Number

[Leetcode - 263]

is 60 ugly

→ 60 → 30 → 15 → 5 → 1 true

→ 70 → 35 → 7 false

→ 10 → 5 → 1 true

$$a \times b = HCF(a,b) \times LCM(a,b)$$

Thank you!

OOPS → Lecture

Harsh Sir → SDE at a MNC