#### EGR 7050 Design and Analysis of Engineering experiments

## Homework 9

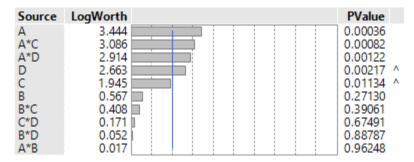
1. Consider the single replicate of the  $2^4$  design in Example 6.2. Suppose that we ran five points at the center (0, 0, 0, 0) and observed the responses 93, 95, 91, 89, and 96. Test for curvature in this experiment. Interpret the results.

Run Number	Factor		Run Label	Filtration		
	Α	В	С	D		Rate (gal/h)
1	-	-	-	-	(1)	45
2	+	-	-	-	а	71
3	-	+	-	-	b	48
4	+	+	-	-	ab	65
5	-	-	+	-	С	68
6	+	-	+	-	ac	60
7	-	+	+	-	bc	80
8	+	+	+	-	abc	65
9	-	-	-	+	d	43
10	+	-	-	+	ad	100
11	-	+	-	+	bd	45
12	+	+	-	+	abd	104
13	-	-	+	+	cd	75
14	+	-	+	+	acd	86
15	-	+	+	+	bcd	70
16	+	+	+	+	abcd	96

Solution:

# Response Filtration Rate (gal/h)

# **Effect Summary**



### **Summary of Fit**

RSquare 0.977698 RSquare Adj 0.933093 Root Mean Square Error 5.055937 Mean of Response 70.0625 Observations (or Sum Wgts) 16

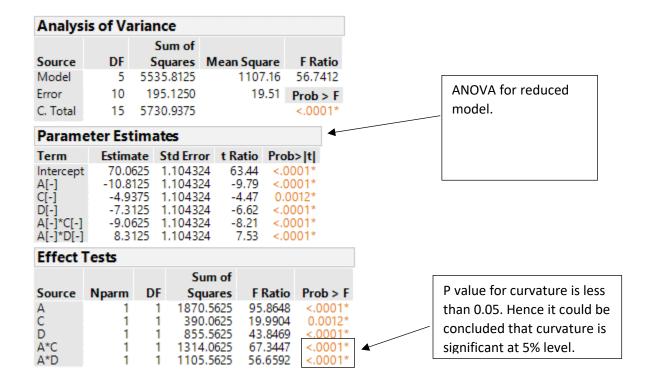
Analysis of Variance						
Source	DF	Sum of Squares	Mean Square	F Ratio		
Model	10	5603.1250	560.313	21.9193		
Error C. Total	5 15	127.8125 5730.9375	25.563	Prob > F 0.0016*		

Parameter Estimates						
Term	Estimate	Std Error	t Ratio	Prob> t		
Intercept	70.0625	1.263984	55.43	<.0001*		
A[+]	10.8125	1.263984	8.55	0.0004*		
B[+]	1.5625	1.263984	1.24	0.2713		
C[+]	4.9375	1.263984	3.91	0.0113*		
D[+]	7.3125	1.263984	5.79	0.0022*		
A[+]*B[+]	0.0625	1.263984	0.05	0.9625		
A[+]*C[+]	-9.0625	1.263984	-7.17	*8000.0		
B[+]*C[+]	1.1875	1.263984	0.94	0.3906		
A[+]*D[+]	8.3125	1.263984	6.58	0.0012*		
B(+)*D(+)	-0.1875	1.263984	-0.15	0.8879		
C[+]*D[+]	-0.5625	1.263984	-0.45	0.6749		

P value for A, C, D, AC and AD are less than 0.05. Therefore, main effects A, C and D and the interaction effects AC and AD are significant at 5% level

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			Sum of			
Source	Nparm	DF	Squares	F Ratio	Prob > F	
Α	1	1	1870.5625	73.1760	0.0004*	
В	1	1	39.0625	1.5281	0.2713	
C	1	1	390.0625	15.2592	0.0113*	
D	1	1	855.5625	33.4694	0.0022*	
A*B	1	1	0.0625	0.0024	0.9625	
A*C	1	1	1314.0625	51.4059	0.0008*	
B*C	1	1	22.5625	0.8826	0.3906	
A*D	1	1	1105.5625	43.2494	0.0012*	
B*D	1	1	0.5625	0.0220	0.8879	
C*D	1	1	5.0625	0.1980	0.6749	



#### Hand calculation:

The sum of squares for the Pure Quadratic (curvature) component

$$SS_{PureQuadratic} = \frac{n_F n_C (\bar{y}_F - \bar{y}_C)^2}{n_F + n_C}$$

$$=\frac{(16)(5)(70.0625-92.8)^2}{16+5}$$

 $SS_{PureQuadratic} = 1969.5$ 

The sum of squares 1 degree of freedom.

$$MS_{PureQuadratic} = 1969.5$$

The mean square for pure error is

$$\begin{split} MS_{PureError} &= \sum_{i \in CenterPoints} (y_i - \bar{y}_c)^2 / (n_c - 1) \\ &= \frac{(93 - 92.8)^2 + (95 - 92.8)^2 + (91 - 92.8)^2 + (89 - 92.8)^2 + (96 - 92.8)^2}{4} \\ &= 8.2 \end{split}$$

#### F-statistic:

$$\mathsf{F} = MS_{PureQuadratic} \ / MS_{PureError} = 1969.5/8.2 = 240.18$$

From F table,  $F_{0.01,1,4}=21.2$ . F statistic is much larger than 21.2. Thus, P-Value is much smaller than 0.01. There is strong evidence that PureQuadratic component of the model is not zero. Thus, there is an indication of curvature.

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#### 3. Factor Definition table

Factor	Catapult Variable	High Level (+)	Low Level (-1)
Α	Cup Position	2	5
В	Tower Pin	2	4
С	Stop Pin	2	5