

**EGR 7050 Design and Analysis of Engineering experiments****Homework 11**

1. The data shown in the Table P11.2 were collected in an experiment to optimize crystal growth as a function of three variables  $x_1$ ,  $x_2$ , and  $x_3$ . Large values of  $y$  (yield in grams) are desirable. Fit a second-order model and analyze the fitted surface. Under what set of conditions is maximum growth achieved?

| <b>x1</b> | <b>x2</b> | <b>x3</b> | <b>y</b> |
|-----------|-----------|-----------|----------|
| -1        | -1        | -1        | 66       |
| -1        | -1        | 1         | 70       |
| -1        | 1         | -1        | 78       |
| -1        | 1         | 1         | 60       |
| 1         | -1        | -1        | 80       |
| 1         | -1        | 1         | 70       |
| 1         | 1         | -1        | 100      |
| 1         | 1         | 1         | 75       |
| -1.682    | 0         | 0         | 100      |
| 1.682     | 0         | 0         | 80       |
| 0         | -1.682    | 0         | 68       |
| 0         | 1.682     | 0         | 63       |
| 0         | 0         | -1.682    | 65       |
| 0         | 0         | 1.682     | 82       |
| 0         | 0         | 0         | 113      |
| 0         | 0         | 0         | 100      |
| 0         | 0         | 0         | 118      |
| 0         | 0         | 0         | 88       |
| 0         | 0         | 0         | 100      |
| 0         | 0         | 0         | 85       |

**Solution:**

**Response Y****Lack Of Fit**

| Source      | DF | Sum of Squares | Mean Square | F Ratio  |
|-------------|----|----------------|-------------|----------|
| Lack Of Fit | 5  | 1001.6142      | 200.323     | 1.1656   |
| Pure Error  | 5  | 859.3333       | 171.867     | Prob > F |
| Total Error | 10 | 1860.9475      |             | 0.4353   |
|             |    |                | Max RSq     | 0.8444   |

**Summary of Fit**

|                            |          |
|----------------------------|----------|
| RSquare                    | 0.663052 |
| RSquare Adj                | 0.359799 |
| Root Mean Square Error     | 13.64166 |
| Mean of Response           | 83.05    |
| Observations (or Sum Wgts) | 20       |

Rsquare value is not very large.  
Therefore a better fitting model  
would be more useful

**Analysis of Variance**

| Source   | DF | Sum of Squares | Mean Square | F Ratio  |
|----------|----|----------------|-------------|----------|
| Model    | 9  | 3662.0025      | 406.889     | 2.1865   |
| Error    | 10 | 1860.9475      | 186.095     | Prob > F |
| C. Total | 19 | 5522.9500      |             | 0.1194   |

**Parameter Estimates**

| Term      | Estimate  | Std Error | t Ratio | Prob> t |
|-----------|-----------|-----------|---------|---------|
| Intercept | 100.66609 | 5.563726  | 18.09   | <.0001* |
| X1        | 1.27146   | 3.691405  | 0.34    | 0.7377  |
| X2        | 1.3612971 | 3.691405  | 0.37    | 0.7200  |
| X3        | -1.494453 | 3.691405  | -0.40   | 0.6941  |
| X1*X2     | 2.875     | 4.823053  | 0.60    | 0.5644  |
| X1*X3     | -2.625    | 4.823053  | -0.54   | 0.5982  |
| X2*X3     | -4.625    | 4.823053  | -0.96   | 0.3602  |
| X1*X1     | -3.767495 | 3.593483  | -1.05   | 0.3191  |
| X2*X2     | -12.42955 | 3.593483  | -3.46   | 0.0061* |
| X3*X3     | -9.601125 | 3.593483  | -2.67   | 0.0234* |

**Effect Tests**

| Source | Nparm | DF | Sum of Squares | F Ratio | Prob > F |
|--------|-------|----|----------------|---------|----------|
| X1     | 1     | 1  | 22.0778        | 0.1186  | 0.7377   |
| X2     | 1     | 1  | 25.3079        | 0.1360  | 0.7200   |
| X3     | 1     | 1  | 30.5011        | 0.1639  | 0.6941   |
| X1*X2  | 1     | 1  | 66.1250        | 0.3553  | 0.5644   |
| X1*X3  | 1     | 1  | 55.1250        | 0.2962  | 0.5982   |
| X2*X3  | 1     | 1  | 171.1250       | 0.9196  | 0.3602   |
| X1*X1  | 1     | 1  | 204.5541       | 1.0992  | 0.3191   |
| X2*X2  | 1     | 1  | 2226.4547      | 11.9641 | 0.0061*  |
| X3*X3  | 1     | 1  | 1328.4560      | 7.1386  | 0.0234*  |

From the effect tests, we  
could see that many terms are  
not significant at  $\alpha = 0.05$ .  
Only X2\*X2 and X3\*X3 are  
significant.

As there are many insignificant factors, the model is not significant and hence few terms needs to be removed.

| Response Y  |    |                |             |          |
|-------------|----|----------------|-------------|----------|
| Lack Of Fit |    |                |             |          |
| Source      | DF | Sum of Squares | Mean Square | F Ratio  |
| Lack Of Fit | 4  | 697.4544       | 174.364     | 1.1400   |
| Pure Error  | 11 | 1682.5000      | 152.955     | Prob > F |
| Total Error | 15 | 2379.9544      |             | 0.3878   |
|             |    |                | Max RSq     | 0.6954   |

| Summary of Fit             |          |
|----------------------------|----------|
| RSquare                    | 0.569079 |
| RSquare Adj                | 0.454167 |
| Root Mean Square Error     | 12.59618 |
| Mean of Response           | 83.05    |
| Observations (or Sum Wgts) | 20       |

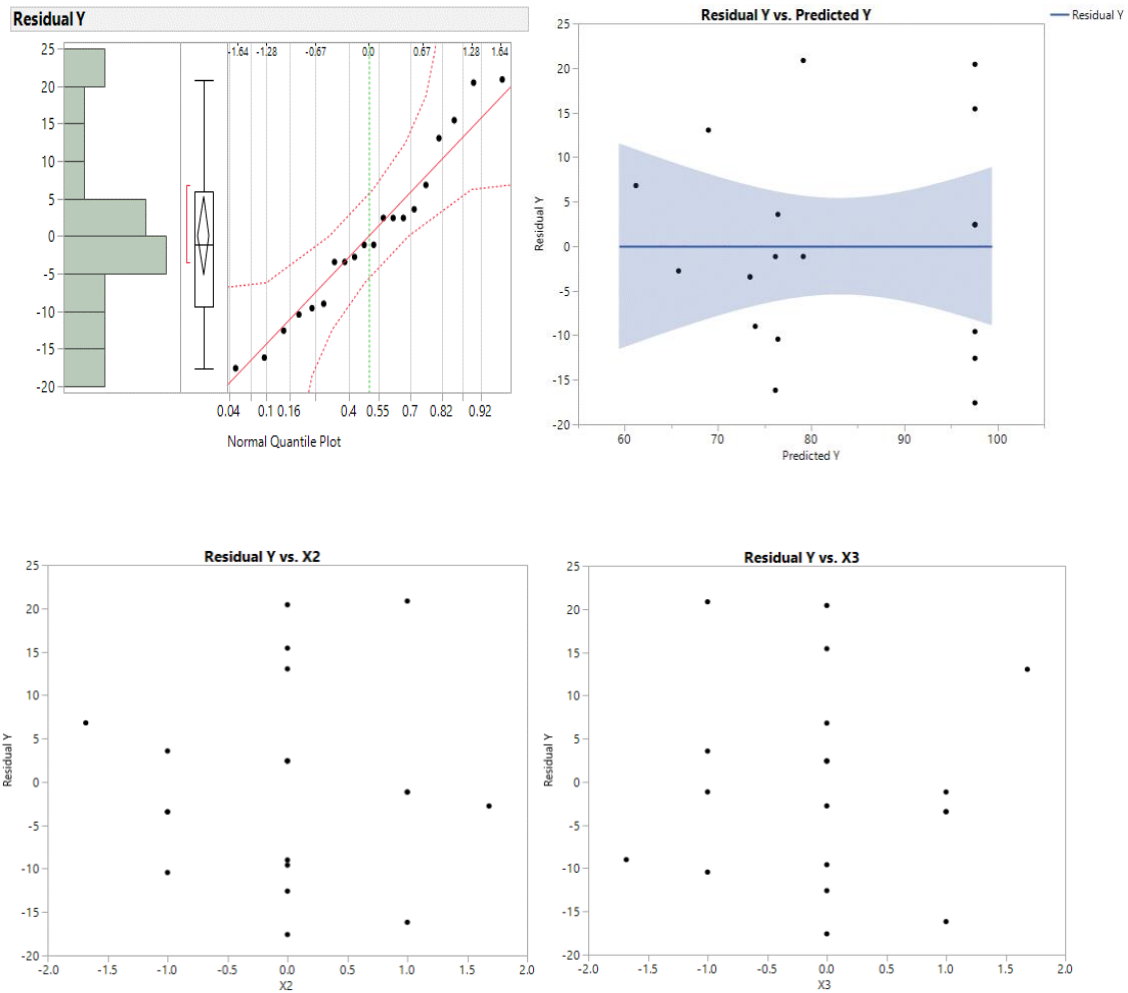
| Analysis of Variance |    |                |             |          |
|----------------------|----|----------------|-------------|----------|
| Source               | DF | Sum of Squares | Mean Square | F Ratio  |
| Model                | 4  | 3142.9956      | 785.749     | 4.9523   |
| Error                | 15 | 2379.9544      | 158.664     | Prob > F |
| C. Total             | 19 | 5522.9500      |             | 0.0095*  |

| Parameter Estimates |           |           |         |         |
|---------------------|-----------|-----------|---------|---------|
| Term                | Estimate  | Std Error | t-Ratio | Prob> t |
| Intercept           | 97.582598 | 4.360876  | 22.38   | <.0001* |
| X2                  | 1.3612971 | 3.4085    | 0.40    | 0.6952  |
| X3                  | -1.494453 | 3.4085    | -0.44   | 0.6673  |
| X2*X2               | -12.05546 | 3.301685  | -3.65   | 0.0024* |
| X3*X3               | -9.227035 | 3.301685  | -2.79   | 0.0136* |

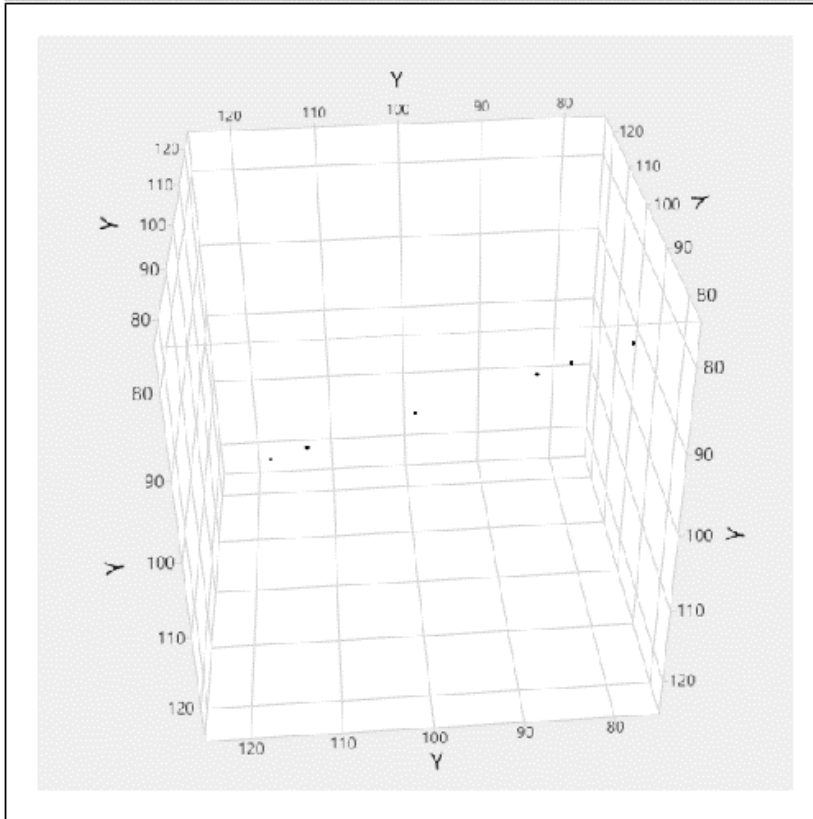
| Effect Tests |       |    |                |         |          |
|--------------|-------|----|----------------|---------|----------|
| Source       | Nparm | DF | Sum of Squares | F Ratio | Prob > F |
| X2           | 1     | 1  | 25.3079        | 0.1595  | 0.6952   |
| X3           | 1     | 1  | 30.5011        | 0.1922  | 0.6673   |
| X2*X2        | 1     | 1  | 2115.3084      | 13.3320 | 0.0024*  |
| X3*X3        | 1     | 1  | 1239.1683      | 7.8100  | 0.0136*  |

This is the final model from this analysis. Although the fit of the model is not great, it is the best information we have from the available data.

$$\hat{y} = 97.582 + 1.361x_2 - 1.494x_3 - 12.055x_2^2 - 9.227x_3^2$$



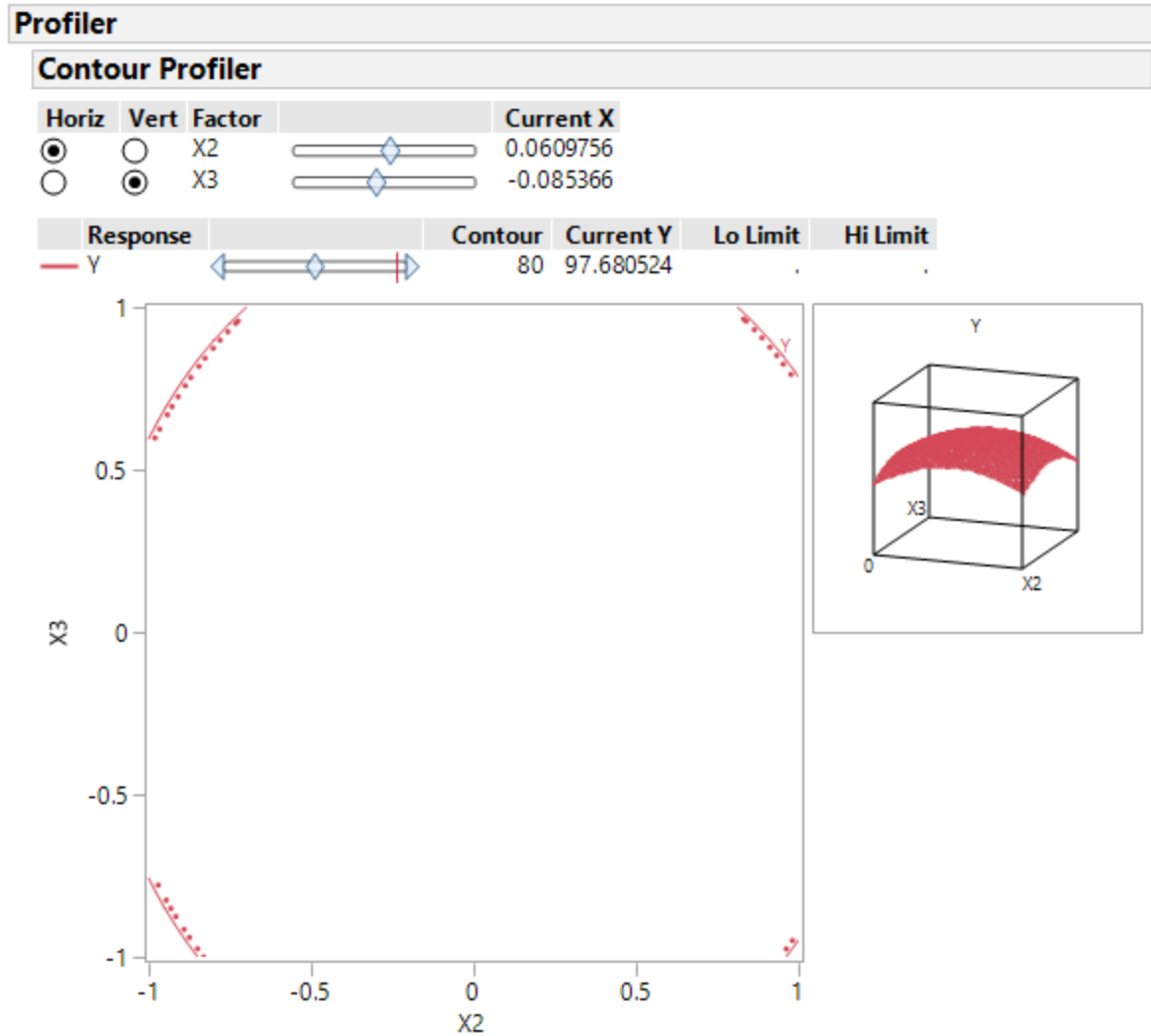
The normal plot has all points close to the line and within the error bounds, indicating no unusual departure from normality. The residual vs. predicted plot and residual vs factors plot show no unusual pattern. This does not violate the model assumptions although the model fit does not great.

**Surface Plot X2=0, X3=0****Dependent Variables**

|                          | Point Response Column | Style  | Surface    | Grid Value |
|--------------------------|-----------------------|--------|------------|------------|
| <input type="checkbox"/> | Y                     | Points | Both sides | 100        |
| <input type="checkbox"/> | none                  |        | Off        |            |
| <input type="checkbox"/> | none                  |        | Off        |            |
| <input type="checkbox"/> | none                  |        | Off        |            |

Grid

This is the maximum yield obtained in this experiment for the input setting x2 and x3 =0 approximately. This can be verified using contour plot.



From the contour plot, we could see that maximum yield can be obtained at  $x_2=0.0609$  and  $x_3=-0.085366$  approximately.