

EGR 7050 Design and Analysis of Engineering experiments**Homework 9**

1. Consider the single replicate of the 2^4 design in Example 6.2. Suppose that we ran five points at the center (0, 0, 0, 0) and observed the responses 93, 95, 91, 89, and 96. Test for curvature in this experiment. Interpret the results.

Run Number	Factor				Run Label	Filtration Rate (gal/h)
	A	B	C	D		
1	-	-	-	-	(1)	45
2	+	-	-	-	a	71
3	-	+	-	-	b	48
4	+	+	-	-	ab	65
5	-	-	+	-	c	68
6	+	-	+	-	ac	60
7	-	+	+	-	bc	80
8	+	+	+	-	abc	65
9	-	-	-	+	d	43
10	+	-	-	+	ad	100
11	-	+	-	+	bd	45
12	+	+	-	+	abd	104
13	-	-	+	+	cd	75
14	+	-	+	+	acd	86
15	-	+	+	+	bcd	70
16	+	+	+	+	abcd	96

Solution:

Response Filtration Rate (gal/h)**Effect Summary**

Source	LogWorth	PValue
A	3.444	0.00036
A*C	3.086	0.00082
A*D	2.914	0.00122
D	2.663	0.00217 ^
C	1.945	0.01134 ^
B	0.567	0.27130
B*C	0.408	0.39061
C*D	0.171	0.67491
B*D	0.052	0.88787
A*B	0.017	0.96248

Summary of Fit

RSquare	0.977698
RSquare Adj	0.933093
Root Mean Square Error	5.055937
Mean of Response	70.0625
Observations (or Sum Wgts)	16

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	10	5603.1250	560.313	21.9193
Error	5	127.8125	25.563	Prob > F
C. Total	15	5730.9375		0.0016*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	70.0625	1.263984	55.43	<.0001*
A[+]	10.8125	1.263984	8.55	0.0004*
B[+]	1.5625	1.263984	1.24	0.2713
C[+]	4.9375	1.263984	3.91	0.0113*
D[+]	7.3125	1.263984	5.79	0.0022*
A[+]*B[+]	0.0625	1.263984	0.05	0.9625
A[+]*C[+]	-9.0625	1.263984	-7.17	0.0008*
B[+]*C[+]	1.1875	1.263984	0.94	0.3906
A[+]*D[+]	8.3125	1.263984	6.58	0.0012*
B[+]*D[+]	-0.1875	1.263984	-0.15	0.8879
C[+]*D[+]	-0.5625	1.263984	-0.45	0.6749

P value for A, C, D, AC and AD are less than 0.05. Therefore, main effects A, C and D and the interaction effects AC and AD are significant at 5% level

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
A	1	1	1870.5625	73.1760	0.0004*
B	1	1	39.0625	1.5281	0.2713
C	1	1	390.0625	15.2592	0.0113*
D	1	1	855.5625	33.4694	0.0022*
A*B	1	1	0.0625	0.0024	0.9625
A*C	1	1	1314.0625	51.4059	0.0008*
B*C	1	1	22.5625	0.8826	0.3906
A*D	1	1	1105.5625	43.2494	0.0012*
B*D	1	1	0.5625	0.0220	0.8879
C*D	1	1	5.0625	0.1980	0.6749

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	5535.8125	1107.16	56.7412
Error	10	195.1250	19.51	Prob > F
C. Total	15	5730.9375		<.0001*

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	70.0625	1.104324	63.44	<.0001*
A[-]	-10.8125	1.104324	-9.79	<.0001*
C[-]	-4.9375	1.104324	-4.47	0.0012*
D[-]	-7.3125	1.104324	-6.62	<.0001*
A[-]*C[-]	-9.0625	1.104324	-8.21	<.0001*
A[-]*D[-]	8.3125	1.104324	7.53	<.0001*

Effect Tests					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
A	1	1	1870.5625	95.8648	<.0001*
C	1	1	390.0625	19.9904	0.0012*
D	1	1	855.5625	43.8469	<.0001*
A*C	1	1	1314.0625	67.3447	<.0001*
A*D	1	1	1105.5625	56.6592	<.0001*

ANOVA for reduced model.

P value for curvature is less than 0.05. Hence it could be concluded that curvature is significant at 5% level.

Hand calculation:

The sum of squares for the Pure Quadratic (curvature) component

$$SS_{PureQuadratic} = \frac{n_F n_C (\bar{y}_F - \bar{y}_C)^2}{n_F + n_C}$$

$$= \frac{(16)(5)(70.0625 - 92.8)^2}{16 + 5}$$

$$SS_{PureQuadratic} = 1969.5$$

The sum of squares 1 degree of freedom.

$$MS_{PureQuadratic} = 1969.5$$

The mean square for pure error is

$$MS_{PureError} = \sum_{i \in \text{CenterPoints}} (y_i - \bar{y}_C)^2 / (n_C - 1)$$

$$= \frac{(93 - 92.8)^2 + (95 - 92.8)^2 + (91 - 92.8)^2 + (89 - 92.8)^2 + (96 - 92.8)^2}{4}$$

$$= 8.2$$

F-statistic:

$$F = MS_{PureQuadratic} / MS_{PureError} = 1969.5/8.2 = 240.18$$

From F table, $F_{0.01,1,4} = 21.2$. F statistic is much larger than 21.2. Thus, P-Value is much smaller than 0.01. There is strong evidence that PureQuadratic component of the model is not zero. Thus, there is an indication of curvature.

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3. Factor Definition table

Factor	Catapult Variable	High Level (+)	Low Level (-1)
A	Cup Position	2	5
B	Tower Pin	2	4
C	Stop Pin	2	5