Meenakshi Nagarajan

U00618268

EGR 7050 Design and Analysis of Engineering experiments

Homework 6

1. *Four different designs for a digital computer circuit are being studied to compare the amount of noise present. The following data have been obtained:*

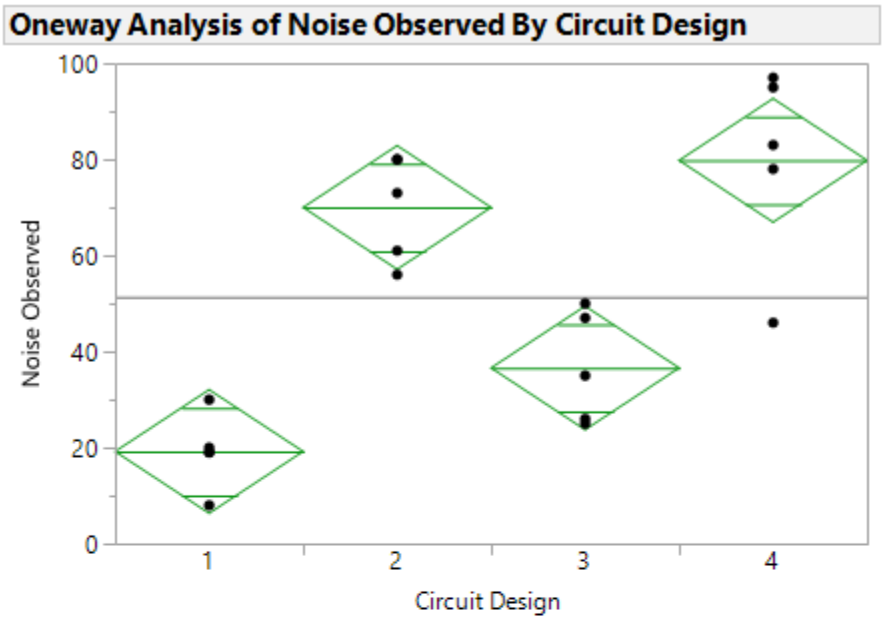
Circuit Design	***Noise Observed***				
1	*19*	*20*	*19*	*30*	*8*
2	*80*	*61*	*73*	*56*	*80*
3	*47*	*26*	*25*	*35*	*50*
4	*95*	*46*	*83*	*78*	*97*

a. *Is the same amount of noise present for all four designs? Use* α *=0.05.*

Solution:

H_0: *amount of noise present in all four designs are same*
H_1: *amount of noise present in at least one design is not same*

Given, $\alpha = 0.05$



Oneway Anova

Summary of Fit

Rsquare	0.803293
Adj Rsquare	0.76641
Root Mean Square Error	13.57571
Mean of Response	51.4
Observations (or Sum Wgts)	20

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Circuit Design	3	12042.000	4014.00	21.7797	<.0001*
Error	16	2948.800	184.30		
C. Total	19	14990.800			

P value is less than the significance level, $\alpha = 0.05$. Thus, null hypothesis could be rejected.

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1	5	19.2000	6.0712	6.330	32.070
2	5	70.0000	6.0712	57.130	82.870
3	5	36.6000	6.0712	23.730	49.470
4	5	79.8000	6.0712	66.930	92.670

Std Error uses a pooled estimate of error variance

Fig. 1 *Oneway ANOVA*

Thus, it could be concluded that amount of noise present in at least one design is not same.

b. *Analyze the residuals from this experiment. Are the analysis of variance assumptions satisfied?*

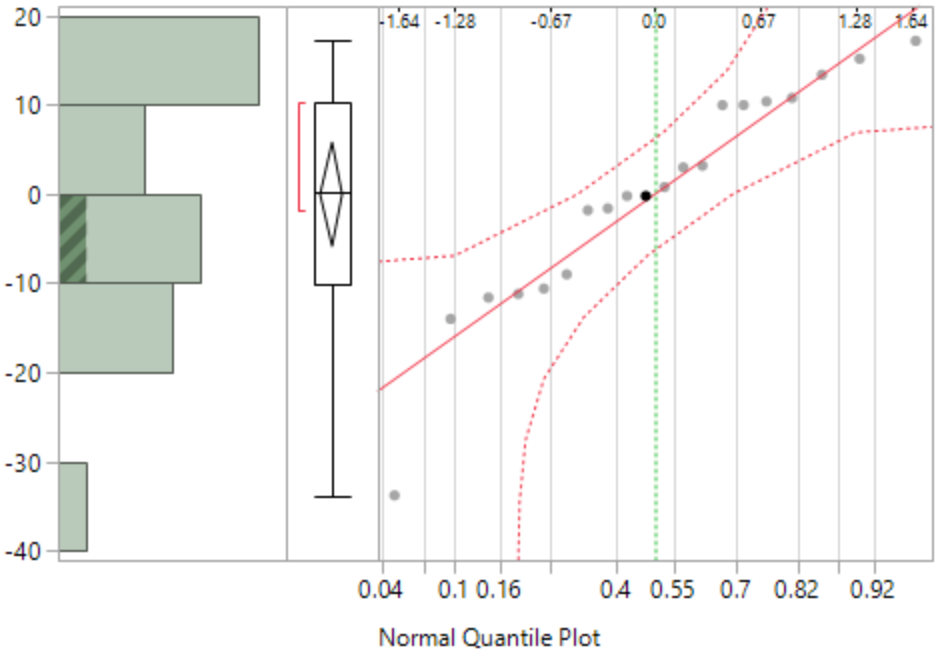


Fig. 2 *Normal quantile plot*

Many points are close to the line and are within the error bounds. There is no significant evidence of deviation from normality for the residuals.

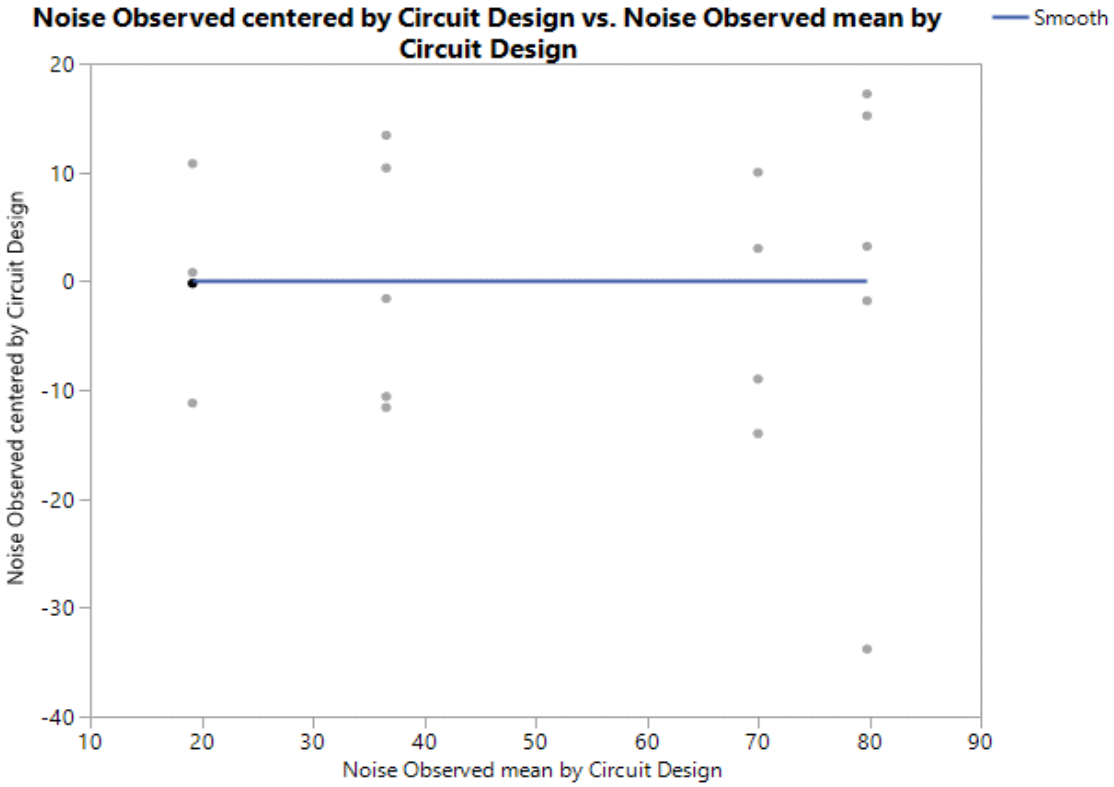


Fig. 3 *Residual vs. Fitted*

There are no outliers in the residual vs. fitted plots. There is a similar range of variation across different fitted values. There is no significant deviation from equal variance assumption.

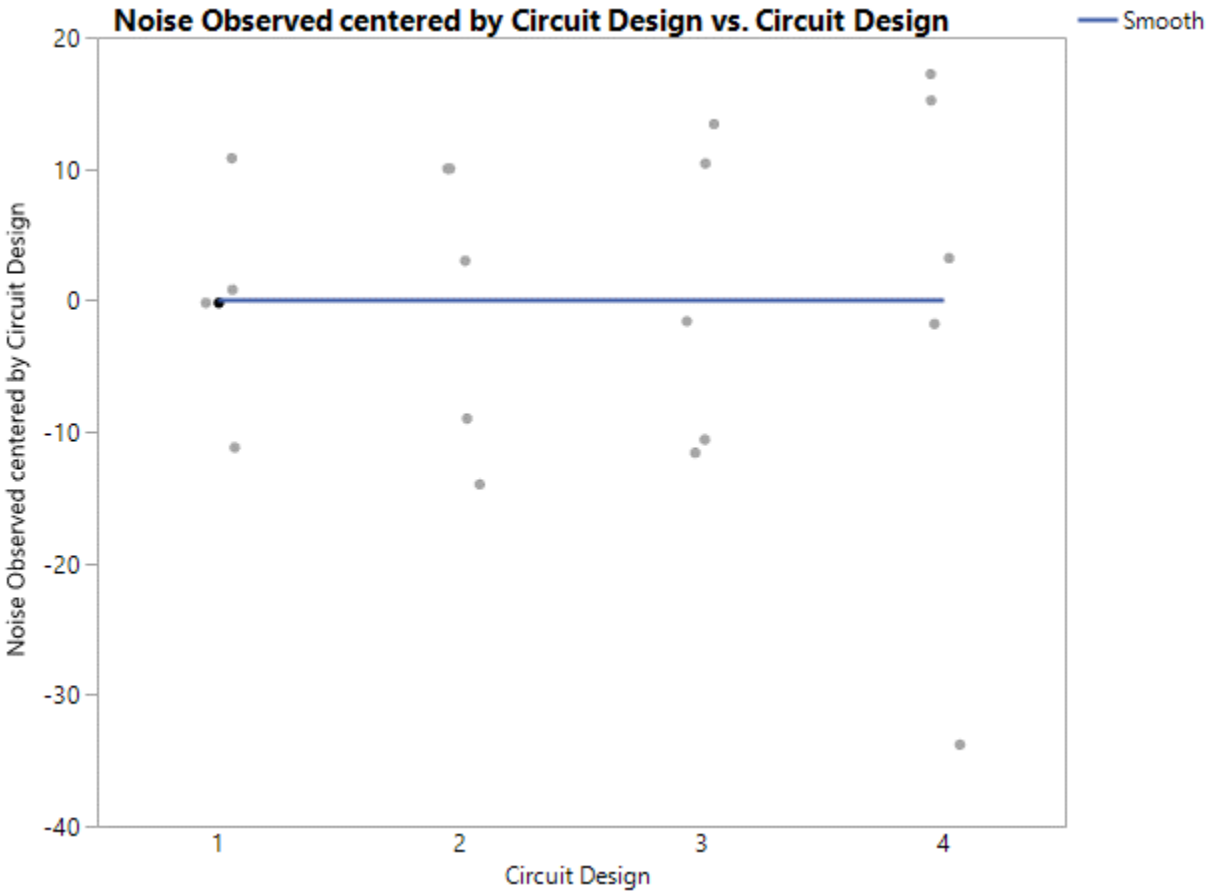


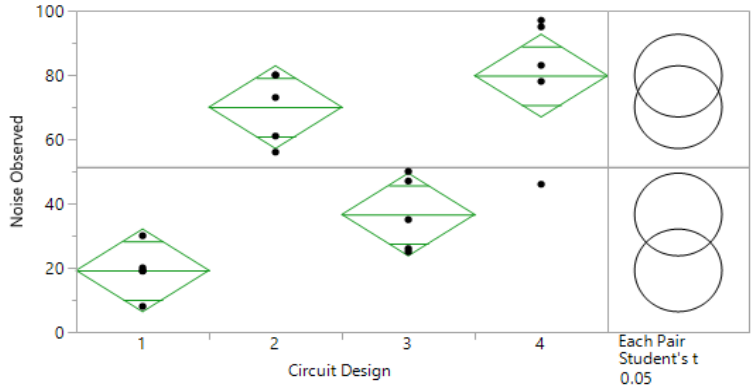
Fig. 4 *Residual vs. circuit design*

The above plot indicates a constant variance.

From the above figures, it could be concluded that analysis of variance assumptions are satisfied.

c. *Which circuit design would you select for use? Low noise is best.*

Oneway Analysis of Noise Observed By Circuit Design



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4	5	79.8000	6.0712	66.930	92.670

Std Error uses a pooled estimate of error variance

Means Comparisons

Comparisons for each pair using Student's t

Confidence Quantile

t	Alpha
2.11991	0.05

LSD Threshold Matrix

Abs(Dif)-LSD

	4	2	3	1
4	-18.202	-8.402	24.998	42.398
2	-8.402	-18.202	15.198	32.598
3	24.998	15.198	-18.202	-0.802
1	42.398	32.598	-0.802	-18.202

Positive values show pairs of means that are significantly different.

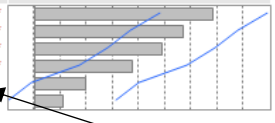
Connecting Letters Report

Level		Mean
4	A	79.800000
2	A	70.000000
3	B	36.600000
1	B	19.200000

Levels not connected by same letter are significantly different.

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
4	1	60.60000	8.586035	42.3984	78.80158	<.0001*
2	1	50.80000	8.586035	32.5984	69.00158	<.0001*
4	3	43.20000	8.586035	24.9984	61.40158	0.0001*
2	3	33.40000	8.586035	15.1984	51.60158	0.0013*
3	1	17.40000	8.586035	-0.8016	35.60158	0.0597
4	2	9.80000	8.586035	-8.4016	28.00158	0.2705



From Fisher's LSD analysis, we see that levels 4,2 are different from levels 1,3. We cannot find a significant difference between levels 4&2 and levels 3&1.

The p value for difference between level 1 & 3 is greater than $\alpha = 0.05$.

Fig. 5 *Oneway ANOVA*

Therefore, it could be concluded that circuit design 1 is having the lower noise.

2. Reconsider the experiment described in Problem 3.24. Suppose that Circuit Design 1 is a control. Use Dunnett's test with α = 0.05 to compare all of the other means with the control.

Solution:

Comparisons with a control using Dunnett's Method

Control Group = 1

Confidence Quantile

|d|	Alpha
2.59232	0.05

LSD Threshold Matrix

Level	Abs(Dif)-LSD	p-Value
4	38.34	<.0001*
2	28.54	<.0001*
3	-4.86	0.1431
1	-22.3	1.0000

Positive values show pairs of means that are significantly different.

Fig. 6 *Dunnett's test*

This shows that, level 4 and 2 treatment means are significantly different from the control group but not level 3.

3. The ANOVA from a randomized complete block experiment output is shown below.

Source	DF	SS	MS	F	P
Treatment	4	1010.56	?	29.84	?
Block	?	?	64.765	?	?
Error	20	169.33	?		
Total	29	1503.71			

Solution:

a. Fill in the blanks. You may give bounds on the P-value.

$$DF_{Error} = (a-1)(b-1) = 20$$
$$4(b-1) = 20$$
$$\boldsymbol{(b-1) = 5}$$

$$SS_E = SS_T - SS_{Treatments} - SS_{Blocks}$$
$$169.33 = 1503.71 - 1010.56 - SS_{Blocks}$$
$$\boldsymbol{SS_{Blocks} = 323.82}$$

$$MS_{Treatment} = SS_{Treatments}/(a-1)$$
$$1010.56/4 = \boldsymbol{252.64}$$

$MS_{Error} = SS_{Error}/(a-1)(b-1)$
$169.33/20$=***8.4665***

F- value of block **=** MS_{Block}/MS_{Error} *= 64.765/8.4665 =* ***7.65***

From the P value calculator, P-value for treatment is ***less than 0.0001***
P-value for block is ***0.0004***

b. *How many blocks were used in this experiment?*

The degrees of freedom for block is 5.

$(b-1) = 5$

No. of blocks $\boldsymbol{b = 6}$

c. *What conclusions can you draw?*

H_0: *All treatment means are same*
H_1: *At least one mean is different*

At significance level $\alpha = 0.05$, *0.0001< 0.05. Therefore, null hypothesis could be rejected. It could therefore be concluded that there exists a difference between treatment means.*

H_0: *No significant difference in blocks*
H_1: *There exists a significant difference in blocks*

At significance level $\alpha = 0.05$, *0.0004 < 0.05. Therefore, null hypothesis could be rejected. It could therefore be concluded that there exists a significant difference in block.*

4. *A consumer products company relies on direct mail marketing pieces as a major component of its advertising campaigns. The company has three different designs for a new brochure and wants to evaluate their effectiveness, as there are substantial differences in costs between the three designs. The company decides to test the three designs by mailing 5000 samples of each to potential customers in four different regions of the country. Since there are known regional differences in the customer base, regions are considered as blocks. The number of responses to each mailing is as follows.*

Design	***Region***			
	NE	***NW***	***SE***	***SW***
1	*250*	*350*	*219*	*375*
2	*400*	*525*	*390*	*580*
3	*275*	*340*	*200*	*310*

Solution:

a. *Analyze the data from this experiment.*

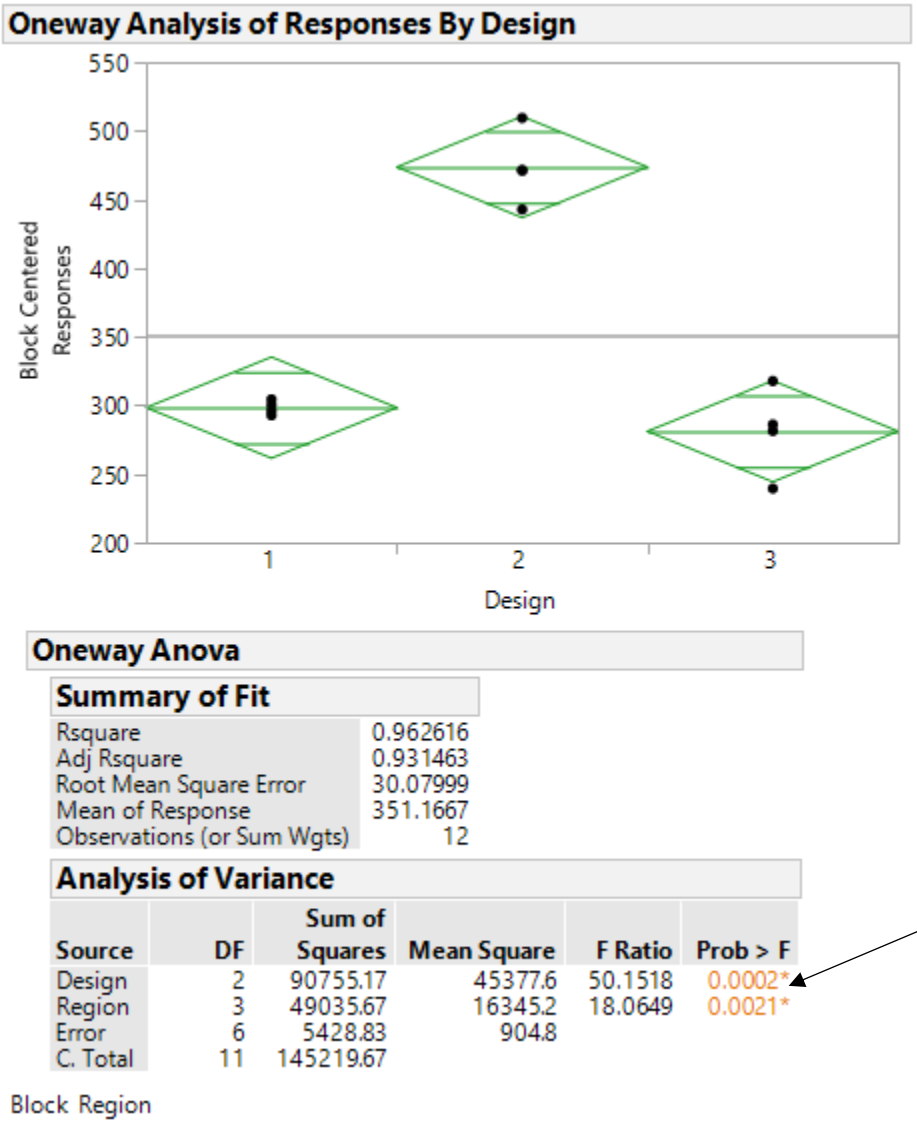


Fig. 7 *ANOVA*

H_0: *No significant difference in design*

H_1: *There exists a significant difference in design*

For both design and region, P-value is less than $\alpha = 0.05$, therefore null hypothesis could be rejected. Thus, it could be concluded that there exists a significant difference in design.

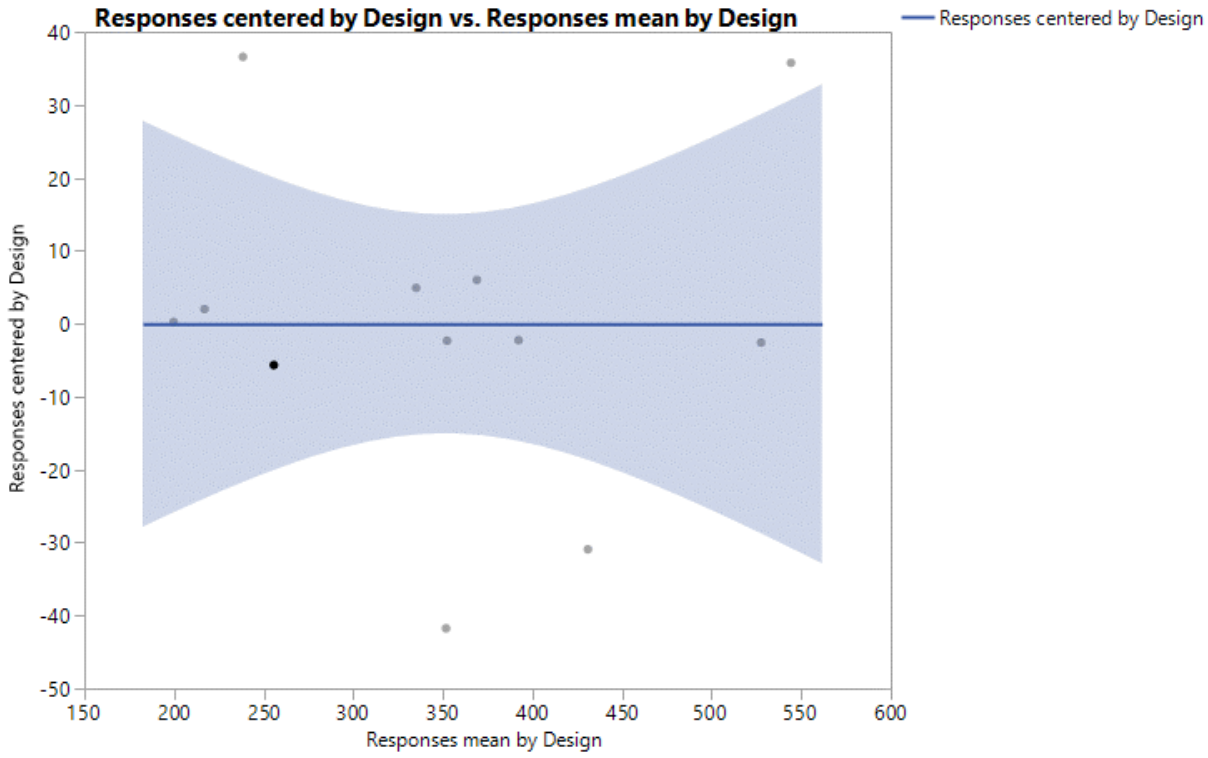


Fig.8 *Residual vs. Predicted*

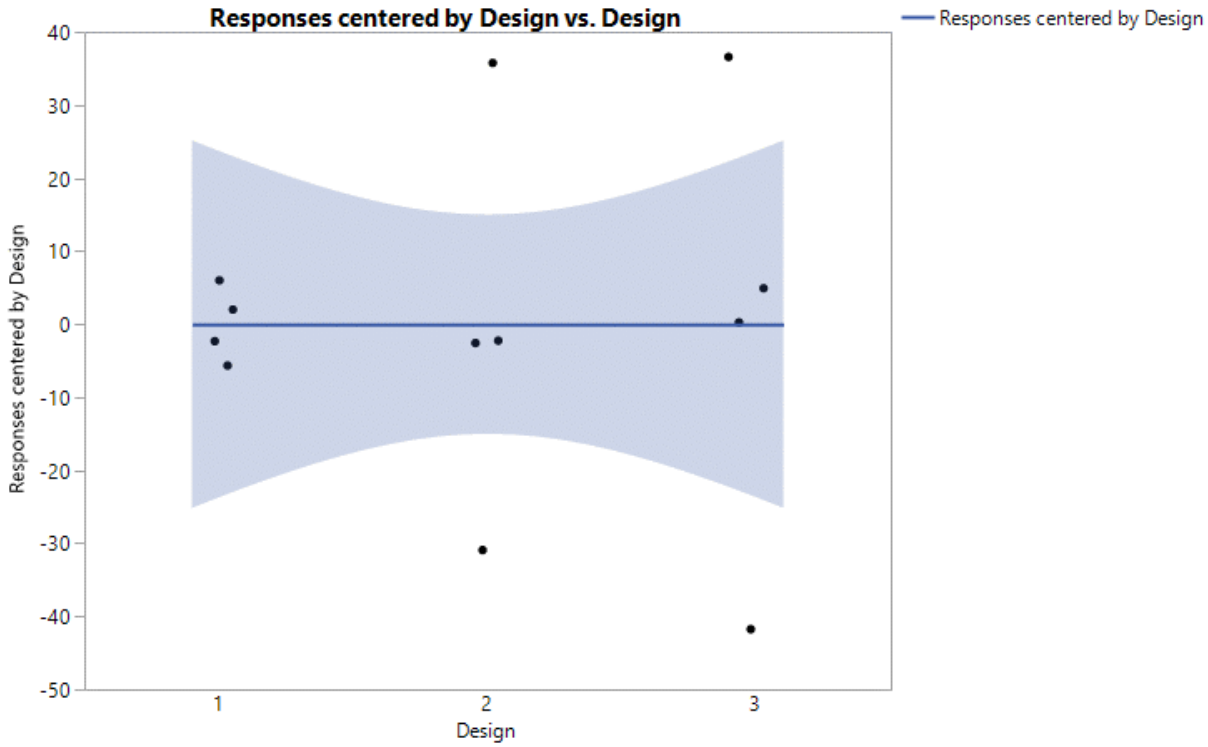


Fig.9 *Residual vs. Design*

The figure shows that variance is not constant for all the predicted values.

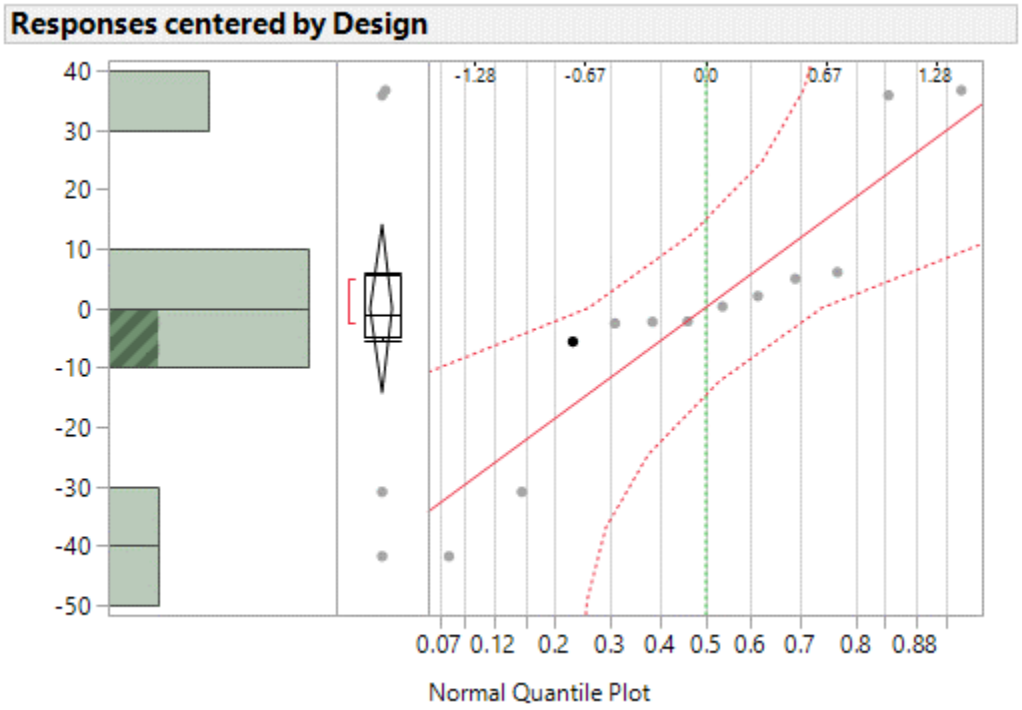
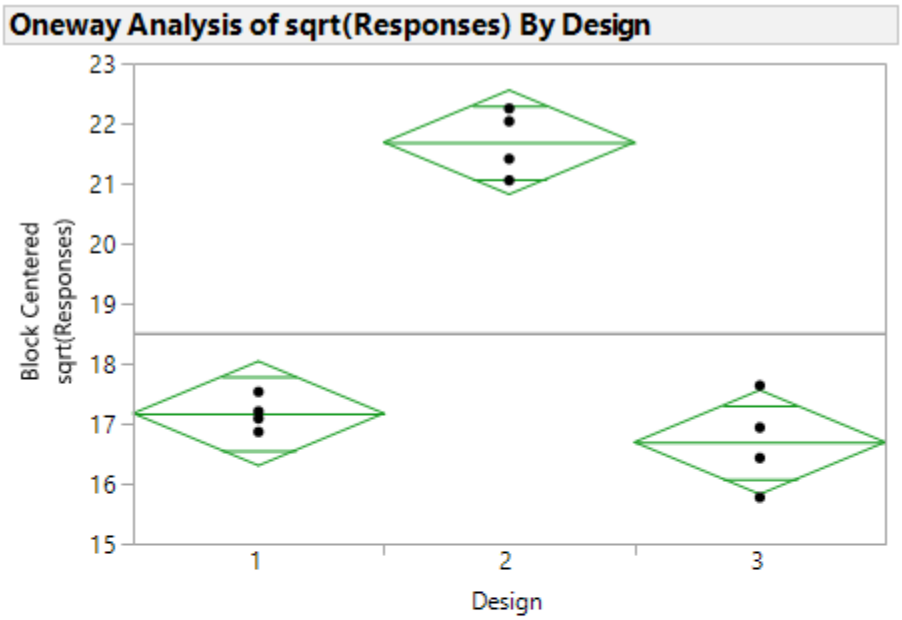


Fig. 10 *Normal quantile plot*

This figure shows that points are not close to the line and outside the error bounds. This shows that normality assumption is not valid.

Since this data deals with counts of occurrences, it is ideal to use square root transformation.



Oneway Anova

Summary of Fit

Rsquare	0.969759
Adj Rsquare	0.944558
Root Mean Square Error	0.708654
Mean of Response	18.51657
Observations (or Sum Wgts)	12

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Design	2	60.733175	30.3666	60.4682	0.0001*
Region	3	35.891407	11.9638	~~23.8232~~	~~0.0010*~~
Error	5 ~~6~~	3.013147	~~0.5022~~		
C. Total	10 ~~11~~	99.637729	0.6026	19.853	0.0033

Block Region

P value is still less than $\alpha = 0.05$ for both the treatment and block. Therefore, reject null hypothesis.

Fig. 11 *Oneway ANOVA of sqrt of responses by design*

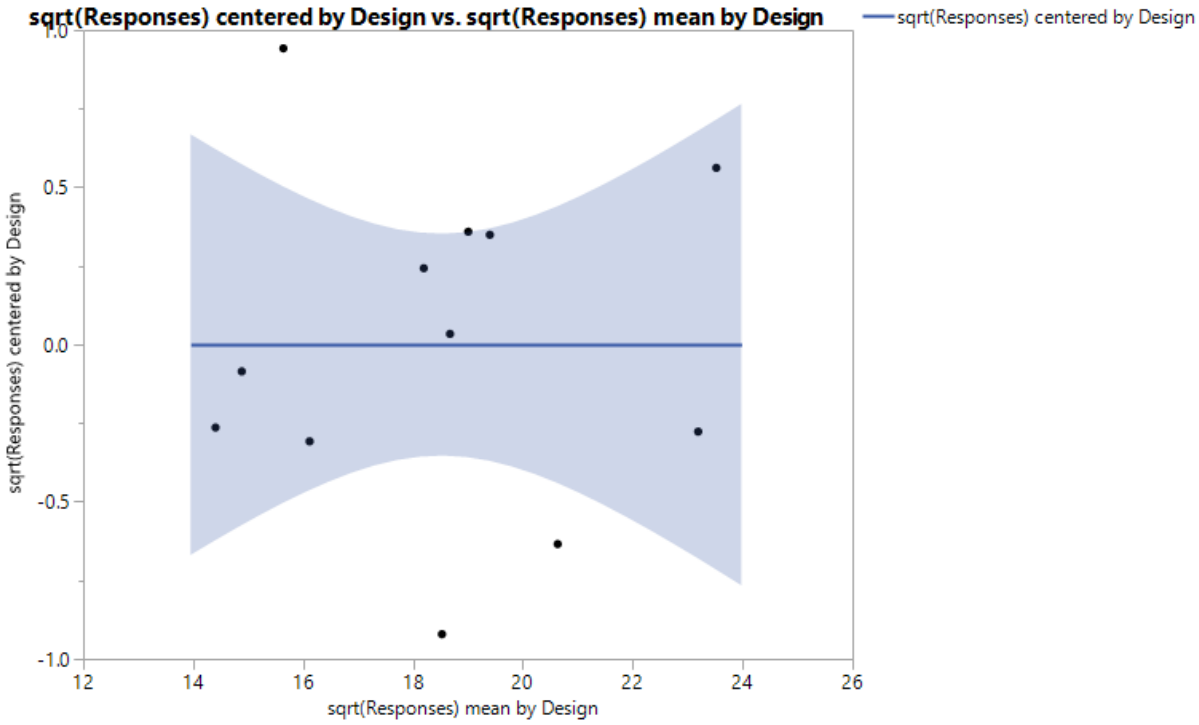


Fig.12 *Residual vs. Predicted*

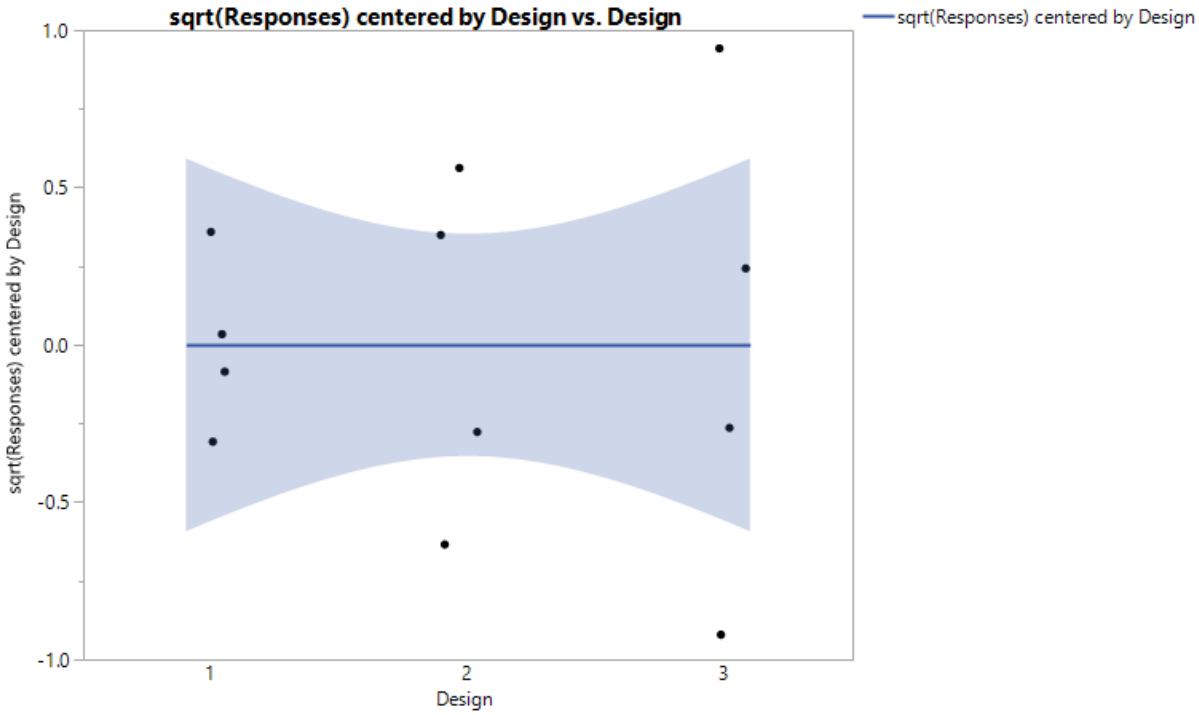


Fig.13 *Residual vs. Design*

This plot has been improved with transformed data although range of variation of one of the materials is lower than others.

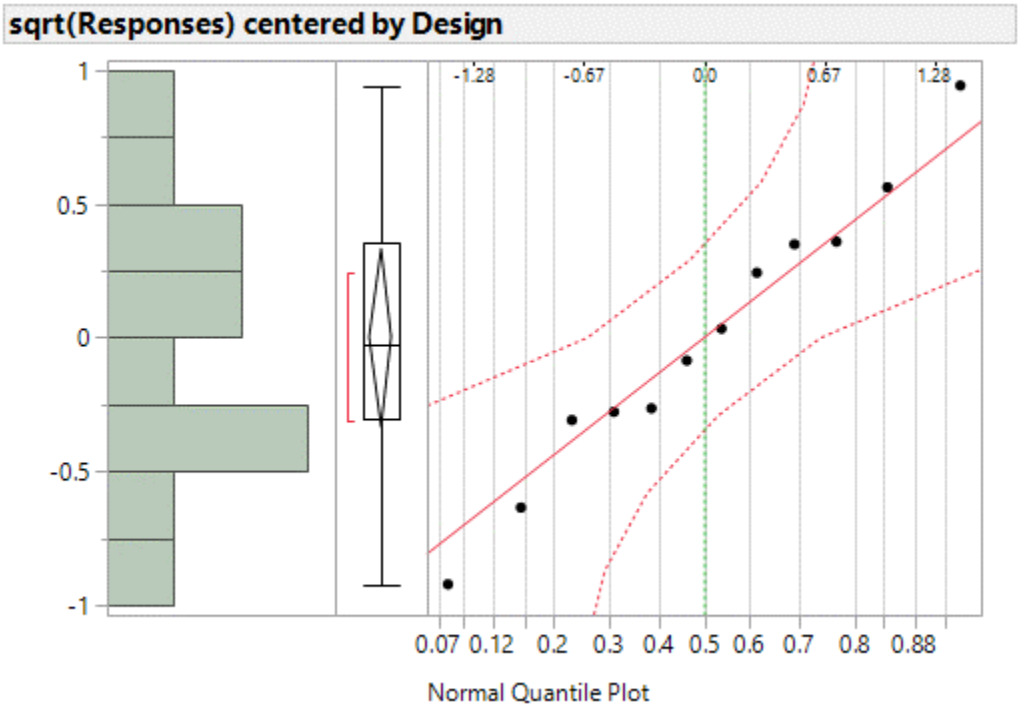
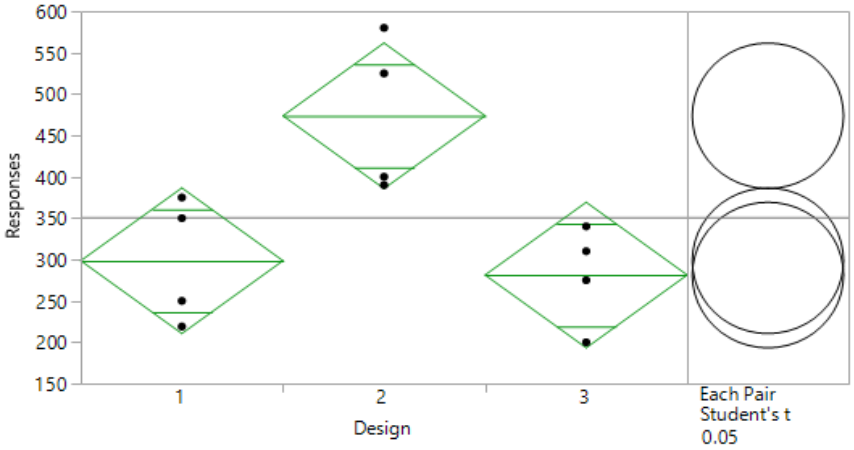


Fig.14 *Normal quantile plot*

This shows that all points lie close to the line and within the error bounds. Hence the assumptions are not violated.

b. *Use the Fisher LSD method to make comparisons among the three designs to determine specifically which designs differ in the mean response rate*

Oneway Analysis of Responses By Design



Oneway Anova

Summary of Fit

Rsquare	0.624951
Adj Rsquare	0.541607
Root Mean Square Error	77.7921
Mean of Response	351.1667
Observations (or Sum Wgts)	12

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Design	2	90755.17	45377.6	7.4984	0.0121*
Error	9	54464.50	6051.6		
C. Total	11	145219.67			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1	4	298.500	38.896	210.51	386.49
2	4	473.750	38.896	385.76	561.74
3	4	281.250	38.896	193.26	369.24

Std Error uses a pooled estimate of error variance

Means Comparisons

Comparisons for each pair using Student's t

Confidence Quantile

t	Alpha
2.26216	0.05

LSD Threshold Matrix

Abs(Dif)-LSD

	2	1	3
2	-124.44	50.81	68.06
1	50.81	-124.44	-107.19
3	68.06	-107.19	-124.44

Positive values show pairs of means that are significantly different.

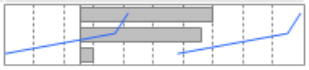
Connecting Letters Report

Level		Mean
2	A	473.75000
1	B	298.50000
3	B	281.25000

Levels not connected by same letter are significantly different.

Ordered Differences Report

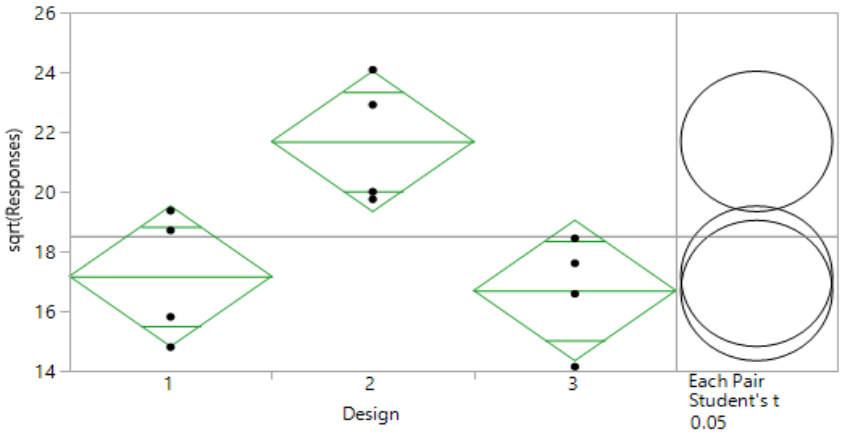
Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
2	3	192.5000	55.00732	68.065	316.9352	0.0067*
2	1	175.2500	55.00732	50.815	299.6852	0.0111*
1	3	17.2500	55.00732	-107.185	141.6852	0.7610



From Fisher's LSD analysis, we see the same result. Design 2 has a significantly different mean than the others. Design 1 and 3 cannot be differentiated for $\alpha = 0.05$

Fig. 15 *Oneway ANOVA of Responses by Design*

Oneway Analysis of sqrt(Responses) By Design



Oneway Anova

Summary of Fit

Rsquare	0.60954
Adj Rsquare	0.522771
Root Mean Square Error	2.079117
Mean of Response	18.51657
Observations (or Sum Wgts)	12

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Design	2	60.733175	30.3666	7.0249	0.0145*
Error	9	38.904554	4.3227		
C. Total	11	99.637729			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1	4	17.1708	1.0396	14.819	19.522
2	4	21.6861	1.0396	19.334	24.038
3	4	16.6928	1.0396	14.341	19.044

Std Error uses a pooled estimate of error variance

Means Comparisons

Comparisons for each pair using Student's t

Confidence Quantile

t	Alpha
2.26216	0.05

LSD Threshold Matrix

Abs(Dif)-LSD

	2	1	3
2	-3.3257	1.1896	1.6676
1	1.1896	-3.3257	-2.8477
3	1.6676	-2.8477	-3.3257

Positive values show pairs of means that are significantly different.

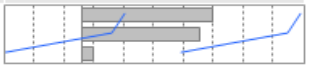
Connecting Letters Report

Level		Mean
2	A	21.686121
1	B	17.170810
3	B	16.692791

Levels not connected by same letter are significantly different.

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
2	3	4.993330	1.470158	1.66760	8.319058	0.0079*
2	1	4.515311	1.470158	1.18958	7.841039	0.0133*
1	3	0.478019	1.470158	-2.84771	3.803747	0.7525



After transformation, from Fisher's LSD analysis, we see the same result. Design 2 has a significantly different mean than the others. Design 1 and 3 cannot be differentiated for $\alpha = 0.05$

Fig. 16 *Oneway ANOVA of of sqrt(Responses) by Design*

c. *Analyze the residuals from this experiment.*

From the residual plots of untransformed data, it could be seen that there are issues with normality and equality of variance. The residual plots of transformed data do not show those issues and do not violate the assumptions of normality.

5. *The effect of five different ingredients (A, B, C, D, E) on the reaction time of a chemical process is being studied. Each batch of new material is only large enough to permit five runs to be made. Furthermore, each run requires approximately* $1\,{}^{1}/_{2}$ *hours, so only five runs can be made in one day. The experimenter decides to run the experiment as a Latin square so that day and batch effects may be systematically controlled. She obtains the data that follow. Analyze the data from this experiment (use,*$\alpha = 0.05$*) and draw conclusions.*

Batch	***Day***				
	1	***2***	***3***	***4***	***5***
1	*A=8*	*B=7*	*D=1*	*C=7*	*E=3*
2	*C=11*	*E=2*	*A=7*	*D=3*	*B=8*
3	*B=4*	*A=9*	*C=10*	*E=1*	*D=5*
4	*D=6*	*C=8*	*E=6*	*B=6*	*A=10*
5	*E=4*	*D=2*	*B=3*	*A=8*	*C=8*

Solution:

Response Reaction time

Effect Summary

Source	LogWorth	PValue
Ingredients	3.312	0.00049
Batch	0.459	0.34762
Day	0.342	0.45501

Summary of Fit

RSquare	0.818428
RSquare Adj	0.636856
Root Mean Square Error	1.768238
Mean of Response	5.88
Observations (or Sum Wgts)	25

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	12	169.12000	14.0933	4.5075
Error	12	37.52000	3.1267	Prob > F
C. Total	24	206.64000		0.0072*

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Batch	4	4	15.44000	1.2345	0.3476
Ingredients	4	4	141.44000	11.3092	0.0005
Day	4	4	12.24000	0.9787	0.4550

Fig. 17 *Fit Least Square*

JMP says that, the P value of ingredients is 0.0005 which is less than $\alpha = 0.05$. This shows that the Ingredients has a significant effect on Response Reaction time.

The other two P values (Batch = 0.3476 and Day=0.4550) are greater than $\alpha = 0.05$. This concludes that there is no significant difference in the batch and no significant difference between the days.