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EGR 7050 Design and Analysis of Engineering experiments

Homework 2

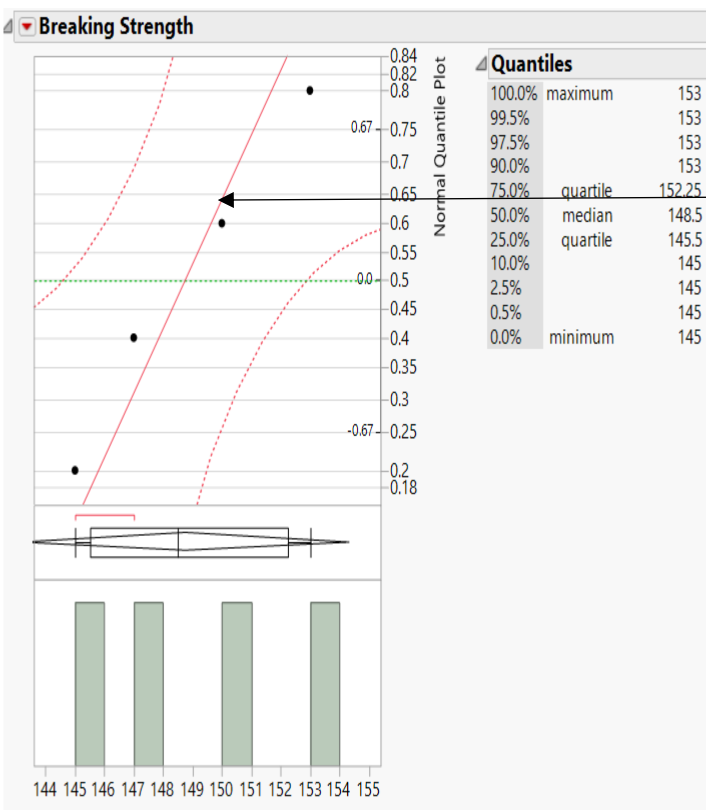
1. *The breaking strength of a fiber is required to be at least 150 psi. Past experience has indicated that the standard deviation of breaking strength is* $\sigma = 3$ *psi. A random sample of four specimens is tested, and the results are y1 =145, y2 =153, y3 = 150, and y4 =147.*
*(**a**) State the hypotheses that you think should be tested in this experiment.*
*(**b**) Test these hypotheses using* α*= 0.05. What are your conclusions?*
*(**c**) Find the P-value for the test in part (b).*
*(**d**) Construct a 95 percent confidence interval on the mean breaking strength.*

Solution:

a) $H_0: \mu = 150 \qquad H_1: \mu > 150$
It is given in the question that, the breaking strength is required to be **at least** 150. Therefore, a test for $H_1: \mu > 150$ has been chosen.

b) α *= 0.05*

Output from JMP:



This Normal probability plot shows that all the points lie close to the line and within the error bounds.

__Fig. 1__ Normal quantile plot

Summary Statistics

Mean	148.75
Std Dev	3.5
Std Err Mean	1.75
Upper 95% Mean	154.31928
Lower 95% Mean	143.18072
N	4

__Fig. 2__ Summary statistics

Test Mean

Hypothesized Value	150
Actual Estimate	148.75
DF	3
Std Dev	3.5
Sigma given	3

	z Test
Test Statistic	-0.8333
Prob > |z|	0.4047
Prob > z	0.7977
Prob < z	0.2023

Sample mean

Standard deviation

2.16 c) Represents P value for $H_1: \mu > 150$. Larger P value shows that there is no strong evidence to reject H_0

__Fig. 3__ Test Mean

Confidence Intervals

Parameter	Estimate	Lower CI	Upper CI	1-Alpha	Sigma
Mean	148.75	145.8101	151.6899	0.950	3.000
Std Dev	3.5	1.982713	13.04992		

__Fig. 4__ Confidence Intervals

2.16 d) 95% confidence interval of mean breaking strength

One sample z test

n = 4, σ=3, $\bar{y} = \frac{145+153+150+147}{4}$ = 148.75, $\mu_0 = 150$

$$Z_0 = \frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}}$$

$$= \frac{148.75-150}{3/2}$$

$= -2.5/3 =$ **- 0.8333**

From z table, $Z_{0.05}$=1.645. Z_0 is not greater than $Z_{0.05}$. Hence, H_0 cannot be rejected

c) P-value is P = 1 – φ(-0.8333) = 1 – 0.2033 = **0.7967**

d) 95% C.I can be found by, $\bar{y} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{y} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$= 148.75 - (1.96)\left(\frac{3}{2}\right) \leq \mu \leq 148.75 + (1.96)\left(\frac{3}{2}\right)$

$= \mathbf{145.81 \leq \mu \leq 151.69}$

2. *The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:*

Days

108	138
124	163
124	159
106	134
115	139

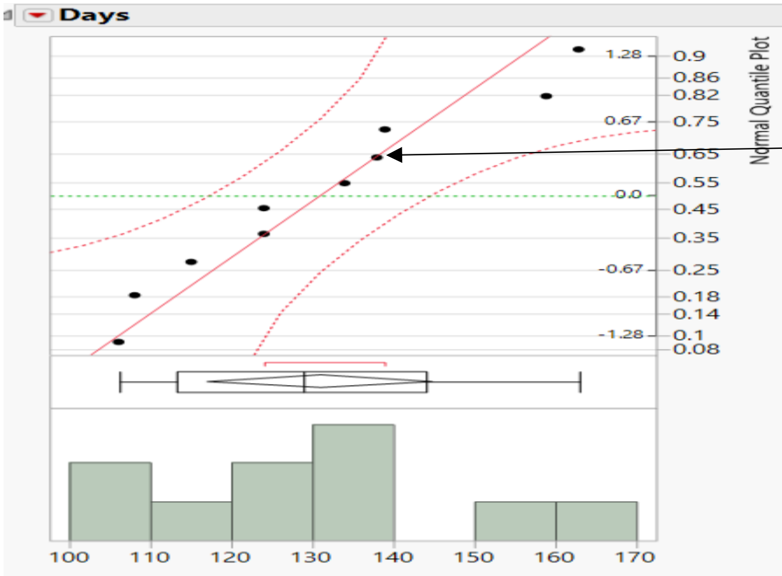
a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.
b) Test these hypotheses using 0.01. What are your conclusions?
c) Find the P-value for the test in part (b).
d) Construct a 99 percent confidence interval on the mean shelf life.

Solution:

a) To find out if mean shelf life exceeds 120 days, the hypotheses would be

$H_0: \mu = 120 \qquad H_1: \mu > 120$

b) $\alpha = 0.01$



Normal quantile plot shows that all points lie close to line of fit and within error bounds.

Fig. 5 Normal quantile plot

Quantiles

100.0%	maximum	163
99.5%		163
97.5%		163
90.0%		162.6
75.0%	quartile	144
50.0%	median	129
25.0%	quartile	113.25
10.0%		106.2
2.5%		106
0.5%		106
0.0%	minimum	106

Summary Statistics

Mean	131
Std Dev	19.54482
Std Err Mean	6.1806149
Upper 95% Mean	144.98152
Lower 95% Mean	117.01848
N	10

Fig. 6 Summary statistics

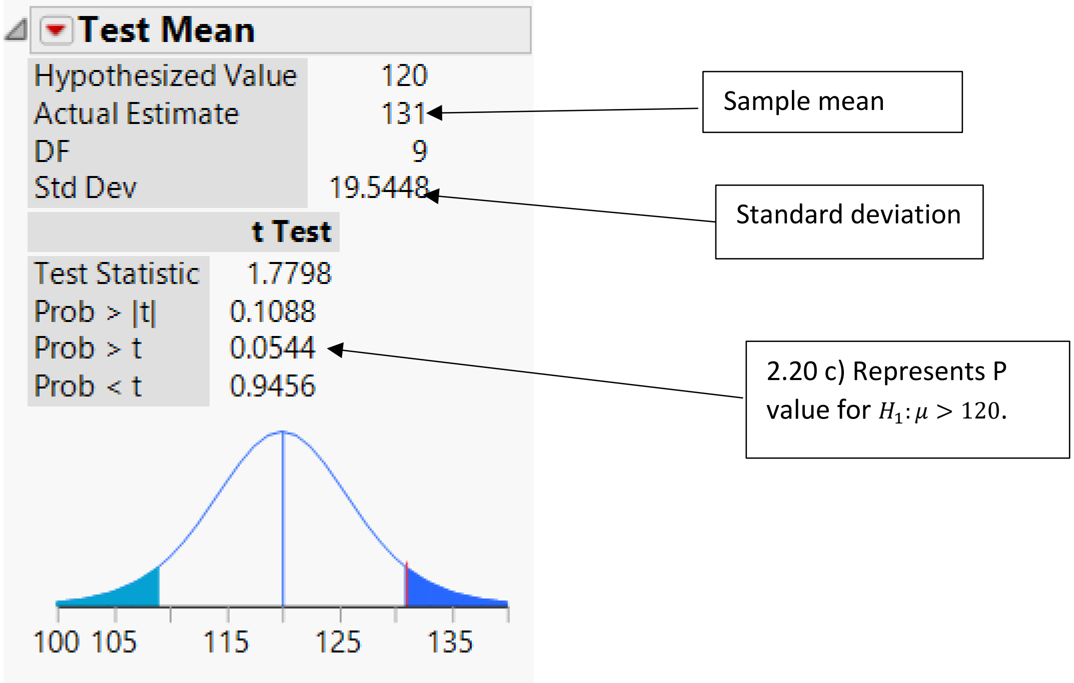


Fig. 7 Test mean

Confidence Intervals

Parameter	Estimate	Lower CI	Upper CI	1-Alpha
Mean	131	110.914	151.086	0.990
Std Dev	19.54482	12.07244	44.51553	0.990

2.20 d) Upper and lower limits of 99% confidence interval

Fig. 8 Confidence intervals

$\bar{y} = 131,$ $\quad S^2 = \sum_{i=1}^{10} \frac{(y_i - \bar{y})^2}{n-1}$ = 529+49+49+625+256+49+1024+784+9+64/9 = 382

S = $\sqrt{382}$ = 19.54

$t_0 = (\bar{y} - \mu_0)/(s/\sqrt{n})$ = 131-120 / (19.54/$\sqrt{10}$)
=11/6.18
= 1.78

$t_{0.01,9} = 2.821$

$\mathbf{t_0}$ **not greater than** $\mathbf{t_{0.01,9}}$**. Therefore,** $\mathbf{H_0}$ **cannot be rejected.**

c) P-value: **P =0.0544** for $H_1: \mu > 120$ (from JMP)

d) The 99% CI is , $\bar{y} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{y} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$

$131 - (3.250)(6.18) \leq \mu \leq 131 + (3.250)(6.18)$ = $\mathbf{110.915 \leq \mu \leq 151.0}$

3. Consider the shelf life data in Problem 2.20. Can shelf life be described or modeled adequately by a normal distribution? What effect would the violation of this assumption have on the test procedure you used in solving Problem 2.15?

Solution:

From the normal quantile plot of the previous problem, it could be seen that all points lie close to the normal line and within the error bounds. Therefore, it can be modeled by a normal distribution.

4. Twenty observations on etch uniformity on silicon wafers are taken during a qualification experiment for a plasma etcher. The data are as follows:

5.34	*6.65*	*4.76*	*5.98*	*7.25*
6.00	*7.55*	*5.54*	*5.62*	*6.21*
5.97	*7.35*	*5.44*	*4.39*	*4.98*
5.25	*6.35*	*4.61*	*6.00*	*5.32*

a) *Construct a 95 percent confidence interval estimate of* σ^2.
b) *Test the hypothesis that* $\sigma^2 = 1.0$. *Use* $\alpha = 0.05$. *what are your conclusions?*
c) *Discuss the normality assumption and its role in this problem.*
d) *Check normality by constructing a normal probability plot. What are your conclusions?*

Solution:

a) $\frac{(n-1)S^2}{\aleph^2_{\alpha/2, n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\aleph^2_{1-(\alpha/2), n-1}}$

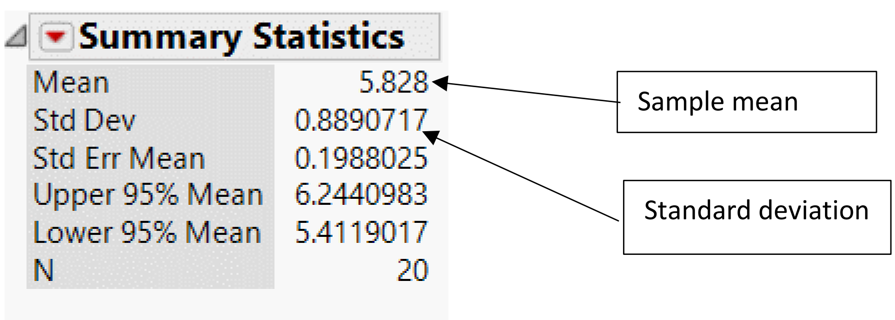


Fig. 9 Summary statistics

From JMP, S = 0.8891, $S^2 = 0.7905$

C.I is $\frac{(n-1)S^2}{\aleph^2_{\alpha/2, n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\aleph^2_{1-(\alpha/2), n-1}}$

$$\frac{(19)0.7905}{32.852} \le \sigma^2 \le \frac{(19)0.7905}{8.907}$$

$$\mathbf{0.457 \le \sigma^2 \le 1.686}$$

b) *Test the hypothesis that* $\sigma^2 = 1.0$. *Use* $\alpha = 0.05$. *what are your conclusions?*

$H_0\!: \sigma^2 = 1 \qquad H_1\!: \sigma^2 \neq 1$

Test statistic:

$$\aleph_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

$$= \frac{19(0.7905)}{1}$$

= **15.0195**

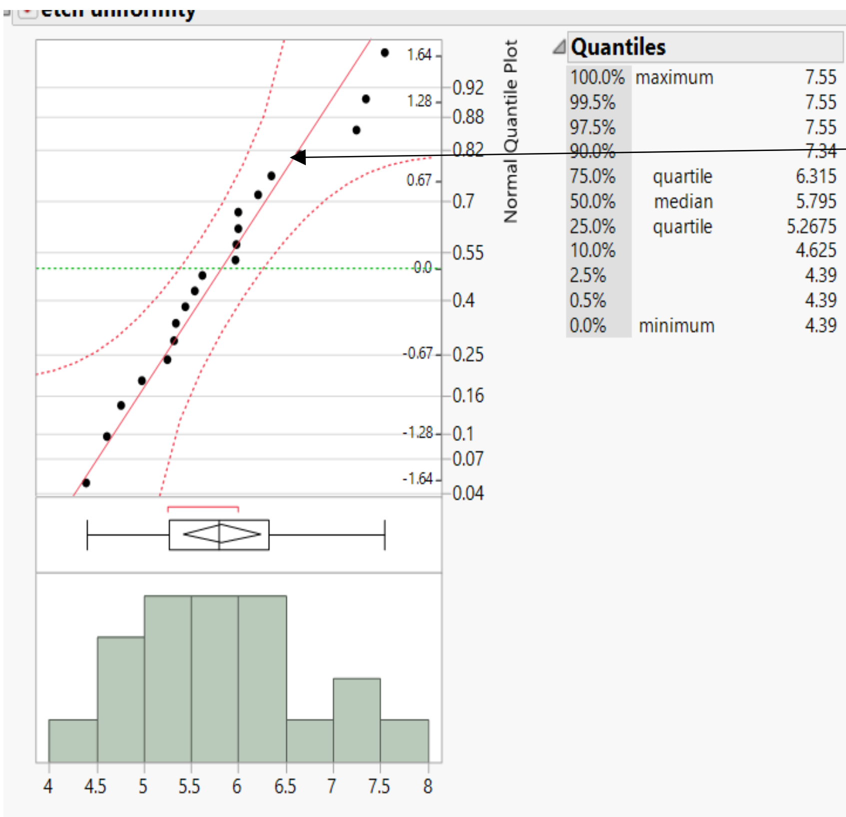
Thus, $\aleph_0^2$ is neither greater than $\aleph^2_{\alpha/2,n-1}$ nor less than $\aleph^2_{1-(\alpha/2),n-1}$. Hence, H_0 cannot be rejected.

It is concluded that $\sigma^2 = 1$

c) *Discuss the normality assumption and its role in this problem.*

Normality assumption is important in variance test. Normality can be assumed only if the data is very close to the line else this might lead to incorrect conclusions.

d) *Check normality by constructing a normal probability plot. What are your conclusions?*



Normal quantile plot all points lie close to line though within the error bounds.

Fig. 10 Normal quantile plot

Test Standard Deviation

Hypothesized Value	1
Actual Estimate	0.88907
DF	19

Test	ChiSquare
Test Statistic	15.01852
Min PValue	0.5572
Prob < ChiSq	0.2786
Prob > ChiSq	0.7214

P value is 0.5572. (greater than 0.05) Hence null hypothesis $H_0: \sigma^2 = 1$

Fig. 11 Test statistic

Confidence Intervals

Parameter	Estimate	Lower CI	Upper CI	1-Alpha
Mean	5.828	5.411902	6.244098	0.950
Std Dev	0.889072	0.67613	1.298553	0.950

95% CI of standard deviation

Fig. 12 Confidence intervals