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EGR 7050 Design and Analysis of Engineering experiments

Homework 5

1. *Four different designs for a digital computer circuit are being studied to compare the amount of noise present. The following data have been obtained:*

Circuit Design	***Noise Observed***				
1	*19*	*20*	*19*	*30*	*8*
2	*80*	*61*	*73*	*56*	*80*
3	*47*	*26*	*25*	*35*	*50*
4	*95*	*46*	*83*	*78*	*97*

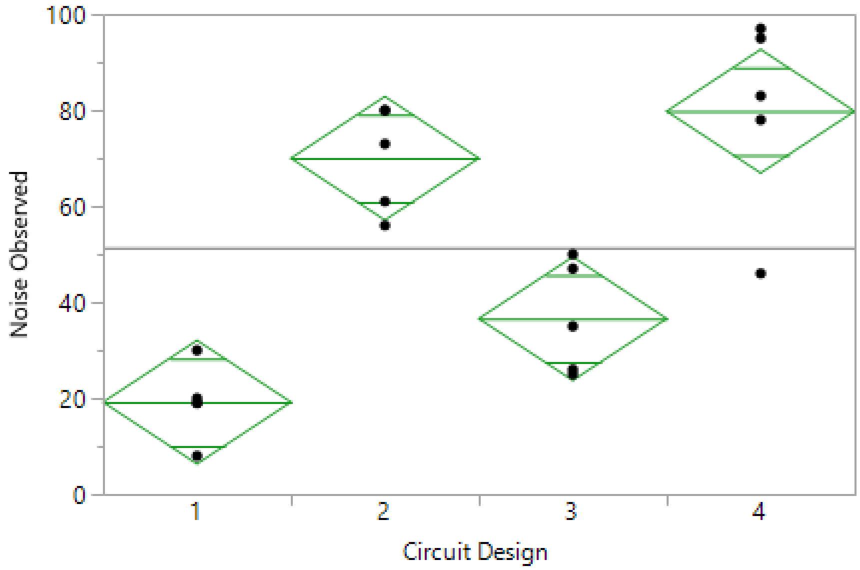
a. *Is the same amount of noise present for all four designs? Use α =0.05.*

Solution:

H_0: *amount of noise present in all four designs are same*
H_1: *amount of noise present in at least one design is not same*

Given, $\alpha = 0.05$

Oneway Analysis of Noise Observed By Circuit Design



Oneway Anova

Summary of Fit

Rsquare	0.803293
Adj Rsquare	0.76641
Root Mean Square Error	13.57571
Mean of Response	51.4
Observations (or Sum Wgts)	20

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Circuit Design	3	12042.000	4014.00	21.7797	<.0001*
Error	16	2948.800	184.30		
C. Total	19	14990.800			

P value is less than the significance level, $\alpha = 0.05$. Thus, null hypothesis could be rejected.

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1	5	19.2000	6.0712	6.330	32.070
2	5	70.0000	6.0712	57.130	82.870
3	5	36.6000	6.0712	23.730	49.470
4	5	79.8000	6.0712	66.930	92.670

Std Error uses a pooled estimate of error variance

***Fig. 1** Oneway ANOVA*

Thus, it could be concluded that amount of noise present in at least one design is not same.

b. *Analyze the residuals from this experiment. Are the analysis of variance assumptions satisfied?*

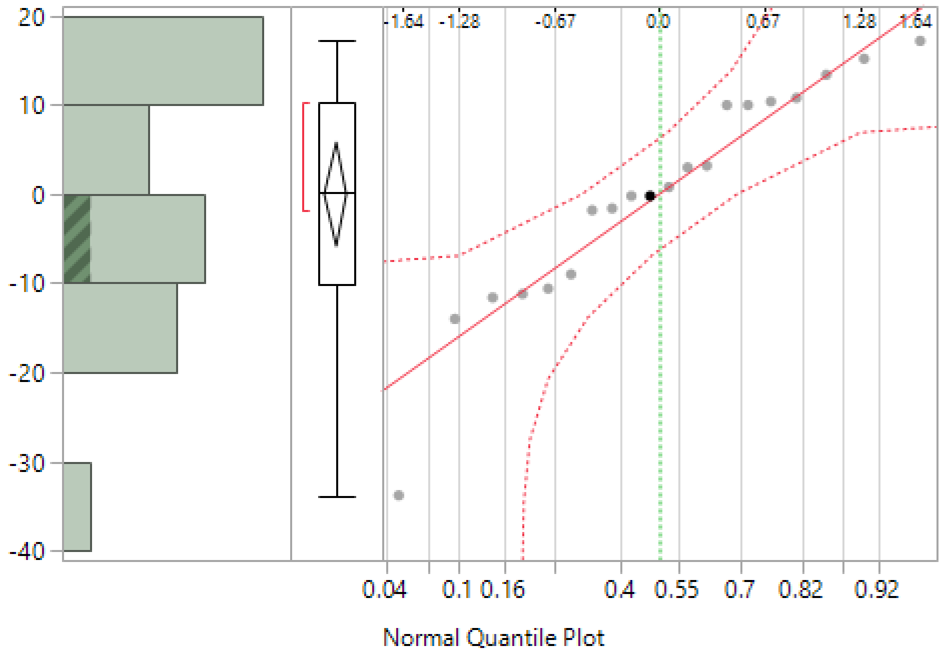


Fig. 2 *Normal quantile plot*

Many points are close to the line and are within the error bounds. There is no significant evidence of deviation from normality for the residuals.

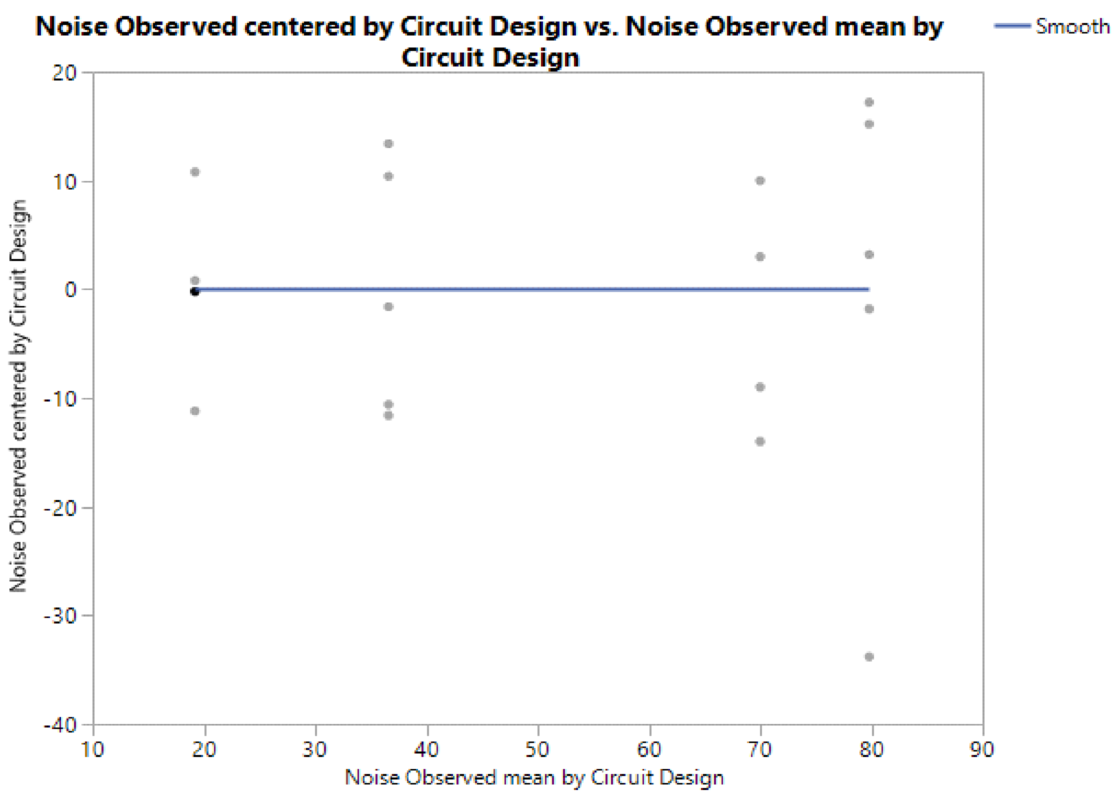


Fig. 3 *Residual vs. Fitted*

There are no outliers in the residual vs. fitted plots. There is a similar range of variation across different fitted values. There is no significant deviation from equal variance assumption.

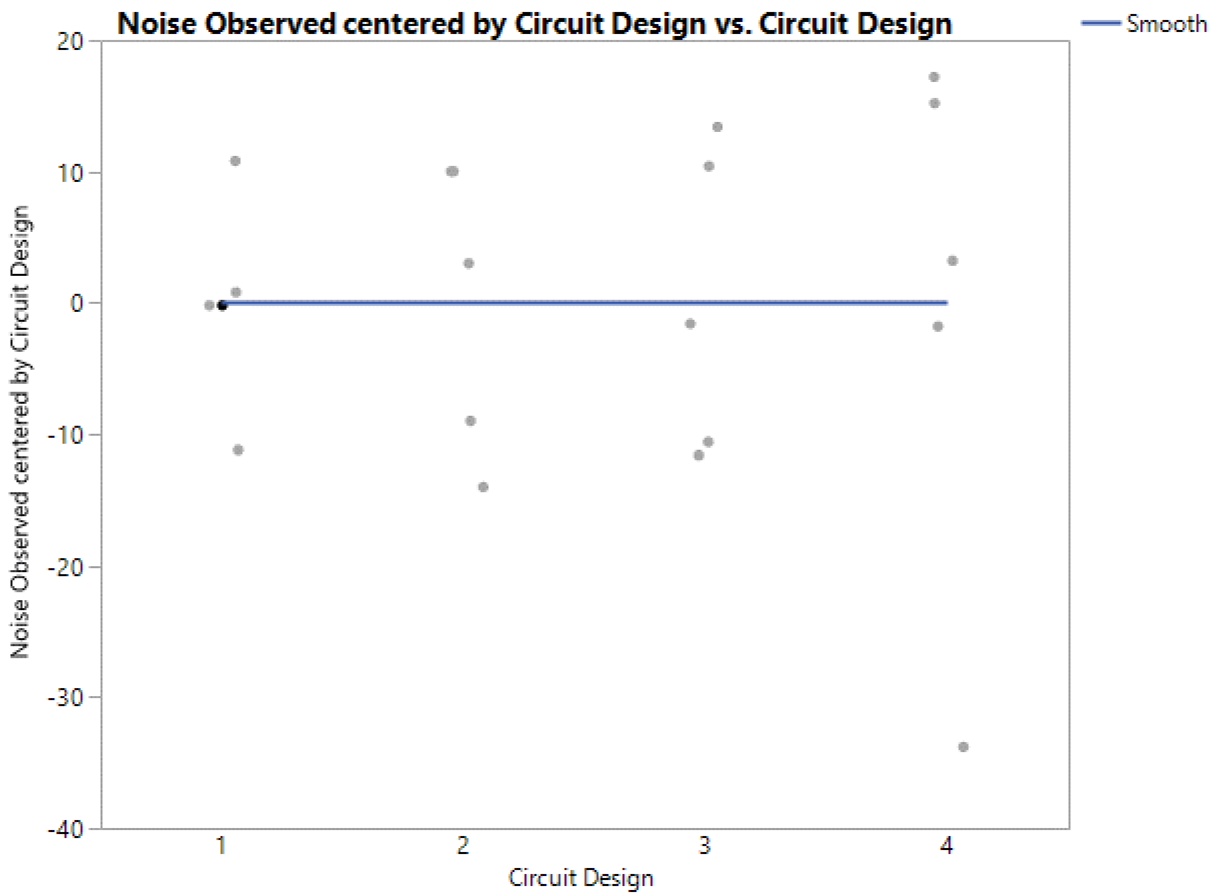


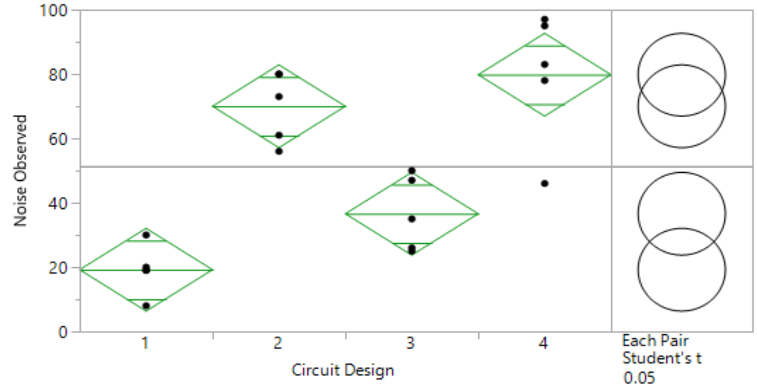
Fig. 4 *Residual vs. circuit design*

The above plot indicates a constant variance.

From the above figures, it could be concluded that analysis of variance assumptions are satisfied.

c. *Which circuit design would you select for use? Low noise is best.*

Oneway Analysis of Noise Observed By Circuit Design



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3	5	36.6000	6.0712	23.730	49.470
4	5	79.8000	6.0712	66.930	92.670

Std Error uses a pooled estimate of error variance

Means Comparisons

Comparisons for each pair using Student's t

Confidence Quantile

t	Alpha
2.11991	0.05

LSD Threshold Matrix

Abs(Dif)-LSD

	4	2	3	1
4	-18.202	-8.402	24.998	42.398
2	-8.402	-18.202	15.198	32.598
3	24.998	15.198	-18.202	-0.802
1	42.398	32.598	-0.802	-18.202

Positive values show pairs of means that are significantly different.

Connecting Letters Report

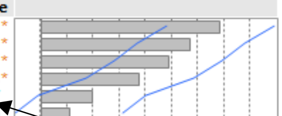
Level			Mean
4	A		79.800000
2	A		70.000000
3		B	36.600000
1		B	19.200000

Levels not connected by same letter are significantly different.

From Fisher's LSD analysis, we see that levels 4,2 are different from levels 1,3. We cannot find a significant difference between levels 4&2 and levels 3&1.

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
4	1	60.60000	8.586035	42.3984	78.80158	<.0001*
2	1	50.80000	8.586035	32.5984	69.00158	<.0001*
4	3	43.20000	8.586035	24.9984	61.40158	0.0001*
2	3	33.40000	8.586035	15.1984	51.60158	0.0013*
3	1	17.40000	8.586035	-0.8016	35.60158	0.0597
4	2	9.80000	8.586035	-8.4016	28.00158	0.2705



The p value for difference between level 1 & 3 is greater than $\alpha = 0.05$.

***Fig. 5** Oneway ANOVA*

Therefore, it could be concluded that circuit design 1 is having the lower noise.

2. Reconsider the experiment described in Problem 3.24. Suppose that Circuit Design 1 is a control. Use Dunnett's test with α = 0.05 to compare all of the other means with the control.

Solution:

Comparisons with a control using Dunnett's Method

Control Group = 1

Confidence Quantile

|d|	Alpha
2.59232	0.05

LSD Threshold Matrix

Level	Abs(Dif)-LSD	p-Value
4	38.34	<.0001*
2	28.54	<.0001*
3	-4.86	0.1431
1	-22.3	1.0000

Positive values show pairs of means that are significantly different.

Fig. 6 *Dunnett's test*

This shows that, level 4 and 2 treatment means are significantly different from the control group but not level 3.

3. The ANOVA from a randomized complete block experiment output is shown below.

Source	DF	SS	MS	F	P
Treatment	4	1010.56	?	29.84	?
Block	?	?	64.765	?	?
Error	20	169.33	?		
Total	29	1503.71			

Solution:

a. Fill in the blanks. You may give bounds on the P-value.

$$DF_{Error} = (a-1)(b-1) = 20$$
$$4(b-1) = 20$$
$$\boldsymbol{(b-1) = 5}$$

$$SS_E = SS_T - SS_{Treatments} - SS_{Blocks}$$
$$169.33 = 1503.71 - 1010.56 - SS_{Blocks}$$
$$\boldsymbol{SS_{Blocks} = 323.82}$$

$$MS_{Treatment} = SS_{Treatments}/(a-1)$$
$$1010.56/4 = \boldsymbol{252.64}$$

$MS_{Error} = SS_{Error}/(a-1)(b-1)$
$169.33/20$=***8.4665***

F- value of block **=** MS_{Block}/MS_{Error} *= 64.765/8.4665 =* ***7.65***

From the P value calculator, P-value for treatment is ***less than 0.0001***
P-value for block is ***0.0004***

b. *How many blocks were used in this experiment?*

The degrees of freedom for block is 5.

$(b-1) = 5$

No. of blocks $\boldsymbol{b = 6}$

c. *What conclusions can you draw?*

$\mathrm{H_0}$: *All treatment means are same*
$\mathrm{H_1}$: *At least one mean is different*

At significance level $\alpha = 0.05$, *0.0001< 0.05. Therefore, null hypothesis could be rejected. It could therefore be concluded that there exists a difference between treatment means.*

$\mathrm{H_0}$: *No significant difference in blocks*
$\mathrm{H_1}$: *There exists a significant difference in blocks*

At significance level $\alpha = 0.05$, *0.0004 < 0.05. Therefore, null hypothesis could be rejected. It could therefore be concluded that there exists a significant difference in block.*

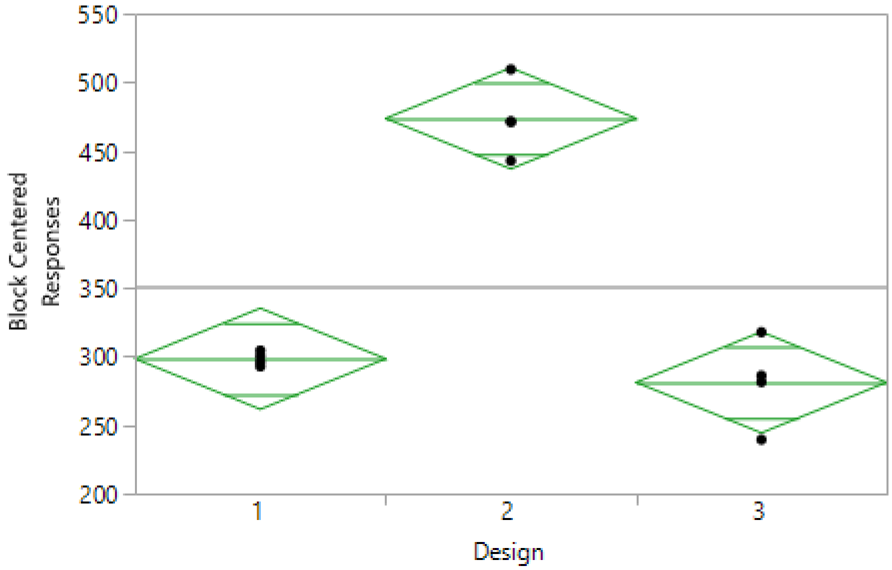
4. *A consumer products company relies on direct mail marketing pieces as a major component of its advertising campaigns. The company has three different designs for a new brochure and wants to evaluate their effectiveness, as there are substantial differences in costs between the three designs. The company decides to test the three designs by mailing 5000 samples of each to potential customers in four different regions of the country. Since there are known regional differences in the customer base, regions are considered as blocks. The number of responses to each mailing is as follows.*

Design	***Region***			
	NE	***NW***	***SE***	***SW***
1	*250*	*350*	*219*	*375*
2	*400*	*525*	*390*	*580*
3	*275*	*340*	*200*	*310*

Solution:

a. Analyze the data from this experiment.

Oneway Analysis of Responses By Design



Oneway Anova

Summary of Fit

Rsquare	0.962616
Adj Rsquare	0.931463
Root Mean Square Error	30.07999
Mean of Response	351.1667
Observations (or Sum Wgts)	12

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Design	2	90755.17	45377.6	50.1518	0.0002*
Region	3	49035.67	16345.2	18.0649	0.0021*
Error	6	5428.83	904.8		
C. Total	11	145219.67			

P value for both the treatment and block is less than $\alpha = 0.05$.

Block Region

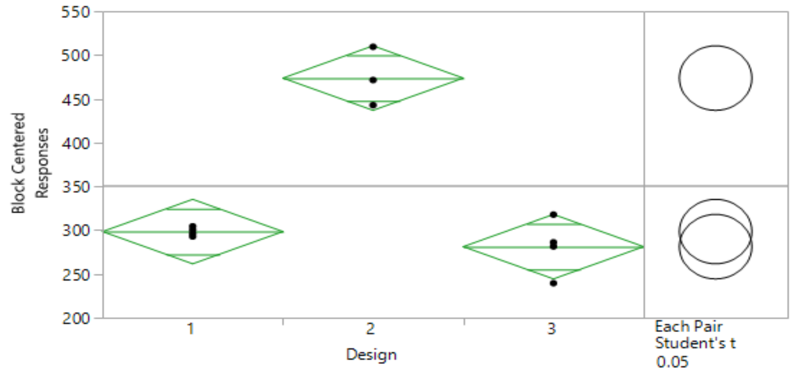
Fig. 7 *ANOVA*

H_0: *No significant difference in design*
H_1: *There exists a significant difference in design*

For both design and region, P-value is less than $\alpha = 0.05$, therefore null hypothesis could be rejected. Thus, it could be concluded that there exists a significant difference in design.

b. *Use the Fisher LSD method to make comparisons among the three designs to determine specifically which designs differ in the mean response rate*

Oneway Analysis of Responses By Design



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Design	2	90755.17	45377.6	50.1518	0.0002*
Region	3	49035.67	16345.2	18.0649	0.0021*
Error	6	5428.83	904.8		
C. Total	11	145219.67			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1	4	298.500	15.040	261.70	335.30
2	4	473.750	15.040	436.95	510.55
3	4	281.250	15.040	244.45	318.05

Std Error uses a pooled estimate of error variance

Block Means

Region	Mean	Number
NE	308.333	3
NW	405.000	3
SE	269.667	3
SW	421.667	3

Means Comparisons

Comparisons for each pair using Student's t

Confidence Quantile

t	Alpha
2.44691	0.05

LSD Threshold Matrix

Abs(Dif)-LSD

	2	1	3
2	-52.05	123.20	140.45
1	123.20	-52.05	-34.80
3	140.45	-34.80	-52.05

Positive values show pairs of means that are significantly different.

Connecting Letters Report

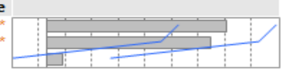
Level			Mean
2	A		473.75000
1		B	298.50000
3		B	281.25000

Levels not connected by same letter are significantly different.

From Fisher's LSD analysis, we see that Design 2 has a significantly different mean than the others. Design 1 and 3 cannot be differentiated for $\alpha = 0.05$

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
2	3	192.5000	21.26976	140.455	244.5452	0.0001*
2	1	175.2500	21.26976	123.205	227.2952	0.0002*
1	3	17.2500	21.26976	-34.795	69.2952	0.4483



Block Region

Fig. 8 *Oneway ANOVA*

c. Analyze the residuals from this experiment.

5. *The effect of five different ingredients (A, B, C, D, E) on the reaction time of a chemical process is being studied. Each batch of new material is only large enough to permit five runs to be made. Furthermore, each run requires approximately* $1\,{}^{1}/{}_{2}$ *hours, so only five runs can be made in one day. The experimenter decides to run the experiment as a Latin square so that day and batch effects may be systematically controlled. She obtains the data that follow. Analyze the data from this experiment (use,*$\alpha = 0.05$*) and draw conclusions.*

Batch	***Day***				
	1	***2***	***3***	***4***	***5***
1	*A=8*	*B=7*	*D=1*	*C=7*	*E=3*
2	*C=11*	*E=2*	*A=7*	*D=3*	*B=8*
3	*B=4*	*A=9*	*C=10*	*E=1*	*D=5*
4	*D=6*	*C=8*	*E=6*	*B=6*	*A=10*
5	*E=4*	*D=2*	*B=3*	*A=8*	*C=8*

Solution:

Response Reaction time

Effect Summary

Source	LogWorth	PValue
Ingredients	3.312	0.00049
Batch	0.459	0.34762
Day	0.342	0.45501

Summary of Fit

RSquare	0.818428
RSquare Adj	0.636856
Root Mean Square Error	1.768238
Mean of Response	5.88
Observations (or Sum Wgts)	25

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	12	169.12000	14.0933	4.5075
Error	12	37.52000	3.1267	Prob > F
C. Total	24	206.64000		0.0072*

Effect Tests

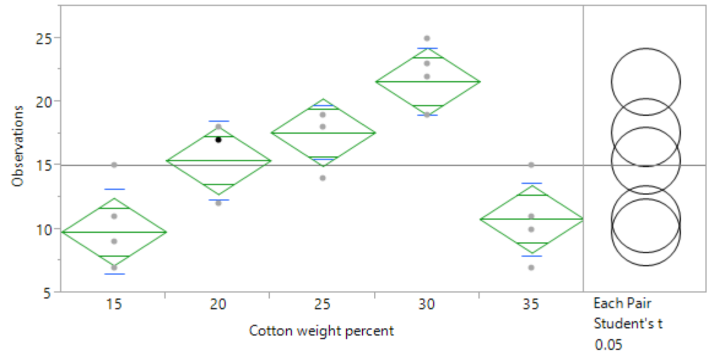
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Batch	4	4	15.44000	1.2345	0.3476
Ingredients	4	4	141.44000	11.3092	$\boxed{0.0005}$
Day	4	4	12.24000	0.9787	0.4550

Fig. *Fit Least Squares*

JMP says that, the P value of ingredients is 0.0005 which is less than $\alpha = 0.05$. This shows that the Ingredients has a significant effect on Response Reaction time.

The other two P values (Batch = 0.3476 and Day=0.4550) are greater than $\alpha = 0.05$. This concludes that there is no significant difference in the batch and no significant difference between the days.

Oneway Analysis of Observations By Cotton weight percent



Oneway Anova

Summary of Fit

Rsquare	0.746923
Adj Rsquare	0.696307
Root Mean Square Error	2.839014
Mean of Response	15.04
Observations (or Sum Wgts)	25

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Cotton weight percent	4	475.76000	118.940	14.7568	<.0001*
Error	20	161.20000	8.060		
C. Total	24	636.96000			

P value is less than significance level $\alpha = 0.05$ and $F_0 > F_{0.05,4,20}$. Therefore, reject null hypotheses.

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
15	5	9.8000	1.2696	7.152	12.448
20	5	15.4000	1.2696	12.752	18.048
25	5	17.6000	1.2696	14.952	20.248
30	5	21.6000	1.2696	18.952	24.248
35	5	10.8000	1.2696	8.152	13.448

Std Error uses a pooled estimate of error variance

Means Comparisons

Comparisons for each pair using Student's t

Confidence Quantile

t	Alpha
2.08596	0.05

LSD Threshold Matrix

Abs(Dif)-LSD

	30	25	20	35	15
30	-3.7455	0.2545	2.4545	7.0545	8.0545
25	0.2545	-3.7455	-1.5455	3.0545	4.0545
20	2.4545	-1.5455	-3.7455	0.8545	1.8545
35	7.0545	3.0545	0.8545	-3.7455	-2.7455
15	8.0545	4.0545	1.8545	-2.7455	-3.7455

Positive values show pairs of means that are significantly different.

Connecting Letters Report

Level				Mean
30	A			21.600000
25		B		17.600000
20		B		15.400000
35			C	10.800000
15			C	9.800000

Levels not connected by same letter are significantly different.

From connecting letters report, we could see that level 30 is different from the rest of the levels. Similarly, levels 25, 20 are different from levels 35, 15. We cannot detect a significant difference between levels 25, 20 and levels 35,15.

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
30	15	11.80000	1.795550	8.05455	15.54545	<.0001*
30	35	10.80000	1.795550	7.05455	14.54545	<.0001*
25	15	7.80000	1.795550	4.05455	11.54545	0.0003*
25	35	6.80000	1.795550	3.05455	10.54545	0.0012*
30	20	6.20000	1.795550	2.45455	9.94545	0.0025*
20	15	5.60000	1.795550	1.85455	9.34545	0.0054*
20	35	4.60000	1.795550	0.85455	8.34545	0.0186*
30	25	4.00000	1.795550	0.25455	7.74545	0.0375*
25	20	2.20000	1.795550	-1.54545	5.94545	0.2347
35	15	1.00000	1.795550	-2.74545	4.74545	0.5838

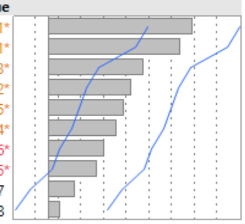


***Fig. 1** OneWay ANOVA*

Thus, it could be concluded that the percentage of cotton used in the fiber affects the mean tensile strength.

a. Analyze the residuals from this experiment and comment on model adequacy.

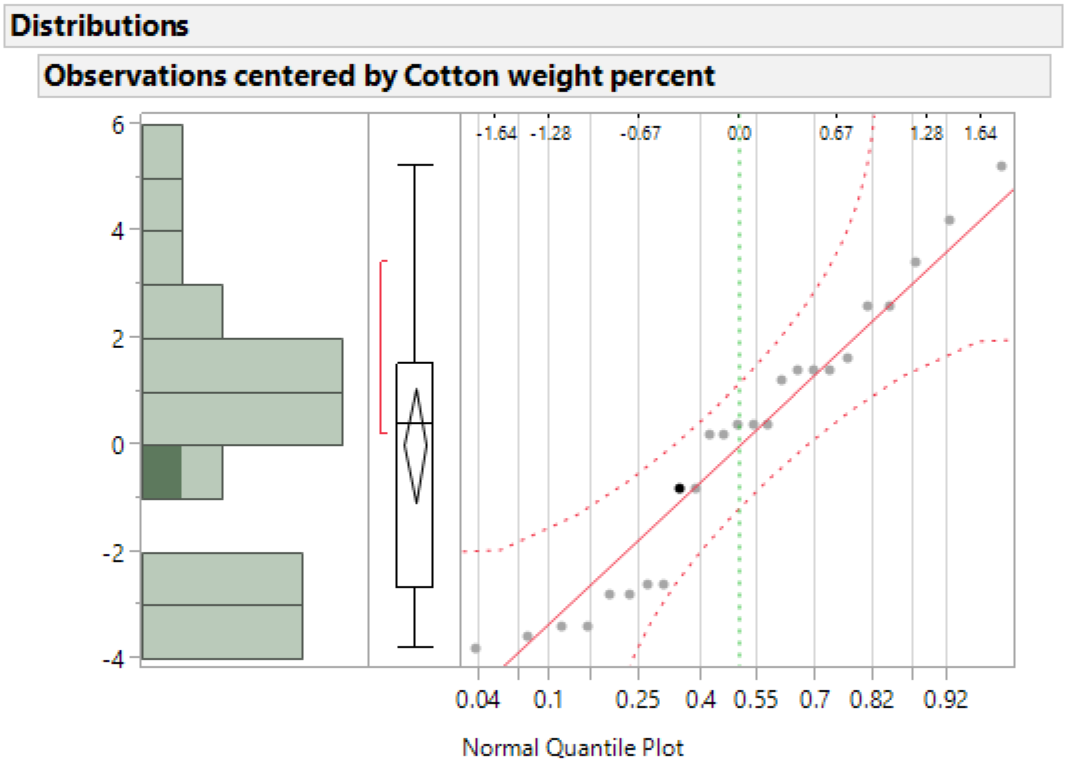


***Fig.2** Normal quantile plot*

Points are close to the line and are within the error bounds. There is no significant evidence of deviation from normality for the residuals.

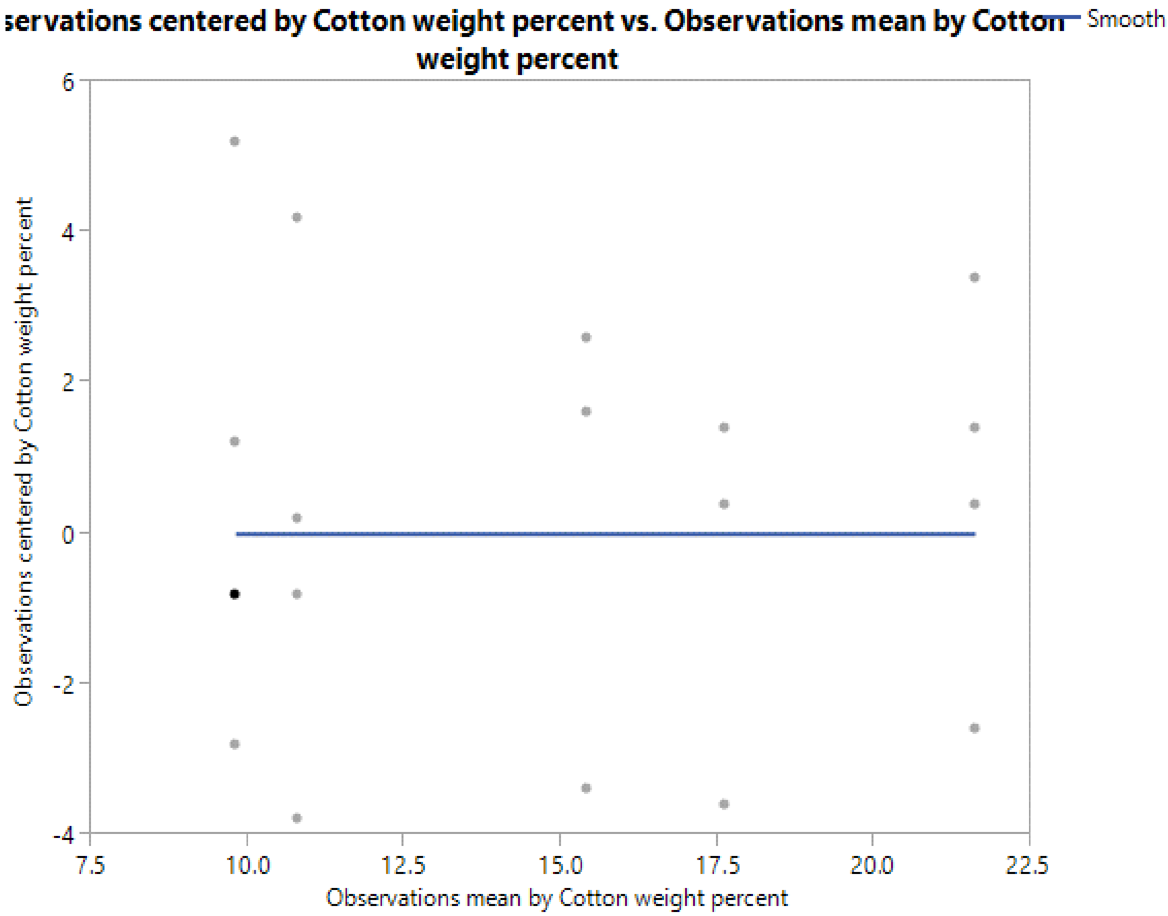


Fig. 3 *Residual vs. Fitted*

There are no outliers in the residual vs. fitted plots. There is a similar range of variation across different fitted values. There is no significant deviation from equal variance assumption.

1. *An experiment was performed to investigate the effectiveness of five insulating materials. Four samples of each material were tested at an elevated voltage level to accelerate the time to failure. The failure times (in minutes) are shown below:*

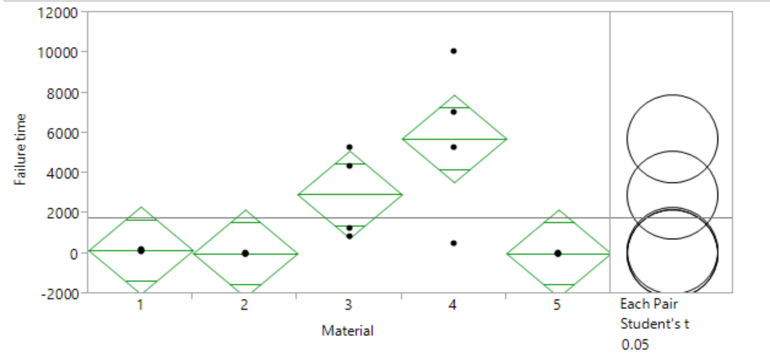
Material	***Failure Time (minutes)***			
1	*110*	*157*	*194*	*178*
2	*1*	*2*	*4*	*18*
3	*880*	*1256*	*5276*	*4355*
4	*495*	*7040*	*5307*	*10050*
5	*7*	*5*	*29*	*2*

Solution:

a. *Do all five materials have the same effect on mean failure time?*

H_0: *Mean failure time is same for all materials*
H_1: *Mean failure time is different for all materials*

Oneway Analysis of Failure time By Material



Oneway Anova

Summary of Fit

Rsquare	0.622772
Adj Rsquare	0.522177
Root Mean Square Error	2041.334
Mean of Response	1768.3
Observations (or Sum Wgts)	20

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Material	4	103191489	25797872	6.1909	0.0038*
Error	15	62505657	4167043.8		
C. Total	19	165697146			

P value is less than significance level $\alpha = 0.05$ and $F_0 > F_{0.05,4,15}$. Therefore, reject null hypothesis.

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1	4	159.75	1020.7	-2016	2335.3
2	4	6.25	1020.7	-2169	2181.8
3	4	2941.75	1020.7	766	5117.3
4	4	5723.00	1020.7	3547	7898.5
5	4	10.75	1020.7	-2165	2186.3

Std Error uses a pooled estimate of error variance

Means Comparisons

Comparisons for each pair using Student's t

Confidence Quantile

t	Alpha
2.13145	0.05

LSD Threshold Matrix

Abs(Dif)-LSD

	4	3	1	5	2
4	-3076.6	-295.4	2486.6	2635.6	2640.1
3	-295.4	-3076.6	-294.6	-145.6	-141.1
1	2486.6	-294.6	-3076.6	-2927.6	-2923.1
5	2635.6	-145.6	-2927.6	-3076.6	-3072.1
2	2640.1	-141.1	-2923.1	-3072.1	-3076.6

Positive values show pairs of means that are significantly different.

Connecting Letters Report

Level			Mean
4	A		5723.0000
3	A	B	2941.7500
1		B	159.7500
5		B	10.7500
2		B	6.2500

Levels not connected by same letter are significantly different.

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
4	2	5716.750	1443.441	2640.13	8793.372	0.0013*
4	5	5712.250	1443.441	2635.63	8788.872	0.0013*
4	1	5563.250	1443.441	2486.63	8639.872	0.0016*
3	2	2935.500	1443.441	-141.12	6012.122	0.0601
3	5	2931.000	1443.441	-145.62	6007.622	0.0604
3	1	2782.000	1443.441	-294.62	5858.622	0.0731
4	3	2781.250	1443.441	-295.37	5857.872	0.0732
1	2	153.500	1443.441	-2923.12	3230.122	0.9167
1	5	149.000	1443.441	-2927.62	3225.622	0.9192
5	2	4.500	1443.441	-3072.12	3081.122	0.9976

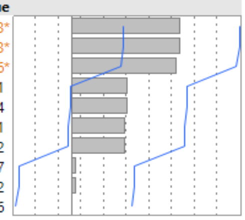


***Fig. 4** OneWay ANOVA*

Therefore, we could conclude that mean failure time is not the same for all materials.

b. *Plot the residuals versus the predicted response. Construct a normal probability plot of the residuals. What information is conveyed by these plots?*

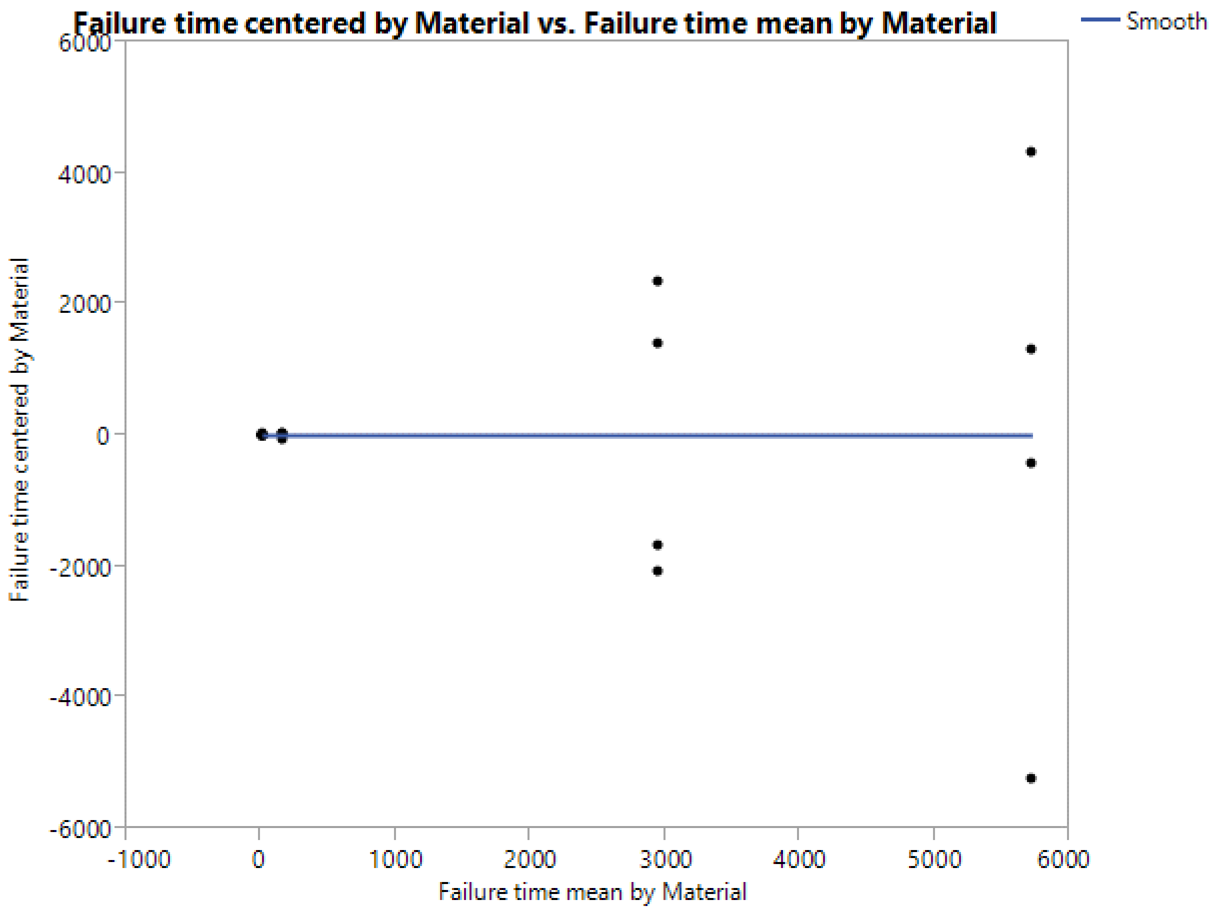


Fig. 5 *Residuals vs. Predicted*

The figure shows that variance is not constant for all the predicted values.

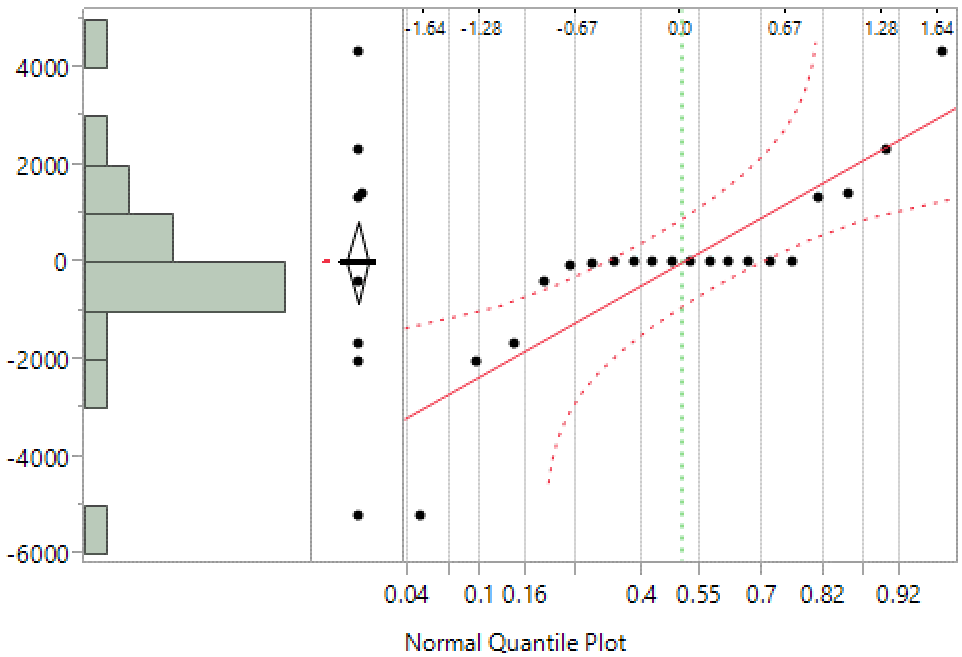


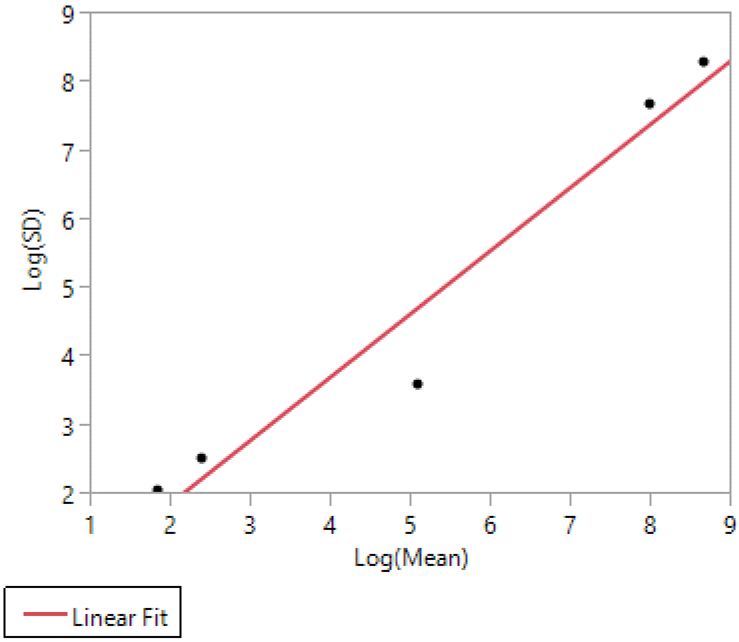
Fig. 6 *Normal quantile plot*

The figure clearly shows that points are not close to the line and outside the error bounds. This shows that normality assumption is not valid.

c. *Based on your answer to part (b) conduct another analysis of the failure time data and draw appropriate conclusions.*

Let us plot a bivariate fit of Log(Mean) and Log(Standard deviation).

Bivariate Fit of Log(SD) By Log(Mean)



Linear Fit

Log(SD) = 0.0591555 + 0.9210885*Log(Mean)

Summary of Fit

RSquare	0.953466
RSquare Adj	0.937955
Root Mean Square Error	0.733858
Mean of Response	4.834098
Observations (or Sum Wgts)	5

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	33.104224	33.1042	61.4694
Error	3	1.615643	0.5385	Prob > F
C. Total	4	34.719867		0.0043*

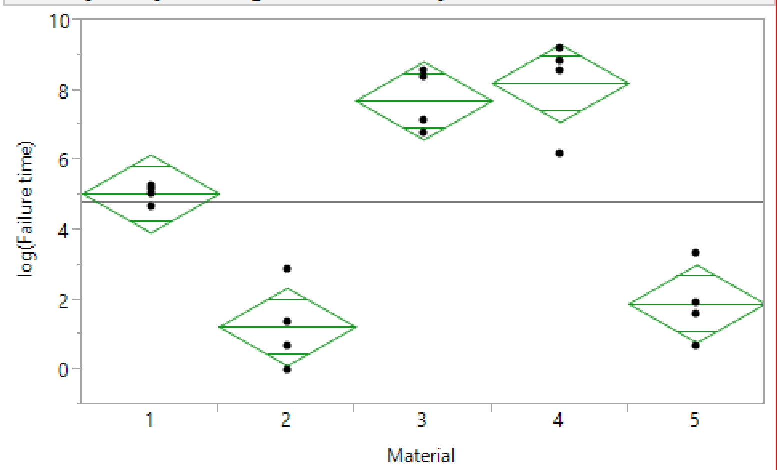
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0591555	0.691829	0.09	0.9372
Log(Mean)	0.9210885	0.117482	7.84	0.0043*

Fig. 7 *Log(Mean) vs. Log(S.D)*

From the plot, we could see that slope of the fit line is 0.9210 which is close to 1. So, from the table 3.9, a log transformation would be appropriate.

Oneway Analysis of log(Failure time) By Material



Oneway Anova

Summary of Fit

Rsquare	0.909435
Adj Rsquare	0.885285
Root Mean Square Error	1.046801
Mean of Response	4.825681
Observations (or Sum Wgts)	20

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Material	4	165.05646	41.2641	~~37.6569~~	<.0001*
Error	14 ~~15~~	16.43689	~~1.0958~~		
C. Total	19	181.49335	1.17406	35.1463	

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1	4	5.05159	0.52340	3.9360	6.1672
2	4	1.24245	0.52340	0.1269	2.3581
3	4	7.71640	0.52340	6.6008	8.8320
4	4	8.21401	0.52340	7.0984	9.3296
5	4	1.90395	0.52340	0.7883	3.0195

Std Error uses a pooled estimate of error variance

P value is less than significance level $\alpha = 0.05$. Therefore, reject null hypotheses.

After changing the error degree of freedom, other values are adjusted. P value is still small.

***Fig. 8** OneWay ANOVA*

Therefore, we could conclude that mean failure time is not the same for all the materials.

Lower t

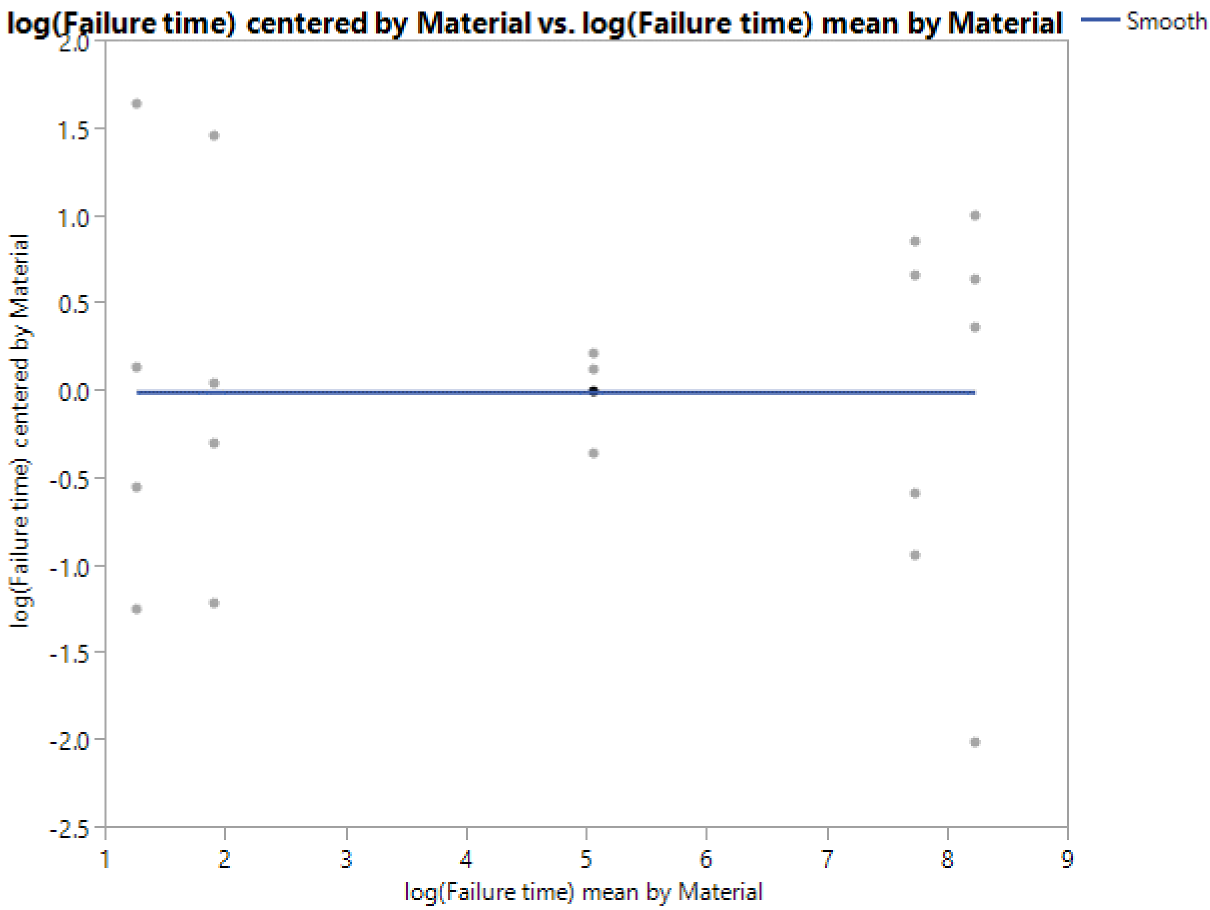


Fig. 9 *Residuals vs. Predicted*

This plot has been improved with transformed data although range of variation of one of the materials is lower than others.

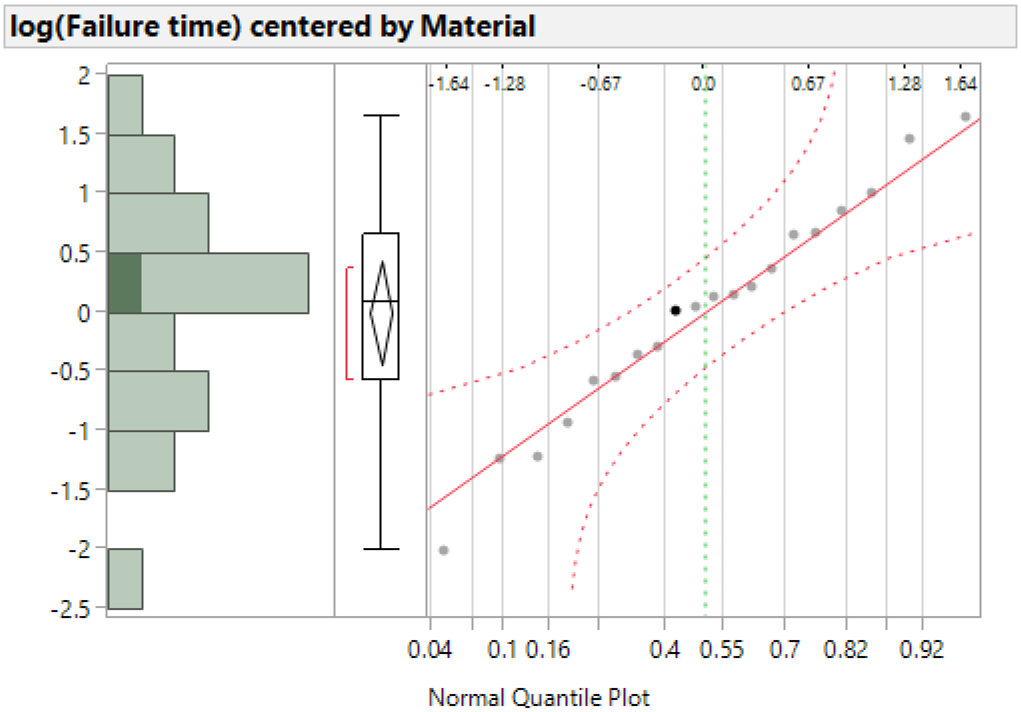


***Fig. 10** Normal quantile plot*

This shows that all points lie close to the line and within the error bounds. Hence the assumptions are not violated.