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EGR 7050 Design and Analysis of Engineering experiments

Homework 7

1. *An engineer is studying the mileage performance characteristics of five types of gasoline additives. In the road test he wishes to use cars as blocks; however, because of a time constraint, he must use an incomplete block design. He runs the balanced design with the five blocks that follow. Analyze the data from this experiment (use, $\alpha = 0.05$) and draw conclusions.*

Additive	Car				
	1	2	3	4	5
1		17	14	13	12
2	14	14		13	10
3	12		13	12	9
4	13	11	11	12	
5	11	12	10		8

Solution:

H_0: *No significant difference in gasoline additives*
H_1: *There is a significant difference in gasoline additives*

Given, $\alpha = 0.05$

Response Response

Summary of Fit

RSquare	0.869829
RSquare Adj	0.775159
Root Mean Square Error	0.954257
Mean of Response	12.05
Observations (or Sum Wgts)	20

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	8	66.933333	8.36667	9.1880
Error	11	10.016667	0.91061	Prob > F
C. Total	19	76.950000		0.0007*

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Additive	4	4	35.733333	9.8103	0.0012*
Car	4	4	35.233333	9.6730	0.0013*

P value is less than α = 0.05. Thus, reject null hypothesis and conclude that there is a significant evidence that at least one additive results in a significant different mean.

Effect Details

Additive

Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
1	14.250000	0.48968656	14.0000
2	12.783333	0.48968656	12.7500
3	11.850000	0.48968656	11.5000
4	11.116667	0.48968656	11.7500
5	10.250000	0.48968656	10.2500

LSMeans Differences Student's t

From the Fishers LSD analysis, we see that adjacent pairs of additive could not differentiated for α = 0.05.

α= 0.050 t= 2.20099

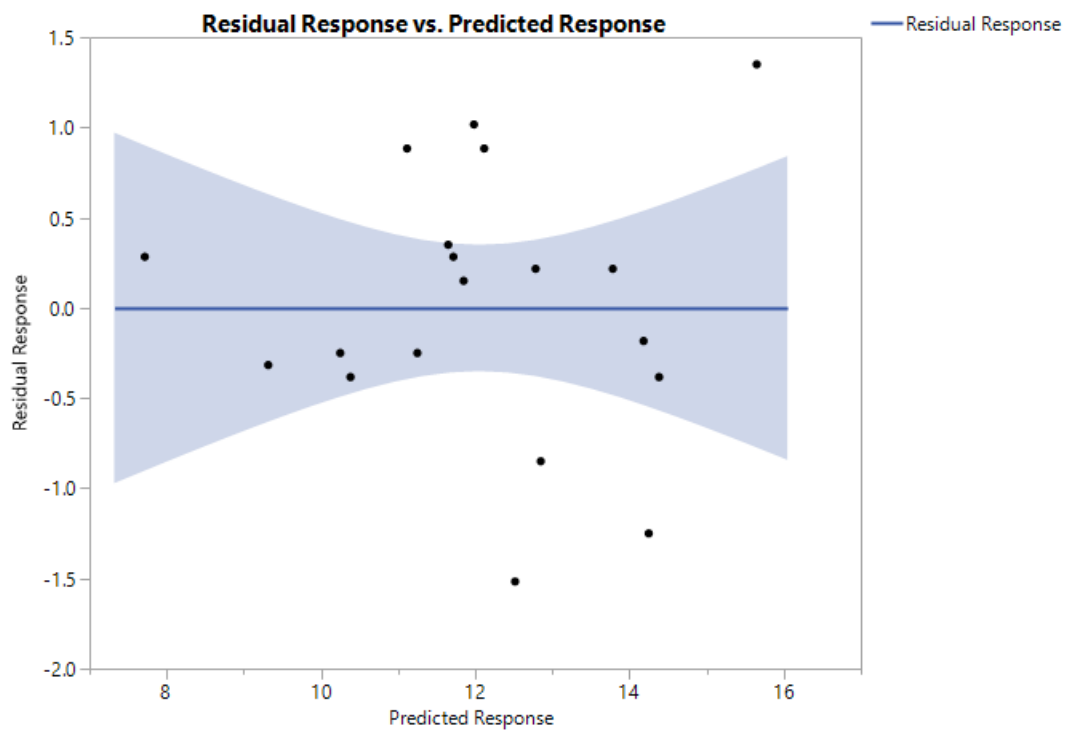| Level				Least Sq Mean
1	A			14.250000
2	A	B		12.783333
3		B	C	11.850000
4			C D	11.116667
5			D	10.250000

Levels not connected by same letter are significantly different.

Car

Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
1	13.050000	0.48968656	12.5000
2	13.450000	0.48968656	13.5000
3	12.183333	0.48968656	12.0000
4	12.050000	0.48968656	12.5000
5	9.516667	0.48968656	9.7500

Residual vs. predicted plot does not show any strong evidence for unequal variance. There is similar range of variation across different fitted values.
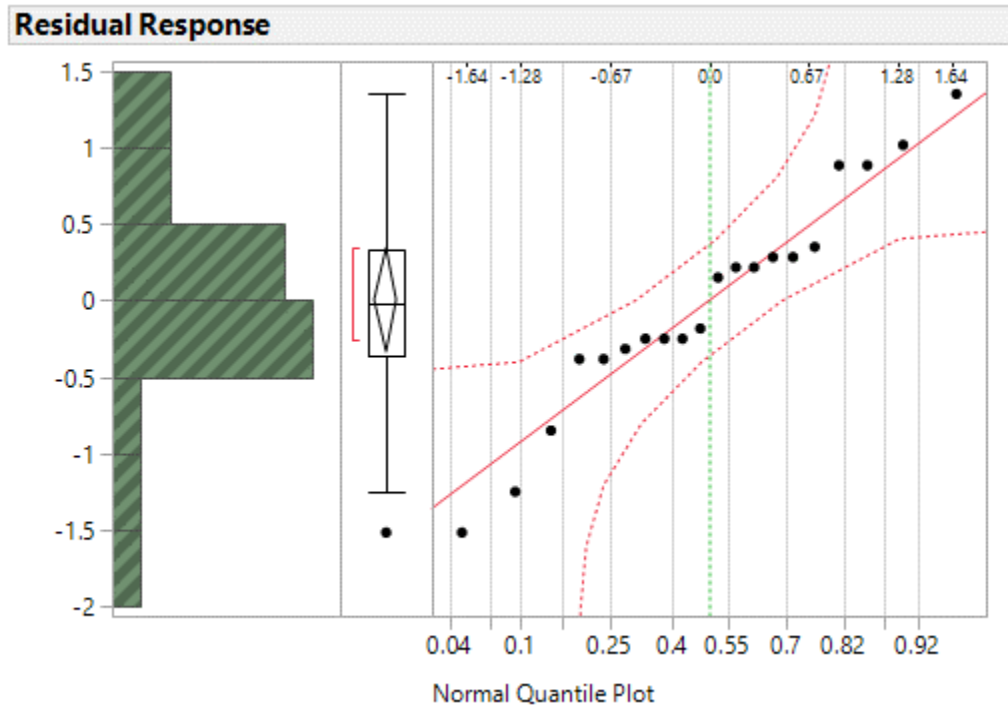
All points are close to the line and lie within the error bounds. Hence, normality assumption is not violated.

Residual Response vs. Additive

Residual Response vs. Car
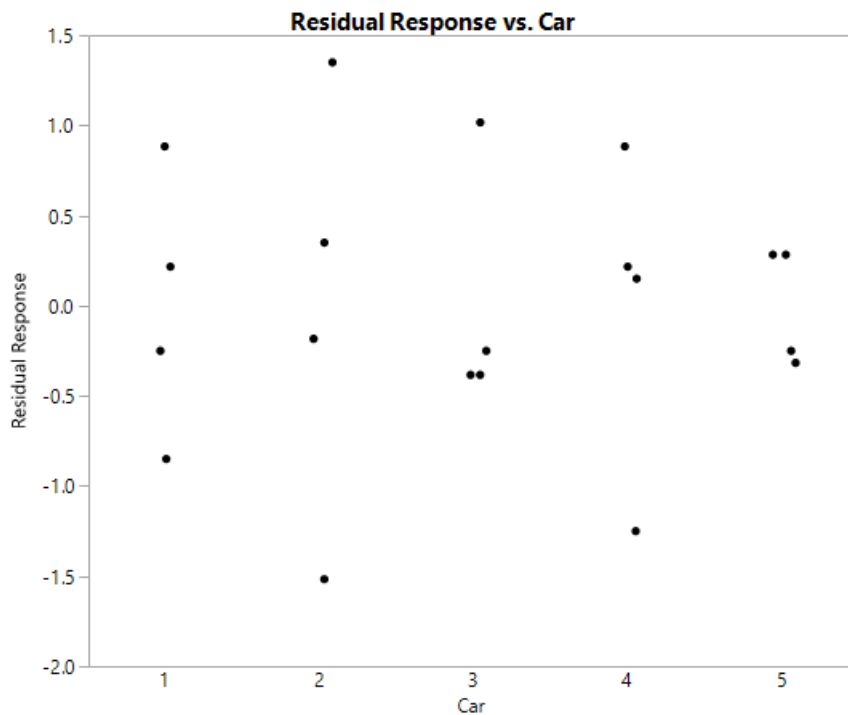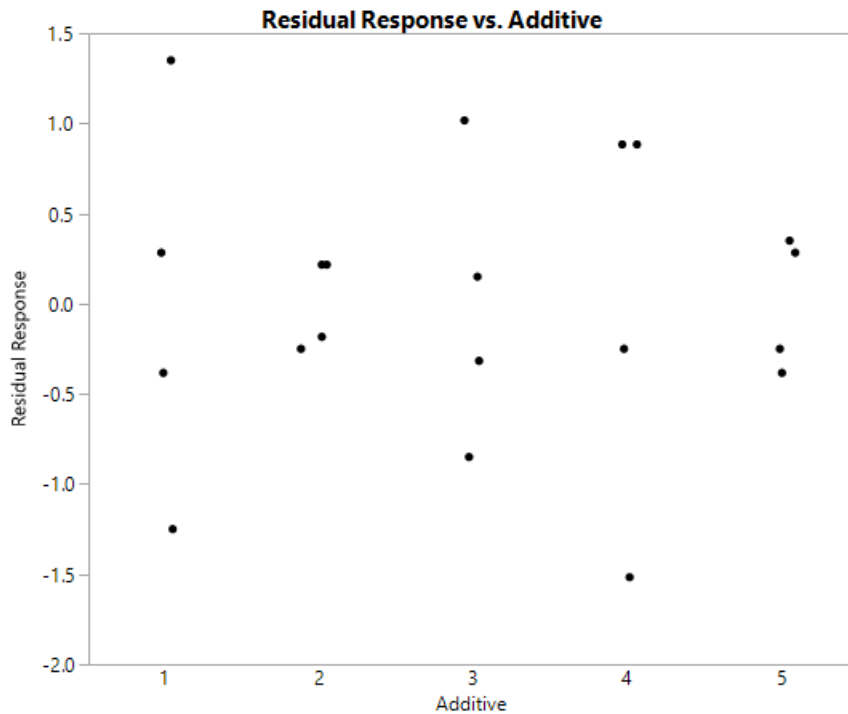
The above plots does not show any pattern in the variance. There is similar range of variation across different fitted values although the range is somewhat smaller for car 3 and 5 and additives 2 and 5.

2. The following output was obtained from a computer program that performed a two-factor ANOVA on a factorial experiment.

Source	DF	SS	MS	F	P
A	1	0.322	?	?	?
B	?	80.554	40.2771	4.59	?
Interaction	?	?	?	?	?
Error	12	105.327	8.7773		
Total	17	231.551			

Solution:

a) Fill in the blanks in the ANOVA table. You can use bounds on the P-values.

$a - 1 = 1$
$\mathbf{a = 2}$

$$MS_A = \frac{SS_A}{a-1}$$
$$= \frac{0.322}{1} = \mathbf{0.322}$$

$$F_0 = \frac{MS_A}{MS_E} = \frac{0.322}{8.7773} = \mathbf{0.0367}$$

$$MS_B = \frac{SS_B}{b-1}$$
$$40.2771 = \frac{80.554}{b-1}$$
$$b - 1 = \frac{80.554}{40.2771} = 1.99 \approx \mathbf{2}$$

$$(a-1)(b-1) = \mathbf{2}$$

$$SS_E = SS_T - SS_{AB} - SS_A - SS_B$$

$$105.327 = 231.551 - SS_{AB} - 0.322 - 80.554$$

$$SS_{AB} = 231.551 - 105.327 - 0.322 - 80.554$$

$$= \mathbf{45.348}$$

$$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)} = 45.348/2 = 22.674$$

$$F_0 = \frac{MS_{AB}}{MS_E} = 22.674/8.7773 = \mathbf{2.583}$$

From P value calculator, P value for A = 0.8513

P value for B = 0.0331

P value for AB = 0.1167

Source	DF	SS	MS	F	P
A	1	0.322	0.322	0.0367	0.8513
B	2	80.554	40.2771	4.59	0.0331
Interaction	2	45.348	22.674	2.583	0.1167
Error	12	105.327	8.7773		
Total	17	231.551			

b) How many levels were used for factor B?

b-1 = 2
   b = 3 levels

c) How many replicates of the experiment were performed?

Total number of observations = 18

a =2, b =3 levels

	1	2	3
1	$y_{111}, y_{112}, y_{113}$	$y_{121}, y_{122}, y_{123}$	$y_{131}, y_{132}, y_{133}$
2	$y_{211}, y_{212}, y_{213}$	$y_{221}, y_{222}, y_{223}$	$y_{331}, y_{332}, y_{333}$

Number of replicates formed is 3

d) What conclusions would you draw about this experiment?

H_0: *No significant difference in A*
H_1: *There is a significant difference in A*

P value for A is 0.8513 which is greater than $\alpha = 0.05$. Therefore, do not reject null. It could therefore be concluded that **there is no significant difference in A.**

H_0: *No significant difference in B*
H_1: *There is a significant difference in B*

P value for B is 0.0331 which is less than $\alpha = 0.05$. Therefore, reject null. It could therefore be concluded that **there is a significant difference in B**.

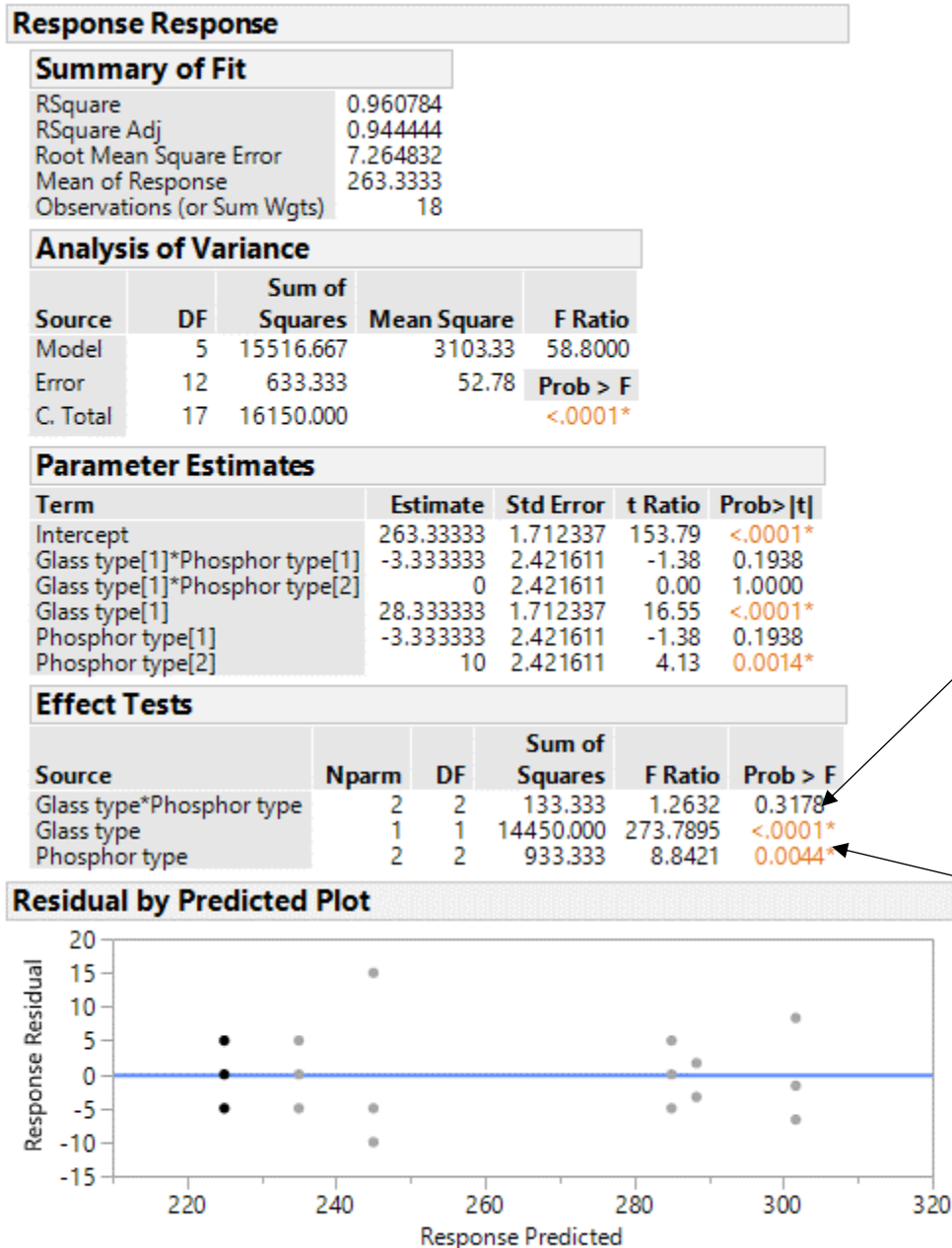H_0: *No significant difference in AB*
H_1: *There is a significant difference in AB*

P value for AB is 0. 0.1167 which is less than $\alpha = 0.05$. Therefore, do not reject null. It could therefore be concluded that **there is no significant difference in AB.**

3. An article in Industrial Quality Control (1956, pp.5‑8) describes an experiment to investigate the effect of the type of glass and the type of phosphor on the brightness of a television tube. The response variable is the current necessary (in microamps) to obtain a specified brightness level. The data are as follows:

Glass type	Phosphor Type		
	1	2	3
1	280	300	290
	290	310	285
	285	295	290
2	230	260	220
	235	240	225
	240	235	230

a) Is there any indication that either factor influences brightness? Use $\alpha = 0.05$

Response Response

Summary of Fit

RSquare	0.960784
RSquare Adj	0.944444
Root Mean Square Error	7.264832
Mean of Response	263.3333
Observations (or Sum Wgts)	18

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	15516.667	3103.33	58.8000
Error	12	633.333	52.78	**Prob > F**
C. Total	17	16150.000		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	263.33333	1.712337	153.79	<.0001*
Glass type[1]*Phosphor type[1]	-3.333333	2.421611	-1.38	0.1938
Glass type[1]*Phosphor type[2]	0	2.421611	0.00	1.0000
Glass type[1]	28.333333	1.712337	16.55	<.0001*
Phosphor type[1]	-3.333333	2.421611	-1.38	0.1938
Phosphor type[2]	10	2.421611	4.13	0.0014*

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Glass type*Phosphor type	2	2	133.333	1.2632	0.3178
Glass type	1	1	14450.000	273.7895	<.0001*
Phosphor type	2	2	933.333	8.8421	0.0044*

b. Interaction effect is not significant at $\alpha = 0.05$.

a. Both glass type and phosphor type are significant at $\alpha = 0.05$.

Residual by Predicted Plot

b) Do the two factors interact? Use $\alpha = 0.05$

> H_0: *No significant difference in interaction*
> H_1: *There is a significant difference in interaction*

Interaction effect is not significant at $\alpha = 0.05$ as p value is greater than $\alpha = 0.05$. Hence, do not reject null hypothesis.

4. The yield of a chemical process is being studied. The two factors of interest are temperature and pressure. Three levels of each factor are selected; however, only nine runs can be made in one day. The experimenter runs a complete replicate of the design on each day. The data are shown in the following table. Analyze the data, assuming that the days are blocks.

Temperature	Day 1 pressure			Day 2 pressure		
	250	**260**	**270**	**250**	**260**	**270**
Low	86.3	84.0	85.8	86.1	85.2	87.3
Medium	88.5	87.3	89.0	89.4	89.9	90.3
High	89.1	90.2	91.3	91.7	93.2	93.7

Solution:

Response Chemical process yield

Summary of Fit

RSquare	0.966554
RSquare Adj	0.928927
Root Mean Square Error	0.728869
Mean of Response	88.79444
Observations (or Sum Wgts)	18

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	9	122.81944	13.6466	25.6877
Error	8	4.25000	0.5313	Prob > F
C. Total	17	127.06944		<.0001*

Effect Tests

| Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Temperature	2	2	99.854444	93.9807	<.0001*
Pressure	2	2	5.507778	5.1838	0.0360
Day	1	1	13.005000	24.4800	0.0011*
Temperature*Pressure	4	4	4.452222	2.0952	0.1733

The temperature and pressure effect are significant at $\alpha = 0.05$.

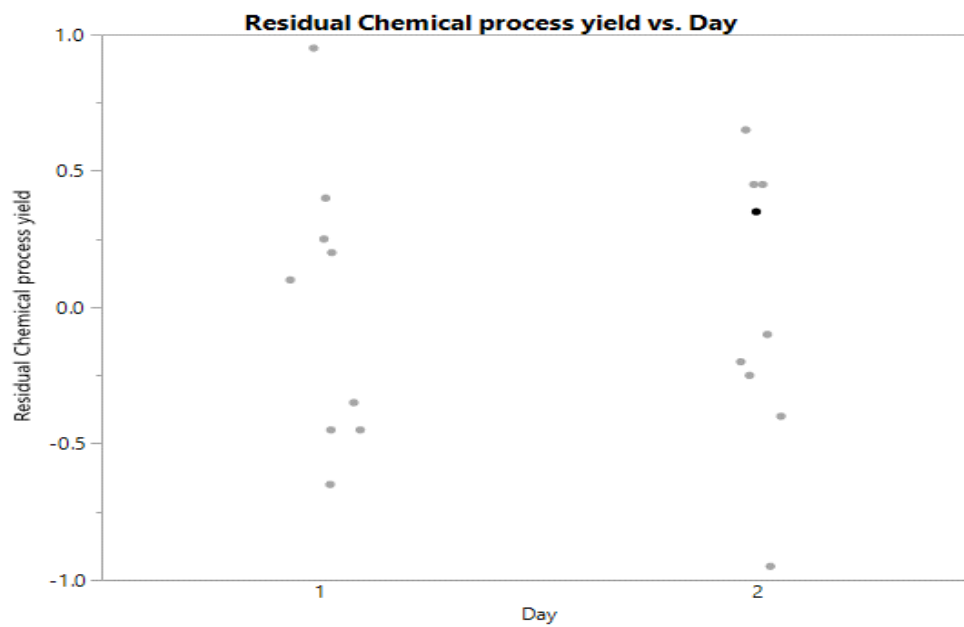The interaction effect is not significant at $\alpha = 0.05$.
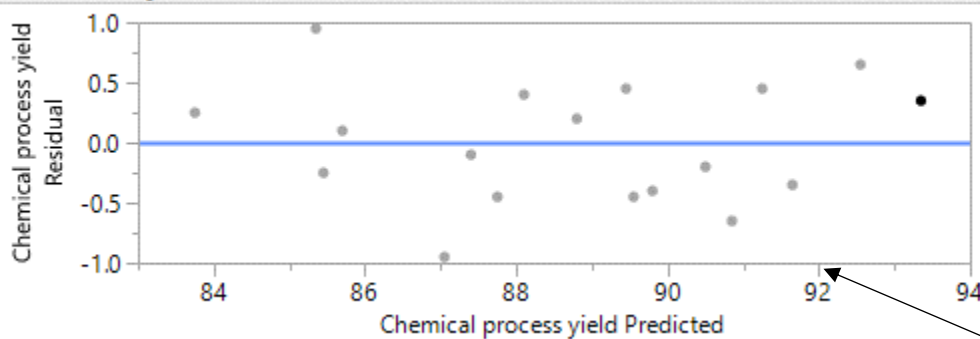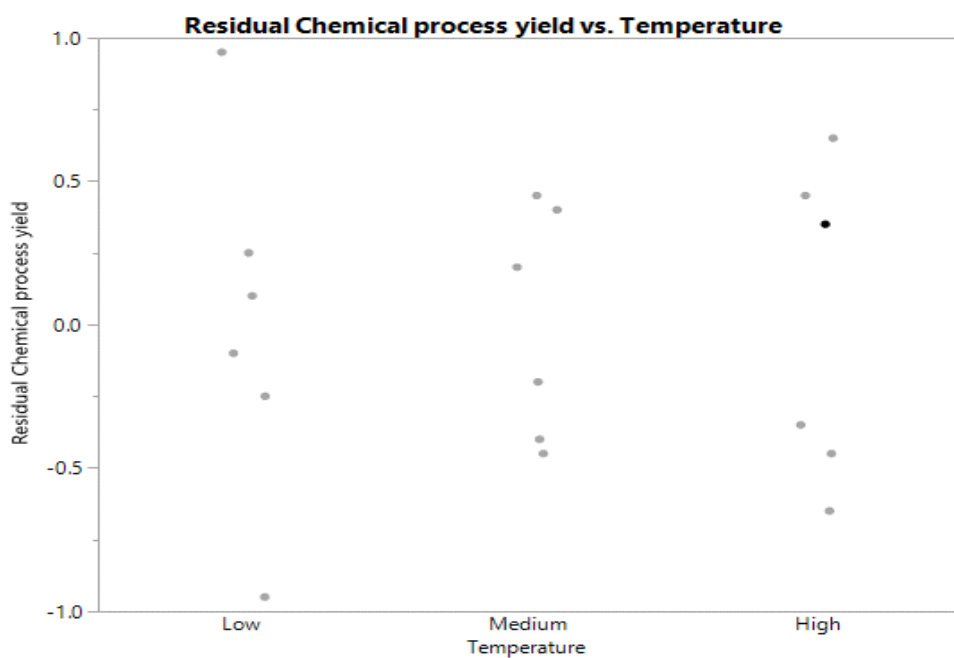
Residual by Predicted Plot

Residual vs. predicted plot does not show any strong evidence for unequal variance. There is similar range of variation across different fitted values.
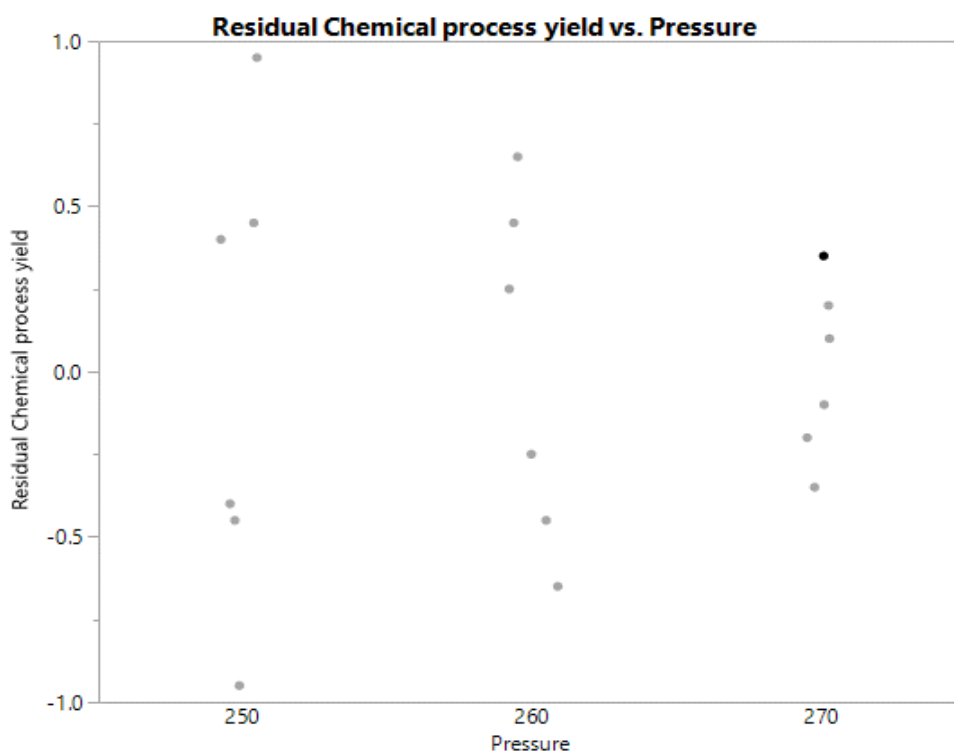
Normal Quantile Plot
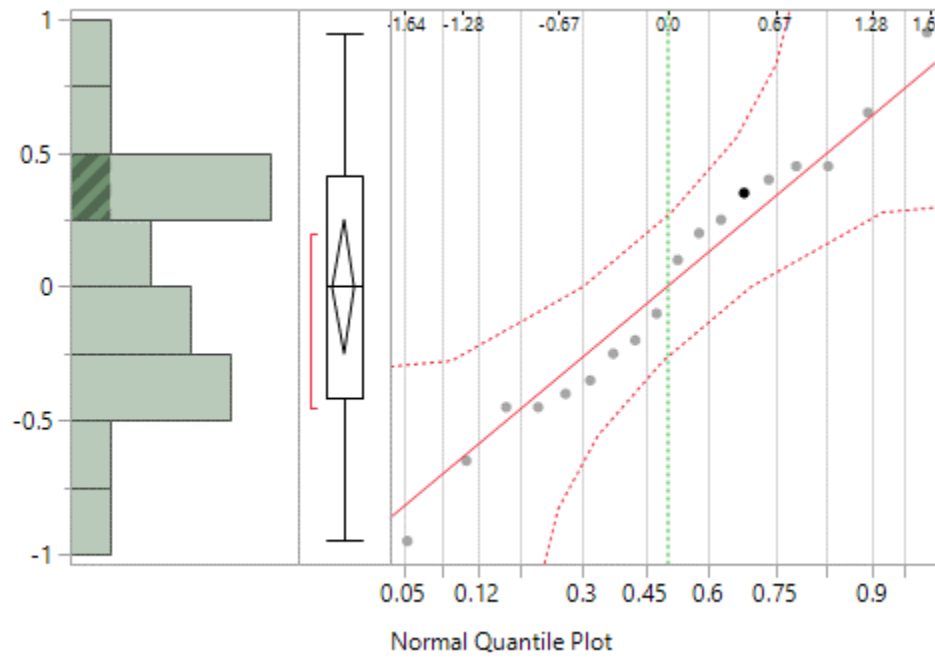
All the points lie close to the line and within error bounds. Therefore, the normality assumption is true.

Residual vs. Day and Residual Vs. Temperature does not show any strong evidence for unequal variance. There is similar range of variation across different fitted values.

There is some decrease in the residual variance as pressure increases. But this doesn't seem to affect the normality of residuals.