Meenakshi Nagarajan

U00618268

EGR 7050 Design and Analysis of Engineering experiments

Homework 3

1. *Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The production engineers are interested in both the mean and the variance of the fill volumes.*

Machine 1		*Machine 2*	
16.03	*16.01*	*16.02*	*16.03*
16.04	*15.96*	*15.97*	*16.04*
16.05	*15.98*	*15.96*	*16.02*
16.05	*16.02*	*16.01*	*16.01*
16.02	*15.99*	*15.99*	*16.00*

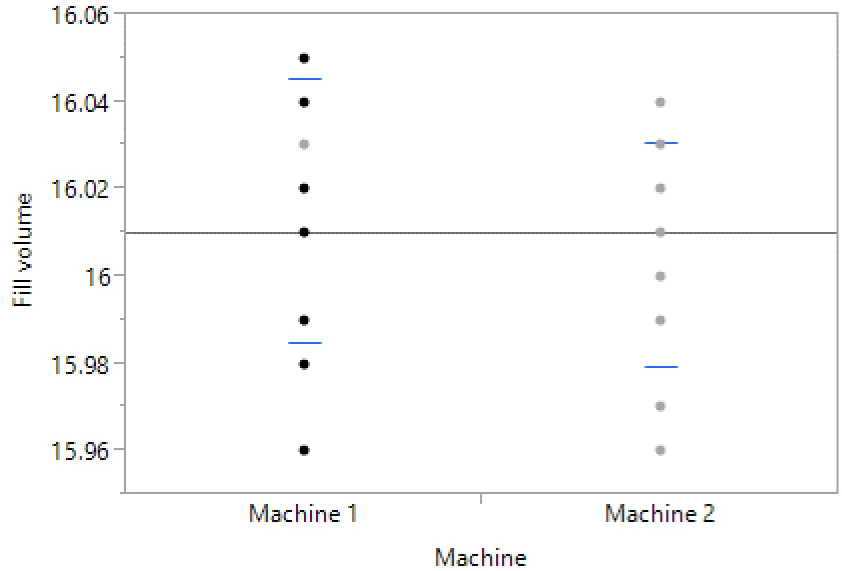
a. *Test the hypothesis that the variances of fill volume are equal for the two machines. Use α = 0.05.*
b. *Using the results of (a) choose an appropriate test, and test whether the two machines have equal mean fill volumes. Use α = 0.05. What is the P-value for this test?*
c. *Check the assumption of normality for each machine.*

Solution:

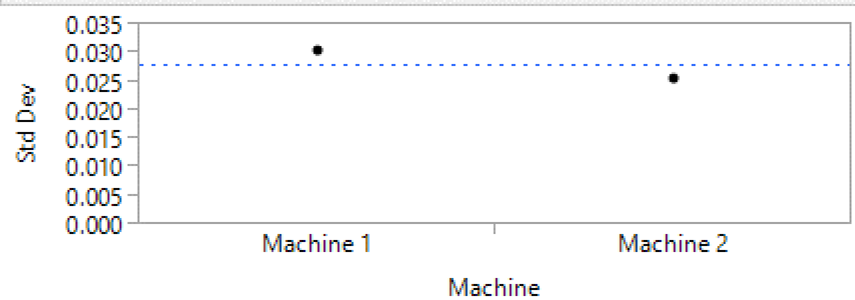
a. $H_0: {\sigma_1}^2 = {\sigma_2}^2 \quad H_1: {\sigma_1}^2 \neq {\sigma_2}^2$

Given, $\alpha = 0.05$

Oneway Analysis of Fill volume By Machine



Tests that the Variances are Equal



Level	Count	Std Dev	MeanAbsDif to Mean	MeanAbsDif to Median
Machine 1	10	0.0302765	0.0240000	0.0230000
Machine 2	10	0.0254951	0.0200000	0.0190000

Test	F Ratio	DFNum	DFDen	p-Value
O'Brien[.5]	0.3858	1	18	0.5423
Brown-Forsythe	0.2526	1	18	0.6213
Levene	0.3318	1	18	0.5717
Bartlett	0.2507	1	.	0.6166
F Test 2-sided	1.4103	9	9	$\boxed{0.6168}$

P value is greater than the significance level 0.05. Thus, there is no strong evidence to reject null hypothesis. So it can be concluded that this set of data can be analyzed with an equal variance Means/ ANOVA/ pooled-t test.

Welch's Test

Welch Anova testing Means Equal, allowing Std Devs Not Equal

F Ratio	DFNum	DFDen	Prob > F
0.6383	1	17.493	0.4350

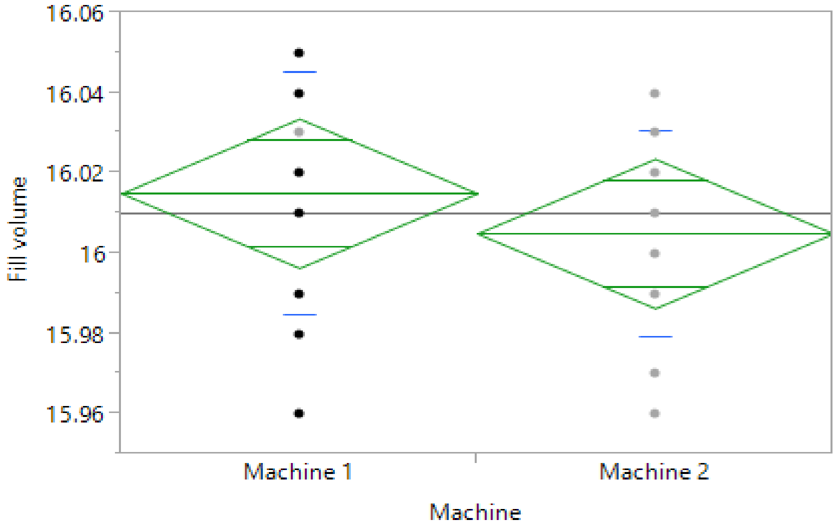
t Test
0.7989

Fig. 1 *One way analysis*

b. $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$

Assuming equal variance,

Oneway Analysis of Fill volume By Machine



Oneway Anova

Summary of Fit

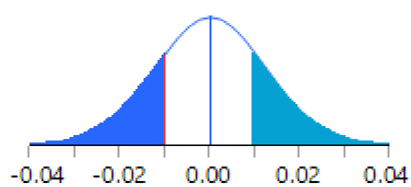
Rsquare	0.034247
Adj Rsquare	-0.01941
Root Mean Square Error	0.027988
Mean of Response	16.01
Observations (or Sum Wgts)	20

t Test

Machine 2-Machine 1

Assuming equal variances

Difference	-0.01000	t Ratio	-0.79894
Std Err Dif	0.01252	DF	18
Upper CL Dif	0.01630	Prob > |t|	0.4347
Lower CL Dif	-0.03630	Prob > t	0.7826
Confidence	0.95	Prob < t	0.2174



Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Machine	1	0.00050000	0.000500	0.6383	$\boxed{0.4347}$
Error	18	0.01410000	0.000783		
C. Total	19	0.01460000			

P value greater than 0.05 shows that there is no enough evidence to reject null hypothesis. Therefore, it can be concluded that there is no difference in group means.

Means for Oneway Anova

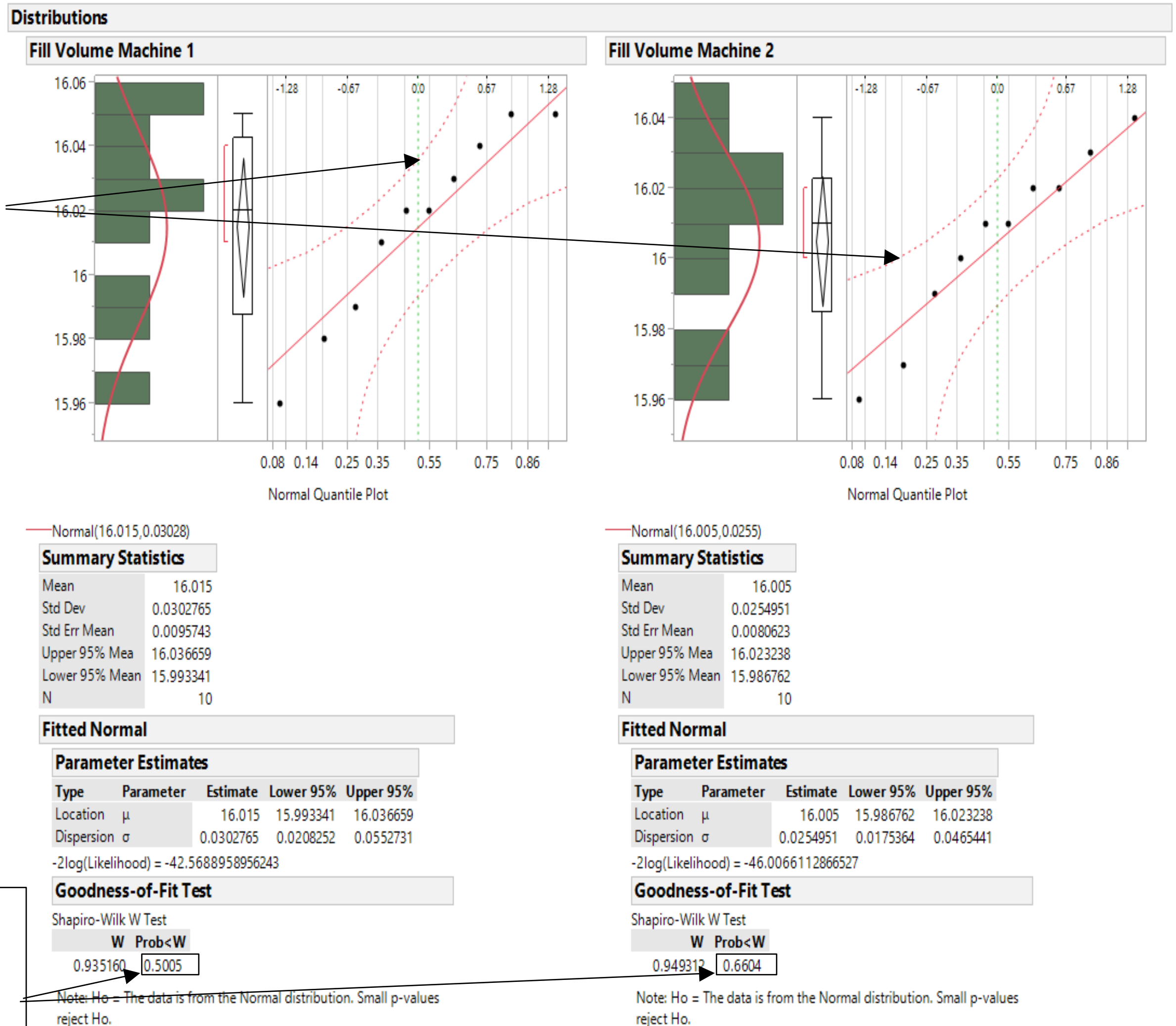
Level	Number	Mean	Std Error	Lower 95%	Upper 95%
Machine 1	10	16.0150	0.00885	15.996	16.034
Machine 2	10	16.0050	0.00885	15.986	16.024

Std Error uses a pooled estimate of error variance

***Fig. 2** Means/ ANOVA/ pooled-t*

c. From JMP,

Normal plot for Machine 1 and 2.



As P value greater than 0.05, it can be assumed that data is from the normal distribution.

Fig. 3 *Distribution of Machine 1 and 2*

This assumption can be checked using the normal quantile plot of two machines. It shows that all points lie close to the line and within the error bounds. Hence, it could be concluded that the data is from a normal distribution.

2. *An article in the Journal of Strain Analysis (vol. 18, no. 2, 1983) compares several procedures for predicting the shear strength for steel plate girders. Data for nine girders in the form of the ratio of predicted to observed load for two of these procedures, the Karlsruhe and Lehigh methods, are as follows:*

Girder	Karlsruhe Method	Lehigh Method
S1/1	*1.186*	*1.061*
S2/1	*1.151*	*0.992*
S3/1	*1.322*	*1.063*
S4/1	*1.339*	*1.062*
S5/1	*1.200*	*1.065*
S2/1	*1.402*	*1.178*
S2/2	*1.365*	*1.037*
S2/3	*1.537*	*1.086*
S2/4	*1.559*	*1.052*

Solution:

a. Is *there any evidence to support a claim that there is a difference in mean performance between the two methods? Use* $\alpha = 0.05$.

H_0: μ_d= 0 H_1: $\mu_d \neq 0$

The test statistic is,

$$t_0 = \frac{\bar{d}}{S_d/\sqrt{n}}$$

Where $\bar{d} = \frac{1}{n}\sum_{j=1}^{n} d_j$ is the sample mean of differences and

$$S_d = \left[\frac{\sum_{j=1}^{n}\left(d_j - \bar{d}\right)^2}{n-1}\right]^{1/2}$$

Degrees of freedom, $n - 1 = 8$

$From\ JMP, t_0 = \frac{0.2739}{0.1351/\sqrt{9}}$ = $\frac{0.2739}{0.045}$ = 6.086

If $t_0 > t_{0.025,8}$, we would reject H_0.

From t distribution, $t_{0.025,8} = 2.306$. Therefore, from the above condition, H_0 can be rejected.

It can be concluded that, there is a difference in mean performance between the two methods.

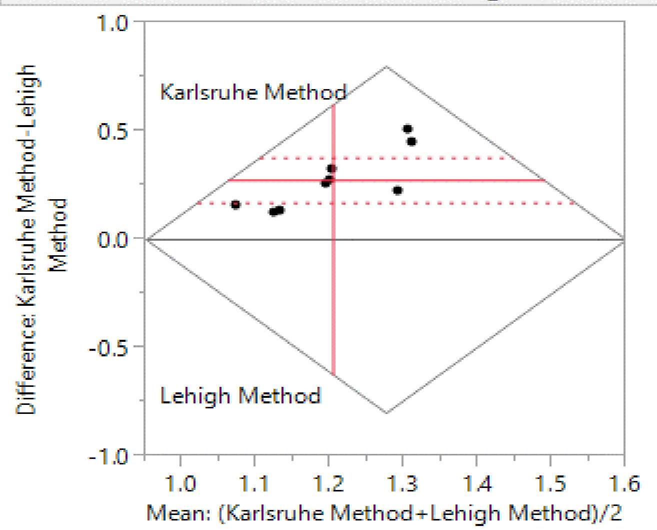
	Girder	Karlsruhe Method	Lehigh Method	Difference	dbar	standard deviation
1	S1/1	1.186	1.061	0.125	0.2738888889	0.135099449
2	S2/1	1.151	0.992	0.159	0.2738888889	0.135099449
3	S3/1	1.322	1.063	0.259	0.2738888889	0.135099449
4	S4/1	1.339	1.062	0.277	0.2738888889	0.135099449
5	S5/1	1.2	1.065	0.135	0.2738888889	0.135099449
6	S2/1	1.402	1.178	0.224	0.2738888889	0.135099449
7	S2/2	1.365	1.037	0.328	0.2738888889	0.135099449
8	S2/3	1.537	1.086	0.451	0.2738888889	0.135099449
9	S2/4	1.559	1.052	0.507	0.2738888889	0.135099449

Fig. 4 *Data table in JMP*

b. What is the P-value for the test in part (a)?

Matched Pairs

Difference: Karlsruhe Method-Lehigh Method



Karlsruhe Method	1.34011	t-Ratio	6.081939
Lehigh Method	1.06622	DF	8
Mean Difference	0.27389	Prob > |t|	0.0003*
Std Error	0.04503	Prob > t	0.0001*
Upper 95%	0.37774	Prob < t	0.9999
Lower 95%	0.17004		
N	9		
Correlation	0.38217		

2.34b) the small P value indicates that this difference is statistically significant

Fig.5 Paired t test

c. Construct a 95 percent confidence interval for the difference in mean predicted to observed load.

95 percent C.I on $\mu_1 - \mu_2$ is $\bar{d} \pm t_{0.025,8}(S_d/\sqrt{n})$

$$0.2739 \pm (2.306)(0.045)$$

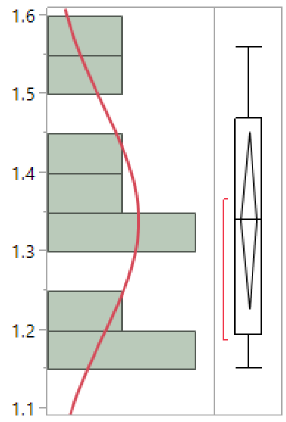
$$0.1703 \leq \mu_d \leq 0.3777$$

95 percent C.I is (0.1703, 0.3777)

d. Investigate the normality assumption for both samples.

Distributions

Karlsruhe Method



Summary Statistics

Mean	1.3401111
Std Dev	0.1460312
Std Err Mean	0.0486771
Upper 95% Mea	1.4523606
Lower 95% Mean	1.2278616
N	9

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	μ	1.3401111	1.2278616	1.4523606
Dispersion	σ	0.1460312	0.0986378	0.2797623

-2log(Likelihood) = -10.0899361795599

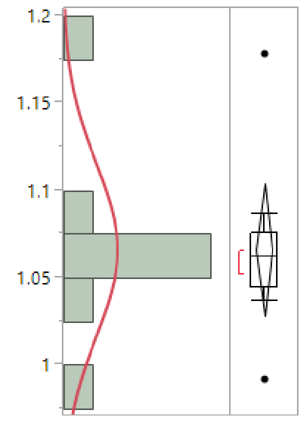
Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.929052	0.4724

Note: Ho = The data is from the Normal distribution. Small p-values reject Ho.

Lehigh Method



Summary Statistics

Mean	1.0662222
Std Dev	0.0493806
Std Err Mean	0.0164602
Upper 95% Mea	1.1041795
Lower 95% Mean	1.0282649
N	9

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	μ	1.0662222	1.0282649	1.1041795
Dispersion	σ	0.0493806	0.0333545	0.0946019

-2log(Likelihood) = -29.6066610940636

Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.841822	0.0605

Note: Ho = The data is from the Normal distribution. Small p-values reject Ho.

In both methods, P value is greater than 0.05. Therefore, it can be assumed that data is from normal distribution.

***Fig. 6** Distribution of Lehigh method and Karlsruhe method*

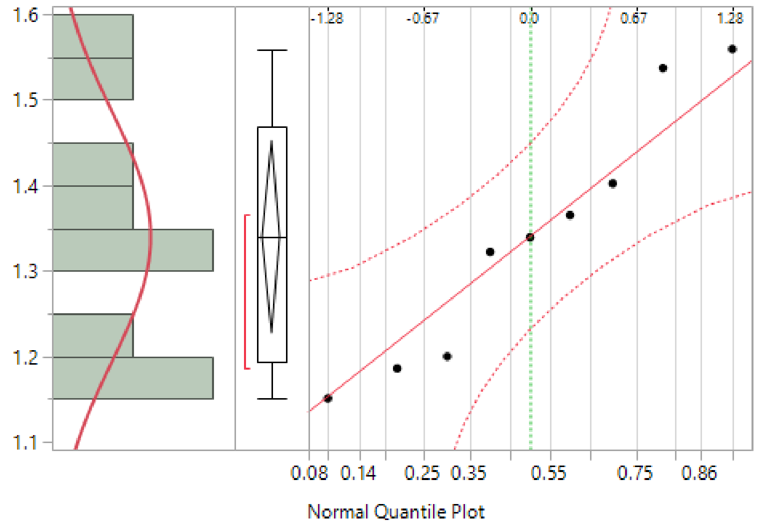


Fig. 7 *Normal quantile plot of Karlsruhe method*

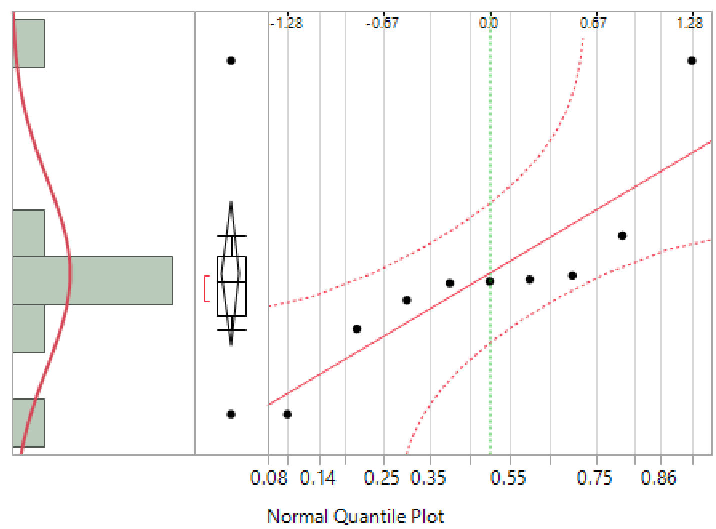


Fig. 8 *Normal quantile plot of Lehigh method*

In Karlsruhe method, all points lie close to the line and within the error bounds. Hence, the data is from a normal distribution whereas in Lehigh method, not all points lie close to the line and within the error bounds. Hence, the data is not from a normal distribution

e. *Investigate the normality assumption for the difference in ratios for the two methods.*

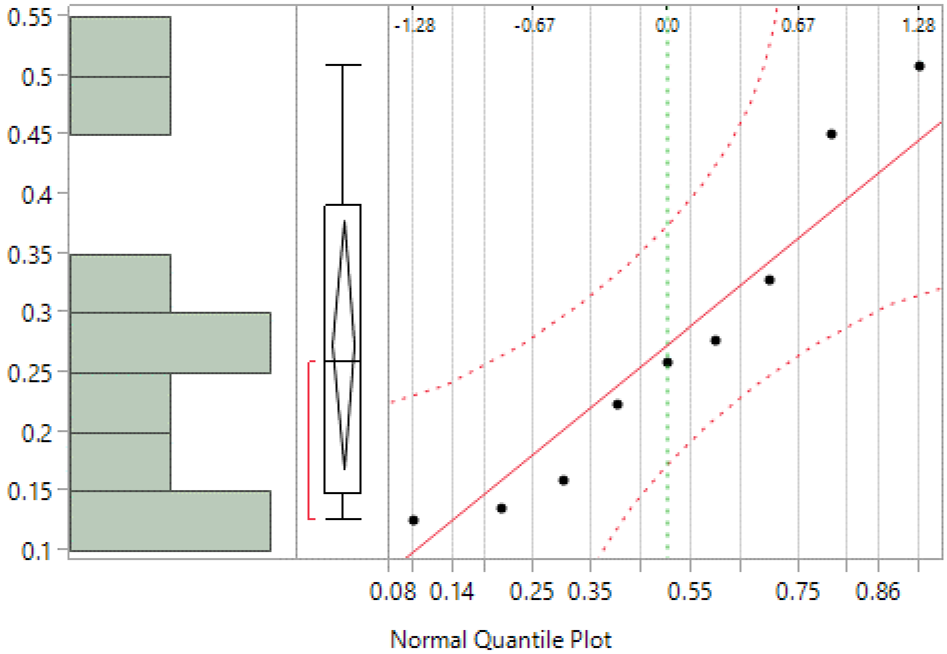


Fig. 9 *Normal quantile plot for difference*

From the above plot, it could be seen that all points lie within the error bounds and close to the line and could be assumed that it is distributed normally.

f. *Discuss the role of the normality assumption in the paired t-test.*

In paired t test, by pairing we could eliminate the additional source of variability. Hence the difference in ratio for two methods will be normally distributed but not the individual samples.

3. *An experimenter has conducted a single-factor experiment with six levels of the factor, and each factor level has been replicated three times. The computed value of the F- statistic is* F_0 *= 5.81. Find bounds on the P-value.*

Solution:

As the experimenter has conducted experiment with 6 levels of factor, a=6. Thus, D.F = 6-1 = 5

As each factor level has been replicated thrice, N = 18
Degrees of freedom of error within treatments is N-a = 18 – 6 = 12.

Given F_0= 5.81 = $\frac{MS_{Treatments}}{MS_E}$

From the P value calculator, it is calculated as **0.00594169**

4. A computer ANOVA output is shown below. Fill in the blanks. You may give bounds on the P-value.
One-way ANOVA:

Source	**DF**	**SS**	**MS**	**F**	**P**
Factor	3	36.15	?	?	?
Error	?	?	?		
Total	19	196.04			

Solution:

$MS_{Factor} = \frac{SS_{Factor}}{a-1}$

= 36.15/3 = ***12.05***

$MS_{Error} = \frac{SS_{Error}}{N-a}$

$SS_{Error} = SS_T - SS_{Factor}$
$= 196.04 - 36.15$ = **159.89**

N – 1 = 19 → N = 20
a – 1 = 3 → a = 4
N – a = **16**
$MS_{Error} = \frac{SS_{Error}}{N-a}$
= 159.89/16 = **9.993**

$F_0 = \frac{MS_{Factor}}{MS_{Error}} = \frac{12.05}{9.993} = \mathbf{1.206}$

From P value calculator, **P = 0.339**

Source	**DF**	**SS**	**MS**	**F**	**P**
Factor	3	36.15	12.05	1.206	0.339
Error	16	159.89	9.993		
Total	19	196.04			

5. *An article in the ACI Materials Journal (Vol. 84, 1987, pp. 213–216) describes several experiments investigating the rodding of concrete to remove entrapped air. A 3-inch * 6-inch cylinder was used, and the number of times this rod was used is the design variable. The resulting compressive strength of the concrete specimen is the response. The data are shown in the following table:*

Rodding Level	***Compressive Strength***		
10	*1530*	*1530*	*1440*
15	*1610*	*1650*	*1500*
20	*1560*	*1730*	*1530*
25	*1500*	*1490*	*1510*

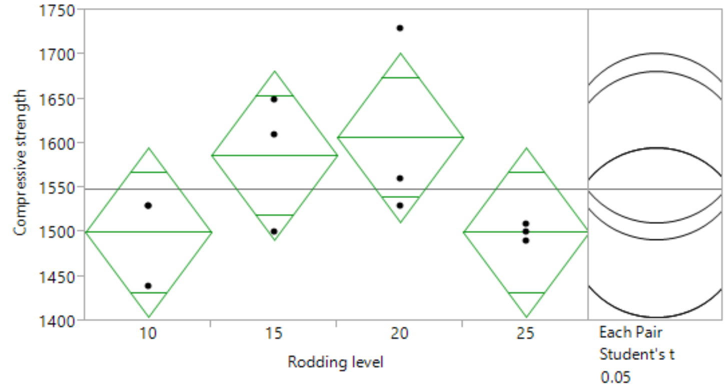
a. *Is there any difference in compressive strength due to the rodding level? Use $\alpha = 0.05$.*

$\mathrm{H_0}$: There is no difference in compressive strength
$\mathrm{H_1}$: There is difference in compressive strength

As P-value 0.2138 (from JMP) is greater than $\alpha = 0.05$. Therefore, we do not reject null hypothesis.

Thus, it can be concluded that there is no difference in compressive strength.

Oneway Analysis of Compressive strength By Rodding level



Oneway Anova

Summary of Fit

Rsquare	0.411596
Adj Rsquare	0.190944
Root Mean Square Error	71.53088
Mean of Response	1548.333
Observations (or Sum Wgts)	12

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Rodding level	3	28633.333	9544.44	1.8654	0.2138
Error	8	40933.333	5116.67		
C. Total	11	69566.667			

P value for rodding level.

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
10	3	1500.00	41.298	1404.8	1595.2
15	3	1586.67	41.298	1491.4	1681.9
20	3	1606.67	41.298	1511.4	1701.9
25	3	1500.00	41.298	1404.8	1595.2

Std Error uses a pooled estimate of error variance

Means Comparisons

Comparisons for each pair using Student's t

Confidence Quantile

t	Alpha
2.30600	0.05

LSD Threshold Matrix

Abs(Dif)-LSD

	20	15	10	25
20	-134.68	-114.68	-28.01	-28.01
15	-114.68	-134.68	-48.01	-48.01
10	-28.01	-48.01	-134.68	-134.68
25	-28.01	-48.01	-134.68	-134.68

Positive values show pairs of means that are significantly different.

Connecting Letters Report

Level		Mean
20	A	1606.6667
15	A	1586.6667
10	A	1500.0000
25	A	1500.0000

Levels not connected by same letter are significantly different.

Levels connected by same letter are not significantly different

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
20	10	106.6667	58.40472	-28.015	241.3482	0.1052
20	25	106.6667	58.40472	-28.015	241.3482	0.1052
15	10	86.6667	58.40472	-48.015	221.3482	0.1761
15	25	86.6667	58.40472	-48.015	221.3482	0.1761
20	15	20.0000	58.40472	-114.682	154.6815	0.7408
25	10	0.0000	58.40472	-134.682	134.6815	1.0000

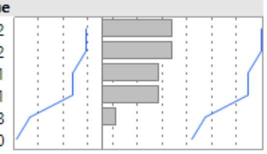


Fig. 10 *One way ANOVA*

b. *Find the P-value for the F statistic in part (a).*

From the JMP output, P value for the F statistic is 0.2138

c. *Analyze the residuals from this experiment. What conclusions can you draw about the underlying model assumptions?*

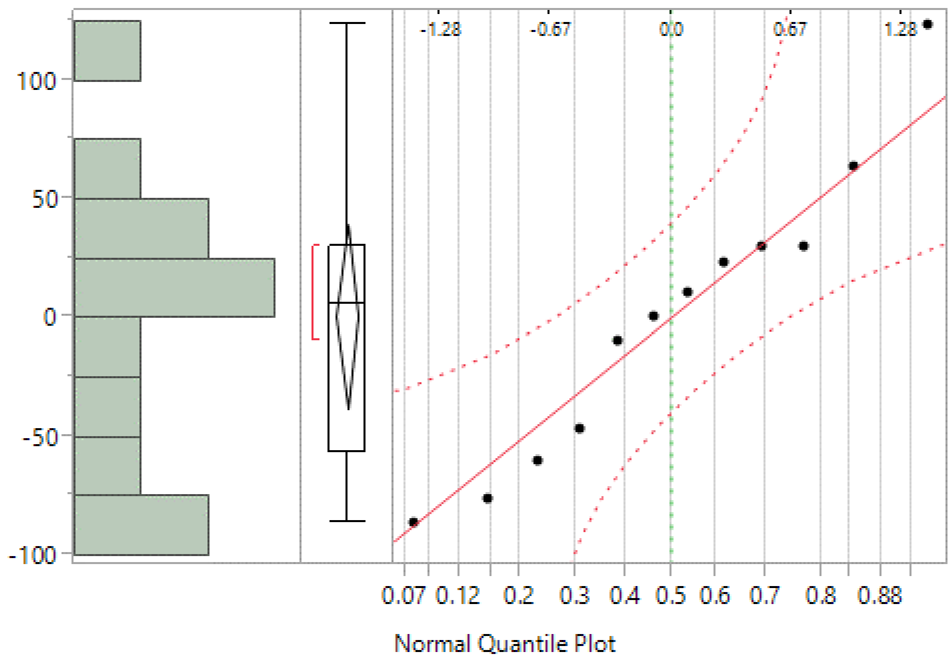


Fig. 11 *Normal quantile plot of residuals*

It shows that there are no outliers and data is close to the line. Therefore, normality assumption is true.

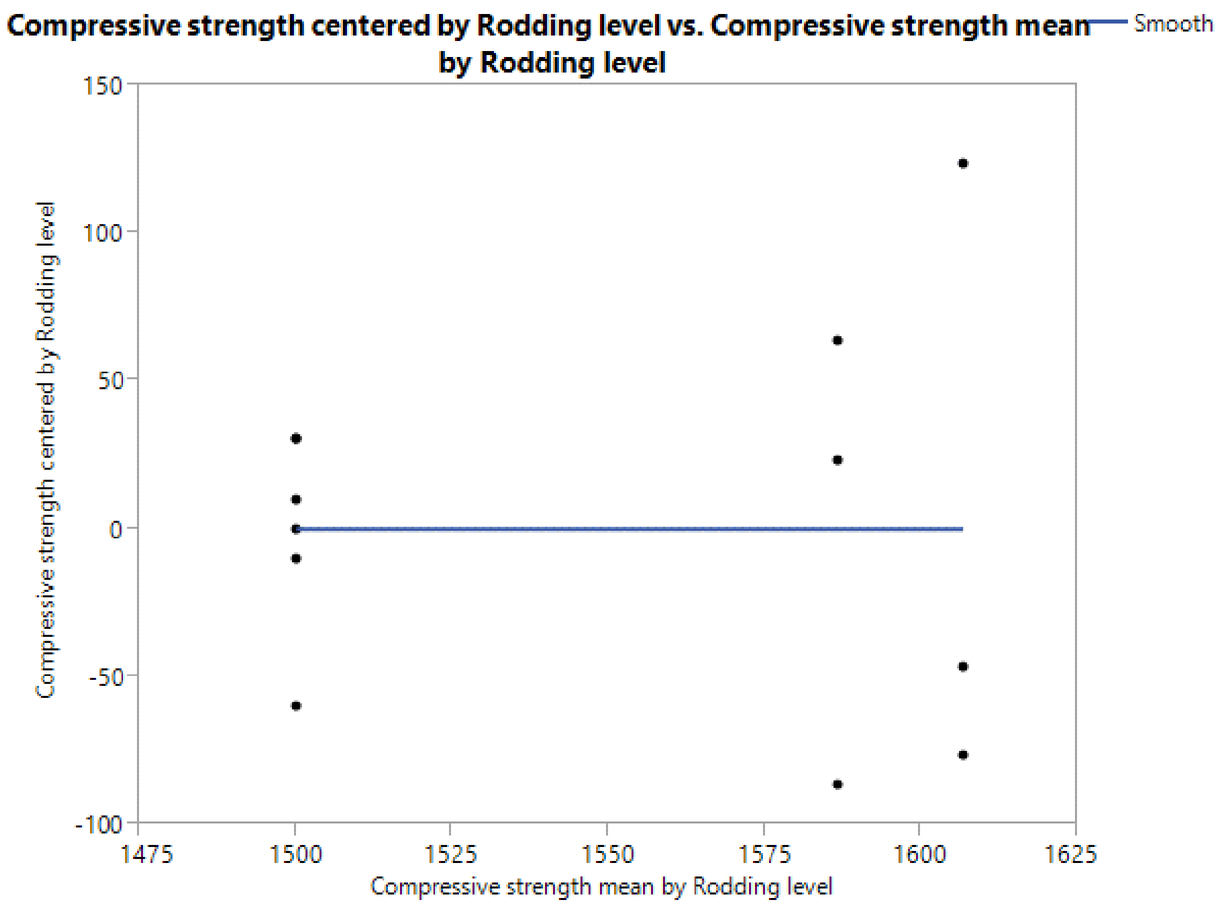


Fig. 12 *Residuals vs. Fitted*

From this graph, it could be concluded that there is no relationship between residuals and the fitted values.

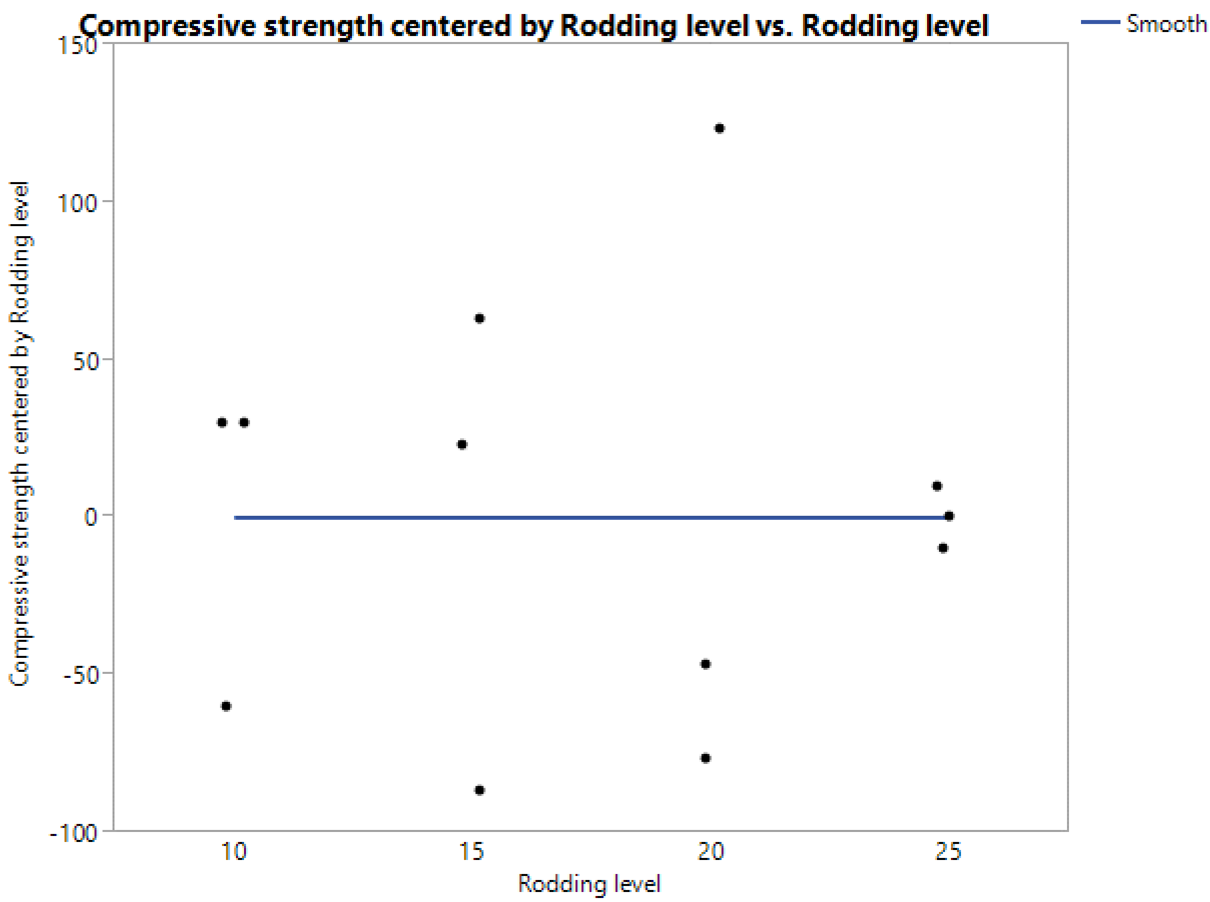


Fig. 13 *Residuals vs. Rodding level*

From this graph, it could be concluded that there is no relationship between residuals and the rodding level.

Therefore, it could be concluded that the underlying model assumptions are not violated.