

## EGR 7050 Design and Analysis of Engineering experiments

## Homework 4

1. A product developer is investigating the tensile strength of a new synthetic fiber that will be used to make cloth for men's shirts. Strength is usually affected by the percentage of cotton used in the blend of materials for the fiber. The engineer conducts a completely randomized experiment with five levels of cotton content and replicates the experiment five times. The data are shown in the following table.

Cotton weight percent	Observations				
15	7	7	15	11	9
20	12	17	12	18	18
25	14	19	19	18	18
30	19	25	22	19	23
35	7	10	11	15	11

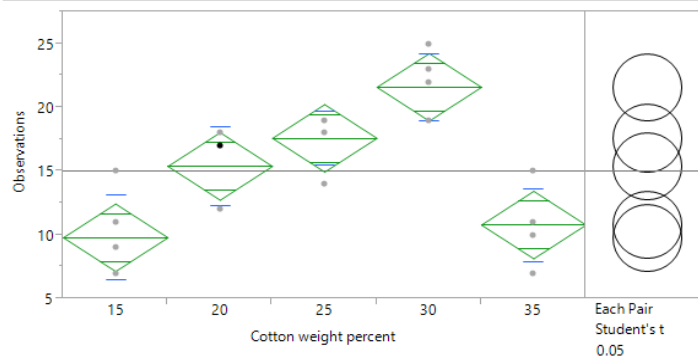
- a. Is there evidence to support the claim that cotton content affects the mean tensile strength? Use  $\alpha = 0.05$ .

**Solution:**

- a.  $H_0$ : Percentage of cotton used in the fiber does not affect mean the tensile strength  
 $H_1$ : Percentage of cotton used in the fiber affects the mean tensile strength

Given,  $\alpha = 0.05$

# Oneway Analysis of Observations By Cotton weight percent



## Oneway Anova

### Summary of Fit

Rsquare	0.746923
Adj Rsquare	0.696307
Root Mean Square Error	2.839014
Mean of Response	15.04
Observations (or Sum Wgts)	25

### Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Cotton weight percent	4	475.76000	118.940	14.7568	<.0001*
Error	20	161.20000	8.060		
C. Total	24	636.96000			

### Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
15	5	9.8000	1.2696	7.152	12.448
20	5	15.4000	1.2696	12.752	18.048
25	5	17.6000	1.2696	14.952	20.248
30	5	21.6000	1.2696	18.952	24.248
35	5	10.8000	1.2696	8.152	13.448

Std Error uses a pooled estimate of error variance

## Means Comparisons

### Comparisons for each pair using Student's t

#### Confidence Quantile

t	Alpha
2.08596	0.05

#### LSD Threshold Matrix

Abs(Dif)-LSD	30	25	20	35	15
30	-3.7455	0.2545	2.4545	7.0545	8.0545
25	0.2545	-3.7455	-1.5455	3.0545	4.0545
20	2.4545	-1.5455	-3.7455	0.8545	1.8545
35	7.0545	3.0545	0.8545	-3.7455	-2.7455
15	8.0545	4.0545	1.8545	-2.7455	-3.7455

Positive values show pairs of means that are significantly different.

### Connecting Letters Report

Level	Mean
30	A
25	B
20	B
35	C
15	C

Levels not connected by same letter are significantly different.

### Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
30	15	11.80000	1.795550	8.05455	15.54545	<.0001*
30	35	10.80000	1.795550	7.05455	14.54545	<.0001*
25	15	7.80000	1.795550	4.05455	11.54545	0.0003*
25	35	6.80000	1.795550	3.05455	10.54545	0.0012*
30	20	6.20000	1.795550	2.45455	9.94545	0.0025*
20	15	5.60000	1.795550	1.85455	9.34545	0.0054*
20	35	4.60000	1.795550	0.85455	8.34545	0.0186*
30	25	4.00000	1.795550	0.25455	7.74545	0.0375*
25	20	2.20000	1.795550	-1.54545	5.94545	0.2347
35	15	1.00000	1.795550	-2.74545	4.74545	0.5838

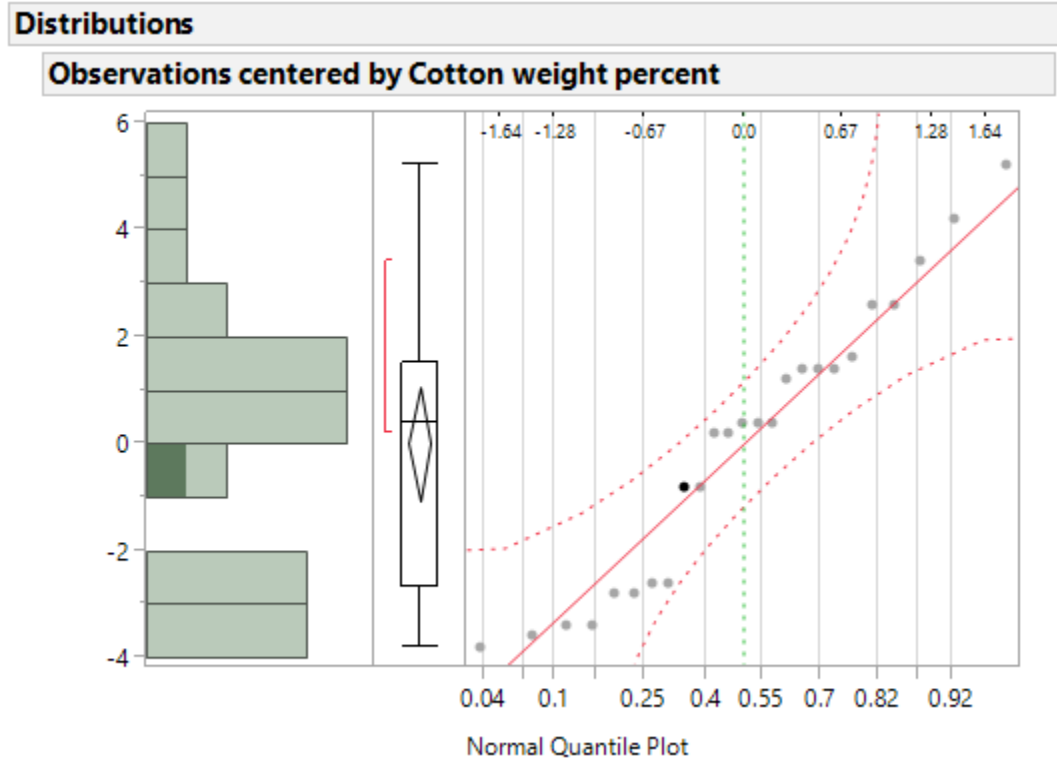
P value is less than significance level  $\alpha = 0.05$  and  $F_0 > F_{0.05,4,20}$ . Therefore, reject null hypotheses.

From connecting letters report, we could see that level 30 is different from the rest of the levels. Similarly, levels 25, 20 are different from levels 35, 15. We cannot detect a significant difference between levels 25, 20 and levels 35,15.

Fig. 1 OneWay ANOVA

Thus, it could be concluded that the percentage of cotton used in the fiber affects the mean tensile strength.

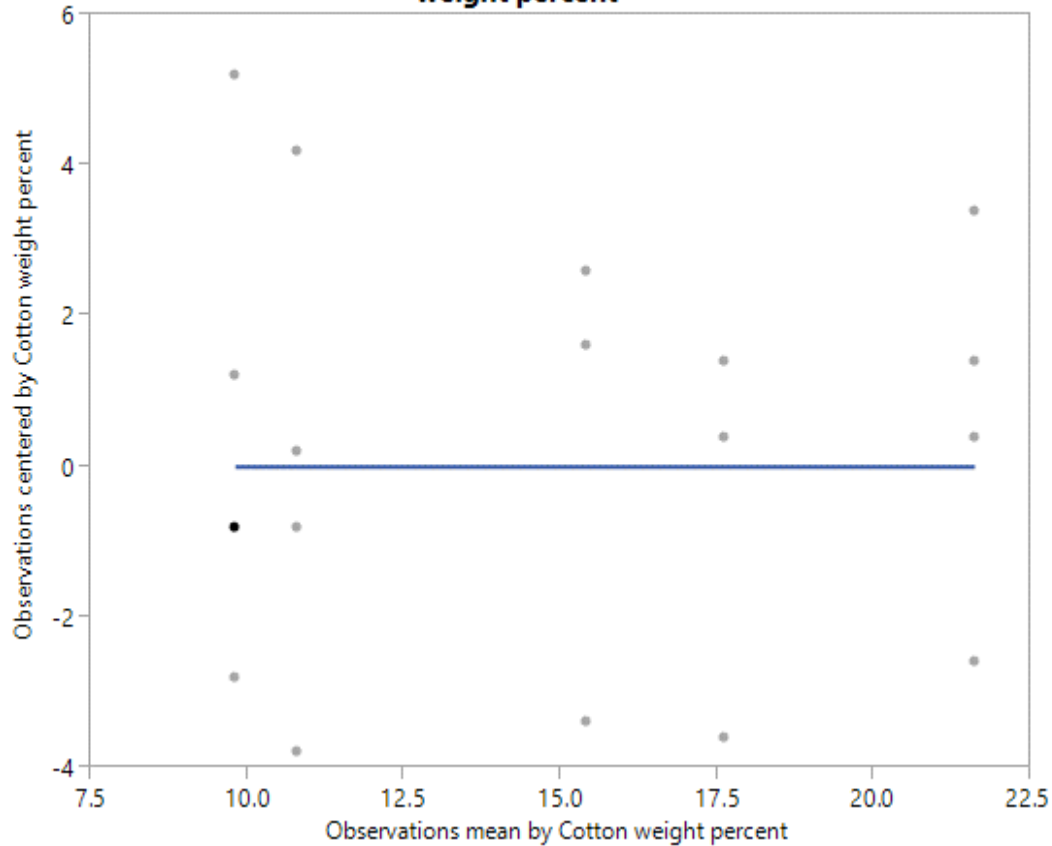
- b. Analyze the residuals from this experiment and comment on model adequacy.



**Fig.2** Normal quantile plot

Points are close to the line and are within the error bounds. There is no significant evidence of deviation from normality for the residuals.

servations centered by Cotton weight percent vs. Observations mean by Cotton weight percent



**Fig. 3** Residual vs. Fitted

There are no outliers in the residual vs. fitted plots. There is a similar range of variation across different fitted values. There is no significant deviation from equal variance assumption.

2. An experiment was performed to investigate the effectiveness of five insulating materials. Four samples of each material were tested at an elevated voltage level to accelerate the time to failure. The failure times (in minutes) are shown below:

Material	Failure Time (minutes)			
1	110	157	194	178
2	1	2	4	18
3	880	1256	5276	4355
4	495	7040	5307	10050
5	7	5	29	2

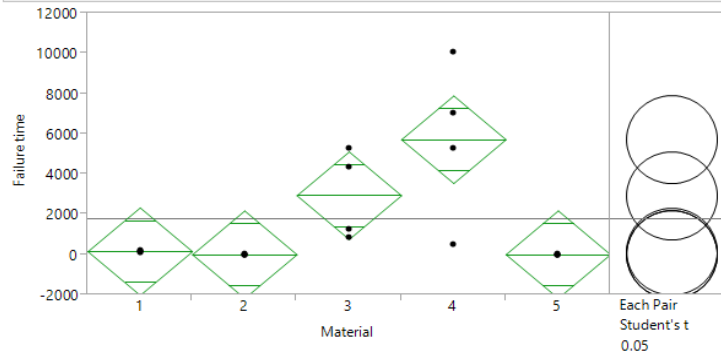
**Solution:**

- a. Do all five materials have the same effect on mean failure time?

$H_0$ : Mean failure time is same for all materials

$H_1$ : Mean failure time is different for all materials

# Oneway Analysis of Failure time By Material



## Oneway Anova

### Summary of Fit

Rsquare	0.622772
Adj Rsquare	0.522177
Root Mean Square Error	2041.334
Mean of Response	1768.3
Observations (or Sum Wgts)	20

### Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Material	4	103191489	25797872	6.1909	0.0038*
Error	15	62505657	4167043.8		
C. Total	19	165697146			

### Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1	4	159.75	1020.7	-2016	2335.3
2	4	6.25	1020.7	-2169	2181.8
3	4	2941.75	1020.7	766	5117.3
4	4	5723.00	1020.7	3547	7898.5
5	4	10.75	1020.7	-2165	2186.3

Std Error uses a pooled estimate of error variance

## Means Comparisons

### Comparisons for each pair using Student's t

#### Confidence Quantile

t	Alpha
2.13145	0.05

#### LSD Threshold Matrix

Abs(Dif)-LSD

	4	3	1	5	2
4	-3076.6	-2954	2486.6	2635.6	2640.1
3	-2954	-3076.6	-2946	-1456	-141.1
1	2486.6	-2946	-3076.6	-2927.6	-2923.1
5	2635.6	-1456	-2927.6	-3076.6	-3072.1
2	2640.1	-141.1	-2923.1	-3072.1	-3076.6

Positive values show pairs of means that are significantly different.

### Connecting Letters Report

Level	Mean
4	A 5723.0000
3	A B 2941.7500
1	B 159.7500
5	B 10.7500
2	B 6.2500

Levels not connected by same letter are significantly different.

### Ordered Differences Report

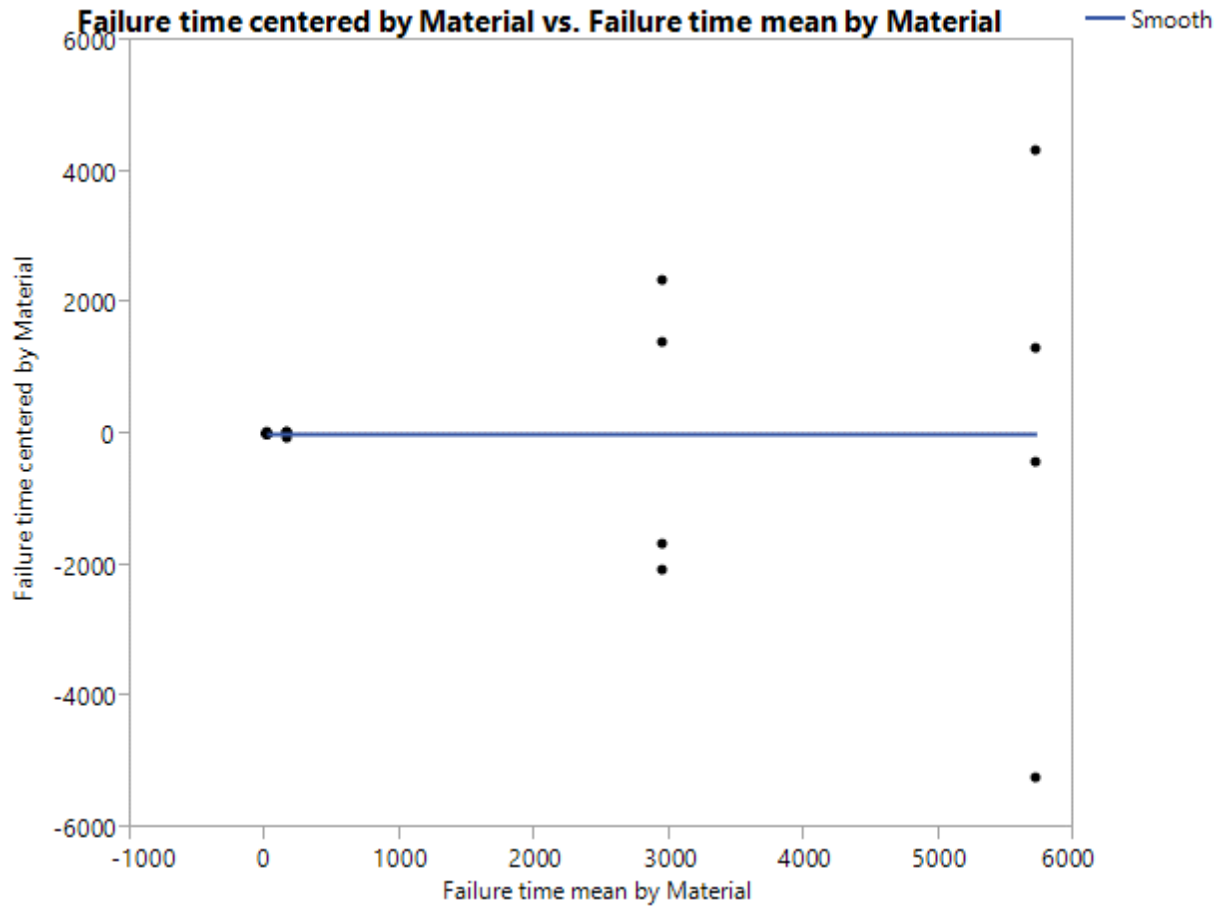
Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
4	2	5716.750	1443.441	2640.13	8793.372	0.0013*
4	5	5712.250	1443.441	2635.63	8788.872	0.0013*
4	1	5563.250	1443.441	2486.63	8639.872	0.0016*
3	2	2935.500	1443.441	-141.12	6012.122	0.0601
3	5	2931.000	1443.441	-145.62	6007.622	0.0604
3	1	2782.000	1443.441	-294.62	5858.622	0.0731
4	3	2781.250	1443.441	-295.37	5857.872	0.0732
1	2	153.500	1443.441	-2923.12	3230.122	0.9167
1	5	149.000	1443.441	-2927.62	3225.622	0.9192
5	2	4.500	1443.441	-3072.12	3081.122	0.9976

P value is less than significance level  $\alpha = 0.05$  and  $F_0 > F_{0.05,4,15}$ . Therefore, reject null hypothesis.

Fig. 4 OneWay ANOVA

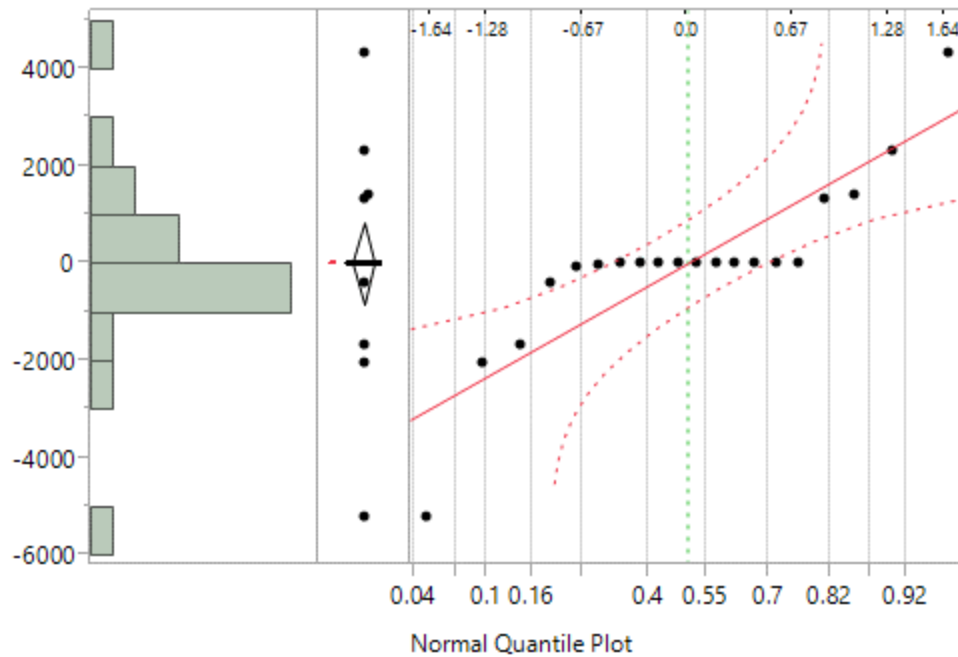
Therefore, we could conclude that mean failure time is not the same for all materials.

- b. Plot the residuals versus the predicted response. Construct a normal probability plot of the residuals. What information is conveyed by these plots?



**Fig. 5** Residuals vs. Predicted

The figure shows that variance is not constant for all the predicted values.



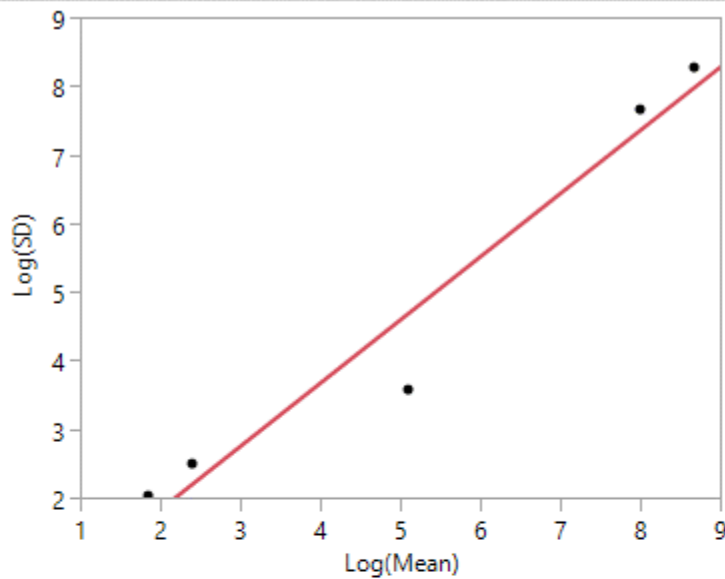
**Fig. 6** Normal quantile plot

The figure clearly shows that points are not close to the line and outside the error bounds. This shows that normality assumption is not valid.

- c. Based on your answer to part (b) conduct another analysis of the failure time data and draw appropriate conclusions.

Let us plot a bivariate fit of Log(Mean) and Log(Standard deviation).

### Bivariate Fit of Log(SD) By Log(Mean)



— Linear Fit

#### Linear Fit

$$\text{Log(SD)} = 0.0591555 + 0.9210885 \cdot \text{Log(Mean)}$$

#### Summary of Fit

RSquare	0.953466
RSquare Adj	0.937955
Root Mean Square Error	0.733858
Mean of Response	4.834098
Observations (or Sum Wgts)	5

#### Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	33.104224	33.1042	61.4694
Error	3	1.615643	0.5385	Prob > F
C. Total	4	34.719867		0.0043*

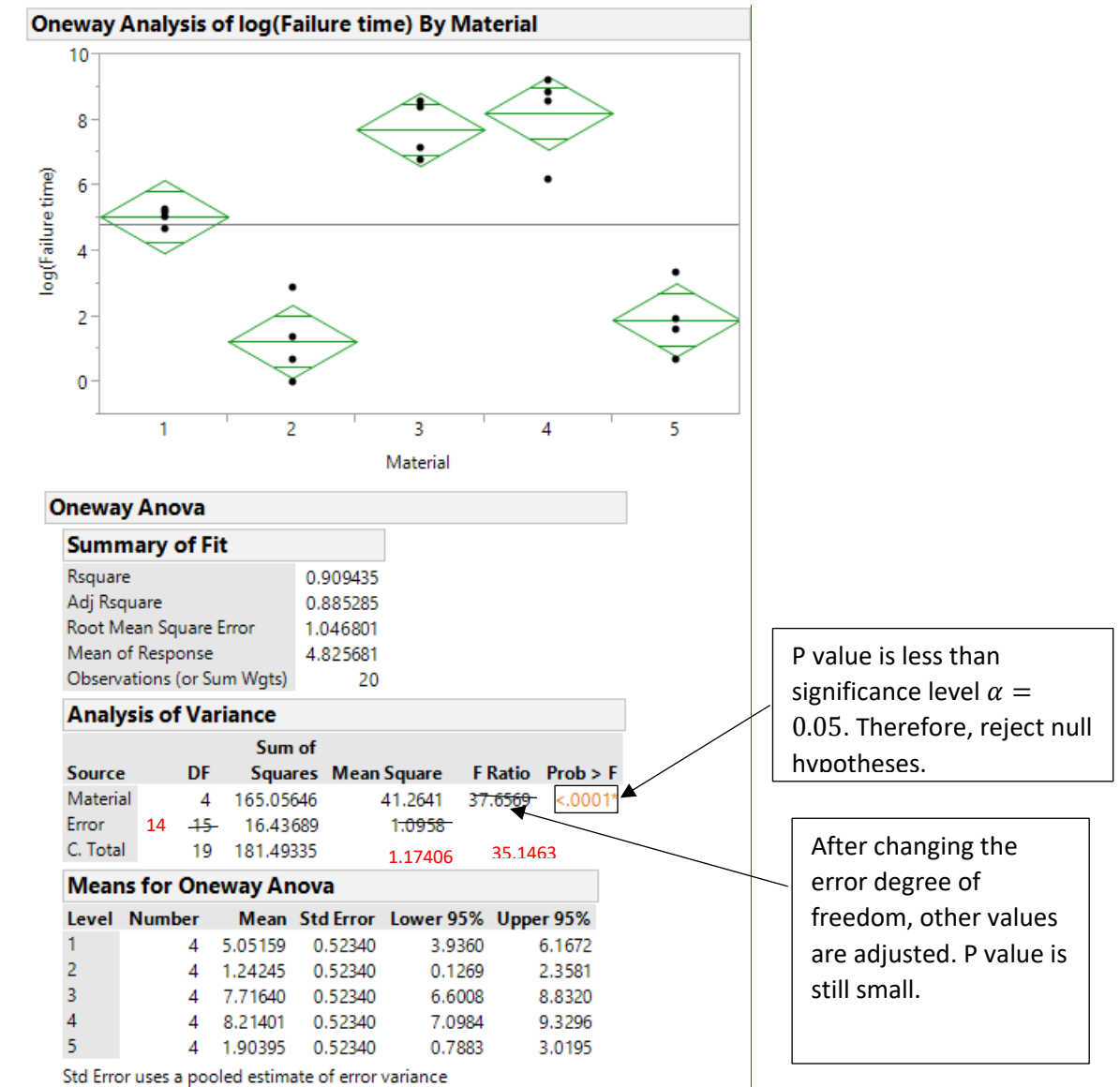
#### Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	0.0591555	0.691829	0.09	0.9372
Log(Mean)	0.9210885	0.117482	7.84	0.0043*

**Fig. 7** Log(Mean) vs. Log(S.D)

From the plot, we could see that slope of the fit line is 0.9210 which is close to 1. So, from the table 3.9, a log transformation would be appropriate.

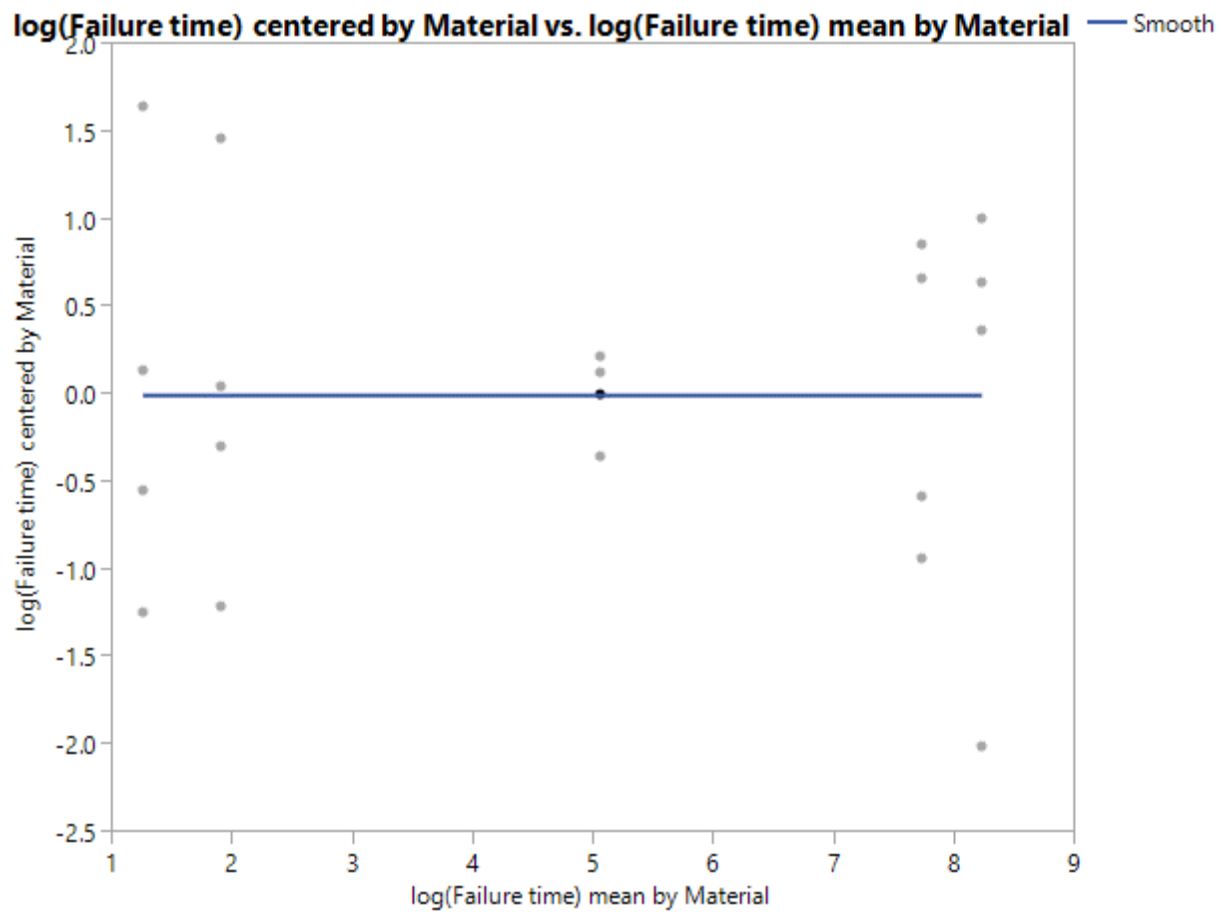




**Fig. 8 OneWay ANOVA**

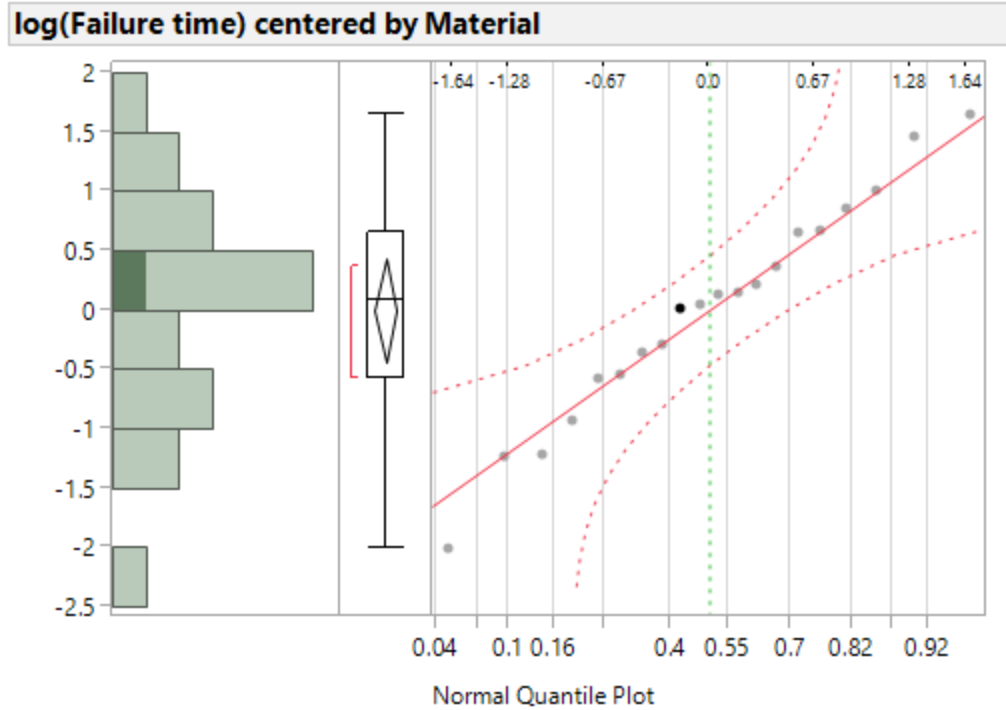
Therefore, we could conclude that mean failure time is not the same for all the materials.

Lower t



**Fig. 9** *Residuals vs. Predicted*

This plot has been improved with transformed data although range of variation of one of the materials is lower than others.



**Fig. 10** Normal quantile plot

This shows that all points lie close to the line and within the error bounds. Hence the assumptions are not violated.