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**Assignment 3**

**STT4110/6110**

1. Sketch the autocorrelation functions for the following MA(2) models with parameters as specified:

Consider the moving average process of order 2:

*Yt* = *et* –θ1*et*–1 –θ2*et*–2

Theoretical autocorrelation function is as follows,

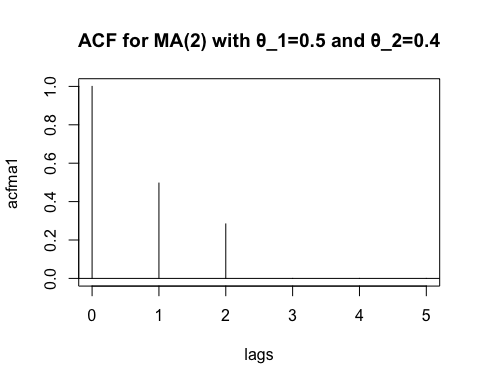
,

Using R-code, we obtain the sketch of ACF functions and the values of theoretical autocorrelation function

library(ggplot2)  
library(grid)  
par(mfrow=c(1,1))  
acfma1 <- ARMAacf(ma=c(0.5,0.4),lag.max=5)  
print(acfma1)

## 0 1 2 3 4 5   
## 1.0000000 0.4964539 0.2836879 0.0000000 0.0000000 0.0000000

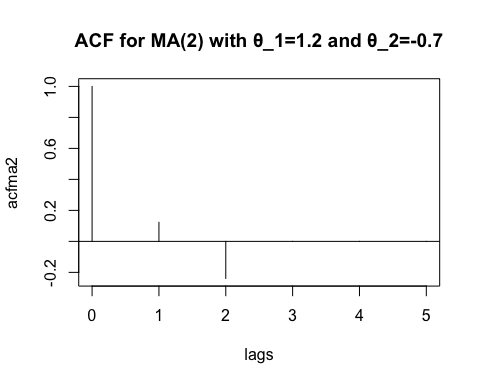
lags <- 0:5  
plot(lags,acfma1,type="h",main="ACF for MA(2) with θ\_1=0.5 and θ\_2=0.4")  
abline(h=0)



par(mfrow=c(1,1))  
acfma2 <- ARMAacf(ma=c(1.2,-0.7),lag.max=5)  
print(acfma2)

## 0 1 2 3 4 5   
## 1.0000000 0.1228669 -0.2389078 0.0000000 0.0000000 0.0000000

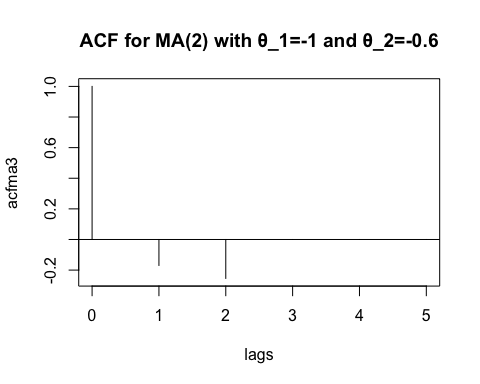
lags <- 0:5  
plot(lags,acfma2,type="h",main="ACF for MA(2) with θ\_1=1.2 and θ\_2=-0.7")  
abline(h=0)



par(mfrow=c(1,1))  
acfma3 <- ARMAacf(ma=c(-1,-0.6),lag.max=5)  
print(acfma3)

## 0 1 2 3 4 5   
## 1.0000000 -0.1694915 -0.2542373 0.0000000 0.0000000 0.0000000

lags <- 0:5  
plot(lags,acfma3,type="h",main="ACF for MA(2) with θ\_1=-1 and θ\_2=-0.6")  
abline(h=0)



1. Show that when θ is replaced by 1/θ, the autocorrelation function for an MA(1) process does not change.

MA(1) model is

Hence proved

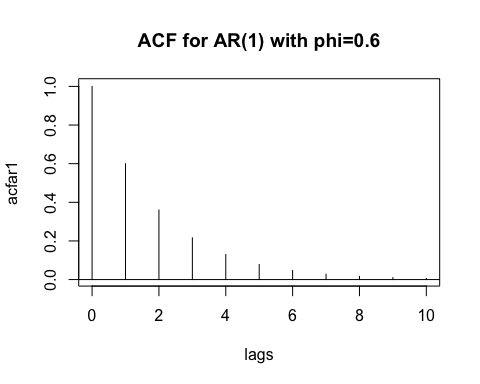
1. Calculate and sketch the autocorrelation functions for each of the following AR(1) models. Plot for sufficient lags that the autocorrelation function has nearly died out.

1. φ1 = 0.6.

library(ggplot2)  
library(grid)  
par(mfrow=c(1,1))  
acfar1 <- ARMAacf(ar=0.6,lag.max=10)  
print(acfar1)

## 0 1 2 3 4 5   
## 1.000000000 0.600000000 0.360000000 0.216000000 0.129600000 0.077760000   
## 6 7 8 9 10   
## 0.046656000 0.027993600 0.016796160 0.010077696 0.006046618

lags <- 0:10  
plot(lags,acfar1,type="h",main="ACF for AR(1) with phi=0.6")  
abline(h=0)

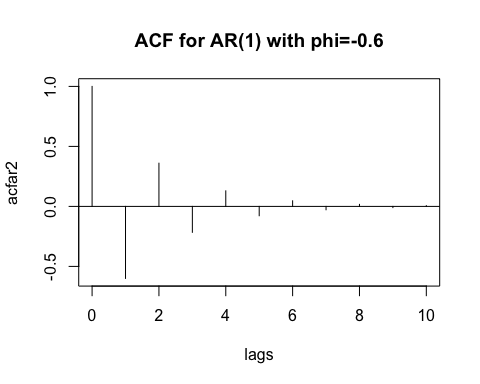


b) φ1 = −0.6.

library(ggplot2)  
library(grid)  
par(mfrow=c(1,1))  
acfar2 <- ARMAacf(ar=-0.6,lag.max=10)  
print(acfar2)

## 0 1 2 3 4   
## 1.000000000 -0.600000000 0.360000000 -0.216000000 0.129600000   
## 5 6 7 8 9   
## -0.077760000 0.046656000 -0.027993600 0.016796160 -0.010077696   
## 10   
## 0.006046618

lags <- 0:10  
plot(lags,acfar2,type="h",main="ACF for AR(1) with phi=-0.6")  
abline(h=0)

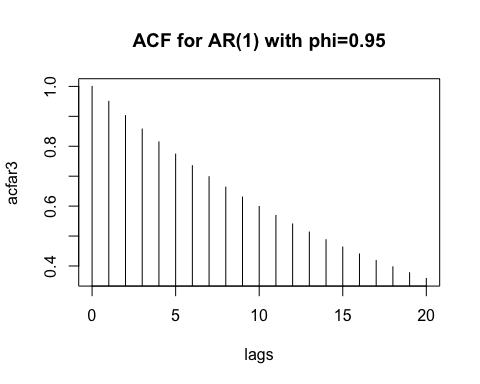


c) φ1 = 0.95. (Do out to 20 lags.)

library(ggplot2)  
library(grid)  
par(mfrow=c(1,1))  
acfar3 <- ARMAacf(ar=0.95,lag.max=20)  
print(acfar3)

## 0 1 2 3 4 5 6   
## 1.0000000 0.9500000 0.9025000 0.8573750 0.8145062 0.7737809 0.7350919   
## 7 8 9 10 11 12 13   
## 0.6983373 0.6634204 0.6302494 0.5987369 0.5688001 0.5403601 0.5133421   
## 14 15 16 17 18 19 20   
## 0.4876750 0.4632912 0.4401267 0.4181203 0.3972143 0.3773536 0.3584859

lags <- 0:20  
plot(lags,acfar3,type="h",main="ACF for AR(1) with phi=0.95")  
abline(h=0)

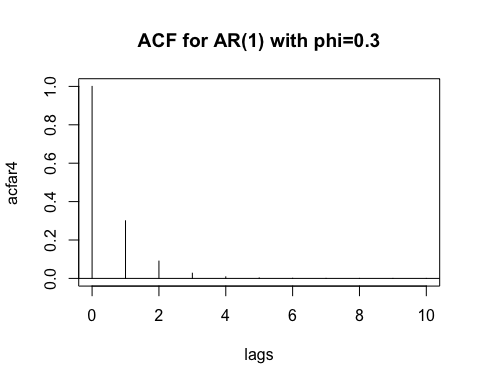


d) φ1 = 0.3.

library(ggplot2)  
library(grid)  
par(mfrow=c(1,1))  
acfar4 <- ARMAacf(ar=0.3,lag.max=10)  
print(acfar4)

## 0 1 2 3 4 5   
## 1.0000e+00 3.0000e-01 9.0000e-02 2.7000e-02 8.1000e-03 2.4300e-03   
## 6 7 8 9 10   
## 7.2900e-04 2.1870e-04 6.5610e-05 1.9683e-05 5.9049e-06

lags <- 0:10  
plot(lags,acfar4,type="h",main="ACF for AR(1) with phi=0.3")  
abline(h=0)



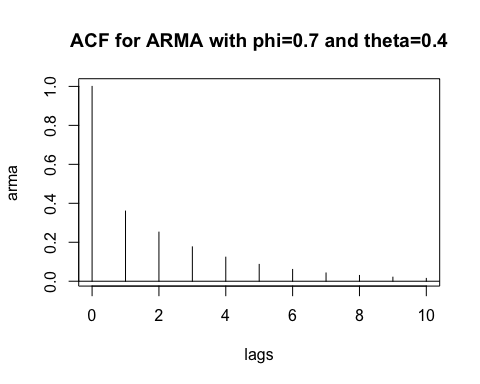
1. Sketch the autocorrelation functions for each of the following ARMA models:
2. ARMA(1,1) with φ = 0.7 and θ = 0.4. **(b)** ARMA(1,1) with φ = 0.7 and θ = −0.4.

1. ARMA(1,1) with φ = 0.7 and θ = 0.4.

library(ggplot2)  
library(grid)  
par(mfrow=c(1,1))  
arma <- ARMAacf(ar=0.7,ma=-0.4,lag.max=10)  
print(arma)

## 0 1 2 3 4 5   
## 1.00000000 0.36000000 0.25200000 0.17640000 0.12348000 0.08643600   
## 6 7 8 9 10   
## 0.06050520 0.04235364 0.02964755 0.02075328 0.01452730

lags <- 0:10  
plot(lags,arma,type="h",main="ACF for ARMA with phi=0.7 and theta=0.4")  
abline(h=0)

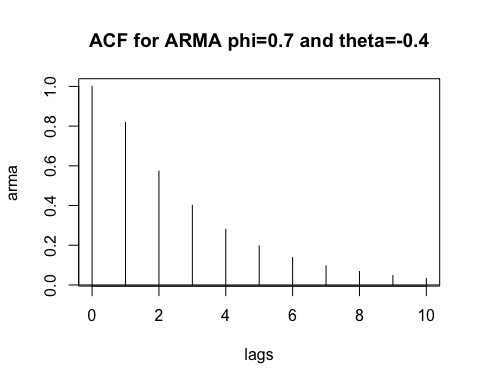


1. ARMA(1,1) with φ = 0.7 and θ = − 0.4.

par(mfrow=c(1,1))  
arma <- ARMAacf(ar=0.7,ma=0.4,lag.max=10)  
print(arma)

## 0 1 2 3 4 5   
## 1.00000000 0.81860465 0.57302326 0.40111628 0.28078140 0.19654698   
## 6 7 8 9 10   
## 0.13758288 0.09630802 0.06741561 0.04719093 0.03303365

lags <- 0:10  
plot(lags,arma,type="h",main="ACF for ARMA phi=0.7 and theta=-0.4")  
abline(h=0)



1. Consider a process that satisfies the zero-mean, “stationary” AR(1) equation *Yt* =  φ*Yt* − 1 + *et* with −1 < φ < +1. Let *c* be any nonzero constant, and define *Wt* = *Yt* + *c t*.
   1. Show that

(Process is zero mean)

1. Show that {*Wt*} satisfies the “stationary” AR(1) equation *Wt* = φ*Wt* − 1 + *et*.

1. Is {Wt} stationary?

is not constant in time, therefore it is not stationary

1. Suppose that {Yt} is an AR(1) process with −1 < φ < +1.
2. Find the auto covariance function for Wt = ∇Yt = Yt − Yt−1 in terms of φ and σe2.

Wt = ∇ = −

auto-covariance is

if = 0

if = 1

1. In particular, show that

1. Suppose that {Yt} is an AR(1) process with ρ1 = φ. Define the sequence {bt} as bt =Yt −φYt+1.
   1. Show that Cov(bt,bt − k) = 0 for all t and k.

* 1. Show that *Cov*(*bt*,*Yt* + *k*) = 0 for all *t* and *k* > 0.

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