**Meenakshi Nagarajan**

**Assignment 4**

**STT4110/6110**

* 1. Sketch the autocorrelation functions for each of the following ARMA models:

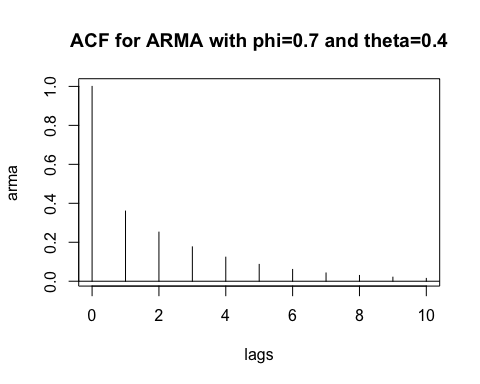
1. ARMA(1,1) with φ = 0.7 and θ = 0.4. **(b)** ARMA(1,1) with φ = 0.7 and θ = −0.4.

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library(ggplot2)  
library(grid)  
par(mfrow=c(1,1))  
arma <- ARMAacf(ar=0.7,ma=-0.4,lag.max=10)  
print(arma)

## 0 1 2 3 4 5   
## 1.00000000 0.36000000 0.25200000 0.17640000 0.12348000 0.08643600   
## 6 7 8 9 10   
## 0.06050520 0.04235364 0.02964755 0.02075328 0.01452730

lags <- 0:10  
plot(lags,arma,type="h",main="ACF for ARMA with phi=0.7 and theta=0.4")  
abline(h=0)

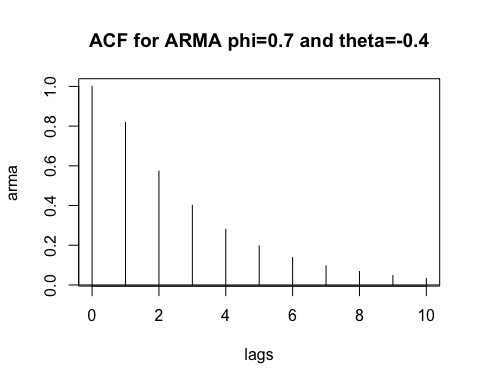


1. ARMA(1,1) with φ = 0.7 and θ = − 0.4.

par(mfrow=c(1,1))  
arma <- ARMAacf(ar=0.7,ma=0.4,lag.max=10)  
print(arma)

## 0 1 2 3 4 5   
## 1.00000000 0.81860465 0.57302326 0.40111628 0.28078140 0.19654698   
## 6 7 8 9 10   
## 0.13758288 0.09630802 0.06741561 0.04719093 0.03303365

lags <- 0:10  
plot(lags,arma,type="h",main="ACF for ARMA phi=0.7 and theta=-0.4")  
abline(h=0)



4.21 Consider the model *Yt*=*et*−1−*et*−2+0.5*et*−3.

**(a)** Find the autocovariance function for this process.

**(b)** Show that this is a certain ARMA(*p*,*q*) process in disguise. That is, identify  values for *p* and *q* and for the θ’s and φ’s such that the ARMA(*p*,*q*) process has the same statistical properties as {*Yt*}.

*Solution:*

1. For a white noise process {

ACF for *Yt*=*et*−1−*et*−2+0.5*et*−3 at lag k is

1. General ARMA (p,q) process is

*Yt* = φ1*Yt*–1 +φ2*Yt*–2 +...+φ*pYt*–*p* +*et* –θ1*et*–1 –θ2*et*–2 – ... – θ*qet* – *q*

Given equation *Yt*=*et*−1−*et*−2+0.5*et*−3.

Comparing this to the general equation we see that it is an ARMA process with p=0 and q=2

Φ=0 and

* **4.25** Consider an “AR(1)” process satisfying *Yt* = φ*Yt* − 1 + *et*, where φ can be ***any*** number and {*et*} is a white noise process such that *et* is independent of the past {*Yt* − 1, *Yt* − 2,...}. Let *Y*0 be a random variable with mean μ0 and variance σ02 .
  1. **(a)** Show that for *t* > 0 we can write  *Yt* =*et* +φ*et*−1 +φ *et*−2 +φ *et*−3 + *+*φ *e*1 +φ*Y*0.
  2. **(b)** Show that for *t* > 0 we have *E*(*Yt*) = φ*t*μ0.
  3. **(c)** Show that for *t* > 0  ⎧ 1–φ2*t*⎪ ------------σ*e*2 + φ2*t*σ02 for φ ≠ 1 *Var*(*Yt*) = ⎨ 1–φ2 ⎪⎩ *t* σ *e*2 + σ 02 f o r φ = 1

**(d)** Suppose now that μ0 = 0. Argue that, if {*Yt*} is stationary, we must have φ ≠ 1.

**(e)** Continuing to suppose that μ0 = 0, show that, if {*Yt*} is stationary, then

*V a r* ( *Y t* ) = σ *e*2 ⁄ ( 1 – φ 2 ) a n d s o w e m u s t h a v e | φ | < 1 .

*Solution:*

1. *Yt* = φ*Yt* − 1 + *et*

Substituting

(

Again, substituting for we get

Again, substituting for we get

……

For a white noise process {

Therefore,

For a stochastic process,



*Since, Var*(*a*+*bX*) = *b*2*Var*(*X*)

For



if

, there {

if

Variance is not free of t if

Therefore, for { to be stationary

1. )

Therefore, we must have

5.4 Suppose that *Yt* = *A* + *Bt* + *Xt*, where {*Xt*} is a random walk. First suppose that *A* and *B* are constants.  **(a)** Is {*Yt*} stationary?  **(b)** Is {∇*Yt*} stationary?

Now suppose that *A* and *B* are random variables that are independent of the random walk {*Xt*}.

*Solution:*

= (Mean of random walk is zero)

, if B

If B=0

b)

+

+

=

Therefore, {∇*Yt*} is stationary

E(∇*Yt) = B*

, , , w=∇*Yt*

c) If A and B are random variables,

=

It is stationary.