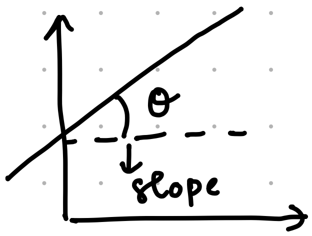
Straight line

Video - Complete linear algebra for ML in hindi by Data Dissection

→ formula = $y = mx + c$

where

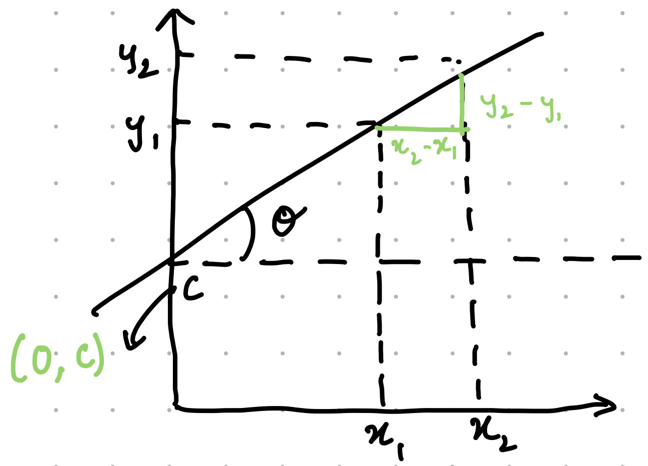


$m \rightarrow$ slope, $c \rightarrow$ intersect } parameters

x, y } variables

→ 2 points are enough to draw a straight line

→ calculating slope



$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y_1 - c}{x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

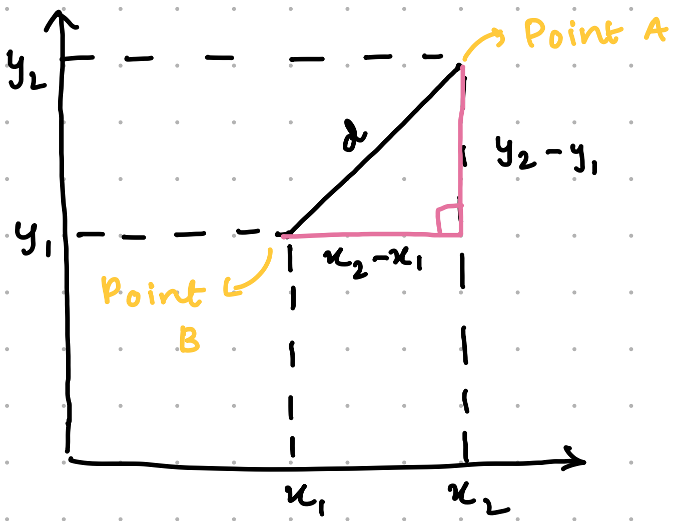
$$\Rightarrow \frac{y_1 - c}{x_1} = m$$

$$\Rightarrow y_1 - c = m(x_1)$$

$$\Rightarrow y_1 = mx_1 + c$$

Co-ordinate Geometry

→ distance b/w 2 points

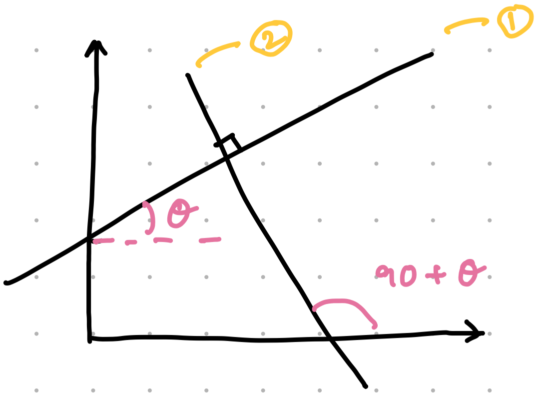


using Pythogoras Theorem -

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Parallel and perpendicular lines

→ parallel - slope is same, intercept differs
→ perpendicular - $m_1 \times m_2 = -1$



① $y = m_1 x + c_1$
② $y = m_2 x + c_2$

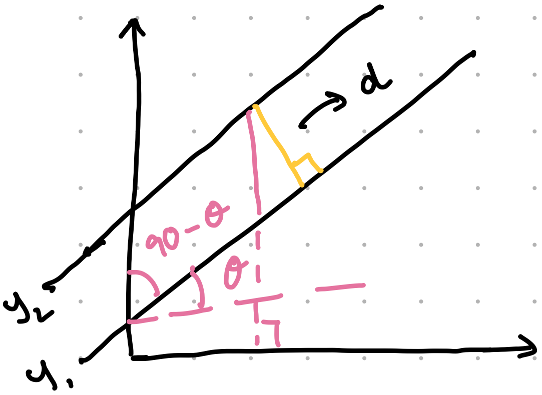
$\tan\theta = m_1$

$\tan(90+\theta) = m_2 = \dfrac{\sin(90+\theta)}{\cos(90+\theta)} = \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1}{m_1}$

Used in KNN

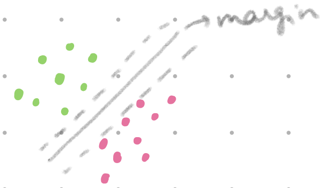
Distance b/w 2 parallel lines



$$d = \frac{|c_2 - c_1|}{\sqrt{1+m^2}}$$

Used in SVM → find best boundary (hyperplane) that separates diff. classes in the data

supporting vectors lie exactly on the margin



Vectors

→ have magnitude and direction

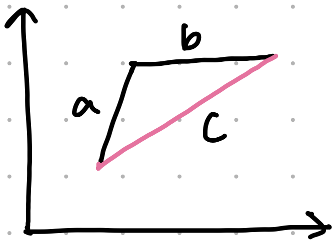
$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

amount · direction

i-hat - x axis
j-hat - y axis
k-hat - z axis

$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

→ **addition**



walking from one end of a to the other end and then covering b. dist covered = c

→ **dot product**

when we multiply a vector by a scalar, the scalar affects the length of the vector (i.e scales it up / down) in a certain direction (same if scalar is pos., opposite if negative)

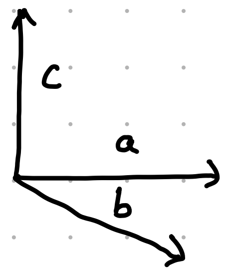
$$(a_1, a_2, a_3) \cdot (b_1, b_2, b_3) = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$\boxed{\text{Used in Neural Network}}$

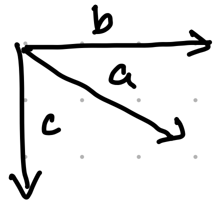
→ **cross product**

a new vector that is perpendicular to both is created.

$\vec{a} \times \vec{b}$



$\vec{b} \times \vec{a}$



Matrix

→ represented as A_{mn} where m = rows, n = col.

→ $A_{mn} + B_{mn} = C_{mn}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 6 & 2 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ 4 & 12 \end{bmatrix}$$

→ **multiplying w/ scalar** - $3 \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 12 & 6 \end{bmatrix}$

→ **multiplying matrix** - $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

no. of columns of A must be equal to no. of rows of B

$$= \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

$$\boxed{A_{mn} \times B_{pq} = C_{xy} = C_{mq}}$$

→ **Transpose**

rows become columns
& vice versa

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

can be used in dot product where cols ≠ row

$$[3 \; 1 \; 2] \cdot [4 \; 1 \; 3]$$

$$= [3 \; 1 \; 2] \cdot \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$$

→ **Determinant**

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} a_{11} & \cancel{a_{12}} & \cancel{a_{13}} \\ \cancel{a_{21}} & a_{22} & a_{23} \\ \cancel{a_{31}} & a_{32} & a_{33} \end{vmatrix}$$

① move element by element in the first row
→ start with a_{11}

② ignore the element's row and column
→ here, ignore $a_{21}, a_{31}, a_{12}, a_{13}$

③ calculate minor of the newly formed matrix
→ here $M_{11} = a_{22}a_{33} - a_{32}a_{23}$

④ multiply selected element w/ result of step ③
→ $a_{11} \times (a_{22}a_{33} - a_{32}a_{23})$

⑤ repeat for other elements in the first row
→ $a_{11} \times M_{11} - a_{12} \times M_{12} + a_{13} \times M_{13} = \det.$

works for both 2 & 3 variables

Use case - if the determinant of a system of 2 equations is not 0, then there exists a unique sol

→ Eigen values and vectors

characteristic equation - $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

where -

trace == sum of eigen values

S_1 = sum of elements of main diagonal

S_2 = sum of minors of diagonal entries

S_3 = determinant of matrix

Solve the so-formed equation to get eigen values

$[A - \lambda]x = 0$ → eigen vector

★ $\lambda_1 \times \lambda_2 \times \lambda_3$ = determinant of matrix

★ for both diagonal & upper triangular matrices, eigen values are the diagonal elements

★ given a square matrix E, eigen values of 2E == 2 x eigen values of E