DSP problems

Q.1 A discrete time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1+\frac{3}{n} & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{else where} \end{cases}$$

a) Determine it's values and sketch the signal $x(n)$.

b) sketch the signals that result if we:

1> fold $x(n)$ and then delay the resulting signal by four samples.

2> first delay $x(n)$ by four samples and then fold the resulting sigl.

(c) sketch the sigl $x(-n+4)$

(d) compare the result in parts (b) and (c) and derive a rule for obtaining the sigl $x(-n+k)$ from $x(n)$.

(e) can you express the sigl $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

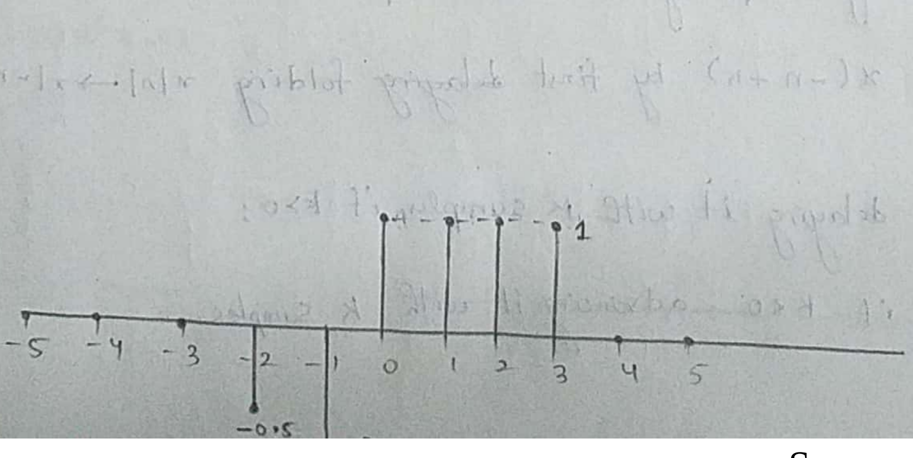
$n = \{-3, -2, -1\}$ then $x(n) = 1+\frac{3}{n}$.

$x(-3) = 1+\frac{3}{-3}$, $x(-2) = 1+\frac{3}{-2}$, $x(-1) = 1+\frac{3}{-1}$

$1+(-1)$ $x(-2) = 1+(-1.5)$ $x(-1) = 1-3$

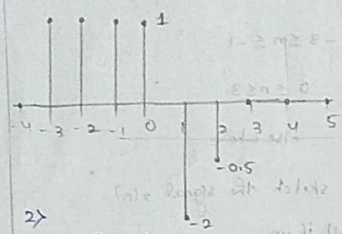
$x(-3) = 0$, $x(-2) = -0.5$, $x(-1) = -2$.

(a)

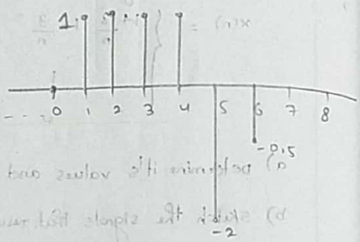


(b) $x(-n)$

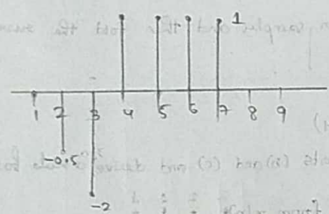
1>



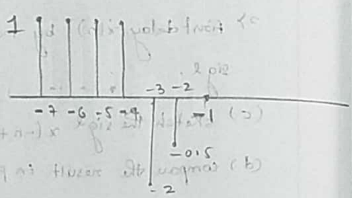
$x(-n-4)$



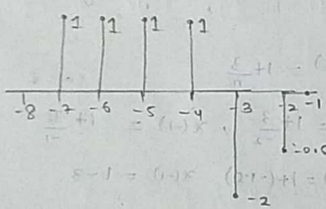
2> $x(n-4)$



$x(-n+4)$



3> $x(-n+4)$



d> By comparing the result in parts (b) and (c) we can get $x(-n+K)$ by first ~~delaying~~ folding $x[n] \rightarrow x[-n]$ and delaying it with K samples if $K>0$;

or if $K<0$, advancing it with K samples.

(e)

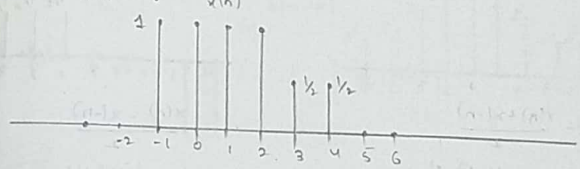
$x(n) = \Sigma$

yes, we can express the signal $x[n]$ in terms of signals $\delta[n]$ and $u[n]$.

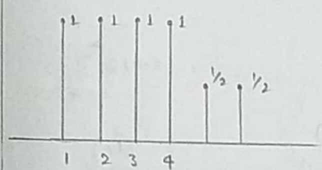
$$x[n] = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u[n] - u[n-4]$$

2.2 A discrete time signal $x(n)$ is defined as is shown in below figure. Sketch and label carefully each of the following signals.

$x(n)$

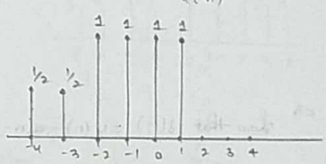


(a) $x(n-2)$

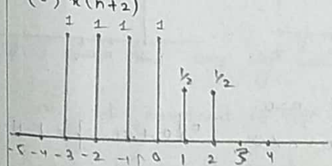


$x(4-n)$

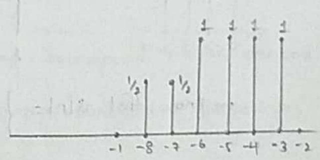
$x(-n)$



(c) $x(n+2)$

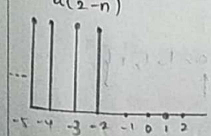


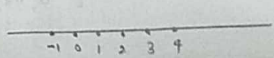
$x(4-n)$



(d) $x(n)u(2-n)$

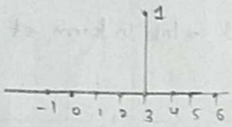
$u(2-n)$



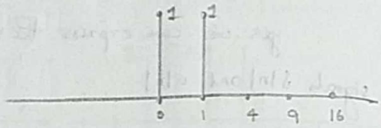


Scanned by CamScanner

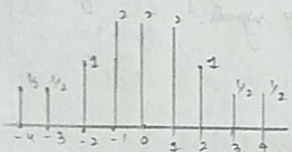
(e) $x(n-1)\,\delta(n-3)$



(f) $x(n^2)$

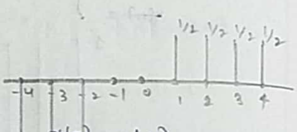


(g) even part of $x(n)$

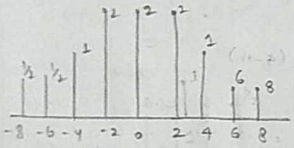


even $= \dfrac{x(n)+x(-n)}{2}$

(h) odd part of $h(n)$.

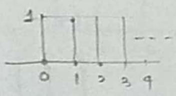


$\dfrac{x(n) - x(-n)}{2}$

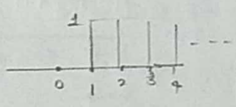


2.3 show that $\delta(n) = u(n) - u(n-1)$

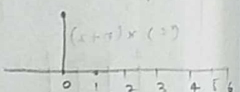
$u(n)$



$u(n-1)$



$\delta(n)$



we know that $\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$ $= \{0, \underset{\uparrow}{1}, 0 \ldots\} = L.H.S$

$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n<0 \end{cases} = \{\cdots 0, 1, 1, 1 \ldots\}$

$u[n-1] = \begin{cases} 1, & n \geq 1 \\ 0, & n<1 \end{cases} = \{\cdots 0, \underset{\uparrow}{0}, 1, 1, 1\}$

$u[n] - u[n-1] = \{\cdots 0, \underset{\uparrow}{1}, 1, 1, \ldots\} - \{\cdots 0, \underset{\uparrow}{0}, 1, 1, 1 \ldots\}$

$R.H.S = \{\cdots 0, 1, 0 \ldots\}$

$R.H.S = \delta[n]$

$L.H.S = R.H.S$

Hence proved. $\delta[n] = u[n] - u[n-1]$

b) $u[n] = \sum_{n=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta[n-k]$

$u[n] = \{\cdots 0, \underset{\uparrow}{1}, 1, 1 \ldots\} = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$

$\sum_{k=-\infty}^{n} \delta[k] = \{\cdots 0, \underset{\uparrow}{1}, 1, 1, 1 \ldots\} = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$

$\sum_{k=0}^{\infty} \delta[n-k] = \{\cdots 0, \underset{\uparrow}{1}, 1, 1, 1 \ldots\} = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$

$\therefore$ Hence proved that $u[n] = \sum_{k=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta[n-k]$.

2.4 show that any sigl can be decomposed into an even and an odd component is the decomposition unique? Illustrate your arguments using the signal.

$x[n] = \{2, 3, 4, 5, 6\}$

By the properties of even and odd signals

we know that $x_e[n] = x_e[-n]$

&

$x_o[-n] = -x_o[n]$.

Scanned by CamScanner

and also let we

$$x_e[n] = \frac{1}{2}[x[n]+x[-n]] \longrightarrow ①$$

$$x_o[n] = \frac{1}{2}[x[n]-x[-n]] — ②$$

add eQ ② & eQ ①

$$x_e[n]+x_o[n] = \frac{1}{2}[x[n]+x[-n]]+\frac{1}{2}[x[n]-x[-n]]$$

$$= \frac{x[n]+\cancel{x[-n]}+x[n]-\cancel{x[-n]}}{2}$$

$$= \frac{\cancel{2}\,x[n]}{\cancel{2}} = x[n]$$

$$x[n] = x_e[n]+x_o[n]$$

The decomposition is unique.

Given $x[n] = \{2,3,\underset{\uparrow}{4},5,6\}$

$$x_e[n] = \frac{1}{2}\{x[n]+x[-n]\}$$

$$= \frac{1}{2}\{8,8,\underset{\uparrow}{8},8,8\}$$

$$x_e[n] = \{4,4,\underset{\uparrow}{4},4,4\}$$

$$x_o[n] = \frac{1}{2}\{x[n]-x[-n]\}$$

$$= \frac{1}{2}\{-4,-2,\underset{\uparrow}{0},2,4\}$$

$$x_o[n] = \{-2,-1,\underset{\uparrow}{0},1,2\}$$

show that the energy (power) of a real valued sig l is equal to the sum of the energies (powers) of its even and odd components.

A: we have to prove that

(energy) power of real valued signal = sum of power (energies) of its even and odd components.

L.H.S = Energy of real valued signal $= \sum_{n=-\infty}^{\infty} x^2[n]$

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty}[x_e[n]+x_o[n]]^2 \quad \left\{ \begin{array}{l} \because \text{we know} \\ x[n] = x_e[n]+x_o[n] \end{array} \right.$$

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty}[x_e[n]]^2 + \sum_{n=-\infty}^{\infty}[x_o[n]]^2 + \sum_{n=-\infty}^{\infty} 2x_e[n]x_o[n] — ①$$

let us simply $2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 2\sum_{n=-\infty}^{\infty} x_e[-n]x_o[-n]$

$\because$ replacing n by $-n$.

$$= -2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n]$$

$$2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 0 — ②$$

substituting eQ ② in eQ ①

$$\underbrace{\sum_{n=-\infty}^{\infty} x^2[n]}_{E_{total}} = \underbrace{\sum_{n=-\infty}^{\infty} x_e^2[n]}_{E_{even}} + \underbrace{\sum_{n=-\infty}^{\infty} x_o^2[n]}_{E_{odd}}$$

Hence proved that,

$$\boxed{E_{total} = E_{even} + E_{odd}}$$

2.6 consider the system

$$y[n] = T[x[n]] = x[n^2]$$

a) determine if the system is time invariant.

i/p : $\longrightarrow x[n]$

o/p : $y[n]$

$y[n] = x[n^2]$

giving delay k samples $\Rightarrow x[n-k]$ to s/m produce $y_1[n]$

$$y_1[n] = x[(n-k)^2]$$

$$y_1[n] = x[n^2+k^2-2nk] \longrightarrow \text{eq } ①$$

giving delay k samples to o/p $\Rightarrow$ produce $y[n-k]$

as $y[n-k] \neq y_1[n]$

The system is not time invariant

$\therefore$ Hence the system is time variant

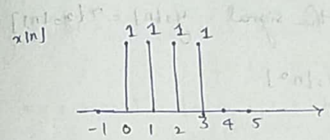
b) To clarify the result in part (a) assume that the signal

$$x[n] = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$ is applied into the s/m.

(i) sketch the signal $x[n]$

$$x[n] = \{ \cdots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \cdots \}$$

A:

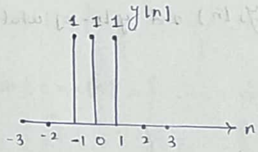


2) determine and sketch the signal $y[n] = T[x[n]]$

$$y[n] = T[x[n]] = x[n^2] = \{ 0, x[(-3)^2], x[(-2)^2]\ x[(-1)^2], \underset{\uparrow}{x[0^2]}, x[1^2], x[2^2], x[3^2], 0 \cdots \}$$

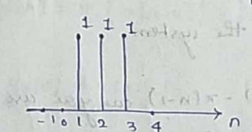
$$= \{ 0, x[9], x[4], x[1], x[0], x[1], x[4], x[9], 0 \cdots \}$$

$$y[n] = \{ \cdots 0, 0, 0, 1, 1, 1, 0, 0, 0 \cdots \}$$



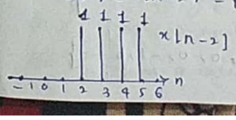
3) sketch the signal $y_2[n] = y[n-2]$

$$y_2[n] = y[n-2] = \{ \cdots 0, \underset{\uparrow}{0}, 1, 1, 1, 0 \cdots \}$$



4) Determine and sketch the signal $x_2[n] = x[n-2]$

$$x_2[n] = x[n-2] = \{ \cdots \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0 \cdots \}$$



Scanned by CamScanner

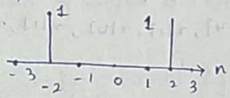
5) Determine and sketch the signal $y_2[n] = T[x_2[n]]$

$$y_2[n] = T[x_2[n]] = x_2[n^2]$$

$$= \{ \cdots 0, x_2[4], x_2[1], x_2[0], x_2[1], x_2[4], 0 \cdots \}$$

$$y_2[n] = \{ \cdots 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0 \cdots \}$$

$$y_2[n] = x_2[n^2]$$



6) compare the signals $y_2[n]$ and $y[n-2]$ what is your conclusion ?

A:- $y_2[n] \neq y[n-2]$

Hence, we can say that the system is <u>time variant</u> and <u>not time invariant</u>

c) Repeat part (b) for the system.

$y(n) = x(n) - x(n-1)$ can you use this result to make any statement about the time invariance of this system, why?

A:- $y[n] = \{ \cdots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \cdots \} - \{ \cdots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \cdots \}$

$$y[n] = \{ \cdots 0, \underset{\uparrow}{1}, 0, 0, 0, -1, 0 \cdots \}$$

$$y_2 = y[n-2] = \{ 0, \underset{\uparrow}{0}, 1, 0, 0, 0, -1 \}$$

$$x_2 = x[n-2] = \{ 0, \underset{\uparrow}{0}, 1, 1, 1, 1, 0 \}$$

$$y_2[n] = \{ \underset{\uparrow}{0}, 0, 1, 0, 0, 0, 0, -1 \}$$

Here $y_2[n] = y[n-2]$, so the system is <u>time invariant</u>

d) Repeat parts (b) and (c) for the s/m

$$y[n] = T[x[n]] = n\,u[n]$$

A:- given s/m $y[n] = n\,x[n]$

$$x[n] = \{ \cdots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \cdots \}$$

$$y[n] = n\,x[n] = \{ \cdots 0, \underset{\uparrow}{1}, 2, 3, 0 \cdots \}$$

$$y_2[n] = y[n-2] = \{ \cdots 0, 0, 0, \underset{\uparrow}{1}, 2, 3, 0 \cdots \}$$

$$x[n-2] = \{ \cdots 0, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0 \cdots \}$$

$$y_2[n] = T[x[n-2]] = \{ \cdots 0, 0, 0, \underset{\uparrow}{2}, 3, 4, 5, 0 \cdots \}$$

$$y_2[n] \neq y[n-2]$$

Hence, the s/m is time variant.

2.7 A discrete time s/m can be

(i) static or dynamic

(ii) linear or nonlinear

(iii) Time invariant or time variant

(iv) Causal or noncausal

(v) stable or unstable

Examine the following systems with respect to the properties above

(a) $y[n] = \cos[x[n]]$

A:- $y[n]$ is static, as it is memoryless

→ $\cos[x_1[n] + x_2[n]] \neq \cos[x_1[n]] + \cos[x_2[n]]$

→ it is non-linear

→ $y[n-k] = \cos[x[n-k]]$

→ it is time invariant

→ only depends on present i/p's it is causal as s/m

→ it is stable

(b) $y[n] = \sum_{n=-\infty}^{\infty\, n+1} x[k]$

A:- It is dynamic because it is depending on future i/p's which needs memory.

→ $\sum_{k=-\infty}^{n+1} [x_1[k] + x_2[k]] = \sum_{k=-\infty}^{n+1} x_1[k] + \sum_{k=-\infty}^{n+1} x_2[k]$

So, it is linear

→ it is time invariant

it is noncausal as it needs future i/p's.

Because for bounded i/p $x[k] = u[k]$

$$y[n] = \sum_{k=-\infty}^{n+1} u[k] = \begin{cases} 0, & n < -1 \\ n+2, & n \geq -1 \end{cases}$$

since $y[n] \to \infty$ as $n \to \infty$, the s/m is unstable.

c. $y[n] = x[n]\cos[\omega_0 n]$

A:- it is static, as $n \to$ i/p values doesn't need memory

→ $\{x_1[n] + x_2[n]\}\cos[\omega_0 n] = x_1[n]\cos[\omega_0 n] + x_2[n]\cos[\omega_0 n]$

so, s/m is linear

$x_1[n-k]\cos[\omega_0 n] = y[n-k] \neq x'_1[n-k]$

so, s/m is time variant

it is causal, no need of future i/p's

it satisfies BIBO stability so, it is stable.

(d) $y[n] = x[-n+2]$

it is dynamic, because it needs past values which requires memory.

$y_1[n] + y_2[n] = x_1[-n+2] + x_2[-n+2]$

so, s/m is linear

$y[n-k] = x[-n+2-k]$

so, s/m is time invariant.

it needs future values because $y[-n] = x[n+2]$ so, system is non-causal.

it is stable.

(e) $y[n] = \text{Trun}[x[n]]$, where $\text{Trun}[x[n]]$ denotes the integer part of $x[n]$, obtained by truncation.

→ it is static, as no need of memory.

→ $Trun[x_1[n]+x_2[n]] \neq Trun[x_1[n]] + Trun[x_2[n]]$

so, s/m is non linear.

$Trun[x[n-k]] = y[n-k]$

so, s/m is time invariant.

as s/m doesn't needs any future values, so, s/m is causal.

for bounded i/p, we will get bounded o/p so,

s/m is stable.

f) $y[n] = Round[x[n]]$, where round $[x[n]]$ denotes the integer part of $x[n]$ obtained by rounding.

A: It is static

nonlinear

time invariant

causal

stable.

(g) $y[n] = |x[n]|$

the s/m is static, no need of memory.

$|x_1[n] + x_2[n]| \neq |x_1[n]| + |x_2[n]|$

so, s/m is non-linear

$y[n-k] = |x[n-k]|$

so, s/m is time invariant.

→ as it not needs future i/p's, s/m is causal

→ stable s/m.

h) $y[n] = x[n]u[n]$

static because of no need of memory

$[x_1[n] + x_2[n]]u[n] = x_1[n]u[n] + x_2[n]u[n]$

so, the s/m is linear.

$y[n-k] = x[n-k]u[n]$ ⇒ so, s/m is time invariant.

as it no needs future i/p's, s/m is causal.

s/m is stable

(i) $y[n] = x[n] + nx[n+1]$

dynamic s/m, as it doesn't need future i/p's which also needs memory & it is non causal.

$x_1[n] + nx_1[n+1] + x_2[n] + nx_2[n+1] = y_1[n] + y_2[n]$

so, s/m is linear

→ $y[n-k] = x[n-k] + (n-k)x[n+1-k] \neq x[n-k] + nx[n+1-k]$

so, s/m is time variant.

for bounded i/p $x[n] = u[n]$

$y[n] = u[n] + n\,u[n+1]$ produces an unbounded o/p

so, the s/m is unstable.

Scanned by CamScanner

(j) $y[n] = x[2n]$

s/m is noncausal and also dynamic as it needs future i/p's.

$\rightarrow x_1[2n] + x_2[2n] = y_1[n] + y_2[n]$

so, s/m is linear

$\rightarrow y[n-k] = x[2n-k] \neq x[2(n-k)]$

so, s/m is time invariant.

s/m is stable.

(k) $y[n] = \begin{cases} x[n], & \text{if } x[n] \geq 0 \\ 0 & \text{if } x[n] < 0 \end{cases}$

s/m is static and also causal as no memory & future i/p's needed.

s/m is nonlinear, time invariant, stable

(l) $y[n] = x[-n]$

$\rightarrow$ s/m is dynamic, noncausal as it needs past & future values and also memory

$\rightarrow$ s/m is linear & time invariant & stable.

(m) $y[n] = \text{sign}[x[n]]$

sol:- s/m is causal and static.

s/m is nonlinear as

$\text{sign}[x_1[n] + x_2[n]] \neq \text{sign}[x_1[n]] + \text{sign}[x_2[n]]$

$y[n-k] = \text{sign}[x[n-k]]$

so, s/m is time invariant

s/m is stable.

(n) the ideal sampling system which i/p $x_a(t)$ and output

$x(n) = x_a(nT), -\infty < n < \infty$.

A: s/m is static & causal as no future, memory needed.

$\rightarrow$ s/m is linear as

$x_a[nT] + x_b[nT] = x_a[nT] + x_b[nT]$

s/m is time invariant & stable.

2.11 The following input-output pairs have been observed using the operation of a linear system.

$$x_1[n] = \{-1, 2, 1\} \xleftrightarrow{T} y_1[n] = \{1, \underset{\uparrow}{2}, -1, 0, 1\}$$

$$x_2[n] = \{1, -1, 1\} \xleftrightarrow{T} y_2[n] = \{-1, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3[n] = \{0, 1, 1\} \xleftrightarrow{T} y_3[n] = \{\underset{\uparrow}{1}, 2, 1\}$$

can you draw any conclusions about the time invariance of the s/m

sol: By observing that

$$x_1[n] + x_2[n] = \delta[n]$$

and the s/m is linear

impulse response of the s/m is

$$y_1[n] + y_2[n] = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$$

if s/m were time invariant, response to $x_3[n]$ would be

$$\{\underset{\uparrow}{3}, 2, 1, 3, 1\}$$

2.12 show that the necessary condition for a relaxed LTI s/m to be BIBO stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| \le M_h < \infty \text{ for some constant } M_h.$$

Ans: An arbitrary relaxed s/m is said to be bounded input-bounded output (BIBO) stable if and only if every bounded input produces a bounded o/p output

mathematically, $|x(n)| \le M_x < \infty$, $|y(n)| \le M_y < \infty$.

we know, $$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$|y[n]| \le \sum_{k=-\infty}^{\infty} |h[k]|\,|x[n-k]|$$

$$\le M_x \sum_{k=-\infty}^{\infty} |h[k]|$$

where $|x(n-k)| \le M_x$. Therefore, $|y[n]| < \infty$ $\forall n$, if and only if

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty.$$

2.14 show that

(a) A relaxed linear s/m is causal if and if for any i/p $x[n]$ such that

$$x[n] = 0 \text{ for } n < n_0 \Rightarrow y[n] = 0 \text{ for } n < n_0.$$

sol: $x[n] = 0$ for $n < n_0$

$y[n] = 0$ for $n < n_0$

if the s/m is causal if the output is non zero of the s/m at any time 'n' depends only on present and past i/p's of the non zero

$x[n] = 0$ for $n < n_0$ is equal to

$y[n] = 0$ for $n < n_0$

(b) A relaxed LTI s/m is causal if and only if

$h[n] = 0$ for $n < 0$.

Ans: $$y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$, where $x[n] = 0$ for $n < 0$

if $h[k] = 0$ for $k < 0$ then

$$y[n] = \sum_{k=0}^{\infty} h[k]\,x[n-k]$$, and hence $y[n] = 0$ for $n < 0$.

Scanned by CamScanner

on the other hand if $y[n]=0$ for $n<0$ then

$$\sum_{k=-\infty}^{n} h[k]\,x[n-k] \Rightarrow h[k]=0,\ k<0.$$

2.15 (a) show that for any real or complex constant a, and any finite integer numbers M and N, we have

$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M - a^{N+1}}{1-a} & ; \text{ if } a \neq 1 \\ N-M+1 & ; \text{ if } a=1 \end{cases}$$

Sol:- for $a=1$, $\sum_{n=M}^{N} a^n = N-M+1$

for $a \neq 1$, $\sum_{n=M}^{N} a^n = a^M + a^{M+1} + \cdots + a^N$

$$(1-a)\sum_{n=M}^{N} a^n = a^M + a^{M+1} - a^{M+1} + \cdots + a^N - a^N - a^{N+1}$$

$$= a^M - a^{N+1}$$

(b) show that if $|a|<1$, then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

Sol:- for $M=0$, $|a|<1$, and $N \rightarrow \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a|<1$$

2.16 (a) If $y[n] = x[n] * h[n]$, show that $\Sigma_y = \Sigma_x \Sigma_n$, where

$$\Sigma_x = \sum_{n=-\infty}^{\infty} x[n].$$

Given $y[n] = x[n] * h[n]$

mean $y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$

$$\sum_{n=-\infty}^{\infty} y[n] = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} h[k]\,x[n-k] = \sum_{k=-\infty}^{\infty} h[k] \sum_{n=-\infty}^{\infty} x[n-k]$$

$$= \sum_{k} h[k] \sum_{n} x[n].$$

$$\Rightarrow \Sigma_y = \Sigma_x \Sigma_n$$

(b) compute the convolution $y[n] = x[n] * h[n]$ of the following sig^{l}s and check the correctness of the results by using the test in (a)

1) $x[n] = [1,2,4]$, $h[n] = \{1,1,1,1,1\}$

$y[n] = h[n] * x[n]$

$= \{1,3,7,7,7,6,4\}$

	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

$\sum_n y[n] = 1+3+7+7+7+6+4$

$= 35$

$\sum_k h[k] = 5$, $\sum_n x[n] = 7$

$$\Rightarrow \Sigma_y = \Sigma_x \Sigma_n$$

2) $x[n] = \{1,2,1\}$, $h[n] = x[n]$

$y[n] = h[n] * h[n]$

$= x[n] * x[n]$

$y[n] = \{1,4,2,-4,1]$

	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$\sum_n y[n] = 4$, $\sum_k h[k] = 2$ $\sum_n x[n] = 2$

$$\Sigma_y = \Sigma_h \Sigma_x$$

3) $x[n] = \{0, 1, -2, 3, -4\}$, $h[n] = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

Sol:- $y[n] = h[n] * x[n]$

$y[n] = \{0, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, \frac{5}{2}, -2\}$

$\sum_n y[n] = -5$, $\sum_k h[k] = 25$

$\sum_n x[n] = -2$

$\sum_y = \sum_h \sum_x$

	0	1	-2	3	-4
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2
1	0	1	-2	3	-4
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2

4) $x[n] = \{1, 2, 3, 4, 5\}$, $h[n] = \{1\}$

Sol:- $y[n] = h[n] * x[n]$ $\quad \sum_n y[n] = 15$ $\quad \sum_k h[k] = 1$

$y[n] = \{1, 2, 3, 4, 5\}$ $\quad \sum_n x[n] = 15$

$\sum_y = \sum_x \sum_h$

5) $x[n] = \{1, -2, 3\}$, $h[n] = \{0, 0, 1, 1, 1, 1\}$

Sol:- $y[n] = h[n] * x[n]$

$y[n] = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y[n] = 8$ $\quad \sum_k h[k] = 4$

$\sum_n x[n] = 2$ $\quad \sum_y = \sum_x \sum_h$

	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

6) $x[n] = \{0, 0, 1, 1, 1, 1\}$, $h[n] = \{1, -2, 3\}$

$y[n] = h[n] * x[n]$

$y[n] = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y[n] = 8$, $\sum_k h[k] = 4$

$\sum_n x[n] = 2$ $\quad \sum_y = \sum_x \sum_n$.

	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

7) $x[n] = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h[n] = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$y[n] = h[n] * x[n]$

$y[n] = \{0, 1, 4, -4, -5, -1, 3\}$

$\sum_n y[n] = -2$, $\sum_k h[k] = -1$

$\sum_n x[n] = 2$ $\quad \sum_y = \sum_x \sum_h$

	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

8) $x[n] = \{\underset{\uparrow}{1}, 1, 2\}$, $h[n] = u[n]$

Sol:- $y[n] = h[n] * x[n]$

$y[n] = u[n] + u[n-1] + 2u[n-2]$

$\sum_n y[n] = \infty$, $\sum_n h[n] = \infty$, $\sum_r x[n] = 4$

9) $x[n] = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h[n] = \{1, -2, -3, \underset{\uparrow}{4}\}$

$y[n] = h[n] * x[n]$

$y[n] = \{1, -1, -5, 2, 3, -5, 1, 4\}$

$\sum_n y[n] = 0$; $\sum_n x[n] = 4$

$\sum_k h[k] = 0$.

	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	4	4	0	4	4

10) $x[n] = \{1, 2, 0, 2, 1\}$, $h[n] = x[n]$

$x[n] = \{1, 2, 0, 2, 1\}$, $h[n] = \{1, 2, 0, 2, 1\}$

$y[n] = h[n] * x[n]$

$y[n] = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$

$\sum_n y[n] = 36$, $\sum_k h[k] = 6$, $\sum_r x[n] = 6$.

	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

11> $x[n] = (\frac{1}{2})^n u(n), h(n) = (\frac{1}{4})^n u[n]$

Sol:- $y[n] = h[n] * x[n]$

$y[n] = (\frac{1}{2})^n u(n) * (\frac{1}{4})^n u[n]$

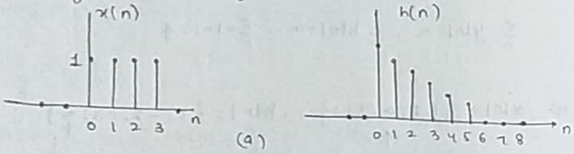
$x[n] = \{0.5, 0.25, 0.125, 0.0625, 0.03125\}$

$h[n] = \{0.25, 0.0625, 0.015625\}$

$h[n] * x[n] \Rightarrow y[n] = \left[2(\frac{1}{2})^n - (\frac{1}{4})^n 4\right] u[n]$

$\sum_n y[n] = \frac{8}{3}, \ \sum_n h[n] = \frac{4}{3}, \ \sum_n x[n] = 2$

2.17 compute and plot the conclusions $x[n] * h[n]$ and $h[n] * x[n]$ for the pair of sig^l^s shown in fig.



(a)

$x[n] = \{1, 1, 1, 1\}$

$h[n] = \{6, 5, 4, 3, 2, 1\}$

$y[n] = \sum_{k=0}^{n} x[k] h[n-k]$

$y[0] = x[0] h[0] = 6$

$y[1] = x[0] h[1] + x[1] h[0] = 11$

$y[2] = x[0] h[2] + x[1] h[1] + x[2] h[0] = 15$

$y[3] = x[0] h[3] + x[1] h[2] + x[2] h[1] + x[3] h[0] = 18$

$y[4] = x[0] h[4] + x[1] h[3] + x[2] h[2] + x[3] h[1] + x[4] h[0] = 14$

$y[5] = x[0] h[5] + x[1] h[4] + x[2] h[3] + x[3] h[2] + x[4] h[1] + x[5] h[0] = 10$

$y[6] = x[1] h[5] + x[2] h[4] + x[3] h[2] = 6$

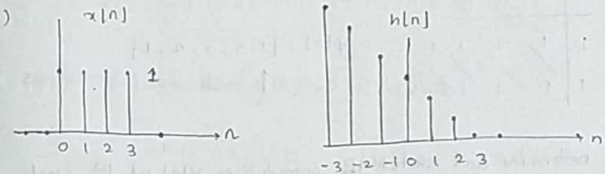
$y[7] = x[2] h[5] + x[3] h[4] = 3$

$y(8) = x[3] h[5] = 1$

$y[n] = 0, \ n \geq 9$

$y[n] = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\}$

(b)

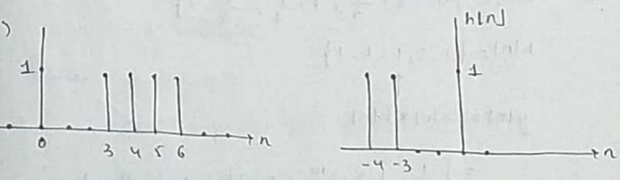


$x[n] = \{1, 1, 1, 1\}$ $h[n] = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1\}$

	1	1	1	1
6	6	6	6	6
5	5	5	5	5
4	4	4	4	4
3	3	3	3	3
2	2	2	2	2
1	1	1	1	1

$y[n] = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$

(c)



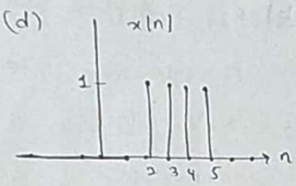
$x[n] = \{1, 1, 1, 1\}$ $y[n] = \{1, 1\}$

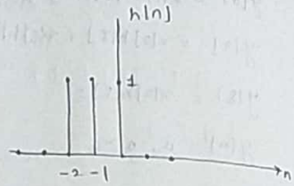
	1	1	1	1
1	1	1	1	1
1	1	1	1	1

$y[n] = \{1, 2, 2, 2, 1\}$

Scanned by CamScanner

(d)





$x[n] = \{1, 1, 1, 1\}$ $h[n] = \{1, 1\}$

	1	1	1	1
1	1	1	1	1
1	1	1	1	1

$y[n] = \{1, 2, 2, 2, 1\}$
$\uparrow$

2.8 Determine and sketch the convolutions $y[n]$ of the signals.

$$x[n] = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases}$$

$$h[n] = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Graphically

(b) Analytically

(a) $x[n] = \{0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

$h[n] = \{1, 1, 1, 1, 1\}$
$\uparrow$

$y[n] = x[n] * h[n]$

$= \{\frac{1}{3}, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

(b) $x[n] = \frac{1}{3}n\,[u[n] - u[n-7]]$

$h[n] = u[n+2] - u[n-3]$

$y[n] = x[n] * h[n]$

$= \frac{1}{3}n\,[u[n] - u[n-7]] * [u[n+2] - u[n-3]]$

$= \frac{1}{3}n\,[u[n] * u[n+2] - u[n] * u[n-3] - u[n-7] * u[n+2]$
$+ u[n-7] * u[n-3]]$.

$y[n] = \frac{1}{3}\delta(n+1) + \delta[n] + 3\delta[n-1] + \frac{10}{3}\delta[n-2] + 5\delta[n-3] + \frac{20}{3}\delta[n-4]$
$+ 6\delta[n-5] + 5\delta[n-6] + 5\delta[n-6] + \frac{11}{3}\delta[n-7] + \delta(n-8)$.

2.19 compute the convolution $y[n]$ of the signals

$$x[n] = \begin{cases} 2^n, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases} \qquad h[n] = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

sol:

$y[n] = \sum_{k=0}^{4} h[k]\,x[n-k]$

$x[n] = \{2^{-3}, 2^{-2}, 2^{-1}, 1, 2, \ldots\ldots 2^5$
$\uparrow$

$h[n] = \{1, 1, 1, 1, 1\}$
$\uparrow$

$y[n] = \sum_{k=0}^{4} x[n-k]$, $-3 \le n \le 9$

$= 0$, otherwise

therefore $y[-3] = 2^{-3}$

$y[-2] = x[-3] + x[-2] = 2^{-3} + 2^{-2}$

$y[-1] = 2^{-3} + 2^{-2} + 2^{-1}$

$y[0] = 2^{-3} + 2^{-2} + 2^{-1} + 1$

$y[1] = 2^{-3} + 2^{-2} + 2^{-1} + 1 + 2$

$y[2] = 2^{-3} + 2^{-2} + 2^{-1} + 1 + 2 + 2^2$

$y[3] = 2^{-1} + 1 + 2 + 2^2 + 2^3$

Scanned by CamScanner

(c) multiply the polynomials: $1+3z+z^2$ and $1+2z+2z^2$.

sol:-

$$(1+3z+z^2)(1+2z+2z^2) = 1+5z+9z^2+8z^3+2z^4.$$

(d) Repeat part (a) for the numbers 1.31 and 12.2

sol:-

$$1.31 \times 12.2 = 15.982$$

(e) comment on your results.

sol:- those are different ways to perform convolution.

2.21 compute the convolution $y[n] = x[n] * h[n]$ of the following pair of signals.

(a) $x[n] = a^n u[n]$, $h[n] = b^n u[n]$ when $a \neq b$ and when $a = b$

sol:-

$$y[n] = \sum_{k=0}^{n} a^k u[k]\, b^{n-k} u[n-k]$$

$$= b^n \sum_{k=0}^{n} (a b^{-1})^k$$

$$y[n] = \begin{cases} \dfrac{b^{n+1} - a^{n+1}}{b-a} u[n], & a \neq b \\ b^n u[n+1] u[n], & a = b \end{cases}$$

(b) $x[n] = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0, & \text{elsewhere} \end{cases}$ $h[n] = \delta[n] - \delta[n-1] + \delta[n-4] + \delta[n-5]$.

sol:-

$$x[n] = \{1, 2, 1, 1\}$$

$$h[n] = \{1, -1, 0, 0, 1, 1\}$$

$$y[n] = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}.$$

Scanned by CamScanner

(c) $x[n] = u[n+1] - u[n-4] - \delta[n-5]$

$h[n] = [u[n+2] - u[n-3]] \cdot (3-|n|)$

$x[n] = \{1, 1, 1, 1, 1, 0, -1\}$

$h[n] = \{1, 2, 3, 2, 1\}$

$y[n] = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, -1\}$
$\uparrow$

(d) $x[n] = u[n] - u[n-5]$; $h[n] = u[n-2] - u[n-8] + u[n-11] - u[n-17]$

let x[n] be the i/p signal to a discrete-time filter with impulse response $h_i[n]$ and let $y_i[n]$ be the corresponding sigl.

sol:

$x[n] = \{1, 1, 1, 1, 1\}$

$h[n] = \{0, 0, 1, 1, 1, 1, 1, 1\}$
$\uparrow$

$h[n] = h'[n] + h'[n-9]$

$y[n] = y'[n] + y'[n-9]$, where

$y'[n] = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$
$\uparrow$

2.22 (a) compare and sketch x[n] and y[n] in the following case, using the same scale in all figures.

$x[n] = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1[n] = \{1, 1\}$

$h_2[n] = \{1, 2, 1\}$

$h_3[n] = \{\frac{1}{2}, \frac{1}{2}\}$

$h_4[n] = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$h_5[n] = \{\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}\}$

sketch $x[n], y_1[n], y_2[n]$ on one graph and $x[n], y_3[n], y_4[n], y_5[n]$ on another graph.

Sol:- $y[n] = h[n] * x[n]$

$y_1[n] = x[n] + x[n-1]$

$= \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 13, 15, 9\}$

$y_2[n] = \{1, 8, 11, 11, 13, 16, 14, 13, 15, 21, 25, 28, 24, 9\}$

$y_3[n] = \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\}$

$y_4[n] = \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 3.75, 5.25, 6.25, 7.6, 2.25\}$

$y_5[n] = \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}$

(b) what is the difference b/w $y_1[n]$ and $y_2[n]$ and b/w $y_3[n]$ & $y_4[n]$.

Sol:- $y_3[n] = \frac{1}{2} y_1[n]$, because

$h_3[n] = \frac{1}{2} h_1[n]$

$y_4[n] = \frac{1}{4} y_2[n]$, because

$h_4[n] = \frac{1}{4} h_2[n]$.

(c) Comment on the smoothness of $y_2[n]$ and $y_4[n]$.

which factors affects the smoothness.

Sol:- $y_2[n]$ and $y_4[n]$ are smoother than $y_1[n]$ but $y_4[n]$ will appear even smoother because of the smaller scale factor.

(d) compare $y_4[n]$ with $y_5[n]$. what is the difference?

can you explain it?

Sol:- System 4 results in a smoother output. The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5[n]$.

(a) let $h_6[n] = \{ \frac{1}{2}, -\frac{1}{2} \}$, compute $y_6[n]$. Sketch $x[n]$, $y_2[n]$ and $y_6[n]$ on the same figure and comment on the results.

sol:- $y_6[n] = \{ \frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{9}{2}, -\frac{9}{2} \}$

$y_2[n]$ is smoother than $y_6[n]$.

2.23 Express the output $y[n]$ of a LTI s/m with impulse response $h[n]$ in terms of it's step response

$s[n] = h[n] * u[n]$ and the input $x[n]$.

sol:- we can express the unit sample in terms of the unit step function as $\delta[n] = u[n] - u[n-1]$

then,
$$h[n] = h[n] * \delta[n]$$
$$= h[n] * [u[n] - u[n-1]]$$
$$= h[n] * u[n] - h[n] * u[n-1]$$
$$= s[n] - s[n-1]$$

using this definition of $h[n]$

$$y[n] = h[n] * x[n]$$
$$= [s[n] - s[n-1]] * x[n]$$
$$= s[n] * x[n] - s[n-1] * x[n].$$

2.24 The discrete time s/m

$$y[n] = ny[n-1] + x[n], \quad n \geq 0$$

is at rest [i.e, $y(-1) = 0$]. check if the s/m is linear time invariant and BIBO stable.

$y_1[n] = n y_1[n-1] + x_1[n]$ and

$y_2[n] = n y_2[n-1] + x_2[n]$ then

$x[n] = a x_1[n] + b x_2[n]$

produces the output

$y[n] = n y[n-1] + x[n]$, where

$y[n] = a\, y_1[n] + b\, y_2[n]$

Hence, the s/m is linear. if the input is $x[n-1]$

we have

$y[n-1] = [n-1]\, y[n-2] + x[n-1]$ but

$y[n-1] = n y[n-2] + x[n-1]$

Hence the s/m is time variant. If $x[n] = u[n]$

then $|x[n]| \le 1$ But for this bounded i/p, the o/p is

$y[0] = 1$, $y[1] = 1+1 = 2$, $y[2] = 2\times 2+1 = 5$ . . . .

which is unbounded. Hence the s/m is unstable.

2.24 consider the sigl $\gamma[n] = a^n u(n)$, $0 < a < 1$

(a) show that any sequence $x(n)$ can be decomposed as

$$x(n) = \sum_{n=-\infty}^{\infty} c_k\, \gamma(n-k)$$

and express c_k in terms of $x(n)$.

sol: $\delta[n] = \gamma[n] - a\gamma[n-1]$ and

$\delta[n-k] = \gamma[n-k] - a\gamma[n-k-1]$. then,

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, \{\delta[n-k] - a\gamma[n-k-1]\}$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] - a\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k-1]$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] - a\sum_{k=-\infty}^{\infty} x[k-1]\,\delta[n-k]$$

$$= \sum_{k=-\infty}^{\infty} [x[k] - a\,x[k-1]]\,\delta[n-k]$$

Thus, $c_k = x[k] - a\,n[k-1]$

(b) use the properties of linearity and time invariance to express the o/p $y[n] = T[x(n)]$ in terms of the i/p $x(n)$ and the sig $g(n) = T[\delta(n)]$, where $T[.]$ is an LTI s/m.

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{k=-\infty}^{\infty} c_k\,\delta[n-k]\right]$$

$$= \sum_{k=-\infty}^{\infty} T[\delta[n-k]]$$

$$= \sum_{k=-\infty}^{\infty} c_k\, g[n-k]$$

(c) express the impulse $h[n] = T[\delta[n]]$ in terms of $g[n]$.

$$h[n] = T[\delta[n]]$$

$$= T[\delta[n] - a\delta[n-1]]$$

$$= g[n] - a\,g[n-1]$$

2.26 Determine the zero-i/p response of the s/m described by the second order difference equation

$$x[n] - 3y[n-1] - 4y[n-2] = 0.$$

with $x[n] = 0$, we have

$$y[n-1] + \frac{4}{3}\,y[n-2] = 0$$

$$y[-1] = -\frac{4}{3}\,y[-2]$$

$$y[0] = \left(\frac{-4}{3}\right)^2 y[-2]$$

$$y[1] = \left(\frac{-4}{3}\right)^3 y[-2]$$

$\vdots$

$$y[k] = \left[\frac{-4}{3}\right]^{k+2} y[-2] \leftarrow \text{zero i/p response.}$$

2.27. Determine the particular solution of the difference equation

$$y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}\,y[n-2] + x[n]$$

when the forcing function is $x[n] = 2^n u[n]$.

Sol: consider the homogeneous equation.

$$y[n] = -\frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 0$$

The characteristic equation is

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0 \cdot \lambda = \frac{1}{2}, \frac{1}{3}$$

Hence,

$$y_h[n] = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$$

The particular solution is

$x[n] = 2^n u[n]$ is

$y_p[n] = K[2^n]\,u[n]$

substitution this solution into the difference equation.

Then we, obtain

$$K[2^n]u[n] - K\left(\frac{5}{6}\right)(2^{n-1})u(n-1) + K\left(\frac{1}{6}\right)(2^{n-1})u[n-2]$$
$$= 2^n u[n]$$

for $n=2$

$$4K - \frac{5K}{3} + \frac{K}{6} = 4 \Rightarrow K = \frac{8}{5}$$

therefore the total solution is

$$y[n] = y_p[n] + y_h[n] = \frac{8}{5}(2^n)u(n) + c_1\left(\frac{1}{2}\right)^n u(n) + c_2\left(\frac{1}{3}\right)^n u(n).$$

to determine c_1 & c_2 assume that $y[-2] = y[-1] = 0$.

Then,

$$y(0) = 1 \text{ and}$$

$$y[1] = \frac{5}{6}y(0) + 2 = \frac{17}{6}$$

Thus,

$$\frac{8}{5} + c_1 + c_2 = 1 \Rightarrow c_1 + c_2 = -\frac{3}{5}$$

$$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6} \Rightarrow 3c_1 + 2c_2 = -\frac{11}{5}$$

and therefore

$$c_1 = -1, c_2 = \frac{2}{5}$$

The total solution is

$$y[n] = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u[n]$$

2.29 Determine the impulse response for the cascade of two linear time-invariant systems having impulse responses.

$$h_1[n] = a^n[u[n] - u[n-N]] \text{ and } h_2[n] = [u[n] - u[n-M]].$$

Sol: $h[n] = h_1[n] * h_2[n]$

$$= \sum_{k=-\infty}^{\infty} a^k [u[k] - u[k-N]][u[n-k] - u[n-k-M]].$$

$$= \sum_{k=-\infty}^{\infty} a^k u[k]\, u[n-k] - \sum_{k=-\infty}^{\infty} a^k u[k]\, u[n-k-M] -$$

$$- \sum_{k=-\infty}^{\infty} a^k u[k-N]\, u[n-k] + \sum_{n=-\infty}^{\infty} a^k u[k-N]\, u[n-k-M]$$

$$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k\right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-M} a^k\right)$$

$$= 0.$$

2.30 Determine the response $y[n], n \geq 0$ of the system described by the second-order difference equation.

$$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$$

to the i/p $x[n] = 4^n u[n]$.

Sol: $y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$

The characteristic equation is

$$\lambda^2 - 3\lambda - 4 = 0$$

hence, $\lambda = 4, -1$ and

$$y_h[n] = c_1[n]4^n + c_2(-1)^n$$

since 4 is a characteristic root and the excitation is

$$x[n] = 4^n u[n]$$

we assume a particular solution of the form

$$y_p[n] = Kn4^n u[n].$$

Then

$$Kn4^n u[n] - 3K[n-1]4^{n-1}u[n-1] - 4K(n-2)4^{n-2}u(n-2)$$

$$= 4^n u[n] + 2(4)^{n-1}u(n-1)$$

for $n=2$,

$$K(32-12) = 4^2 + 8 = 24 \longrightarrow K = \frac{6}{5}$$

the total solution is

$$y[n] = y_p[n] + y_h[n]$$

$= \left[\frac{6}{5} n 4^n + c_1 4^n + c_2 (-1)^n \right] u(n)$

To solve for c_1 and c_2, we assume $y(-1) = y(-2) = 0$

Then, $y[0] = 1$ and

$y[1] = 3y[0] + 4 + 2 = 9$

Hence,

$c_1 + c_2 = 1$ and

$\frac{24}{5} + 4c_1 - c_2 = 9$

$4c_1 - c_2 = \frac{21}{5}$

Therefore $c_1 = \frac{21}{25}$ and $c_2 = \frac{-1}{25}$

The total solution is

$$y[n] = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n \right] u[n]$$

2.31 determine the impulse response of the following causal s/m:

$$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1].$$

Ans: The characteristic values from previous problem are

$\lambda = 4, -1$ Hence

$$y_h[n] = c_1 4^n + c_2 (-1)^n$$

when $x[n] = \delta[n]$, we find that

$y[0] = 1$ and

$y(1) - 3y[0] = 2$ or

$y[1] = 5$.

Hence, $c_1 + c_2 = 1$ and $4c_1 - c_2 = 5$

This yeilds $c_1 = \frac{6}{5}$ and $c_2 = \frac{-1}{5}$ therefore,

$$h[n] = \left[\frac{6}{5} 4^n - \frac{1}{5} (-1)^n \right] u[n].$$

Scanned by CamScanner

2.9 Let T be an LTI, relaxed, BIBO stable s/m with i/p $x(n)$ and o/p $y(n)$. show that:

(a) if $x(n)$ is periodic with period N (i.e., $x(n) = x(n+N)$ for all $n \geq 0$], the output $y(n)$ tends to a periodic sig with the same period.

Sol: i/p: $x[n]$

impulse response of LTI s/m: $h[n]$

$$y[n] = \sum_{k=-\infty}^{n\,\infty} h[k]\, x[n-k]$$

advancing o/p through period N

$$y[n+N] = \sum_{n=-\infty}^{n+N} h[k]\, x[n+N-k] = \sum_{k=-\infty}^{n+N} h[k]\, x[n-k]$$

$$= \sum_{k=-\infty}^{n} h[k]\, x[n-k] + \sum_{k=n+1}^{n+N} h[k]\, x[n-k]$$

$$= y[n] + \sum_{k=n+1}^{n+N} h[k]\, x[n-k]$$

Given, is BIBO stable s/m $\Rightarrow \lim_{n\to\infty} |h[n]| = 0$.

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h[k]\, x[n-k] = 0$$

$$\lim_{n\to\infty} y[n+N] = y[N]$$

$$\therefore\ y[n+N] = y[n]$$

$\therefore$ o/p is also periodic

(b) If $x(n)$ is bounded and tends to a constant, the output will also tend to a constant.

Ans: let $x[n] = x_0[n] + a_0[n]$ $a \to$ constant

$x_0[n]$ is bounded sig with $\lim_{n\to\infty} x_0[n] = 0$.

$$\therefore y[n] = a \sum_{k=0}^{\infty} h[k]\, x[n-k] + \sum_{K=0}^{\infty} h[k]\, x_0[n-k]$$

$$= a \sum_{K=0}^{\infty} h[k] + y_0[n]$$

$$\Rightarrow \sum_n x_0^2[n] < \infty \Rightarrow \sum_n y_0^2[n] < \infty.$$

$$\lim_{n\to\infty} |y_0(n)| = 0$$

$$y(n) = y_0(n) + \underbrace{a \sum_{K=0}^{\infty} h[k]}_{\text{constant} \to c}$$

$$y[n] = y_0[n] + c$$

$$\lim_{n\to\infty} y[n] = c.$$

(c) if $x[n]$ is an energy sigl the o/p $y[n]$ will also be an energy signal.

$$y[n] = \sum_{K=0}^{\infty} h[k]\, x[n-k]$$

$$\text{energy} \Rightarrow \sum_{n=-\infty}^{\infty} y^2[n] = \sum_{n=-\infty}^{\infty} \left[\sum_K h[k]\, x(n-k) \right]^2$$

$$\sum_{K=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} h[k]\, h[l] \sum_{n=-\infty}^{\infty} x[n-k]\, x[n-l]$$

but

$$\sum_n x[n-k]\, x[n-l] \leq \sum_n x^2[n] = E_x$$

$$\sum_n y^2[n] \leq E_x \sum_K |h[k]| \sum_l |h(l)|$$

for a BIBO stable s/m

$$\sum_{K=-\infty}^{\infty} |h[k]| \leq M \qquad E_y \leq M^2 E_x, \text{ so that}$$

$E_y < 0$ if $E_x < 0$.

2.10 The following input-output pairs have been observed during the operation of a time-invariant system.

$$x_1[n] = \{1, 0, 2\} \xleftrightarrow{T} y_1(n) = \{0, 1, 2\}$$

$$x_2[n] = \{0, 0, 3\} \xleftrightarrow{T} y_2(n) = \{0, 1, 0, 2\}$$

$$x_3[n] = \{0, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, 2, 1\}$$

can you draw any conclusions regarding the linearity of the s/m. what is the impulse response of the s/m?

Sol:- If you assume s/m is time invariant, then a shift of x_2 say

$$x_2 = \{0, \underset{\uparrow}{0}, 0, 3\}$$

should give the shifted version of $y_2[n]$.

$$y_2[n] = \{0, 0, \underset{\uparrow}{1}, 0, 2\}$$

but $x_2 = 3x_3$ and y_2 is not a multiple of y_3 linearity is not preserved.

→ If the s/m were linear, then since

$$x_2[n] = 3x_3[n+1], \text{ then } y_2[n] = 3y_3[n+1]$$

impulse respone of the s/m is

$$x_3[n+3] = \{0, 0, 0, \underset{\uparrow}{1}, 0\} \rightarrow y_3[n] = \{1, 3, 1, 0, 0, 0\}$$

2.32. let $x[n]$, $N_1 \le n \le N_2$ and $h[n]$, $M_1 \le n \le M_2$ be two finite duration signals.

(a) Determine the range $L_1 \le n \le L_2$ of their convolution, in terms of N_1, N_2, M_1 and M_2.

sol: $L_1 = N_1 + M$ and $L_2 = N_2 + M_2$

(b) Determine the limits of the cases of particular overlap from the left full overlap, and particular overlap from the right. for convenience, assume that $h[n]$ has shorter duration than $x[n]$.

sol: partial overlap from left:

low $N_1 + M_1$ high $N_1 + M_2 - 1$

full overlap: low $N_1 + M_2$ high $N_2 + M_1$

partial overlap from right:

low $N_2 + M_1 + 1$ high $N_2 + M_2$

(c) Illustrate the validity of your results by computing the convolution of the sig^l^s:

$$x[n] = \begin{cases} 1, & -2 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases} \qquad h[n] = \begin{cases} 2; & -1 \le n \le 2 \\ 0; & \text{elsewhere} \end{cases}$$

Sol: $x[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$ $N_1 = -2$ $M_1 = -1$

$N_2 = 4$ $M_2 = 2$

$h[n] = \{2, 2, 2, 2\}$

partial overlap from left $n = -3$ $n = -1$, $L_1 = -3$

full overlap: $n = 0$ $n = 3$

partial overlap from right: $n = 4$ $n = 6$ $L_2 = 6$

2.33 Determine the impulse response and the unit step response of the s/m described by the difference equation

(a) $y[n] = 0.6\,y[n-1] - 0.08\,y[n-2] + x[n]$.

Sol:- $y[n] - 0.6\,y[n-1] + 0.08\,y[n-2] = x[n]$

The characteristic equation is

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$\lambda = 0.2, 0.4$ hence,

$$y_\lambda[n] = c_1 \frac{1}{5}^n + c_2 \frac{2}{5}^n$$

with $x[n] = \delta[n]$, the initial conditions are

$y[0] = 1$

$y[1] - 0.6\,y[0] = 0 \Rightarrow y[1] = 0.6$

hence, $c_1 + c_2 = 1$ and

$\frac{1}{5}c_1 + \frac{2}{5} = 0.6 \Rightarrow c_1 = -1, c_2 = 3$

Therefore $h[n] = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]u[n]$

The step response is

$$s[n] = \sum_{k=0}^{n} h[n-k],\ n \geq 0$$

$$= \sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= \left\{\frac{1}{0.12}\left[\left(\frac{2}{5}^{n+1} - 1\right)\right] - \frac{1}{0.16}\left[\left(\frac{1}{5}^{n+1} - 1\right)\right]\right\}u[n]$$

(b) $y[n] = 0.7\,y[n-1] - 0.1\,y[n-2] + 2x[n] - x[n-2]$

The characteristic eq is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ hence

$$y_n[n] = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$$

with $x[n] = \delta[n]$, we have

$y[0] = 2$

$y[1] - 0.7\,y[0] = 0 \Rightarrow y[1] = 1.4$

hence, $c_1 + c_2 = 2$ and

$\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$

$\Rightarrow c_1 + \frac{2}{5}c_2 = \frac{14}{5}$ the equations yeild $c_1 = \frac{10}{3}$, $c_2 = \frac{-4}{3}$

$$h[n] = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u[n]$$

The step response is

$$s[n] = \sum_{k=0}^{n} h[n-k]$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left[\left(\frac{1}{2}\right)^n\left(2^{n+1} - 1\right)\right]u(n) - \frac{4}{3}\left(\frac{1}{5}\right)^n\left[5^{n+1} - 1\right]u(n).$$

2.34 consider a s/m with impulse response

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \leq n \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

determine the i/p $x[n]$ for $0 \leq n \leq 8$ that will generate the o/p sequence $y[n] = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0, \ldots\}$

$h[n] = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$

$y[n] = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0\}$

$x[0]h[0] = y[0] \Rightarrow x[0] = 1$

$\frac{1}{2}x[0] + x[1] = y[1] \Rightarrow x[1] = \frac{3}{2}$

by counting this process, we obtain

$x[n] = \{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, \ldots\}$

2.35 consider the interconnection of LTI s/m as shown in fig. P2.35

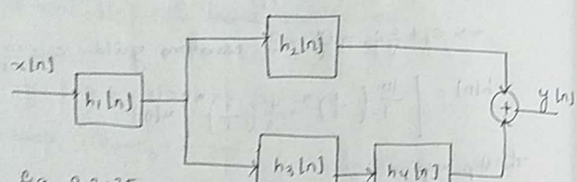


fig P 2.35

(a) express the overall impulse response in terms of $h_1[n]$, $h_2[n]$, $h_3[n]$ and $h_4[n]$.

$h[n] = h_1[n] * [h_2[n] - h_3[n] * h_4[n]]$

(b) determine $h[n]$ when

$h_1[n] = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\}$

$h_2[n] = h_3[n] = [n+1]u[n]$

$h_4[n] = \delta[n-2]$

sol:- $h_3[n] * h_4[n] = [n-1]u[n-2]$

$h_2[n] - h_3[n] * h_4[n] = 2u[n] - \delta[n]$

$h_1[n] = \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{2}\delta[n-2]$

hence $h[n]$ $[\frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{2}\delta[n-2]] * [2u[n] - \delta[n]]$

$= \frac{1}{2}\delta[n] + \frac{5}{4}\delta[n-1] + 2\delta[n-2] + \frac{5}{2}u[n-3]$

(c) determine the response of the s/m in part (b) if

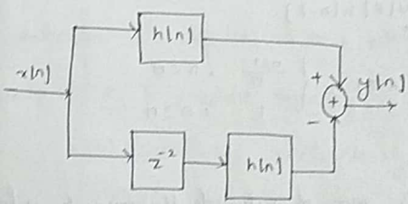
$x[n] = \delta[n+2] + 3\delta[n-1] - 4\delta[n-3]$

sol:- $x[n] = \{1, \underset{\uparrow}{0}, 0, 3, 0, -4\}$

$y[n] = \{\frac{1}{2}, \frac{5}{4}, 2, \frac{25}{4}, 5, 2, 0, 0, \ldots\}$

2.36 consider the s/m in given fig. with $h[n] = a^n u[n]$; $-1 < a < 1$. Determine the response $y[n]$ to the s/m to the excitation

$x[n] = u[n+5] - u[n-10]$



sol:- first we determine

$s[n] = u[n] * h[n]$

$s[n] = \sum_{k=0}^{\infty} u[k]\, h[n-k]$

$= \sum_{k=0}^{n} h[n-k]$

$= \sum_{k=0}^{\infty} a^{n-k} = \frac{a^{n+1} - 1}{a - 1}, n \geq 0.$

Scanned by CamScanner

for $x[n] = u[n+5] - u[n-10]$, we have the response

$$s[n+5] - s[n-10] = \frac{a^{n+6}-1}{a-1} u[n+5] - \frac{a^{n-9}-1}{a-1} u[n-10]$$

from figure

$$y[n] = x[n] * h[n] - x[n] * h[n-2]$$

hence,
$$y[n] = \frac{a^{n+6}-1}{a-1} u[n+5] - \frac{a^{n-9}-1}{a-1} u[n-10]$$
$$- \frac{a^{n+4}-1}{a-1} u[n-3] + \frac{a^{n-11}-1}{a-1} u[n-12]$$

2.37 compute and sketch the step response of the s/m

$$y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x[n-k]$$

Sol:- $h[n] = [u[n] - u[n-M]]/M$

$$s[n] = \sum_{k=-\infty}^{\infty} u[k]\, h[n-k]$$

$$= \sum_{k=0}^{\infty} h[n-k] = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}$$

2.38 Determine the range of values of the parameter a for which the linear time invariant S/m with impulse response

$$h[n] = \begin{cases} a^n, & n \geq 0,\ n \text{ even} \\ 0, & \text{otherwise} \end{cases}$$ is stable

Sol:-
$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0,\ n \text{ is even}}^{\infty} |a|^n = \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$ stable if $|a| < 1$.

2.39 Determine the response of the system with impulse response $h[n] = a^n u[n]$ to the i/p sig^l $x[n] = u[n] - u[n-10]$

[Hint: The solution can be obtained easily & quickly by applying the linearity and time invariance properties to the result in ex 2.3.5].

$h[n] = a^n u(n)$. The response to $u[n]$ is

$$y_1[n] = \sum_{k=0}^{\infty} u[k]\, h[n-k]$$

$$= \sum_{k=0}^{n} a^{n-k} = a^n \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1-a^{n+1}}{1-a} u[n]$$

then, $y[n] = y_1[n] - y_1[n-10]$

$$= \frac{1}{1-a}\left[(1-a^{n+1})\, u[n] - (1-a^{n-9})\, u[n-10]\right]$$

2.40 Determine the impulse response of the (relaxed) s/m characterized by the impulse response

$h[n] = [\frac{1}{2}]^n u[n]$ to the i/p sig^l

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

result from previous $a = \frac{1}{2}$

so, $y[n] = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u(n) - 2\left[1 - \left(\frac{1}{2}\right)^{n-9}\right] u(n-10)$.

2.41 Determine the response of the (relaxed) s/m characterized by the impulse response

$$h[n] = \left(\tfrac{1}{2}\right)^n u[n] \text{ to the i/p signals}$$

(a) $x[n] = 2^n u[n]$

(b) $x[n] = u[n]$

Sol:-

a) $$y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

$$= \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} \left(\tfrac{1}{4}\right)^k = 2^n\left[1-\left(\tfrac{1}{4}\right)^{n+1}\right]\left(\tfrac{4}{3}\right)$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}\right] u[n]$$

(b) $$y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

$$= \sum_{k=0}^{\infty} h[k] = \sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right)^k = 2 \quad n<0$$

$$y[n] = \sum_{k=n}^{\infty} h[k] = \sum_{k=n}^{\infty} \left(\tfrac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\tfrac{1}{2}\right)^k$$

$$= 2 - \frac{\left(1-\left(\tfrac{1}{2}\right)^n\right)}{\tfrac{1}{2}}$$

$$= 2\left(\tfrac{1}{2}\right)^n, \quad n \geq 0.$$

2.42 Three s/m's with impulse responses $h_1[n] = \delta[n] - \delta[n-1]$, $h_2[n] = h[n]$ and $h_3[n] = u[n]$, are connected in cascade.

a) what is the impulse response, $h_c[n]$ of the overall s/m?

sol:- $h_c[n] = h_1[n] * h_2[n] * h_3[n]$

$= [\delta[n] - \delta[n-1]] * u[n] * h[n]$

$= [u[n] - u[n-1]] * h[n]$

$= \delta[n] * h[n]$

$h_c[n] = h[n]$

2.43 prove and explain graphically the difference b/w the relation $x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$ and $x[n] * \delta[n-n_0] = x[n-n_0]$.

sol:- $x[n]\,\delta[n-n_0] = x[n_0]$. Thus only the value of $x[n]$ at $n=n_0$ is of interest.

$x[n] * \delta[n-n_0] = x[n-n_0]$. Thus, we obtain the shifted version of the sequence $x[n]$.

(b) show that a discrete time s/m, which is described by a convolution summation, is LTI and relaxed.

sol:- $y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$

$= h[n] * x[n]$

linearity, $x_1[n] \rightarrow y_1[n] = h[n] * x_1[n]$

$x_2[n] \rightarrow y_2[n] = h[n] * x_2[n]$

then $x[n] = a x_1[n] + B x_2[n] \rightarrow y[n] = h[n] * x[n]$

$y[n] = h[n] * [a x_1[n] + B x_2[n]]$

$= a h[n] * x_1[n] + B h[n] * x_2[n]$

$= a y_1[n] + B y_2[n]$.

Time invariance:

$$x[n] \longrightarrow y[n] = h[n] * x[n]$$

$$x[n-n_0] \longrightarrow y_1[n] = h[n] * x[n-n_0]$$

$$= \sum_k h[k]\, x[n-n_0-k]$$

$$= y[n-n_0]$$

c) what is the impulse response of the s/m described by

$$y[n] = x[n-n_0] ?$$

sol:- $h[n] = \delta[n-n_0]$

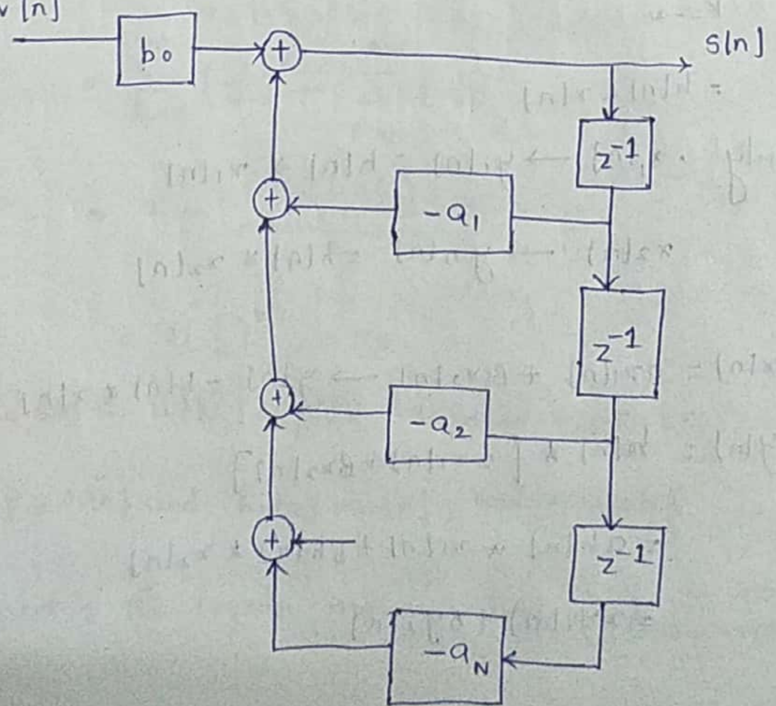
2.44 Two sig's $s[n]$ and $v[n]$ are related through the following difference equations.

$$s[n] + a_1 s[n-1] + \cdots + a_N s(n-N) = b_0 v[n].$$

Design the block diagram realization of

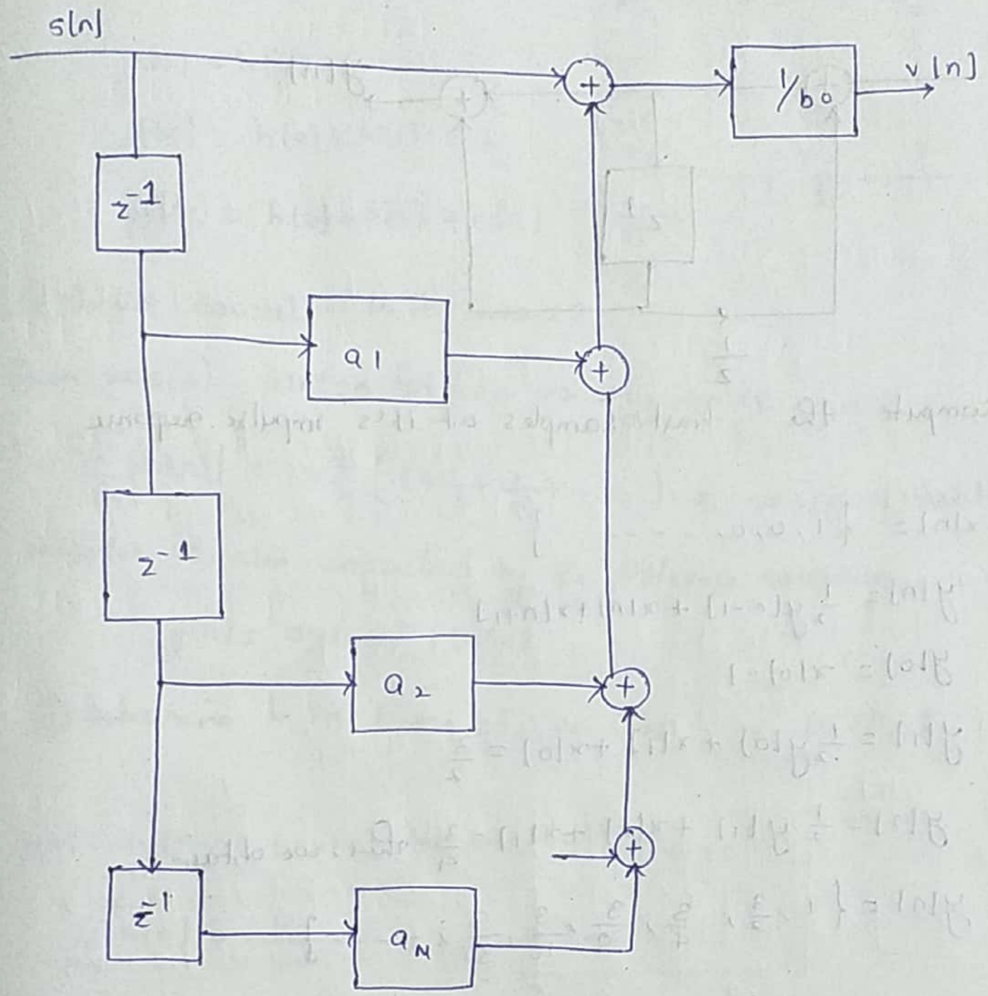
(a) the s/m that generates $s[n]$ when excited by $v[n]$.

sol:- $s[n] = -a_1 s[n-1] - a_2 s[n-2] - \cdots - a_N s(n-N) + b_0 v[n]$



b) the s/m that generates v(n) when excited by s[n].

sol: $v[n] = \frac{1}{b_0}\left[s[n] + a_1 s[n-1] + a_2 s[n-2] + \cdots + a_N s[n-N] \right]$



.45 compute the zero state response of the s/m described by the difference equation.

$$y[n] = +\frac{1}{2} y[n-1] = x[n] + 2n[x-2]$$

to the i/P $x[n] = \{1, 2, 3, 4, 2, 1\}$

$\uparrow$

by solving the difference equation recursively.

$$y[n] = -\frac{1}{2} y[n-1] + x[n] + 2x[n-2]$$

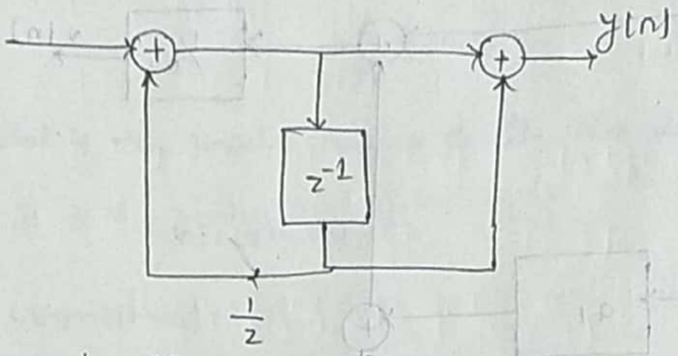
$$y[-2] = -\frac{1}{2} y[-3] + x[-2] + 2x[-4] = 1$$

$$y[-1] = -\frac{1}{2} y[-2] + x[-1] + 2x[-3] = \frac{3}{2}$$

$y[0] = -\frac{1}{2}y[-1] + 2x[-2] + x[0] = \frac{17}{4}$

$y[1] = -\frac{1}{2}y[0] + x[1] + 2x[-1] = \frac{47}{8}$, etc . .

2.47 consider the discrete s/m shown in figure



a) compute the first 10 samples of it's impulse response

sol:- $x[n] = \{1, 0, 0, - - - - \}$

$y[n] = \frac{1}{2}y[n-1] + x[n] + x[n+1]$

$y[0] = x[0] = 1$

$y[1] = \frac{1}{2}y[0] + x[1] + x[0] = \frac{3}{2}$

$y[2] = \frac{1}{2}y[1] + x[2] + x[1] = \frac{3}{4}$ thus, we obtain

$y[n] = \{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, - - - \}$

(b) compare the i/p - o/p relation

$y[n] = \frac{1}{2}y[n-1] + x[n] + x[n-1]$

(c) Apply the input $x[n] = \{1, 1, 1, - - - \}$ and compute the first 10 samples of the o/p.

sol:- As in part (a), we obtain

$y[n] = \{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16} - - - \}$

(d) compute the first 10 samples of the o/p for the i/p given in part (c) by using convolution.

$y[n] = u[n] * h[n]$

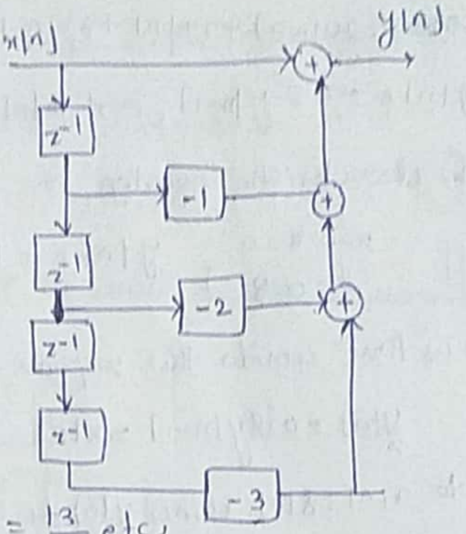
$= \sum_{k} u[k]\, h[n-k]$

$= \sum_{k=0}^{n} h[n-k]$

$y[0] = h[0] = 1$

$y[1] = h[0] + h[1] = \frac{5}{2}$

$y[2] = h[0] + h[1] + h[2] = \frac{13}{4}$ etc.



(c) is the s/m causal ? is it stable?

sol:- from part (a), $h[n] = 0$ for $n<0$ $\Rightarrow$ the s/m is causal

$\sum_{n=0}^{\infty} |h[n]| = 1 + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right) = 4$ $\Rightarrow$ s/m is stable

2.48 consider the s/m described by the difference equation

$y(n) = ay[n-1] + bx[n]$

(a) Determine b in terms of a so that $\sum_{n=-\infty}^{\infty} h[n] = 1$

sol:- $y[n] = ay[n-1] + bx[n] \Rightarrow h[n] = ba^n u[n]$

$\sum_{n=0}^{\infty} h[n] = \frac{b}{1-a} = 1$, $b \Rightarrow 1-a$

(b) compute the zero state step response s[n] of the s/m and choose b so that $s[\infty] = 1$

$s[n] = \sum_{k=0}^{n} h[n-k] = b\left[\frac{1-a^{n+1}}{1-a}\right] u[n]$

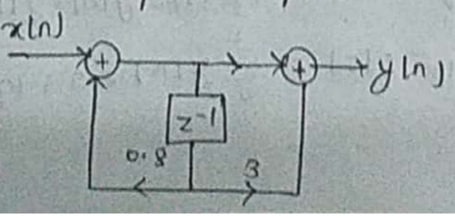
$s[\infty] = \frac{b}{1-a} = 1 \Rightarrow b = 1-a$

(c) compare the value of b obtained in parts (a) and (b) what did you notice?

sol:- $b = 1-a$ in both cases.

2.49 A discrete time s/m is realized by the structure shown in fig.

(a) determine the impulse response



Sol:- $y[n] = 0.8y[n-1] + 2x[n] + 3x[n-1]$

$y[n] - 0.8\,y[n-1] = 2x[n] + 3x[n-1]$

The characteristic equation is

$r - 0.8 = 0 \qquad y[n] = c\,[0.8]^n$

$r = 0.8$

Let us first consider the response of the s/m

$y[n] - 0.8y[n-1] = x[n]$

to $x[n] = \delta[n]$, since $y[0] = 1$. It follows that $c = 1$, then the impulse response of the original s/m is $h[n] = 2[0.8]^n u[n] + 3[0.8]^{n-1} u[n-1]$

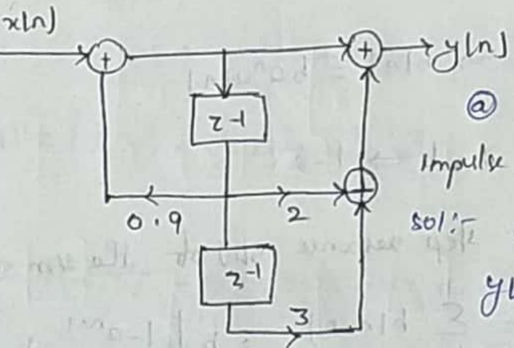
$= 2\delta[n] + 4.8\,[0.8]^{n-1} u[n-1]$

(b) Determine a realization for it's inverse s/m, that is the s/m which produces $x[n]$ as an o/p when $y[n]$ is used as an i/p.

Sol:- The inverse s/m is characterized by the difference equation

$x[n] = 1.5x[n-1] + \frac{1}{3}y[n] - 0.4y[n-1]$

2.50. Consider the discrete-time s/m shown in fig.



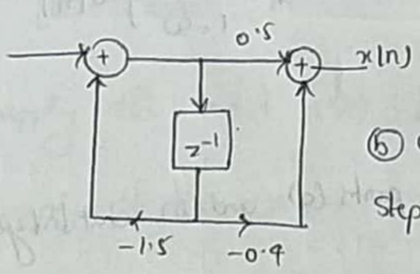
(a) Compute the first 6 values of the impulse response of the s/m.

Sol:- For $x[n] = \delta[n]$, we have

$y[0] = 1$, $y[1] = 2.9$, $y[2] = 5.6$

$y[3] = 5.049$, $y[4] = 4.544$,

$y[5] = 4.090$ ....



(b) Compute the first 6 values of the zero-state step response of the s/m.

Sol:- $s[0] = y[0] = 1$, $s[1] = y[0] + y[1] = 3.91$

$s[2] = y[0] + y[1] + y[2] = 9.51$, $s[3] = y[0] + y[1] + y[2] + y[3] = 14.56$

$s[4] = \sum_{0}^{4} y[n] = 19.10$, $s[5] = \sum_{0}^{5} y[n] = 23.19$

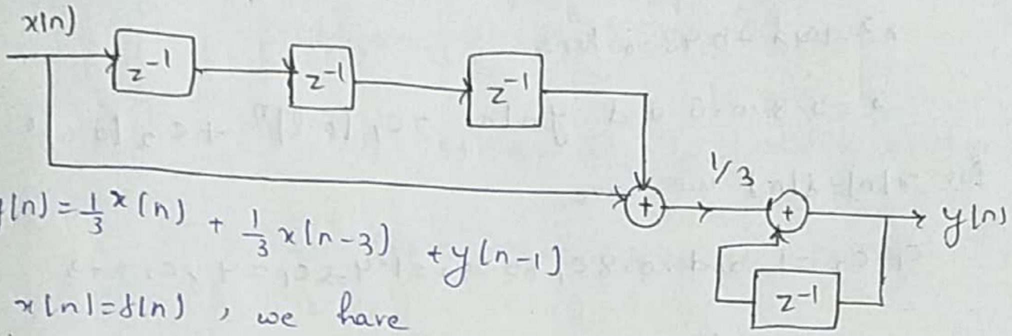
(c) determine an analytical expression for the impulse response of the sm.

sol: $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} + 3(0.9)^{n-2} u(n-2)$.

$\Rightarrow \delta(n) + 2.9\delta(n-1) + 5.61(0.9)^{n-2} u(n-2)$

.51 Determine and sketch the impulse response of the following sm's for $n = 0, 1, ---- 9$.

(a)

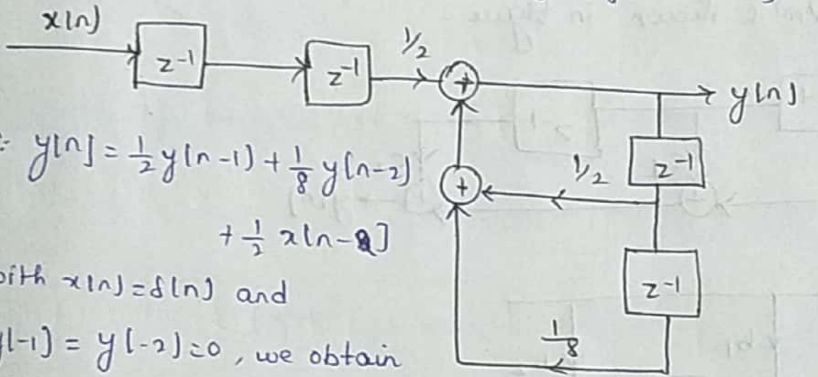


$y(n) = \frac{1}{3} x(n) + \frac{1}{3} x(n-3) + y(n-1)$

for $x(n) = \delta(n)$, we have

$h(n) = \{ \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, ---\}$

(b)



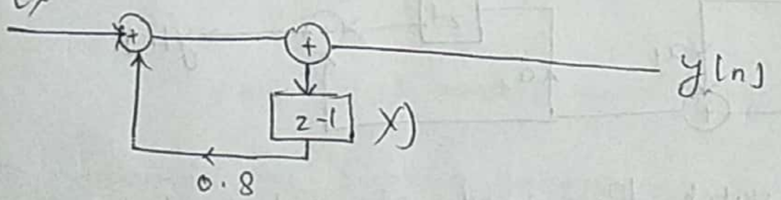
sol: $y(n) = \frac{1}{2} y(n-1) + \frac{1}{8} y(n-2) + \frac{1}{2} x(n-2)$

with $x(n) = \delta(n)$ and

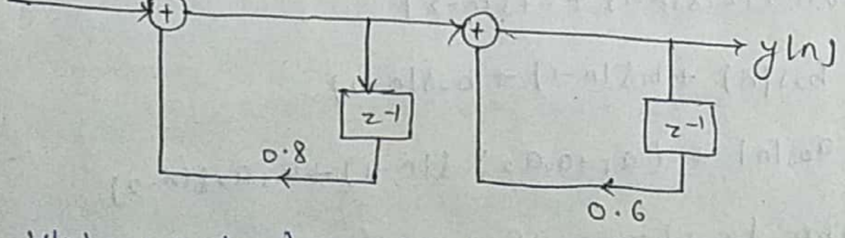
$y(-1) = y(-2) = 0$, we obtain

$h(n) = \{0, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{108}, \frac{15}{256}, \frac{41}{1024}, ----\}$

(c) [X



(c)



$y(n) = 1.4 y(n-1) - 0.48 y(n-2) + x(n)$ with $x(n) = \delta(n)$, and

$y(-1) = y(-2) = 0$ we obtain

$h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086, ----\}$

(d) classify the s/m's above as FIR or IIR

sol:- All 3 s/m's are ~~IIR~~ IIR

(e) Find an explicit expression for the impulse response of the s/m in part (c)

$y[n] = 1.4y[n-1] - 0.48y[n-2] + x[n]$

The characteristic equation is

$\lambda^2 - 1.4\lambda + 0.48 = 0$ hence

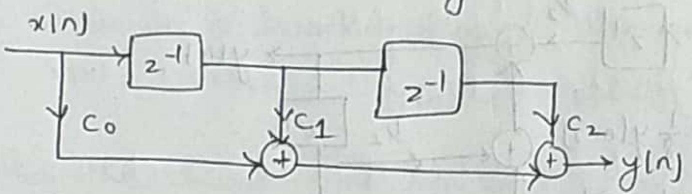
$\lambda = 0.8, 0.6$ and $y_h[n] = c_1 [0.8]^n + c_2 [0.6]^n$

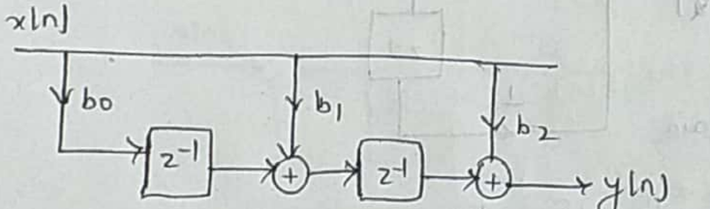
for $x[n] = \delta[n]$ we have

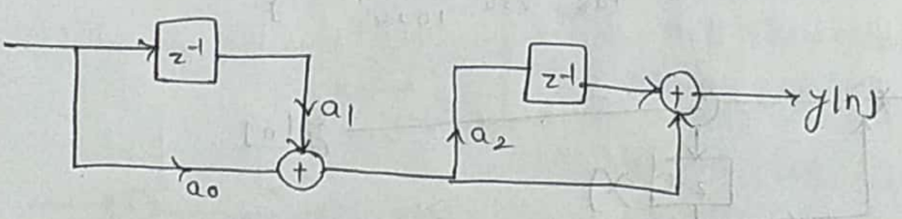
$c_1 + c_2 = 1$ and $0.8c_1 + 0.6c_2 = 1.4 \Rightarrow c_1 = 4, c_2 = -3.$

$\therefore \quad h[n] = [4(0.8)^n - 3(0.6)^n]u[n]$

2.52 consider the s/m's shown in figure.







(a) Determine and sketch these impulse response $h_1[n], h_2[n]$ and $h_3[n]$.

$h_1[n] = c_0\delta[n] + c_1\delta[n-1] + c_2\delta[n-2]$

$h_2[n] = b_2\delta[n] + b_1\delta[n-1] + b_0\delta[n-2]$

$h_3[n] = a_0\delta[n] + (a_1 + a_0a_2)\delta[n-1] + a_1a_2\delta[n-2]$

(b) Is it possible to choose the coefficients of these s/m's in such a way that $h_1[n] = h_2[n] = h_3[n]$

sol:-

$h_3[n] = h_2[n] = h_1[n]$

let $a_0 = c_0$,

$a_1 + a_2 c_0 = c_1$

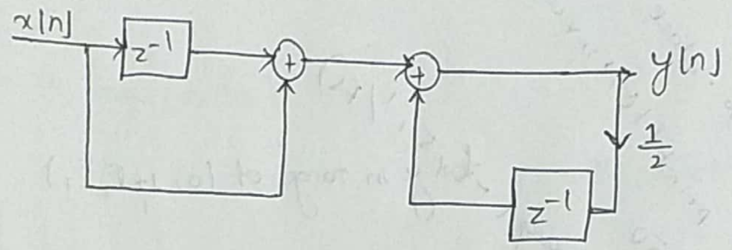
$a_2 a_1 = c_2$

Hence $\frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$

$\Rightarrow c_0 a_2^2 - c_1 a_2 + c_2 = 0$

$c_0 \neq 0$, the quadratic has a real solution if and only if $c_1^2 - 4c_0c_2 \geq 0$

2.53 consider the s/m shown in fig



(a) Determine the impulse response $h[n]$.

Sol:- $y[n] = \frac{1}{2} y[n-1] + x[n] + x[n-1]$

for $y[n] - \frac{1}{2} y[n-1] = \delta[n]$, the solution is

$h[n] = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u[n-1]$

(b) show that $h[n]$ is equal to the convolution of the following sig

$h_1[n] = \delta[n] + \delta[n-1]$

$h_2[n] = \left(\frac{1}{2}\right)^n u[n]$

Sol:- $h_1[n] * [\delta[n] + \delta[n-1]] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1]$

2.54 compare and sketch the convolution $y_1[n]$ and correlation $r_1[n]$ sequences for the following pair of sig's and comment on the results obtained. a] $x_1[n] = \{\underset{\uparrow}{1}, 2, 4\}$ $h_1[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

sol':

	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

<u>convolution</u>:

$$\Rightarrow y(n) = \{1, 3, 7, 7, 7, 6, 4\}$$
$$\uparrow$$

<u>correlation</u>:

$$r_1(n) = \{1, 3, 7, 7, 7, 6, 4\}$$

(b) $x_2(n) = \{0, 1, -2, 3, -4\}$ $\quad h_2(n) = \{\frac{1}{2}, 1, 2, 1, \frac{1}{2}\}$

sol': convolution $y_1(n) = \{\frac{1}{2}, 0, \frac{3}{2}, -2, \frac{1}{2}, -6, \frac{-5}{2}, -2\}$

correlation : $r_1(n) = \{\frac{1}{2}, 0, \frac{3}{2}, -2, \frac{1}{2}, -6, \frac{-5}{2}, 2\}$

Note that $y_2(n) = r_2(n)$, because $h_2(-n) = h_2(n)$

(c) $x_3(n) = \{1, 2, 3, 4\}$ $\quad h_3(n) = \{4, 3, 2, 1\}$

sol': convolution, $y_3(n) = \{4, 11, 20, 30, 20, 11, 4\}$

correlation, $r_3(n) = \{1, 4, 0, 20, 25, 24, 16\}$

(d) $x_4(n) = \{1, 2, 3, 4\}$ $\quad h_4(n) = \{1, 2, 3, 4\}$

sol': convolution $y_4(n) = \{1, 4, 20, 20, 25, 24, 16\}$

correlation $r_4(n) = \{4, 11, 20, 30, 20, 11, 4\}$

Note that $h_3(-n) = h_4(n+3)$

hence $r_3(n) = y_4(n+3)$ & $y_4(-n) = h_3(n+3)$

$\Rightarrow r_4(n) = y_3(n+3)$

2.55 The zero-state response of a causal LTI s/m to the i/p $x(n) = \{1, 3, 3, 1\}$ is $y(n) = \{1, 4, 6, 4, 1\}$ determine it's impulse response.

sol': obviously, the length of $h(n)$ is 2,

i.e; $h(n) = \{h_0, h_1\}$ $\Rightarrow h_0 = 1$ $\quad 3h_0 + h_1 = 4$ $\Rightarrow h_0 = 1, h_1 = 1$

2.56 prove by direct substitution the equivalence of equations

$w[n] = -\sum_{k=1}^{N} a_k w[n-k] + x[n]$ and $y[n] = \sum_{k=0}^{M} b_k w[n-k]$, which

describe the direct form II structure, to the relation

$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$, which describes the direct form I structure.

sol:- $y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k] \longrightarrow (1)$

$w[n] = -\sum_{k=1}^{N} a_k \cancel{kw}\, w[n-k] + x[n] \longrightarrow (2)$

$y[n] = \sum_{k=0}^{M} b_k w[n-k] \longrightarrow (3)$

from eq (2) we obtain

$x[n] = w[n] + \sum_{k=1}^{N} a_k w[n-k] \longrightarrow (4)$

By substituting eq (3) for $y[n]$ and eq (4) into (1) we obtain L.H.S = R.H.S.

2.67 Determine the response $y[n]$, $n \geq 0$ of the s/m described by the second-order difference eq. $y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$

when the i/p is $x[n] = (-1)^n u[n]$, and the initial conditions are

$y[-1] = y[-2] = 0$

sol:- $y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$

The characteristic equation is

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$ hence $y_h[n] = c_1 2^n + c_2 n 2^n$

The particular solution is $y_p[n] = k(-1)^n u[n]$.

substituting this solution into the difference equation we obtain

$\cancel{of}\; k(-1)^n u[n] - 4k(-1)^{n-1} u[n-1] + 4k(-1)^{n-2} u[n-2]$

$= (-1)^n u[n] - (-1)^{n-1} u[n-1]$

for $n=2$, $k(1+4+4) = 2$, $k = \frac{2}{9}$ the total solution is

$y[n] = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u[n]$

from the initial conditions, we obtain $y[0] = 1$, $y[1] = 2$

Then, $c_1 + \frac{2}{9} = 1$

$\Rightarrow c_1 = \frac{7}{9}$ $\quad 2c_1 + 2c_2 - \frac{2}{9} = 2$

$\frac{14}{9} - \frac{2}{9} + 2c_2 = 2 \Rightarrow 2c_2 = 2 - \frac{12}{9}$, $2c_2 = \frac{6}{9}$

$c_2 = \frac{3}{9} = \frac{1}{3}$

2.58 Determine the impulse response $h[n]$ for the s/m described by the second-order differential equation.

$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$

sol:- from previous problem

$h[n] = [c_1 2^n + c_2 n 2^n] u[n]$

with $y[0]=1$, $y[1]=3$, we've $c_1 = 1$

$2c_1 + 2c_2 = 3 \Rightarrow c_2 = \frac{1}{2}$

Thus $h[n] = [2^n + \frac{1}{2} n 2^n] u[n]$.

2.59 show that any discrete time sig $x[n]$ can be expressed as

$x[n] = \sum_{k=-\infty}^{\infty} [x[k] - x[k-1]] u[n-k]$ where $u[n-k]$ is a unit step described by k units in time, that is

$u[n-k] = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise.} \end{cases}$

sol:- $x[n] = x[n] * \delta[n]$

$= x[n] * [u[n] - u[n-1]]$

$= [x[n] - x[n-1]] * u[n]$

$= \sum_{k=-\infty}^{\infty} [x[k] - x[k-1]] u[n-k]$

2.60. show that the o/p of an LTI s/m can be expressed in terms of it's unit step response $s[n]$ as follows.

$y[n] = \sum_{k=-\infty}^{\infty} [s[k] - s[k-1]] x[n-k]$

$= \sum_{k=-\infty}^{\infty} [x[k] - x[k-1]] s[n-k]$

sol:- let $h[n]$ be the impulse response of the s/m $y[n] = \sum_{k=-\infty}^{\infty} [s[k] - s[k-1]] x[n-k]$.

$s[k] = \sum_{m=-\infty}^{k} h[m]$

$h[k] = s[k] - s[k-1]$, $y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$

$y[n] = \sum_{k=-\infty}^{\infty} [s[k] - s[k-1]] x[n-k]$

2.61 Compute the correlation sequences $r_{xx}[l]$ and $r_{xy}[l]$ for the following signal sequences.

$$x[n] = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{otherwise} \end{cases} \quad y[n] = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise.} \end{cases}$$

Sol: $x(n) = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{otherwise} \end{cases} \quad y[n] = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$

$$r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]\, x[n-l]$$

The range of non-zero value of $r_{xx}[l]$ is determined by

$$n_0 - N \le n \le n_0 + N, \quad n_0 - N \le n - l \le n_0 + N$$

which implies $-2N \le l \le 2N$

for a given shift l, the number of term in the summation for which both $x[n]$ and $x[n-l]$ are be non-zero is $2N-1-|l|$, and the value of each term is 1. Hence,

$$r_{xx}[l] = \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

for $r_{xy}[l]$ we have

$$r_{xy}[l] = \begin{cases} 2N+1-|l-n_0|, & n_0 - 2N \le l \le n_0 + 2N \\ 0, & \text{otherwise} \end{cases}$$

2.62 Determine the autocorrelation sequences of the following signals.

(a) $x[n] = \{\underset{\uparrow}{1}, 2, 1, 1\}$

$$r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]x[n-l] \Rightarrow r_{xx}[-3] = x[0]x[3] = 1$$

$r_{xx}[-2] = x[0]x[2] + x[1]x[3] = 3$

$r_{xx}[-1] = x[0]x[1] + x[1]x[2] + x[2]x[3] = 5$

$r_{xx}[0] = \sum_{n=0}^{3} x^2[n] = 7$, also $r_{xx}[-l] = r_{xx}[l]$

$\therefore r_{xx}[l] = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$

ⓑ $y[n] = \{1, 1, 2, 1\}$

sol: $r_{yy}(l) = \sum_{n=-\infty}^{\infty} y[n] y[n-l]$ we obtain $r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

we observe that $y[n] = x[-n+3]$ which is equivalent to reversing the sequence $x[n]$. This has not changed the autocorrelation sequence.

2.63 what is the normalized acceleration sequence of the sigl $x[n]$ given

$$x[n] = \begin{cases} 1 & , -N \le n \le N \\ 0 & , \text{otherwise} \end{cases}$$

sol: $$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x[n] x[n-l] = \begin{cases} 2N+1-|l| , & -2N \le l \le 2N \\ 0 , & \text{otherwise} \end{cases}$$

$r_{xx}(0) = 2N+1$

∴ the normalized autocorrelation is

$$r_{xx}(l) = \begin{cases} \frac{1}{2N+1} [2N+1-|l|] , & -2N \le l \le 2N \\ 0 , & \text{otherwise} \end{cases}$$

2.64 An audio sigl $s(t)$ generated by a loudspeaker is reflected at two different walls with reflection coeff r_1 & r_2. The sigl $x(t)$ recorded by a microphone close to the loudspeaker, after sampling is

$x[n] = s[n] + r_1 s[n-k_1] + r_2 s[n-k_2]$ where k_1 and k_2 are the delays of the two echoes.

ⓐ Determine the autocorrelation $r_{xx}(l)$ of the sigl $x[n]$.

sol: $$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x[n] x[n-l]$$

$$= \sum_{n=-\infty}^{\infty} [s[n] + r_1 s[n-k_1] + r_2 s[n-k_2]] * [s[n-l] + r_1 s[n-l-k_1] + r_2 s[n-l-k_2]]$$

$$= [1+r_1^2+r_2^2] r_{xx}(l) + r_1[r_{xx}[l+k_1] + r_{xx}[l-k_1]] + r_2[r_{xx}[l+k_2] + r_{xx}[l-k_2]] + r_1 r_2 [r_{xx}[l+k_1-k_2] + r_{xx}[l+k_2-k_1]].$$

(b) can we obtain r_1, r_2, k_1 and k_2 by observing $r_{xx}[l]$?

Sol: $r_{xx}[l]$ has peaks at $l=0, \pm k_1, \pm k_2$ and $\pm [k_1+k_2]$.

Suppose that $k_1<k_2$. then, we've determined r_1 and k_1.

The problem is to be determine r_2 and k_2 from the other peak.

(c) what happens if $r_2=0$?

Sol: if $r_2=0$, the peaks occur at $l=0$ and $l=\pm k_1$. then, it is easy to obtain r_1 and k_1.

Scanned by CamScanner