Session 12

Additional Exercise

Problem Statement 1:

A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Solution:

1 . Probability:

p = Success = 0.3 ; q = Failure = 0.7

This is a binomial distribution:

$P(x: = 2) = 6C_2 \text{ X } (0.3)^2 \text{ X } (1\text{-}0.3)^{x-2}$

$= \frac{6 \text{ X } 5 \text{ X } 4\,!}{2\,! \text{ X } 4\,!}$

= 0.3241

2. Calculate Mean

μ = E(x) = n. p

Where:

n = No. of Trials = 6

p = Success ration = 0.3

∴ Mean = 6*0.3

= 1.8

3. Variance

Var = npq

Where:

n = No. of Trials = 6

p = Success ratio = 0.7

q = Failure ratio= 0.3

= 6*0.7*0.3

= 1.26

4. Standard Deviation

$\sqrt{npq}$

<u>Where:</u>

n = No. of Trials = 6

p = Success ratio = 0.7

q = Failure ratio= 0.3

∴ Standard Deviation = $\sqrt{6*0.7*0.3}$

= 1.12

Problem Statement 2:

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Solution:

2. Solution:

(i) Probability of Gaurav solving 5 correctly

$P(G)\ x = 5$

$$\Rightarrow {}^8C_5 \times (0.75)^5 \times (0.25)^3$$

$$= \frac{8 \times 7 \times 6 \times 5!}{3 \times 2 \times 1 \times 5!} \times 0.2373 \times 0.0156$$

$$= 56 \times 0.2373 \times 0.0156$$

$$= 0.2076.$$

Probability of Barakha solving 5 correctly

$P(B) = 0.45$

$n = 12$

$P(B)(x=5)$

$= {}^{12}C_5 \times (0.45)^5 \times (0.55)^7$

$= 792 \times (0.45)^5 \times (0.55)^7$

$= 0.22249$

(ii) Probability of Gaurav solving 4 correctly

$P(G)(x=4)$

$= {}^{8}C_4 \times (0.75)^4 \times (0.25)^4$

$$= \frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!} \times (0.75)^4 \times (0.25)^4$$

$= 70 \times 0.31640 \times 0.0039$

$= 0.865$

(ii) Probability of Barakha solving 6 corre

$P(B)(x=6)$

$\Rightarrow {}^{12}C_6 \times (0.45)^6 \times (0.55)^6$

$= 924 \times (0.45)^6 \times (0.55)^6$

$= 0.21237$

Problem Statement 3:

Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes? a) 5 customers, b) not more than 3 customers, c) more than 3 customers. Give a pictorial representation of the same to validate your answer.

3. Solution

customer visit rate = 72 persons/hr

$\Rightarrow \frac{72}{60} = 1.2$ persons/min

(i) 4 minutes customer rate = $1.2 \times 4 = 4.8$

(a) 5 customers in min :-

$$= \frac{e^{(-4.8)} \times (4.8)^5}{5!}$$

$$= \frac{0.0082297 \times (4.8)^5}{5!}$$

$$= 0.1747$$

(b) not more than 3 customers :-

(i) $x = 0 \Rightarrow \frac{e^{(-4.8)} \times (4.8)^0}{0!} = 0.00829$

(ii) $x = 1 \Rightarrow \frac{e^{(-4.8)} \times (4.8)^1}{1!} = 0.0394$

(iii) $x = 2 \Rightarrow \frac{e^{(-4.8)} \times (4.8)^2}{2!} = 0.09679$

(iv) $x = 3 \Rightarrow \frac{e^{(-4.8)} \times (4.8)^3}{3!} = 0.151690$

$\Rightarrow$ Total probability of 3 customers

$= 0.00829 + 0.0394 + 0.9498 +$

0.151690

$= 0.2941$

(c) More than 3 customers

$= 1 - P(\text{less than 3 customers})$ from b

$= 1 - 0.2941$

$= 0.70591$

Problem Statement 4:

I work as a data analyst in Aeon Learning Pvt. Ltd. After analyzing data, I make reports, where I have the efficiency of entering 77 words per minute with 6 errors per hour. What is the probability that I will commit 2 errors in a 455-word financial report?

What happens when the no. of words increases/decreases (in case of 1000 words, 255 words)? How is the λ affected? How does it influence the PMF? Give a pictorial representation of the same to validate your answer.

Solution

Errors per hour = 6

Words per hour $\Rightarrow 77 \times 60 = 4620$ words/hr

$\Rightarrow$ Errors per word $= \frac{6}{4620} = \frac{1}{770}$

Probability of 2 errors in 455 words :-

$\lambda = \frac{455}{770} = 0.591$

$\Rightarrow P(x=2) = \frac{e^{-(0.591)} \times (0.591)^2}{2!}$

$= 0.09671$

Errors in 1000 words :-

$\lambda = \frac{1000}{770} = 1.2987$

$\Rightarrow P(x=2) = \frac{e^{-(1.2987)} \times (1.2987)^2}{2!}$

924 x

0.2123

$= 0.2303$

(iii) Errors in 255 words :-

$\lambda = \frac{255}{770} = 0.3311$

$P(x=2) = \frac{e^{-(0.311)} \times (0.3311)^2}{2!}$

$= 0.0393$

Problem Statement 5: [100 marks]

The current measured in a copper wire is modelled by a continuous random variable X. X is in milliamperes. Assume that the range of X is [0, 20 mA]. The probability density function is given by, $f(x)$ = 0.05 for $0 \le x \le 20$. What is the probability that a current measurement is less than 10 milliamperes? Draw the PDF and the CDF diagrams as well.

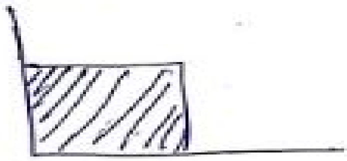
5. Solution

$$f(x) = 0.05 \quad \text{for} \quad 0 \le x \le 20$$

$$\int_{i=0}^{\infty} f(x).dx = \int_{0}^{10} 0.005$$

$$\Rightarrow (0.05)^{10} = 0.5 - 0 = 0.5$$

PDF



CDF