Session 15

Assignment 1 Question

Problem Statement 1:

You survey households in your area to find the average rent they are paying. Find the standard deviation from the following data:

$1550, $1700, $900, $850, $1000, $950.

<u>Solution:</u>

<u>A: Calculate Mean</u>

Mean = (Σ xi) / n

$$\Sigma \text{ of } x_i = \frac{\$1550 + \$1700 + \$900 + \$850 + \$1000 + \$950}{6} = \$1158.33$$

The Mean is $1158.33

<u>B: Calculate Standard Deviation</u>

$$\mathrm{SD} = \sqrt{\frac{\sum |x - \bar{x}|^2}{n}}$$

<u>Where:</u>

SD = Standard Deviation
x = each value in the data set
$\bar{x}$ = Mean is the data set
n = number of values in the data set

Step 1: Calculate mean

$$\Sigma \text{ of } x_i = \underline{\$1550 + \$1700 + \$900 + \$850 + \$1000 + \$950} \quad = \$1158.33$$

Step 2: Subtract the mean calculated from step 1 from each value. This gives you the differences:

$1550 – $1158.33 = $391.67

$1700 – $1158.33 = $541.67

$900 – $1158.33 = -$258.33

$850 – $1158.33 = -$308.33

$1000 – $1158.33 = $158.33

$950 – $1158.33 = $208.33

Step 3: Square the differences you found in Step 3:

$391.672 = 153405.3889

$541.672 = 293406.3889

-$258.332 = 66734.3889

-$308.332 = 95067.3889

$158.332 = 25068.3889

$208.332 = 43401.3889

Step 4: Add up all of the squares you found in Step 3 and divide by 5 (which is 6 – 1):

(153405.3889 + 293406.3889 + 66734.3889 + 95067.3889 + 25068.3889 + 43401.3889) / 5 = 135416.66668

Step 5: Find the square root of the number you found in Step 4 (the variance):

√135416.66668 = 367.99

The standard deviation is 367.99.

Problem Statement 2:

Find the variance for the following set of data representing trees in California (heights in feet):

3, 21, 98, 203, 17, 9

Variance Formula:

$$S^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}$$

Step 1: Add up the numbers in your given data set.

3 + 21 + 98 + 203 + 17 + 9 = 351

Step 2: Square your answer:

351 × 351 = 123,201

…and divide by the number of items. We have 6 items in our example so:

123,201 / 6 = 20,533.5

Step 3: Square each item in the data set & get the sum of squares

3 × 3 + 21 × 21 + 98 × 98 + 203 × 203 + 17 × 17 + 9 × 9

Add those numbers (the squares) together:

9 + 441 + 9604 + 41209 + 289 + 81 = 51,633

Step 4: Subtract the value calculated in Step 2 from the the value of Step 3.

51,633 – 20,533.5 = 31,099.5

Step 5: Subtract 1 from the number of items in your data set*. For our example:

6 – 1 = 5

Step 6: Divide the number in Step 4 by the number in Step 5. This gives you the variance:

31,099.5 / 5 = 6,219.9

Problem Statement 3:

In a class on 100 students, 80 students passed in all subjects, 10 failed in one subject, 7 failed in two subjects and 3 failed in three subjects. Find the probability distribution of the variable for number of subjects a student from the given class has failed in.

Solution:

For a random student,

The probability of failing in 0 subjects, P(X=0) = 0.8

The probability of failing in 1 subjects, P(X=1) = 0.1

The probability of failing in 2 subjects, P(X=2) = 0.07

The probability of failing in 3 subjects, P(X=3) = 0.03

The probability distribution can be shown as:

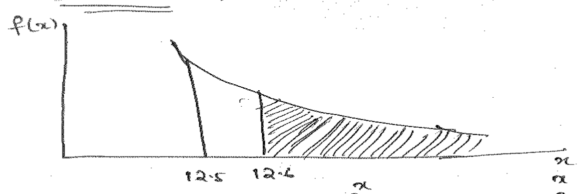
X	0	1	2	3
P(X)	0.8	0.1	0.07	0.03

Problem Statement 4:

Let the continuous random variable D denote the diameter of the hole drilled in an aluminum sheet. The target diameter to be achieved is 12.5mm. Random disturbances in the process often result in inaccuracy. Historical data shows that the distribution of D can be modelled by the PDF f(d) = 20e −20(d−12.5) , d ≥ 12.5. If a part with diameter > 12.6 mm needs to be scrapped, what is the proportion of those parts? What is the CDF when the diameter is of 11 mm? What is your conclusion regarding the proportion of scraps?

Solution:

Solution :-



$$P(x \geq 12.60) = \int_{12.6}^{\infty} f(x)\,dx = \int_{12.6}^{\infty} 20\, e^{-20(x-12.5)}\,dx = -e^{-20(x-12.5)}\Big|_{12.6}^{\infty}$$

$$= 0.135$$

~~total~~

∴ Proportion of parts between 12.5 and 12.6 mm

$$= P(12.5 \leq x < 12.6) = \int_{12.5}^{12.6} f(x)\,dx = -e^{20(x-12.5)}\Big|_{12.5}^{12.6}$$

$$= 0.865$$

[OR]

$$P(12.5 < x < 12.6) = 1 - P(x > 12.6)$$

$$= 1 - 0.135$$

$$= 0.865.$$

Hence, Total units to be scrapped = 0.865.