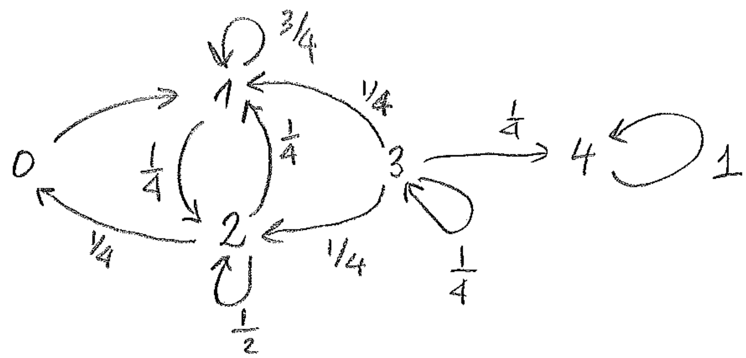
1. (a)



- Closed irreducible recurrent sets: $\{0, 1, 2\}, \{4\}$
- Transient: $\{3\}$

(b)

$$P_3(T_4 < \infty) = \sum_{k=1}^{\infty} P_3(T_4 = k) = \sum_{k=1}^{\infty} P_3(x_1 = ... = x_{k-1} = 3,\ x_k = 4)$$
$$= \sum_{k=1}^{\infty} \left(\frac{1}{4}\right)^k = \frac{1}{3}$$

(c)
- $\lim_{n\to\infty} p^n(2,1) = \pi(1) = \frac{8}{13}$
- $\lim_{n\to\infty} p^n(1,2) = \pi(2) = \frac{4}{13}$
- $E_0[T_0] = 1/\pi(0)$

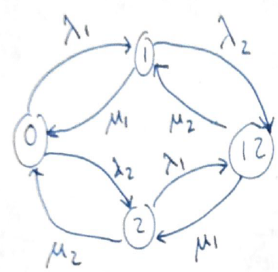
(d) $\lim_{n\to\infty} p^n(3,1) = (1 - 1/3) \cdot \pi(1) = \frac{2}{3} \cdot \pi(1)$

2. - The distribution of patrol's cycle

$$g(z) = \frac{P(t_i > z)}{E[t_i]} = 1/2 \qquad \text{if } 0 \le z \le 2$$

$$P(T > R + 2) = P(R < T - 2) = P(2 \le T \le 4,\ 0 < R < T - 2)$$
$$= \int_2^4 \int_0^{t-2} \frac{1}{4}\frac{1}{2} dr dt = \int_2^4 \frac{1}{8}(t-2)dt$$
$$= \frac{1}{8}\left[\frac{t^2}{2} - 2t\right]_2^4 = \frac{1}{4}$$

3.



$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 12 \end{array} \\ \begin{bmatrix} -(\lambda_1+\lambda_2) & \lambda_1 & \lambda_2 & 0 \\ \mu_1 & -(\lambda_2+\mu_1) & 0 & \lambda_2 \\ \mu_2 & 0 & -(\lambda_1+\mu_2) & \lambda_1 \\ 0 & \mu_2 & \mu_1 & -(\mu_1+\mu_2) \end{bmatrix} \end{array}$$

$$\begin{cases} \pi(0)\lambda_2 = \pi(2)\mu_2 \\ \pi(0)\lambda_1 = \pi(1)\mu_1 \\ \pi(12)\mu_1 = \pi(2)\lambda_1 \\ \pi(12)\mu_2 = \pi(1)\lambda_2 \end{cases} \quad \begin{cases} \pi(2) = \frac{\lambda_2}{\mu_2}\pi(0) \\ \pi(1) = \frac{\lambda_2}{\mu_1}\pi(0) \\ \pi(12) = \frac{\lambda_1}{\mu_1}\pi(2) = \frac{\lambda_1\lambda_2}{\mu_1\mu_2}\pi(0) \\ \pi(12) = \frac{\lambda_2}{\mu_2}\pi(1) = \frac{\lambda_1\lambda_2}{\mu_1\mu_2}\pi(0) \end{cases}$$

$$\pi(0)+\pi(1)+\pi(2)+\pi(12) = \pi(0)\left(1+\frac{\lambda_2}{\mu_1}+\frac{\lambda_2}{\mu_2}+\frac{\lambda_1\lambda_2}{\mu_1\mu_2}\right) = 1$$

$$\pi(0) = \frac{1}{\left(1+\frac{\lambda_2}{\mu_1}+\frac{\lambda_2}{\mu_2}+\frac{\lambda_1\lambda_2}{\mu_1\mu_2}\right)} = \frac{1}{\left(1+\frac{\lambda_1}{\mu_1}\right)\left(1+\frac{\lambda_2}{\mu_2}\right)}$$

$$\pi(12) = \frac{\lambda_1\lambda_2}{\mu_1\mu_2}\cdot\frac{1}{\left(1+\frac{\lambda_1}{\mu_1}\right)\left(1+\frac{\lambda_2}{\mu_2}\right)} = \frac{\lambda_1\lambda_2}{(\mu_1+\lambda_1)(\mu_2+\lambda_2)}$$

4.

$$\begin{aligned} P(\text{I leave first}) &= \sum_{k=0}^{\infty} P(k \text{ customers present}, T_0 < \min_{1\le i\le k} T_i) \\ &= \sum_{k=0}^{\infty} \frac{-e^{-\frac{\lambda}{\mu}}\left(\frac{\lambda}{\mu}\right)^k}{k!}\cdot\frac{1}{k+1} = e^{-1/\mu}\frac{\mu}{\lambda}\sum_{k=0}^{\infty}\frac{\left(\frac{\lambda}{\mu}\right)^{k+1}}{(k+1)!} \\ &= e^{-\frac{\lambda}{\mu}}\frac{\mu}{\lambda}\left(e^{\lambda/\mu}-1\right) = \frac{\mu}{\lambda}\left(1-e^{-\lambda/\mu}\right) \end{aligned}$$