Kidney Exchanges as a Matching Problem to Maximize Successful Matches

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I. Abstract

The widely preferred treatment for kidney failure in humans is transplantation. However, the demand for donor kidneys is far greater than supply. In the US, roughly 35,000 people are added to a waiting list of approximately 100,000 each year. Of the 35,000, only 16,000 leave due to receiving a kidney. Transplants only recently became ubiquitous. In the US, the National Organ Transplant Act allowed for kidney transplants to become legal in 1990. This resulted in more transplant centers to arise.

A healthy person has two kidneys, but can survive with just one. This creates the opportunity for humans to be living organ donors. However, having a living kidney donor is not sufficient. Sometimes, a patient-donor pair is incompatible, meaning that the donor's kidney is unlikely to function properly inside the patient. Factors such as blood type, status of sickness, and genetic information affect kidney exchanges.

Kidney exchange is a recent innovation that allows patients who suffer from kidney failure, and have been lucky enough to find a willing but incompatible kidney donor, to swap donors. The goal of an exchange is to maximize the number and quality of exchanges, taking into account the genetic, medical and psychological factors of the situation. These exchanges and the algorithms behind them have been successful thus far, enabling lives to be saved. However, there is still research currently being done regarding improving the efficiency of kidney exchanges to create successful transplants.

II. Model

A paired kidney exchange can be modeled as a graph, G(V,E) whose vertices are incompatible patient-donor pairs that are mutually compatible and connected by edges. A matching M is a collection of edges such that no vertex is covered more than once. This model can be used in two algorithms: the top trading cycle algorithm and the matching algorithm.

III. Scoring

There are many factors that affect how successful a kidney transplant will be. The success of a transplant is contingent on the matching of blood type and tissue type as well as size of the organ. Other factors that improve the success of a transplant include how long the patient is willing to wait, his or her severity of sickness, and his or her mental health status. A patient who is advanced renal failure will receive a higher priority. On the other hand, if the patient is young and not in imminent danger, he or she can wait for a better matching kidney and therefore receive less prioritization.

Studies show that a patient's mental representations of the transplanted organ may have a significant effect on the healing process. There are many moral issues attached to transplantations. A transplanted organ, sometime comes from cadavers. The idea that one life ends in order to save another life, can affect patients' mindset and cause them to have doubts about their surgeries. This has been shown to cause the patient to reject the organ, or cause the healing process to prolong. To prevent rejection of the organ, the psychological status of a patient is considered.

There are factors on the donor’s side that can affect the success of a transplant as well.The donor’s willingness to donate and their health are important. If a donor is fearful of surgery, he or she is more likely to back out and cause a transplantation failure. To prevent such failures, the motivation of the donor to donate is important. Donors who want to help save someone and are not coerced are more preferred because they are less likely to backout. Moreover, a donor who has proper genetical background, who has a proper and healthy lifestyle consisting of minimal drug use is preferred for the patient. By considering all the factors closely, the chance of postoperative complications will be minimal.

In summary, the greater the number of factors met, the higher the chance that the transplanted kidney will have better and longer functionality. Every patient-donor match will be associated with a score or probability that the transplanted kidney will be successful. This is vital because as the pool of patients and donors is changing constantly, a unique score for each edge will result in better matches. This idea of scoring will be considered in the two algorithms studied: matching and top trading cycle algorithms.

IV. Matching

The matching algorithm uses an undirected graph (V,E), where V represents incompatible patient-donor pairs, and E represents an undirected edge between vertices. An edge only exists between two patient-donor pairs if and only if patient 1 and donor 2 are compatible and patient 2 and donor 1 are compatible. This will result in a maximum cardinality matching in regards to the fact that as many compatible kidney transplants can be completed. Although multiple way exchanges are feasible, exchanges involving four or more patients do not really help matching more patients.

The matching algorithm is done by first reading in a list of patient and donor names and randomly assigning blood types to each patient and donor according the the frequency they appear in the human population. Blood type O is frequent in 44% of the population, blood type A is present in 42% of the population, blood type B is present in 10% of the population and blood type AB is present in only 4% of the population. This result in many to many matching as seen in figure 1.

Patients Donors

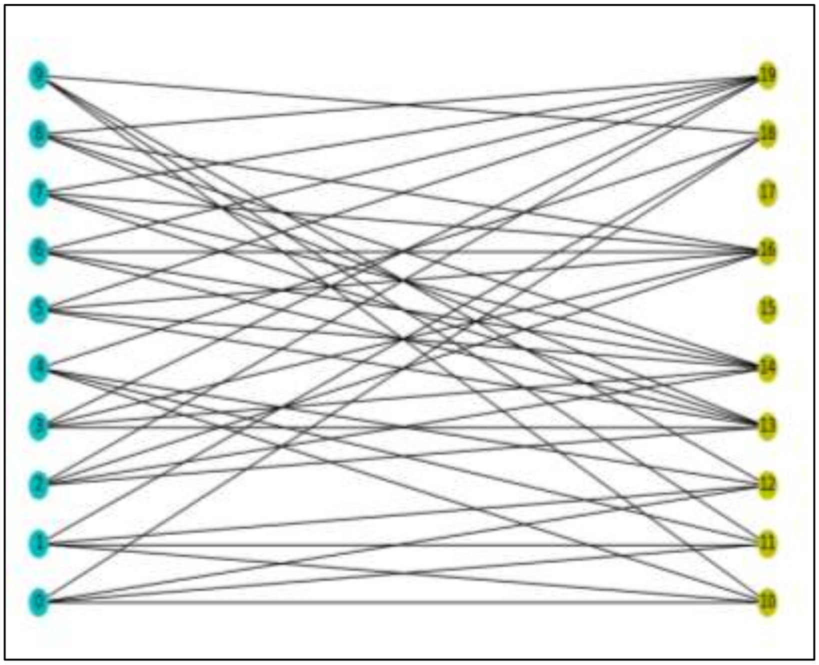


Figure 1: Many-to-many matching: matches patients to donors according to blood type

First, the donor factors, which include the donor’s willingness to donate, their health, and geographical distance is calculated randomly and assigned as an edge weight for each pair donor match. Therefore, each edge extending from a patient node will have a score. The maximum score is computed and assigned. Now, there will only be one edge extending from a patient node because all the non-maximum edge weights have been filtered out, as seen in figure 2.

Patients Donors

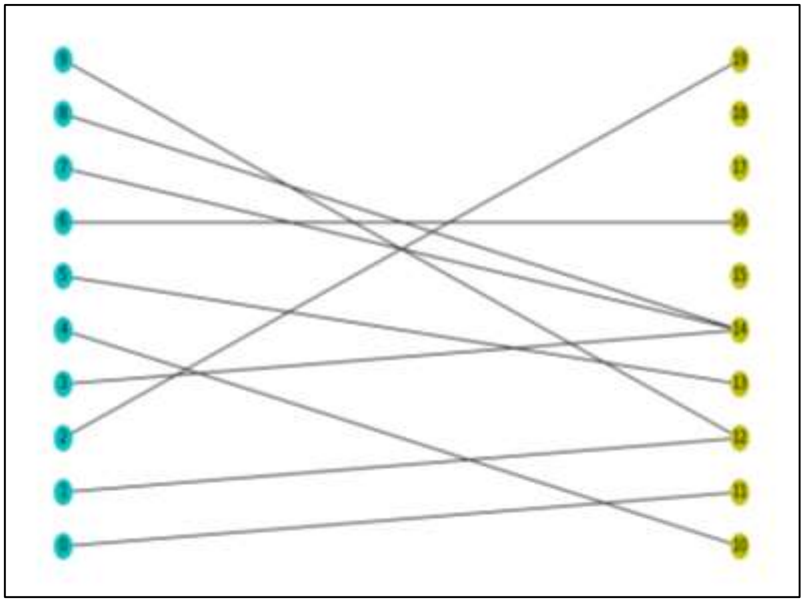


Figure 2: Result after non-maximum donor scores have been filtered.
(only one edge extends from each patient)

There are still multiple edges that extend from a donor node. Therefore, each of these edges is assigned a patient score, which comprises of the patient’s physical health, mental health, and how long the patient is willing to wait. The maximum of the scores is calculated and the remaining edges are filtered out. The remaining graph represented in figure 2 is a one to one matching of patients and donors.

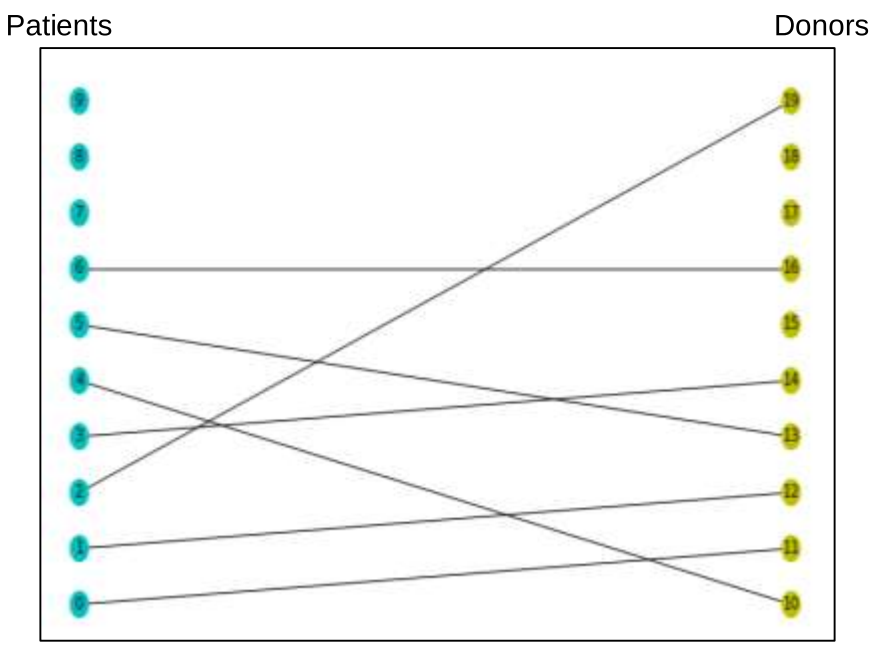


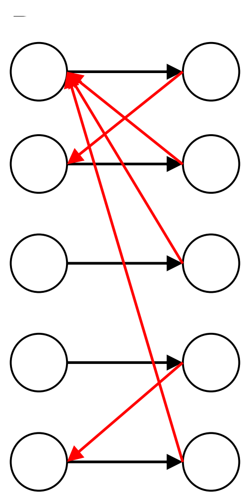
Figure 3: One-to-One Matching: Result after non-maximum patient scores are filtered
(only one edge extends from each patient and donor)

There are some patients and donors that remain without a match. If these patients do not want to wait and are matched with any of the remaining donors, the probability of success could be low and the transplant may be rejected. This will result in a kidney that has gone to waste. However, if the patient remains unmatched and waits for new donor with a better match, the patient's health may deteriorate. Next, the top trading cycle algorithm is studied.

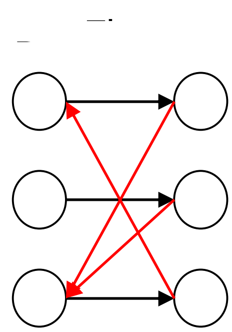
V. Top Trading Cycle Algorithm

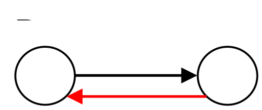
The algorithm was originally used in pairing up prospective buyers and houses in housing markets. Each buyer has preference over the house they want, and sometimes it is possible for buyers to prefer someone else's house. In the housing market, this may lead to mutually beneficial exchanges.

In a kidney exchange problem, top trading cycle algorithm functions in a similar manner. A patient-donor pair is added to the waiting list. The patients rank their preferred donor according to the donor factors described in this paper. An example ranking is shown in figure 4C. The donor who receives the maximum score will be the patient's most preferred donor. Each patient points to his or her preferred donor and each donor points to his or her initial patient, as shown in figure 4A. The algorithm searches the graph for cycles. When a cycle is found, the corresponding pairs are matched and the cycle is removed from the graph, as shown in figure 4B and 4C. This algorithm continues until no patients or donors remain and all the patients have found their matched kidney.

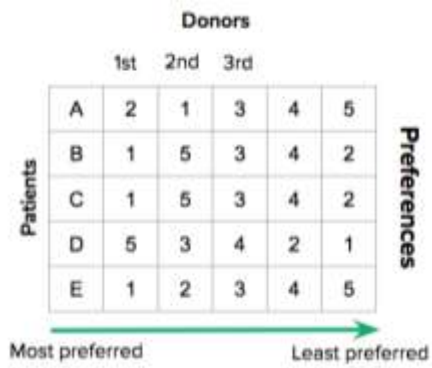


The black arrow: donor pointing to his or her initial





Final matching:



Patients	1st	2nd	3rd		
A	2	1	3	4	5
B	1	5	3	4	2
C	1	5	3	4	2
D	5	3	4	2	1
E	1	2	3	4	5

Figure 4D: Example patient preference table for each patient

In general, the top trading cycle algorithm is found to be a useful technique in kidney matching problem. On average, it could help 75% of patients to match to their top three choices. However, realistically, patients do not seem to care regarding which donor they are matched to as long as they have received a compatible kidney.

It is also concerning that the algorithm continuously looks for a cycle over a waitlist as it gets updated periodically. It has the possibility of forming a long cycle, as shown in figure 5. A long cycle is not optimal because a cycle of length n corresponds to 2n surgeries. Multiple surgeries are dangerous because the surgeries require many medical personnel, equipment and have possible complications, that could compromise the transplant.

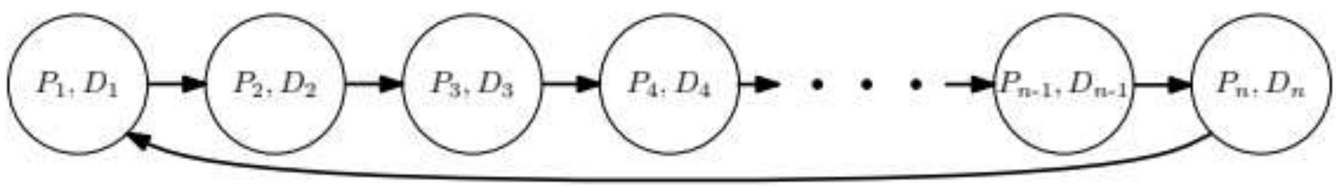


FIgure 5: Example of Long Cycle over N Pairs of Patient-Donor Vertices

VI. Conclusions and Extension

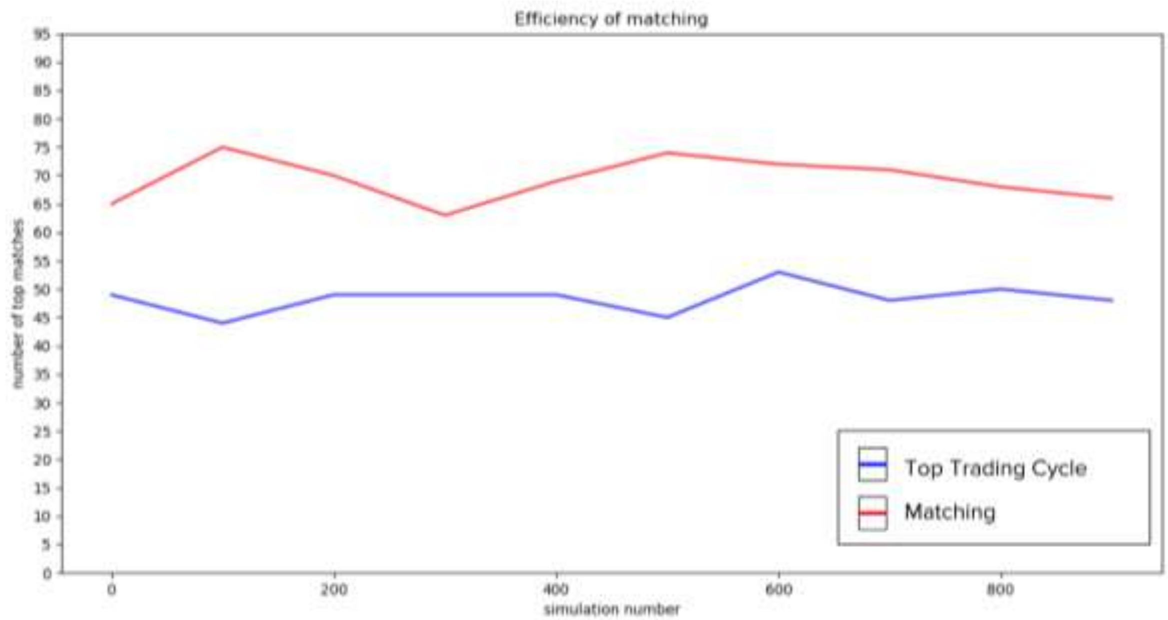


Figure 6: Top matches of the TTCA and one-to-one matches of matching algorithm
Number of matches per simulation cycle

A simulation consisting of a thousand cycles over one hundred patients and donors is run. When considering the top choice for the top trading cycle algorithm and the number of one to one matches in the matching algorithm, it can be seen that the matching algorithm performs much better than the top trading cycle algorithm. The matching algorithm found 70% matches while the top trading cycle algorithm only found approximately 50% matches.

Since the top trading cycle ranks the preferences of one hundred patients, looking at the top three is significant. According to figure 7, it can be seen that when the top three matches are considered, there is about 80% matchings. So, 80% of the patients found a match who was in their top three preferences. This shows that this algorithm is still valid and finds good matches.

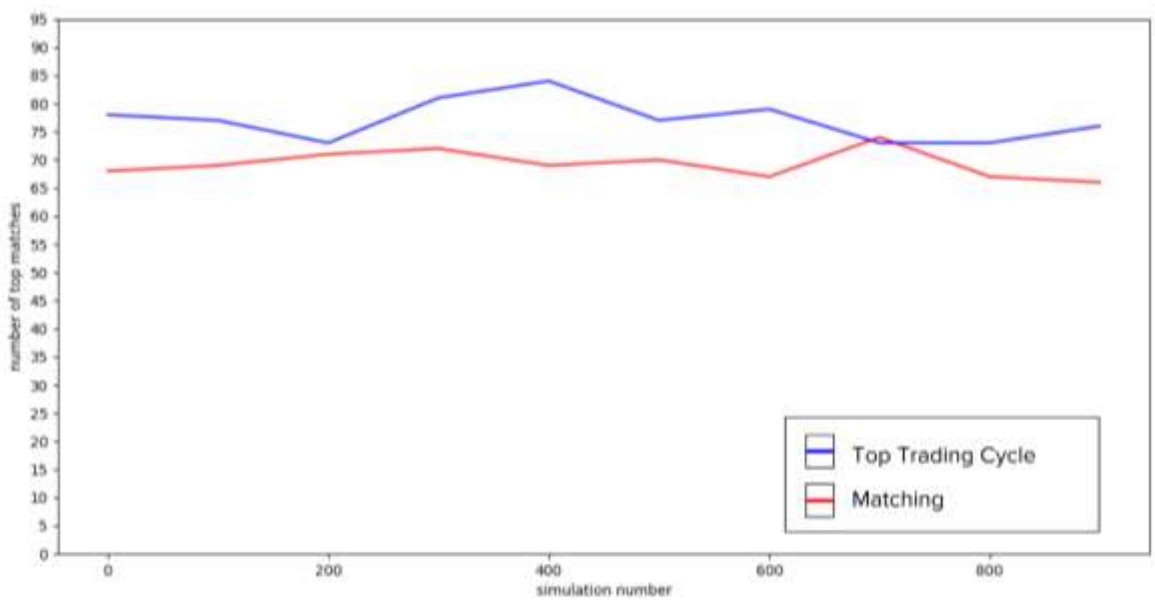


Figure 7: Top three matches of the TTCA and one-to-one matches of matching algorithm
Number of matches per simulation cycle

When running both algorithms simultaneously, it can be seen from figure 8 that the top trading cycle algorithm is much smoother and much faster.It takes nearly 0.07 seconds for it to run once. On the other hand, the matching algorithm is slower and unstable.The graph of this algorithm fluctuates. The fluctuation can be credited to the type of container used in the matching algorithm. An adjacency matrix is used here. In the top trading cycle algorithm, a dictionary is used. The type of container affects the time it takes to access each element, a node, and thus causes fluctuations.

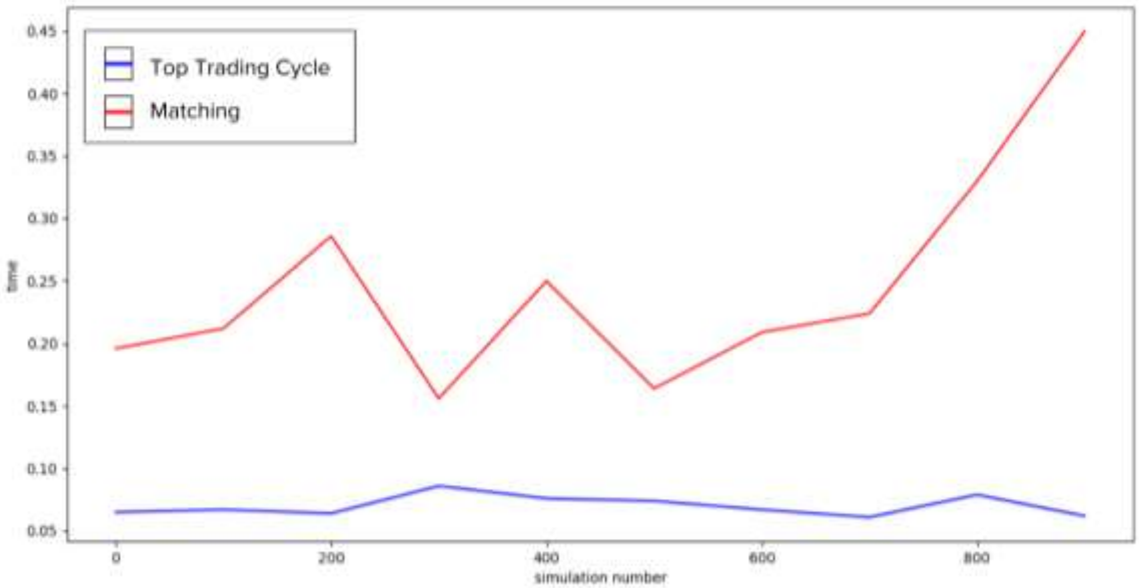


Figure 8: Time in seconds of both algorithms per simulation cycle

Both algorithms find at least 50% matching.The top trading cycle algorithm, although faster and finds more matches in top three preferences, finds less matches in the top preference. Also, in this algorithm, if a donor or patient backs out, then the whole preferences list needs to be updated for each patient, which will take longer time. On the other hand, the matching algorithm, although much slower, finds more good matches because it looks at all the factors for patients and donors, instead of just donor factors as the top trading cycle does. This ensures a better and longer functioning transplanted kidney. Also, it is easier to accommodate new donors and patients. This is important because in real life, the number of patients grows each year, so it is vital to accommodate for this.

The model can be further extended by restricting the lengths of cycles to some amount. This is a common assumption made in many kidney exchange problems because certain countries and hospitals strongly enforce this limitation. It may be too complicated to organize exchanges of longer cycles due to capacity constraints in transplant centers. Including cycles larger than size three will be an NP-hard problem.

Also, extending the model to include cadaver donors and altruistic donors can be useful in studying how important is it to donate organs post-life. This can help encourage more donors to donate. By altering the models as well as the algorithms associated, more lives can be saved.

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