UPPSALA UNIVERSITET

Institutionen för informationsteknologi

Besöksadress:
MIC, Polacksbacken
Lägerhyddvägen 2

Postadress:
Box 337
751 05 Uppsala

Telefon:
018–471 0000 (växel)

Telefax:
018–51 19 25

Hemsida:
http:/www.it.uu.se/

Department of Information Technology

Visiting address:
MIC, Polacksbacken
Lägerhyddvägen 2

Postal address:
Box 337
SE-751 05 Uppsala
SWEDEN

Telephone:
+46 18–471 0000 (switch)

Telefax:
+46 18–51 19 25

Web page:
http:/www.it.uu.se/

Assignment - Monte Carlo Simulation to price European Options

Assignment

- Implement Euler's method in MATLAB to price a European call option when the underlying stock follows the dynamics

$$\mathrm{d}S_t = rS_t\mathrm{d}t + \sigma S_t^\gamma \mathrm{d}W_t, \tag{1}$$

  The diffusion coefficient, σS_t^γ, models the hypothesis that the volatility depends on the stock price. If $\gamma < 1$, the volatility and the price are inversely related, which has been suggested by empirical evidence. Make use of your results from the Seminar. You find help in MATLAB on the use of normally distributed random numbers by typing `help randn`. As an example you can use the following parameters: $S_0 = 14$, $K = 15$, $r = 0.1$, $\sigma = 0.25$, $T = 0.5$, $\gamma = 0.8$.

- The error in the computed solution has two parts:
  - *The sample error* which is due to the fact that you can't use an ∞ number of sample paths.
  - *The discretization error/bias* which is due to the fact that you can't use an ∞ number of time-steps.

  Use your code to make experiments and measure the error in the computed solution. Try to say something about how the error depends on the number of sample paths and on the time-step. You can use the code `bsexact.m` to compute the exact value of the option for $\gamma = 1$. This code uses the fact that the exact value of the option $V(s,t)$ for this value of γ is given by

$$\begin{aligned} V(s,t) &= sN(d_1) - Ke^{-r(T-t)}N(d_2), \\ d_1 &= \frac{ln\left(\frac{s}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \\ d_2 &= d_1 - \sigma\sqrt{T-t}. \end{aligned}$$

  Here $N(x)$ denotes the standard normal cumulative distribution function and is given by $N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz$.

  It might also be helpful in these experiments to note that for $\gamma = 1$ you can integrate (1) exactly in time.

- Now try to reduce the error in your computed solution. Read about antithetic variates on p.243 in Hirsa (section 6.9.2) and use this method to try to reduce the sample error. Then go on reading section 6.6 and try to reduce the bias.
- Vary γ between 0.5 and 1. What happens to the value of the option?
- You don't have to write a full report, but you should hand in your program code and the following plots:
  - The sample error as a function of the number of sample paths.
  - The discretization error as a function of the time step.
  - A plot showing what happens when using antithetic variates.
  - A plot of V as a function of γ.

  All plots should be *commented* with respect to its behavior. How do your obtained results relate to the theory?

You have the opportunity to have tutoring 10 min/group. Fill in the doodle here https://doodle.com/poll/f9qqxsyq4hauguup to make a reservation for a time-slot.

You should hand in your results to Studentportalen no later than 10/9 at noon.

During 10-12/9 there will be a questionnaire open in the Student Portal that you all have to answer individually.

For all extra questions feel free to email slobodan.milovanovic@it.uu.se.