UPPSALA
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Modelling complex systems

Project 2

Population Dynamics
Groups of Friends
Network Epidemics
Flocks and Predators

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1 Population Dynamics

In this part, we model the population with a stochastic model. There are n resource sites in the model world, and at time t =0, the population is A_0 and they are assigned randomly to one resource site. At each t step, the population rules are, if there are exactly two individuals on the same site. They reproduce b offsprings and these offsprings are assigned randomly to resources sites. If the number of individuals on a resources sites is any number other than 2, no offspring will be reproduced.

1.1 Matlab model

To begin, we can run this with different parameters and simulate in Matlab and observe the total number of population at different time step. The different parameters are:

1. b: number of offspring if reproduce
2. n: total of number of resource sites
3. A_0: initial population
4. t: the time steps we want to simulate the model

When set the initial population to a small number relative to the resource site, it would be difficult to have 2 individuals at the same site for reproduce, and even if they reproduce a large number of offspring the population dies out very quick. below are some plots showing the total number of population at different time steps with different parameters.

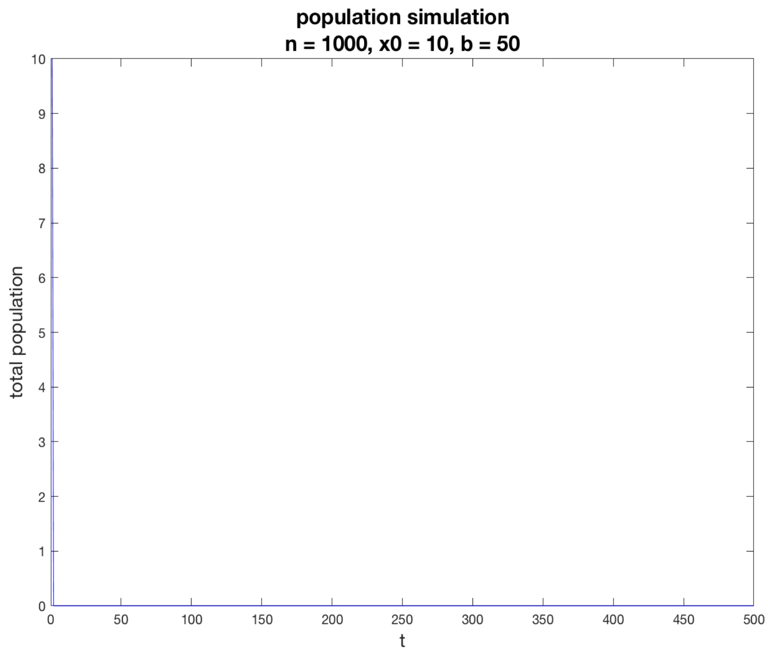

Figure 1: initial population of 10 b = 50 not reproducing

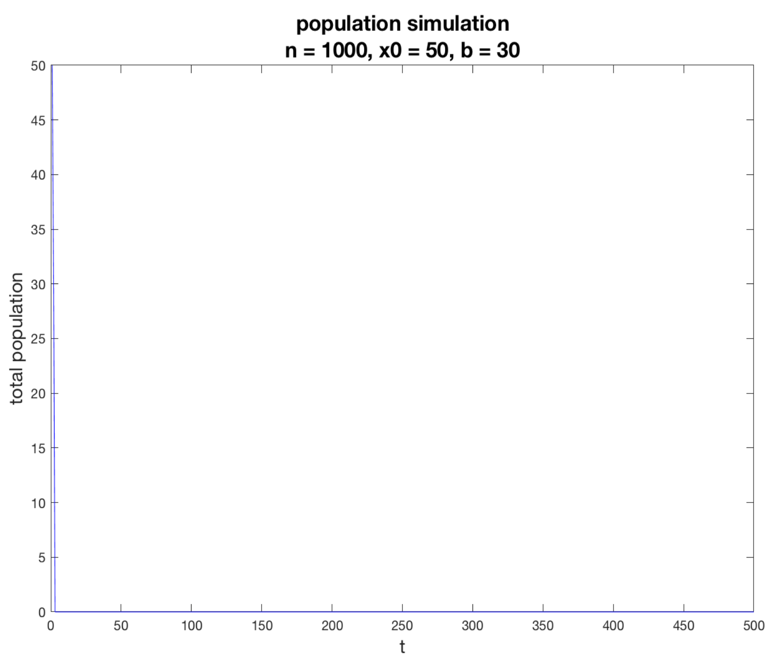

Figure 2: initial population of 50 and b =30 not reproducing

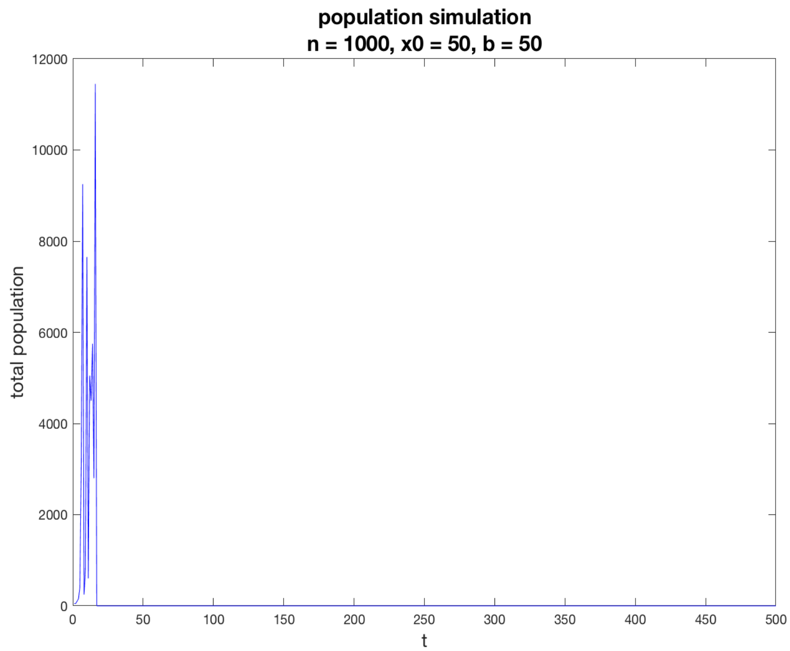

Figure 3: initial population of 50 and b = 50, in the beginning reproduce but quickly dies

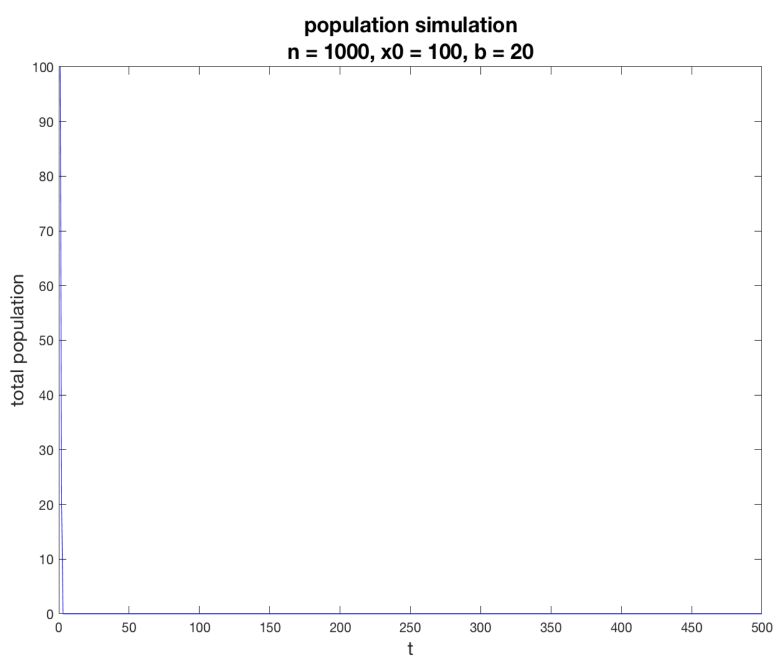

Figure 4: initial population of 100 and b = 20, it dies

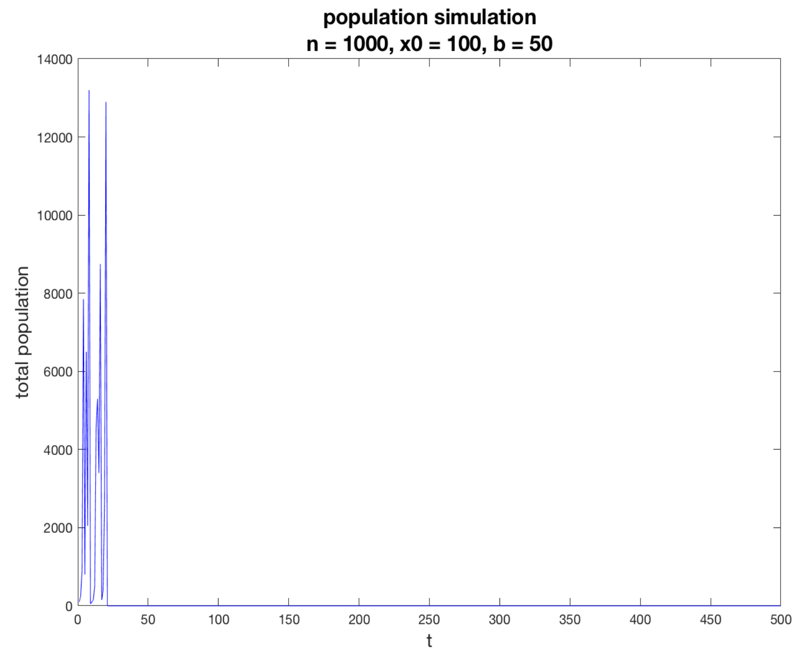

Figure 5: initial population of 100 and b = 50, it reproduce in the beginning but quickly dies

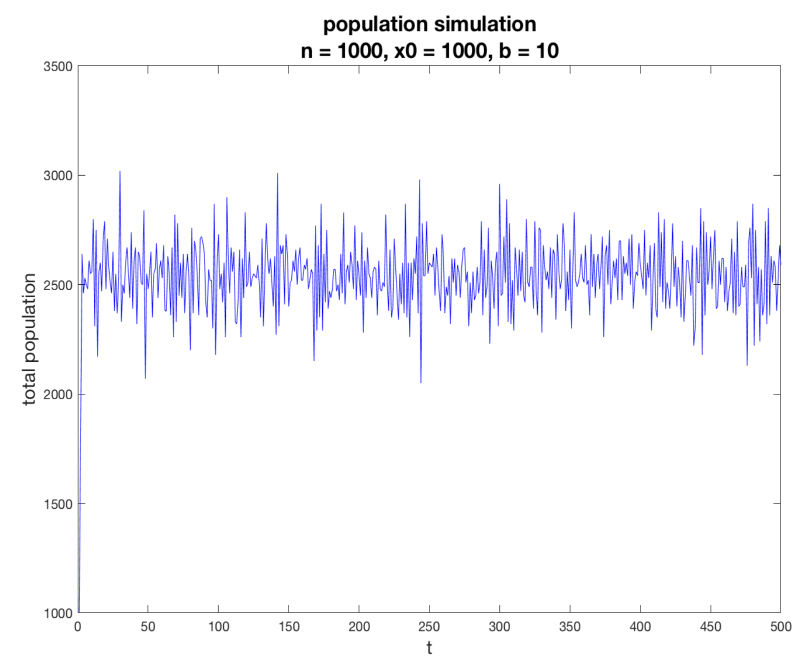

Figure 6: initial population of 1000 and b = 10, it show that the population oscillates around 2500

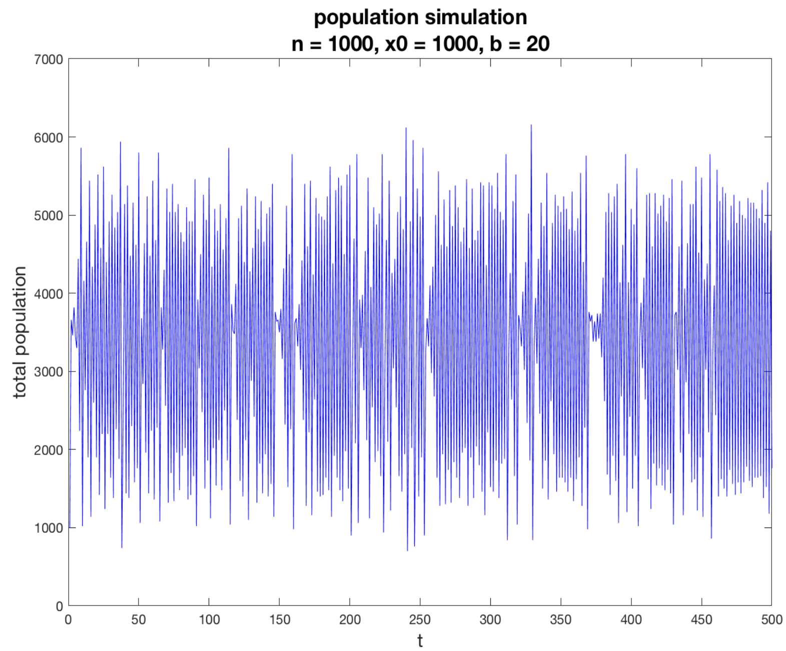

Figure 7: initial population of 1000 and b = 20, the populations starts to get chaotic

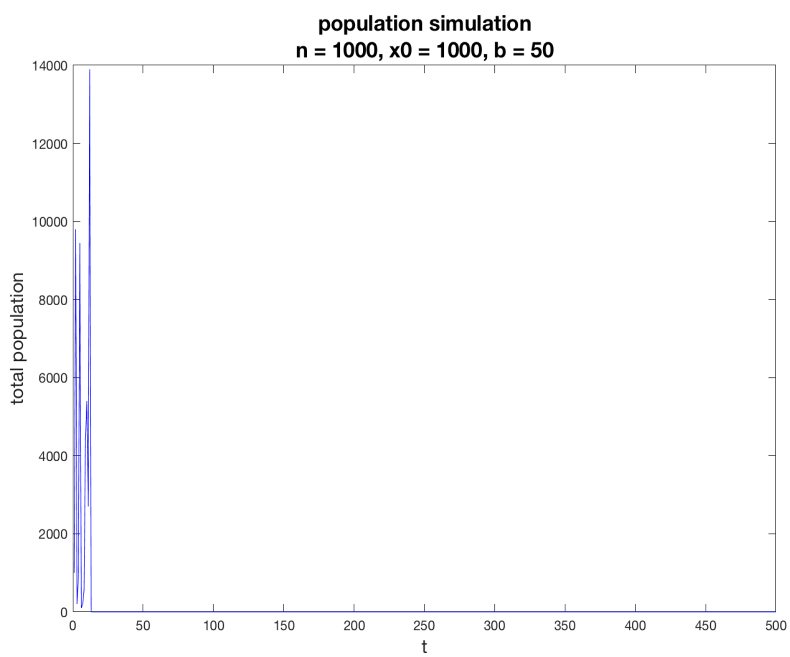

Figure 8: initial population of 1000 and b = 500, the populations dies very quick

To further study this model, we run it with $A_0 = 1000$, n= 1000, b = 1, 2, 3, 4, 5,...,48, 49, 50. and at each b value, run the simulation 100 times and plot a phase transition diagram. and we can see that when b = 5-15, the population increases steady and oscillates around a number. When $15 < b < 35$, the population will get more chaotic and at one time a lot of sites are reproducing, and then the next time steps, they dies because of overcrowding. When $b > 35$ the population will dies very quick due to overcrowding.

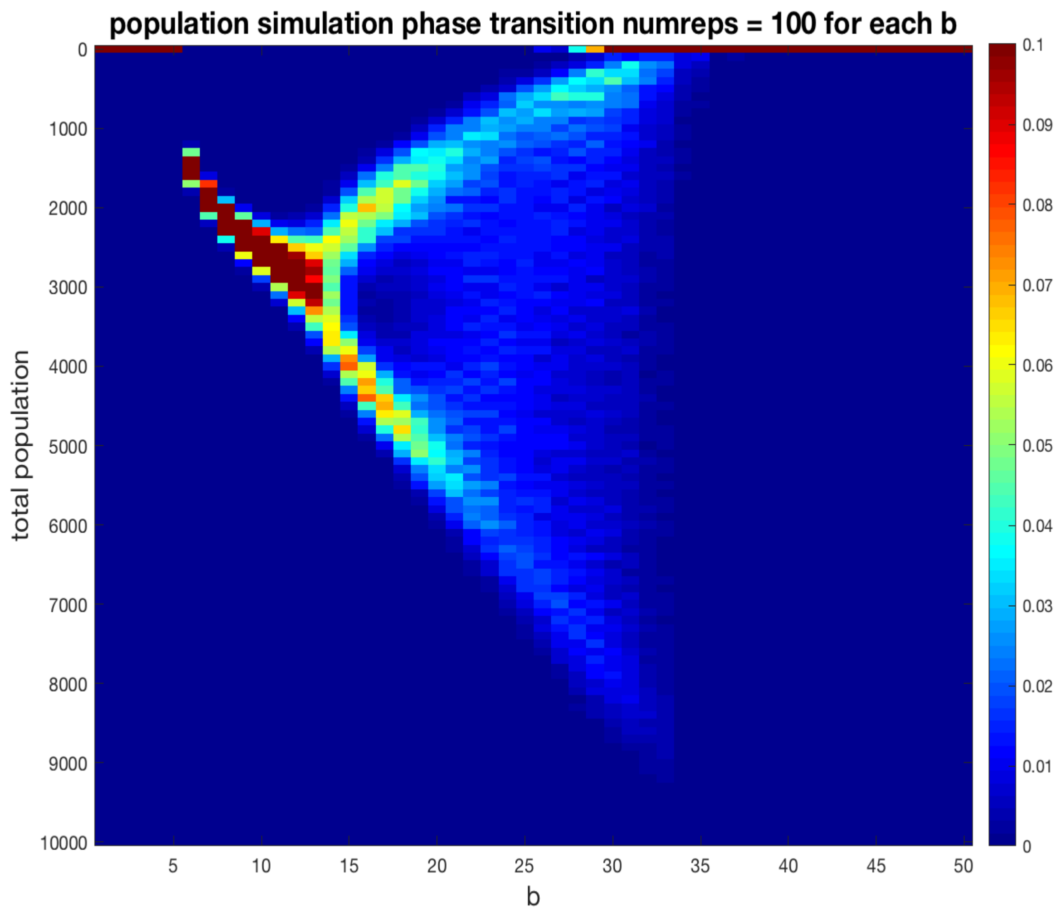

Figure 9: initial population of 1000 and b = 500, the populations dies very quick

1.2 Mean field model

We assume the sites are independent and number of individuals at sites is poisson distributed. I plot selected mean-field model here for b = 5, 10, 15, 20, 35, 50. The model are after the plot

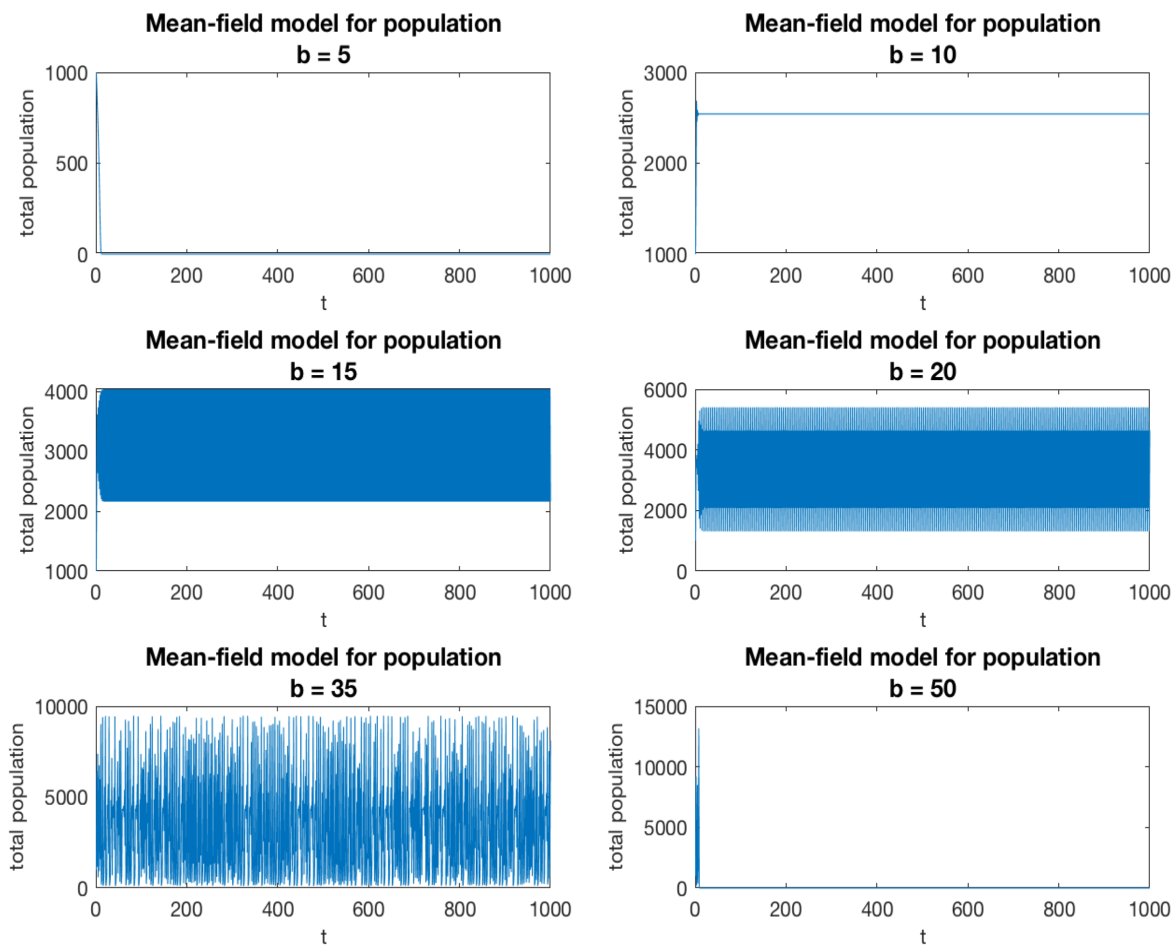

Figure 10: mean field model with selected b value

$$\begin{cases} E(A_{t+1}|A_t) = n \times \sum_{k=0}^{n} P_k \times \phi_k & (1) \\ \phi_k = \begin{cases} b, if k = 2 \\ 0, if k \neq 2 \end{cases} & (2) \end{cases}$$

following the mean field model, and the population only reproduce if there are exactly 2 individuals at same site, we can write the following:

$$A_{t+1} = n \times P(2 at site, A_t) \times b \tag{3}$$

$$A_{t+1} = n \times \frac{(\frac{A_t}{n})^2 \times e^{-\frac{A_t}{n}}}{2} \times b \tag{4}$$

For steady state, we have $A_{t+1} = A_t$.

$$A_t = n \times \frac{(\frac{A_t}{n})^2 \times e^{-\frac{A_t}{n}}}{2} \times b \tag{5}$$

When $A_t = 0$, left hand side always equal to right hand side, they are both 0. When $A_t \neq 0$, we have

$$1 = \frac{b \times A_t}{2n} \times e^{-\frac{A_t}{n}} \tag{6}$$

Let's rearrange equation (6) and let $\frac{A_t}{n} = x$, we have:

$$1 = \frac{b}{2} \times x \times e^{-x} \tag{7}$$

$$x \times e^{-x} = \frac{b}{2} \tag{8}$$

To find the conditions in terms of b for the existence of two further non-zero steady states, we need to find that $f1(x) = x \times e^{-x}$ has intersection with a horizontal line. b need to fulfill the condition that $\frac{2}{b} < 0.3679$, therefore we get: $b > 5.4366$, as b needs to be integer, we get $b > 5$.

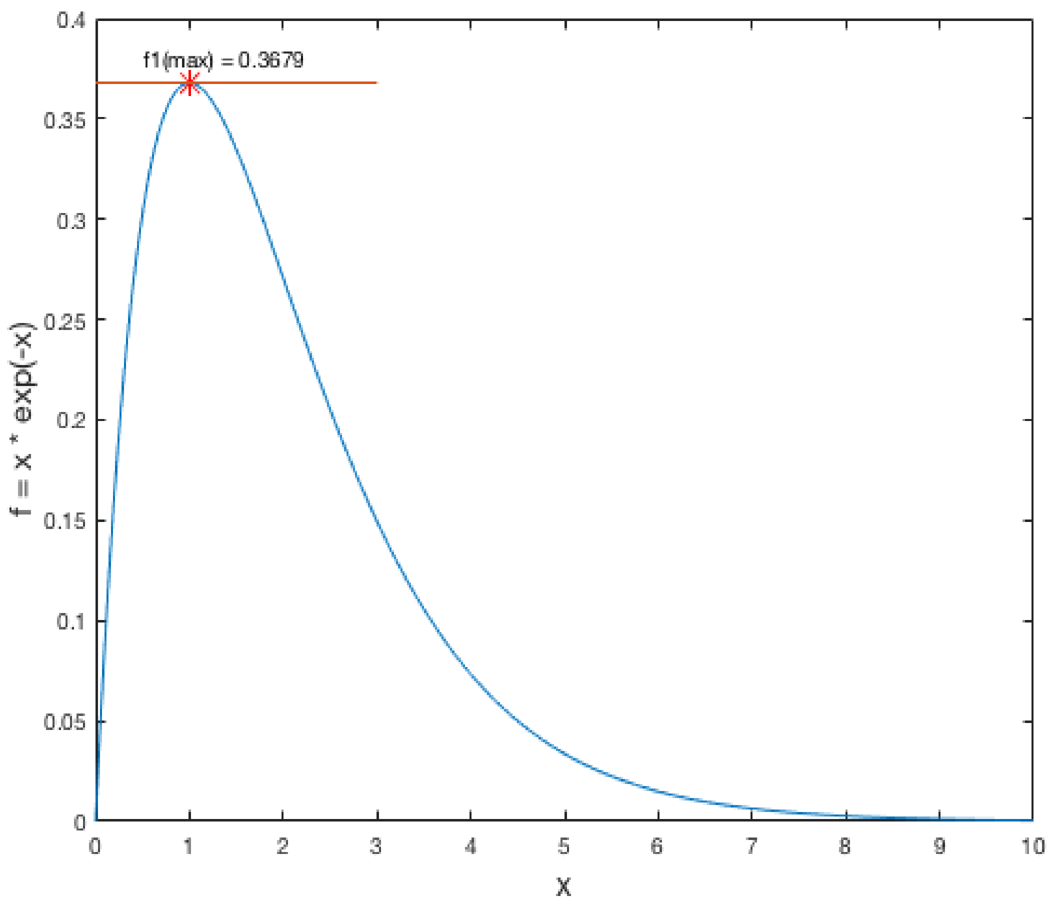

Figure 11: plot of f1(x) = x * exp(-x)

1.3 Lyapunov exponent

From the previous section, we have derived the equation for A_t. that we have

$$f(A_t) = n \times \frac{(\frac{A_t}{n})^2 \times e^{-\frac{A_t}{n}}}{2} \times b \tag{9}$$

$$f(a) = \frac{b}{2n} \times a^2 \times e^{-\frac{a}{n}} \tag{10}$$

$$f'(a) = \frac{b}{2n} \times (2a \times e^{-\frac{a}{n}} + a^2 \times e^{-\frac{a}{n}} \times (-\frac{1}{n})) \tag{11}$$

$$f'(a) = \frac{b}{2n} \times (2a \times e^{-\frac{a}{n}} - \frac{a^2}{n} \times e^{-\frac{a}{n}}) \tag{12}$$

$$|\Delta a_n| = |\Delta a_0| e^{\lambda n} \tag{13}$$

to compute Lyapunov exponent λ numerically, we use the following:

$$\lambda = \frac{1}{n} \sum_{t=0}^{n-1} ln|f'(a_t)| \tag{14}$$

The Lyapunov exponent are plotted below: we can see that when $\lambda > 0$ the diverges, that's when b is between 21-30, the population become chaotic. When $b \leq 5$ and $b \geq 31$ λ is -infinity that means the system converge very fast, and this explains why the population become extinct very fast for large b.

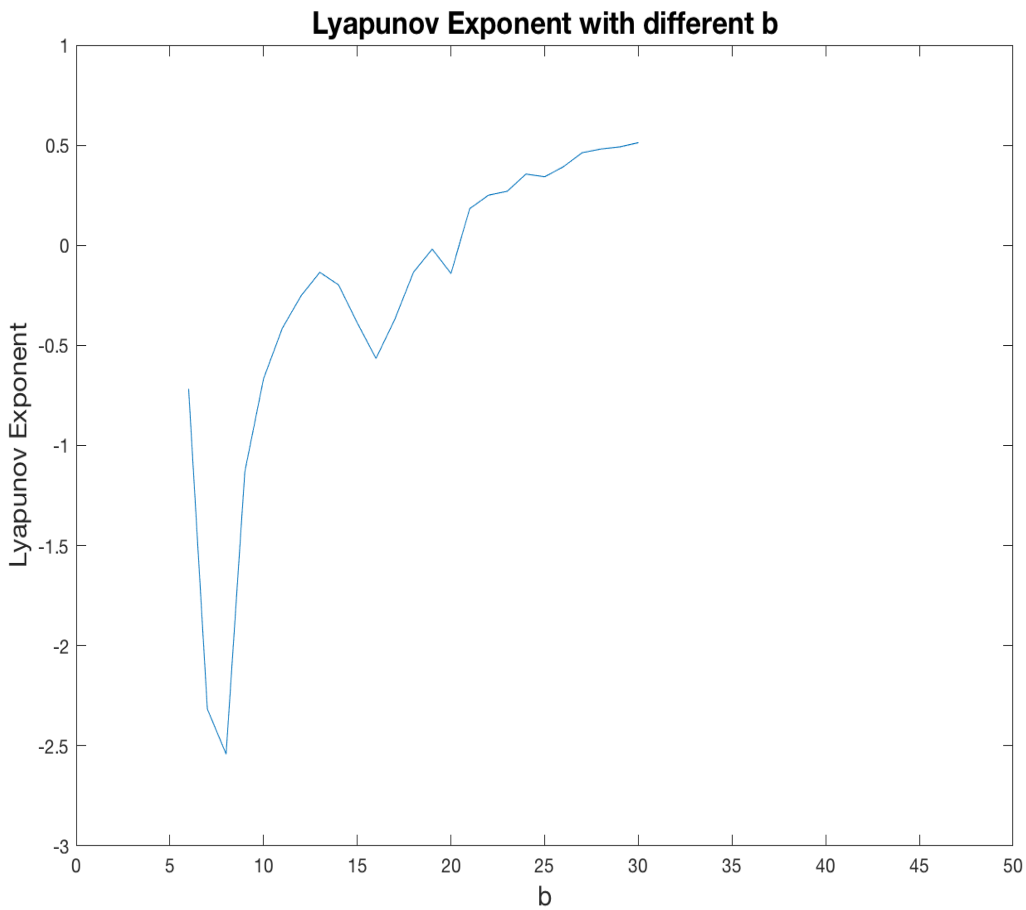

Figure 12: plot of Lyapunov exponent

2 Groups of friends

In this part, we used a model to simulate how students make friends. Initially, there are N students, and they are all alone in a group contains only one student. At each time step, a group is picked:

1. If the picked group contains only one student, this students will join a group with size k with probability of k/N.
2. if the picked group size i ($i > 1$), then this group will split up with probability of ri.

2.1 simulations in Matlab

First, we simulate the above model with 100 students, r = 0.01 and 100 replications. First, we plot the group size vs frequence on a log log scale. Second, we get the relative frequency with group size histogram taken on a log scale that is we take groups size [1,2,4,8,16,32,64,128] and plot the group size vs relative frequency on a log log plot. in figure 14, we can observe that the group distribution following the power law.

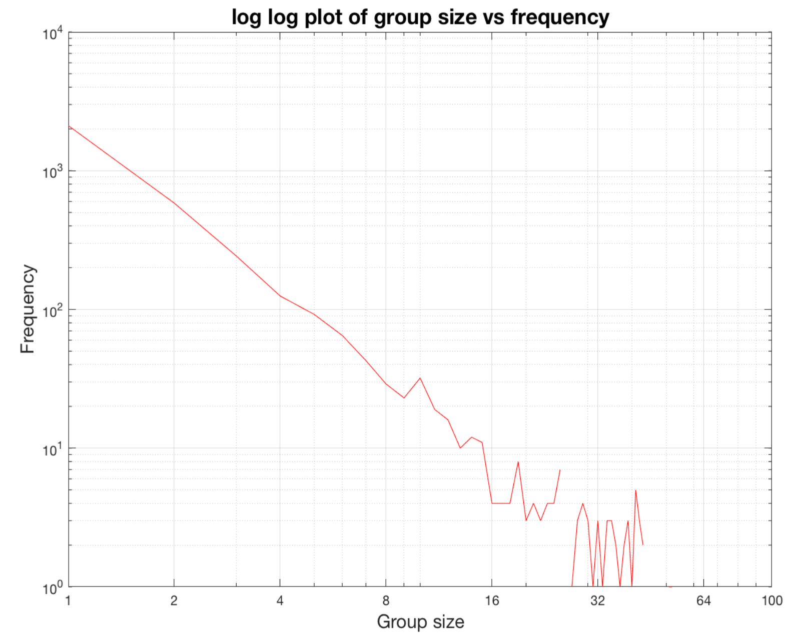

Figure 13: log log plot of group size vs frequency

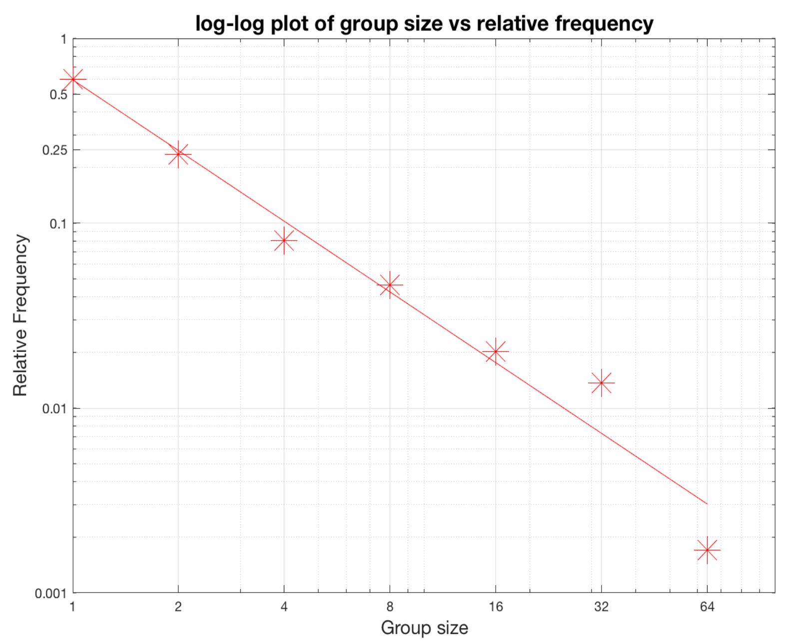

Figure 14: log log plot of group size vs relative frequency

To investigate the how the group size distribution changes with r, we run the model with r from 0.001:0.001:0.1 with 100 replications. In 15, I choose some r values to plot shows how the group size changes, and in 16, the whole group distribution is shown on a 3d plot, it might be a little difficult to from the colourful map, but we can observe the z-axis value that the relative frequency of the group.

When r is small at 0.001 we have groups of size 1 and 2 around $\frac{1}{4}$ of the population and some large groups. when we increase the r, then the number of groups of 1 become the dominant group in the distribution and large group appear less and less. With a bigger r, the larger group will split up if it was chosen at time t. For example, setting r = 0.1, it means that groups of size 10 + will slip up at the probability of 1 if chosen at time t.

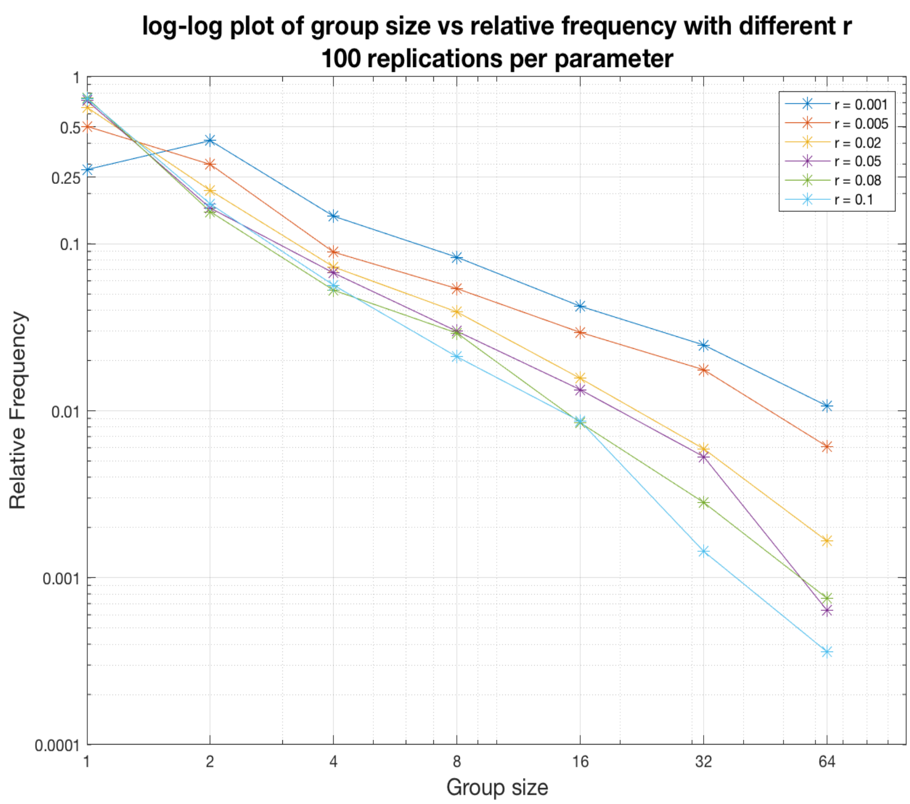

Figure 15: log log plot of group size vs relative frequency with different r

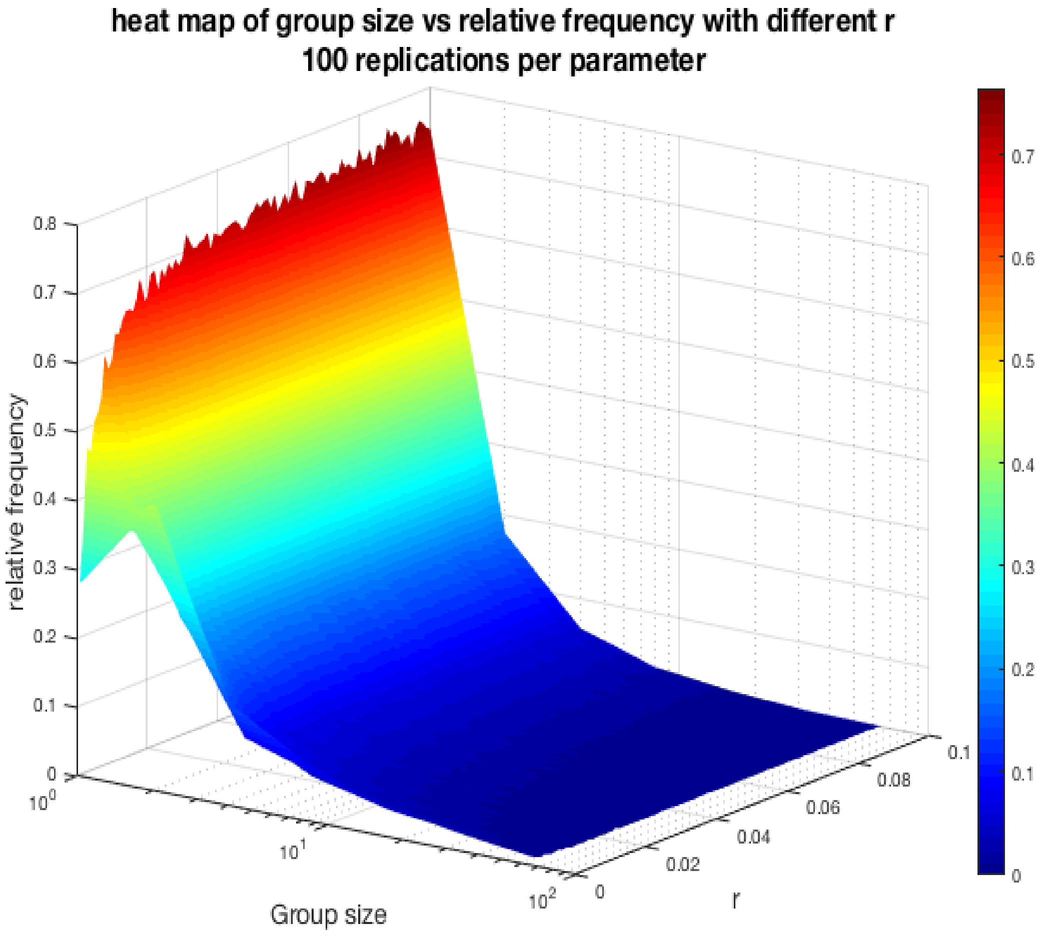

Figure 16: the distribution of groups with different r showing on a 3d plot

2.2 Master Equation

3 Network Epidemics

3.1 random undirected social network

An undIrected networking was made with 5000 students and a link density of 0.0016. we use matrix A to represent the network, as the network is undirected, we have A as a symmetric matrix, and with a link density of 0.0016, we generate A and plot the histogram of the degree distribution. The average degree of this network is around 8 and the degree follows a binomial distribution.

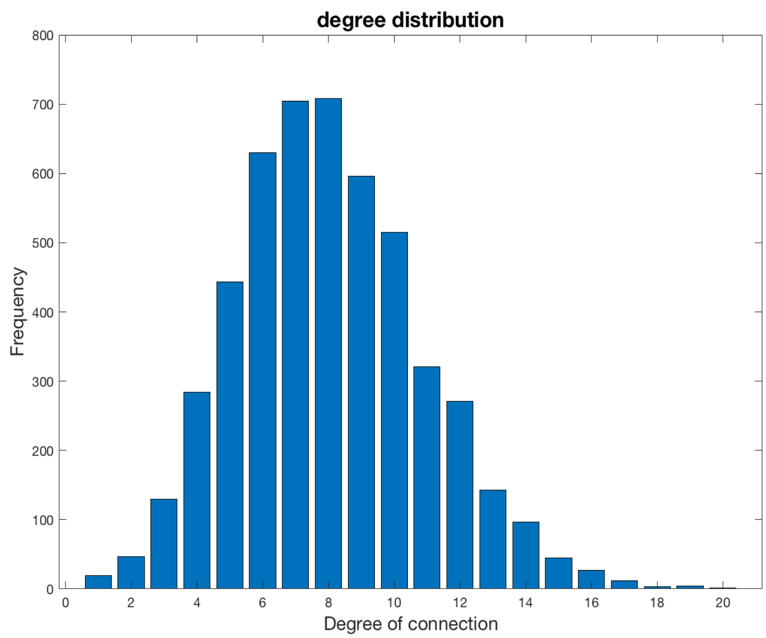

Figure 17: Frequency of the connection degree

3.2 random undirected social network

4 Appendix

4.1 1firing brain code in matlab

- simulate single time of fire brain
- transition function
- initial state
- simulate 100 times of firing brain
- cell that move forward at one cell per time preserving the same shape
- cell that move forwad at one cell per time, launching other shapes behind them
- move forward at a rate of less than one cell per time step
- oscillate shape
- my cellular automata
- my cellular automata transit

4.2 Spread of memes

- simulation of spread of memes
- run single spread of memes
- mean field model
- phase transition
- probability for at least 25% are sharing
- simulation of spread of memes with new rules
- run single spread of memes
- mean field model
- phase transition for new rules
- lattice simulation for memes
- transition function for memes