UPPSALA
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Modelling complex systems

Project 2

Population Dynamics
Groups of Friends
Network Epidemics
Flocks and Predators

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1 Population Dynamics

In this part, we model the population with a stochastic model. There are n resource sites in the model world, and at time t =0, the population is A_0 and they are assigned randomly to one resource site. At each t step, the population rules are, if there are exactly two individuals on the same site. They reproduce b offsprings and these offsprings are assigned randomly to resources sites. If the number of individuals on a resources sites is any number other than 2, no offspring will be reproduced.

1.1 Matlab model

To begin, we can run this with different parameters and simulate in Matlab and observe the total number of population at different time step. The different parameters are:

1. b: number of offspring if reproduce
2. n: total of number of resource sites
3. A_0: initial population
4. t: the time steps we want to simulate the model

When set the initial population to a small number relative to the resource site, it would be difficult to have 2 individuals at the same site for reproduce, and even if they reproduce a large number of offspring the population dies out very quick. below are some plots showing the total number of population at different time steps with different parameters.

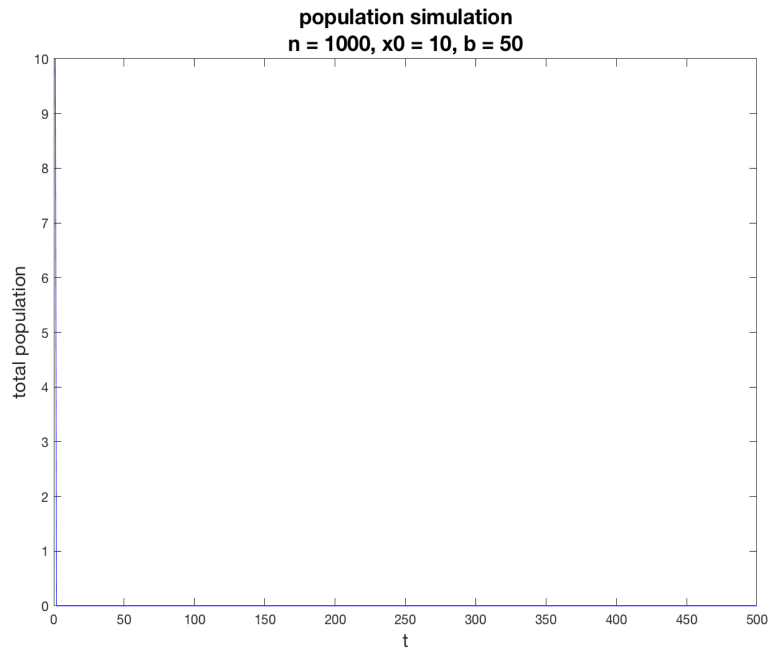

Figure 1: initial population of 10 b = 50 not reproducing

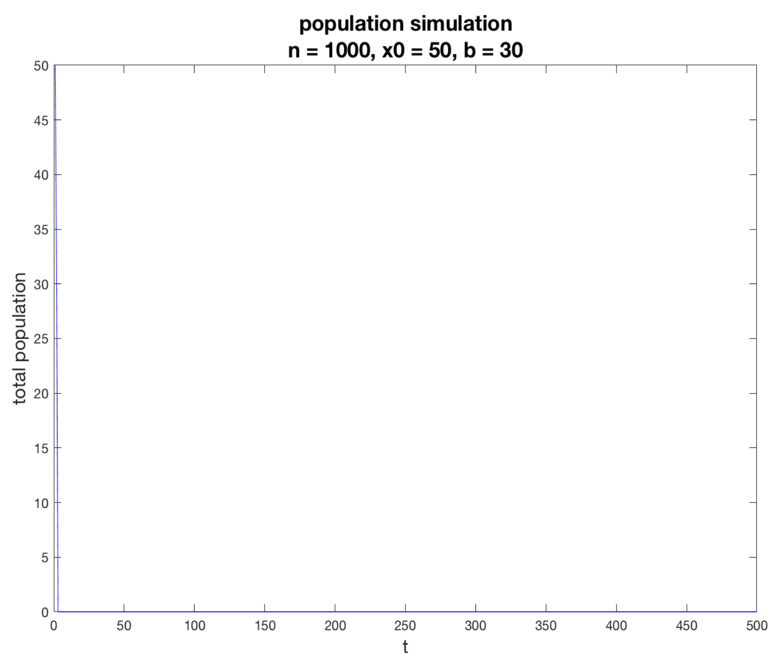

Figure 2: initial population of 50 and b =30 not reproducing

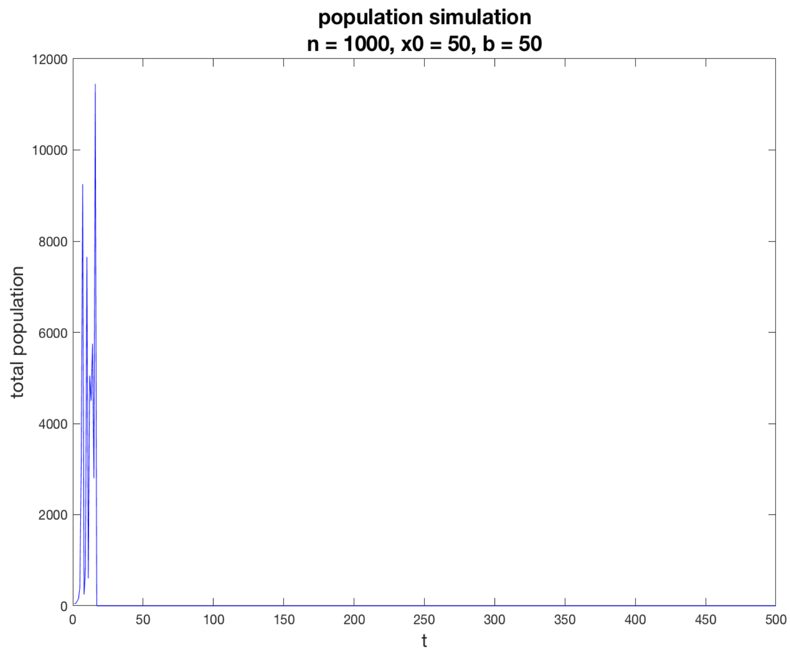

Figure 3: initial population of 50 and b = 50, in the beginning reproduce but quickly dies

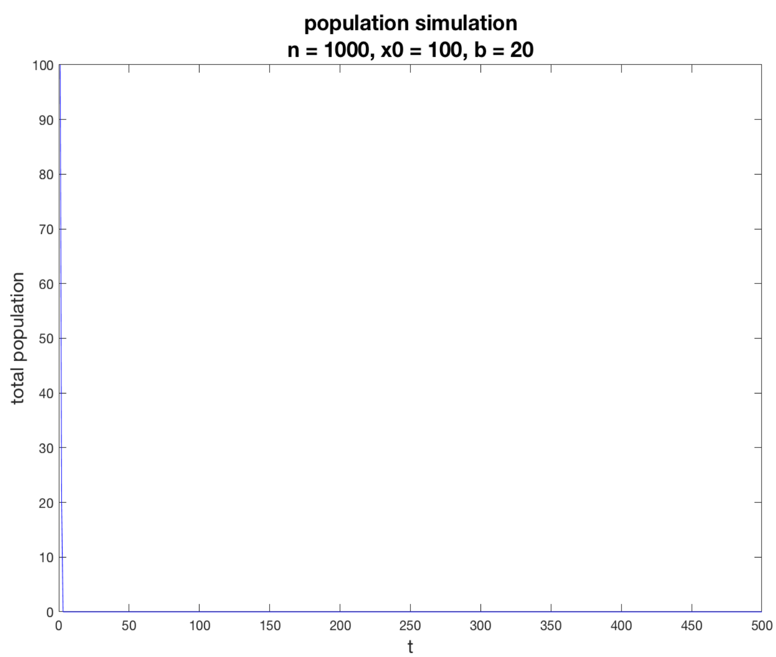

Figure 4: initial population of 100 and b = 20, it dies

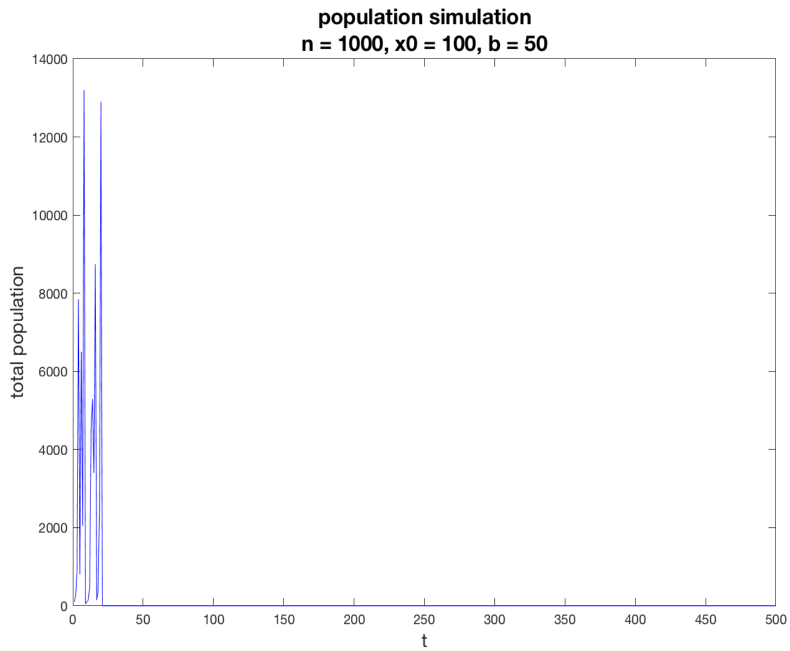

Figure 5: initial population of 100 and b = 50, it reproduce in the beginning but quickly dies

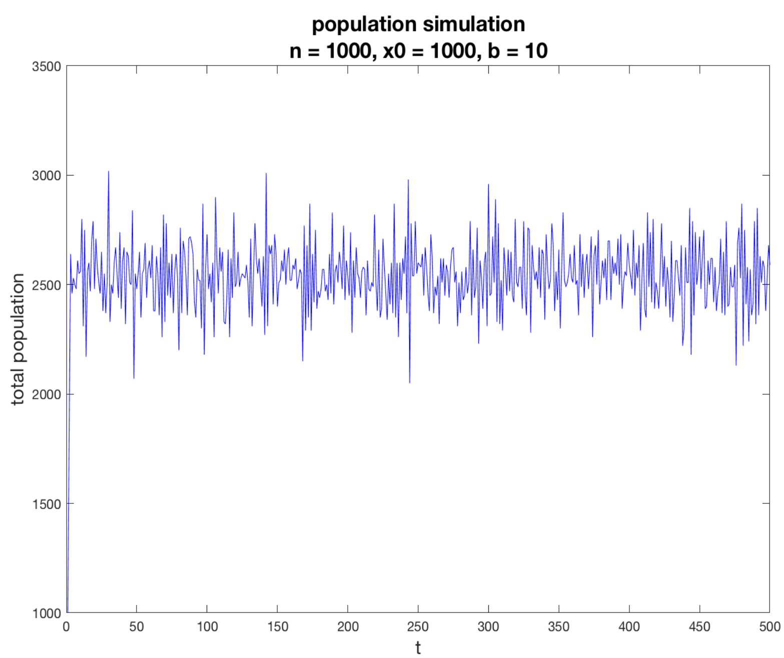

Figure 6: initial population of 1000 and b = 10, it show that the population oscillates around 2500

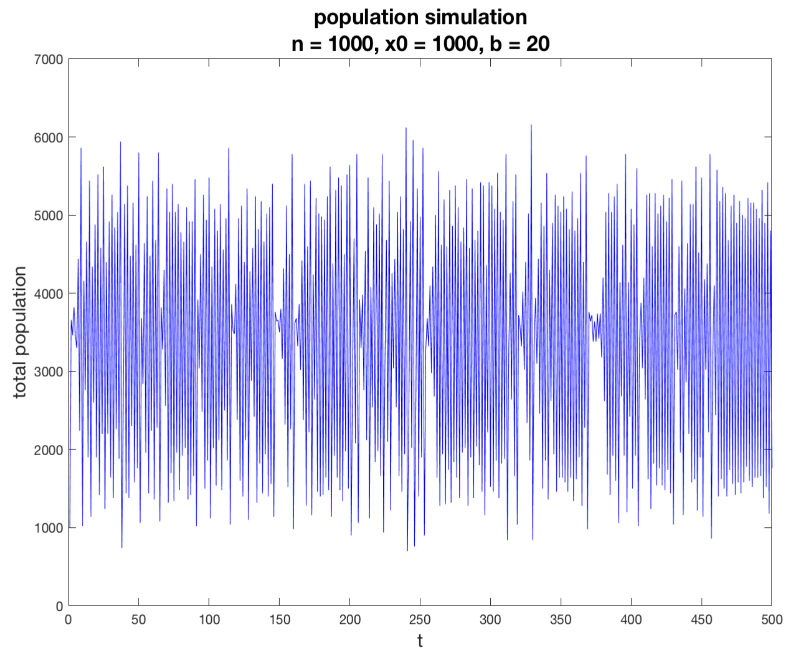

Figure 7: initial population of 1000 and b = 20, the populations starts to get chaotic

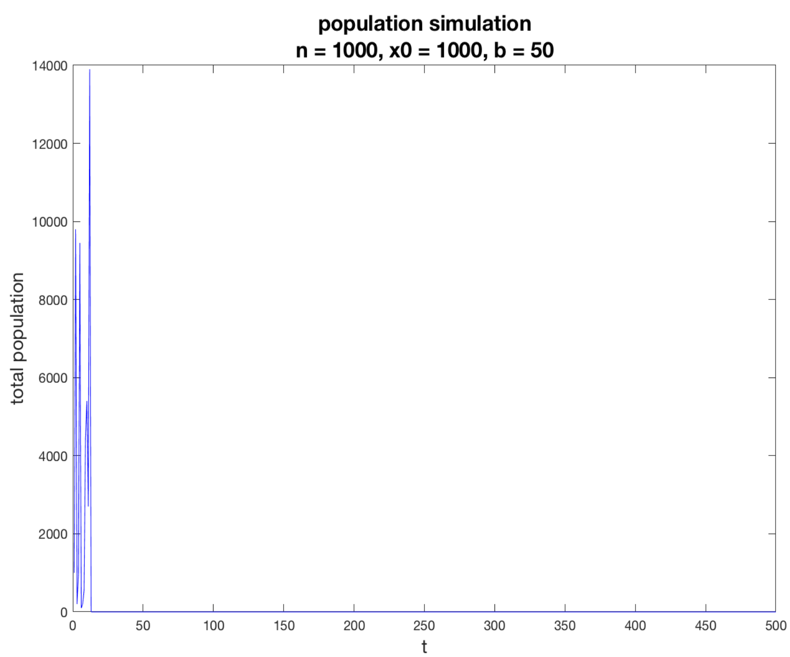

Figure 8: initial population of 1000 and b = 500, the populations dies very quick

To further study this model, we run it with $A_0 = 1000$, n= 1000, b = 1, 2, 3, 4, 5,...,48, 49, 50. and at each b value, run the simulation 100 times and plot a phase transition diagram. and we can see that when b = 5-15, the population increases steady and oscillates around a number. When $15 < b < 35$, the population will get more chaotic and at one time a lot of sites are reproducing, and then the next time steps, they dies because of overcrowding. When $b > 35$ the population will dies very quick due to overcrowding.

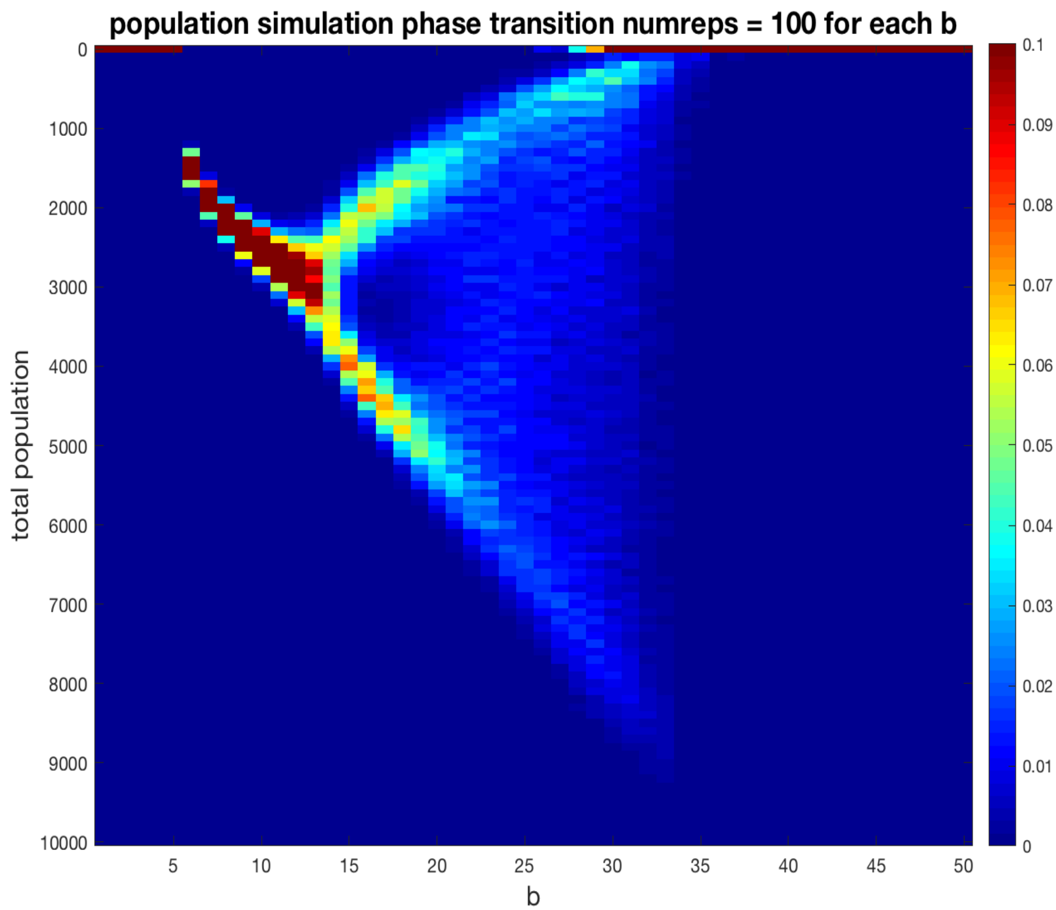

Figure 9: initial population of 1000 and b = 500, the populations dies very quick

1.2 Mean field model

We assume the sites are independent and number of individuals at sites is poisson distributed. I plot selected mean-field model here for b = 5, 10, 15, 20, 35, 50. The model are after the plot

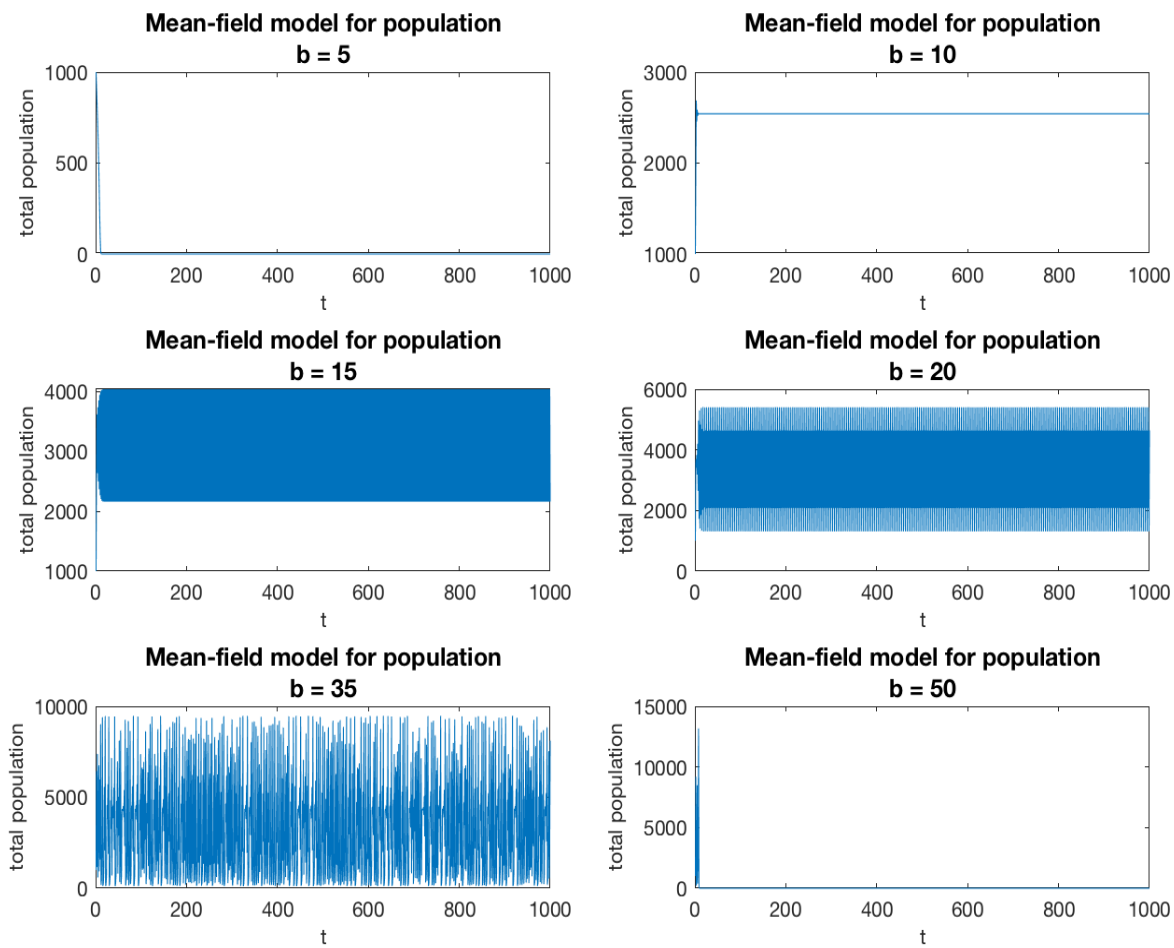

Figure 10: mean field model with selected b value

$$\begin{cases} E(A_{t+1}|A_t) = n \times \sum_{k=0}^{n} P_k \times \phi_k & (1) \\ \phi_k = \begin{cases} b, if k = 2 \\ 0, if k \neq 2 \end{cases} & (2) \end{cases}$$

following the mean field model, and the population only reproduce if there are exactly 2 individuals at same site, we can write the following:

$$A_{t+1} = n \times P(2 at site, A_t) \times b \tag{3}$$

$$A_{t+1} = n \times \frac{(\frac{A_t}{n})^2 \times e^{-\frac{A_t}{n}}}{2} \times b \tag{4}$$

For steady state, we have $A_{t+1} = A_t$.

$$A_t = n \times \frac{(\frac{A_t}{n})^2 \times e^{-\frac{A_t}{n}}}{2} \times b \tag{5}$$

When $A_t = 0$, left hand side always equal to right hand side, they are both 0. When $A_t \neq 0$, we have

$$1 = \frac{b \times A_t}{2n} \times e^{-\frac{A_t}{n}} \tag{6}$$

Let's rearrange equation (6) and let $\frac{A_t}{n} = x$, we have:

$$1 = \frac{b}{2} \times x \times e^{-x} \tag{7}$$

$$x \times e^{-x} = \frac{b}{2} \tag{8}$$

To find the conditions in terms of b for the existence of two further non-zero steady states, we need to find that $f1(x) = x \times e^{-x}$ has intersection with a horizontal line. b need to fulfill the condition that $\frac{2}{b} < 0.3679$, therefore we get: $b > 5.4366$, as b needs to be integer, we get $b > 5$.

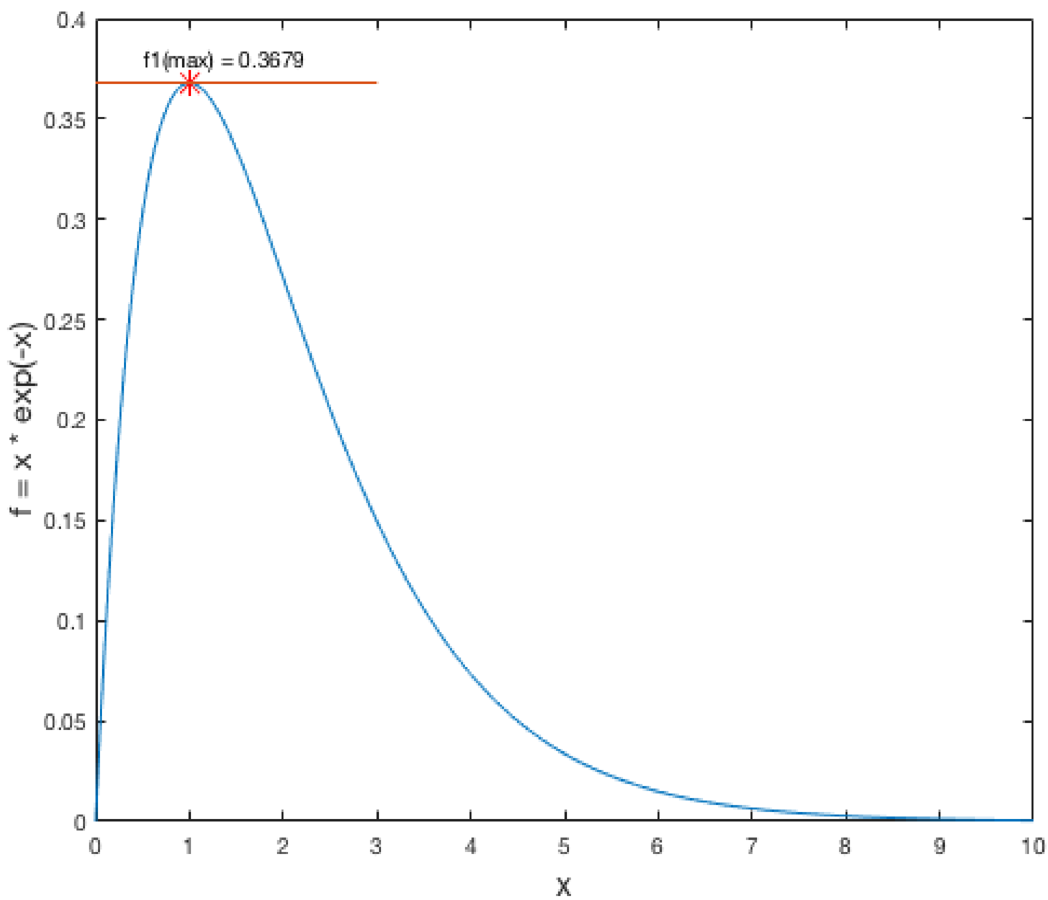

Figure 11: plot of f1(x) = x * exp(-x)

1.3 Lyapunov exponent

From the previous section, we have derived the equation for A_t. that we have

$$f(A_t) = n \times \frac{(\frac{A_t}{n})^2 \times e^{-\frac{A_t}{n}}}{2} \times b \tag{9}$$

$$f(a) = \frac{b}{2n} \times a^2 \times e^{-\frac{a}{n}} \tag{10}$$

$$f'(a) = \frac{b}{2n} \times (2a \times e^{-\frac{a}{n}} + a^2 \times e^{-\frac{a}{n}} \times (-\frac{1}{n})) \tag{11}$$

$$f'(a) = \frac{b}{2n} \times (2a \times e^{-\frac{a}{n}} - \frac{a^2}{n} \times e^{-\frac{a}{n}}) \tag{12}$$

$$|\Delta a_n| = |\Delta a_0| e^{\lambda n} \tag{13}$$

to compute Lyapunov exponent λ numerically, we use the following:

$$\lambda = \frac{1}{n} \sum_{t=0}^{n-1} ln|f'(a_t)| \tag{14}$$

The Lyapunov exponent are plotted below: we can see that when $\lambda > 0$ the diverges, that's when b is between 21-30, the population become chaotic. When $b \leq 5$ and $b \geq 31$ λ is -infinity that means the system converge very fast, and this explains why the population become extinct very fast for large b.

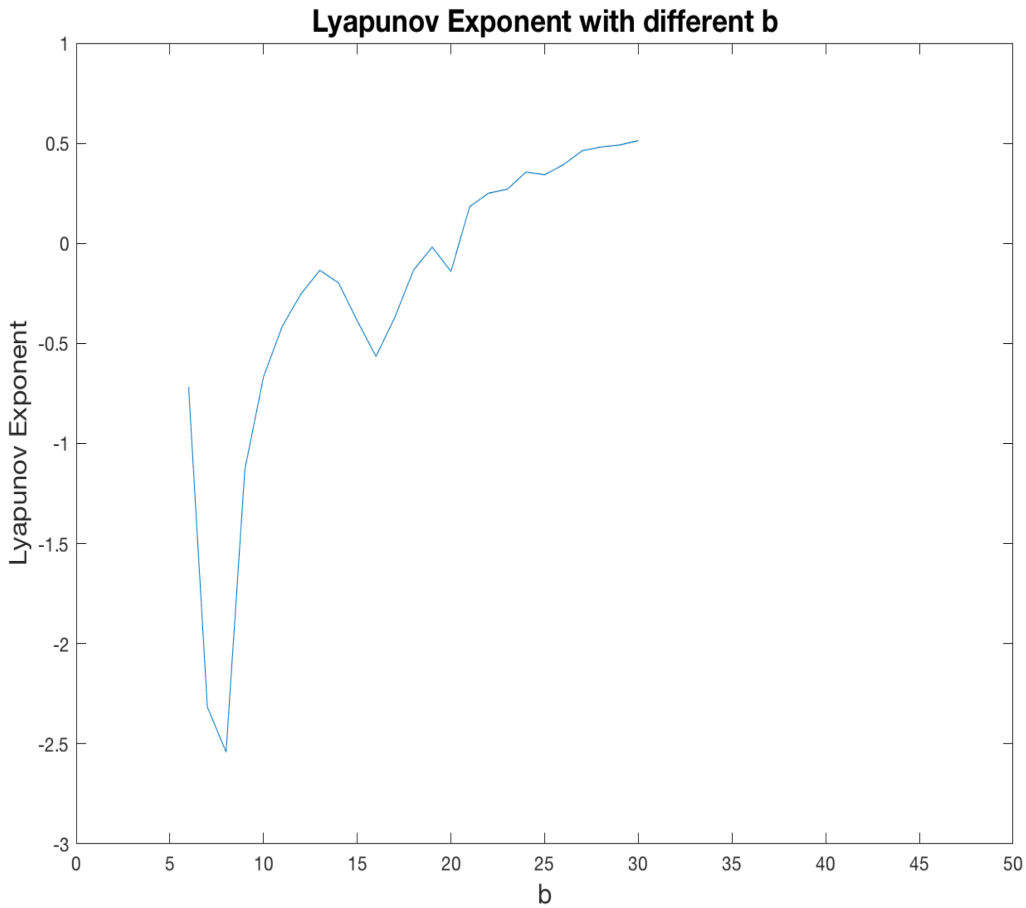

Figure 12: plot of Lyapunov exponent

2 Groups of friends

In this part, we used a model to simulate how students make friends. Initially, there are N students, and they are all alone in a group contains only one student. At each time step, a group is picked:

1. If the picked group contains only one student, this students will join a group with size k with probability of k/N.
2. if the picked group size i ($i > 1$), then this group will split up with probability of ri.

2.1 simulations in Matlab

First, we simulate the above model with 100 students, r = 0.01 and 100 replications. First, we plot the group size vs frequence on a log log scale. Second, we get the relative frequency with group size histogram taken on a log scale that is we take groups size [1,2,4,8,16,32,64,128] and plot the group size vs relative frequency on a log log plot. in figure 14, we can observe that the group distribution following the power law.

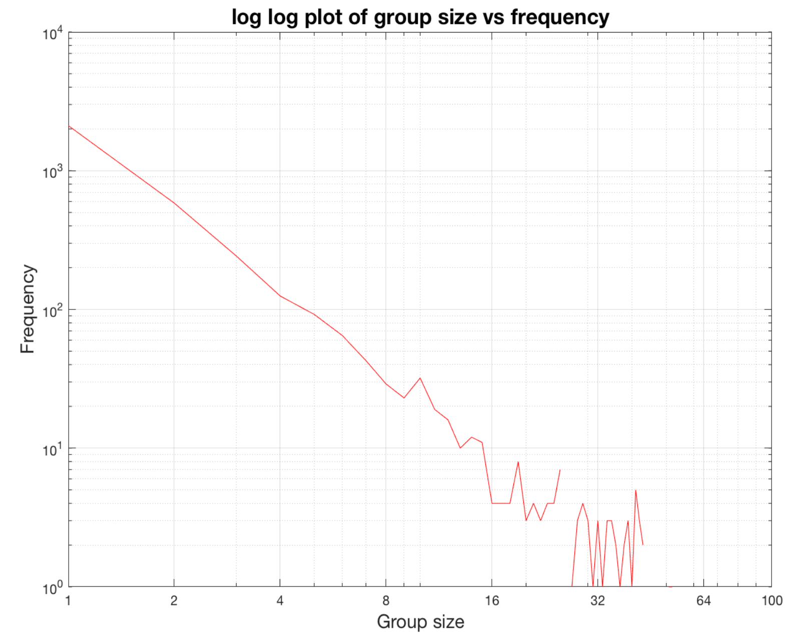

Figure 13: log log plot of group size vs frequency

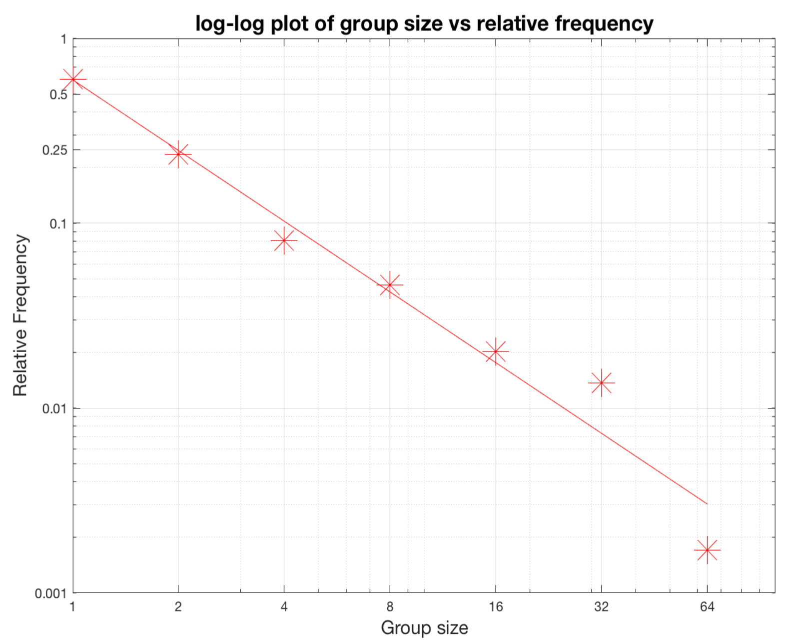

Figure 14: log log plot of group size vs relative frequency

To investigate the how the group size distribution changes with r, we run the model with r from 0.001:0.001:0.1 with 100 replications. In 15, I choose some r values to plot shows how the group size changes, and in 16, the whole group distribution is shown on a 3d plot, it might be a little difficult to from the colourful map, but we can observe the z-axis value that the relative frequency of the group.

When r is small at 0.001 we have groups of size 1 and 2 around $\frac{1}{4}$ of the population and some large groups. when we increase the r, then the number of groups of 1 become the dominant group in the distribution and large group appear less and less. With a bigger r, the larger group will split up if it was chosen at time t. For example, setting r = 0.1, it means that groups of size 10 + will slip up at the probability of 1 if chosen at time t.

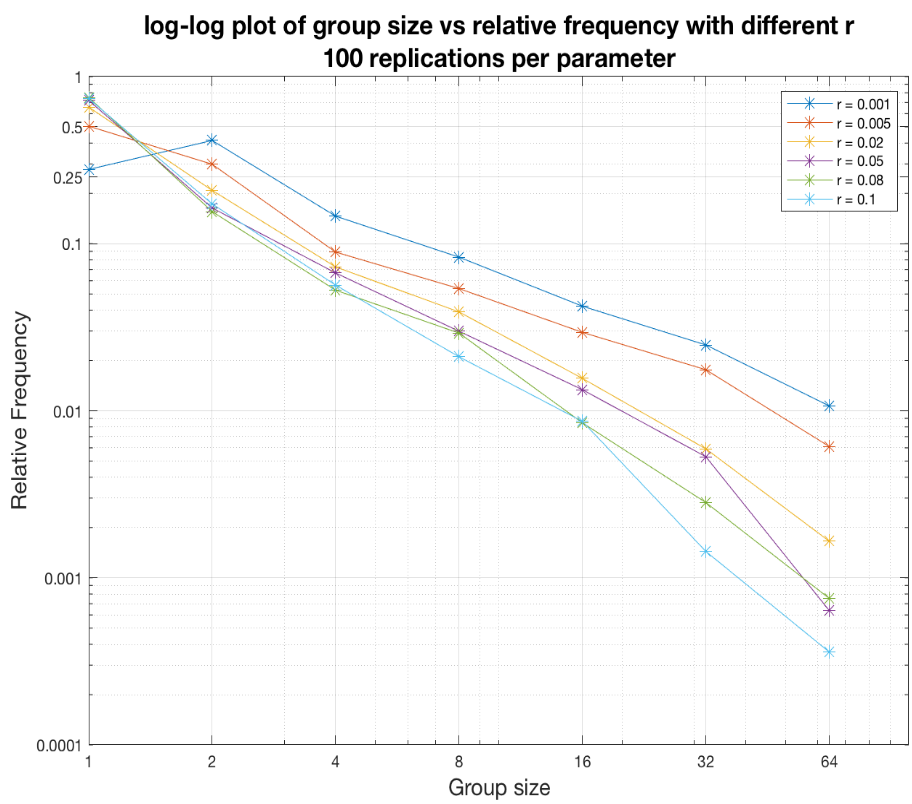

Figure 15: log log plot of group size vs relative frequency with different r

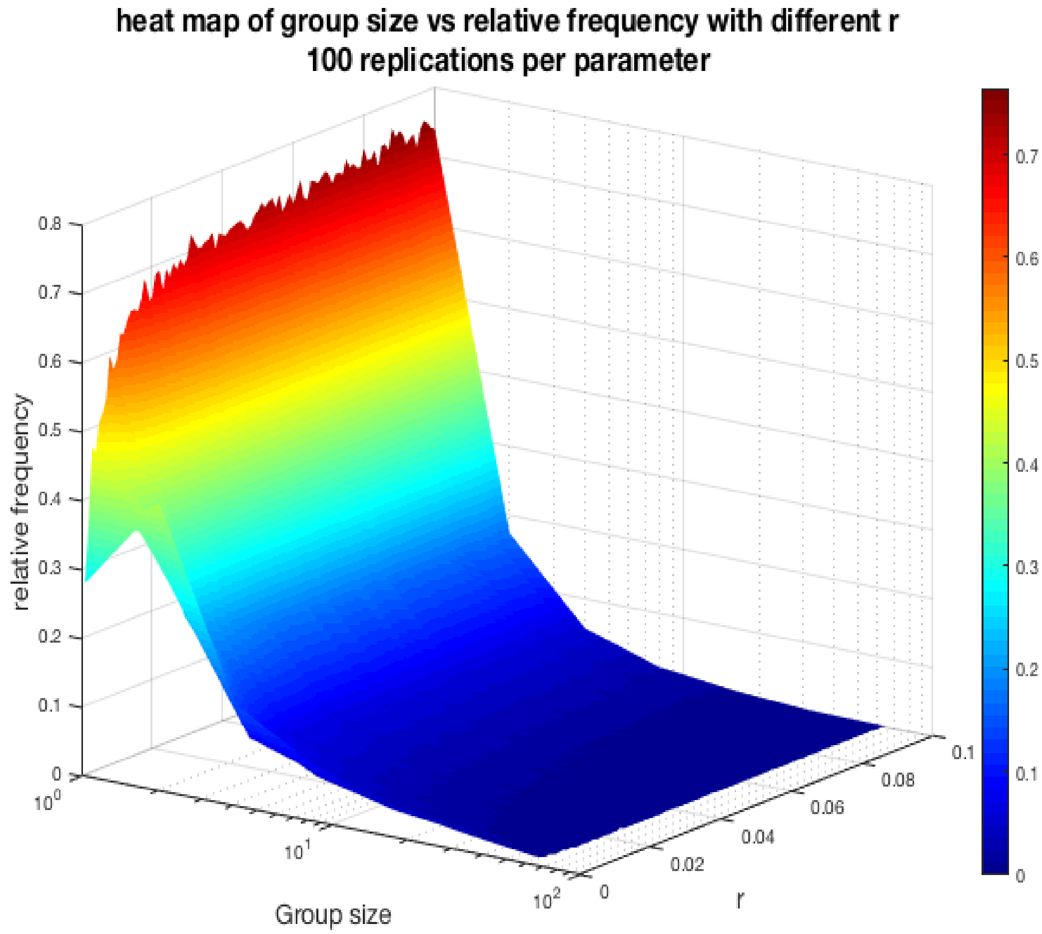

Figure 16: the distribution of groups with different r showing on a 3d plot

2.2 Master Equation

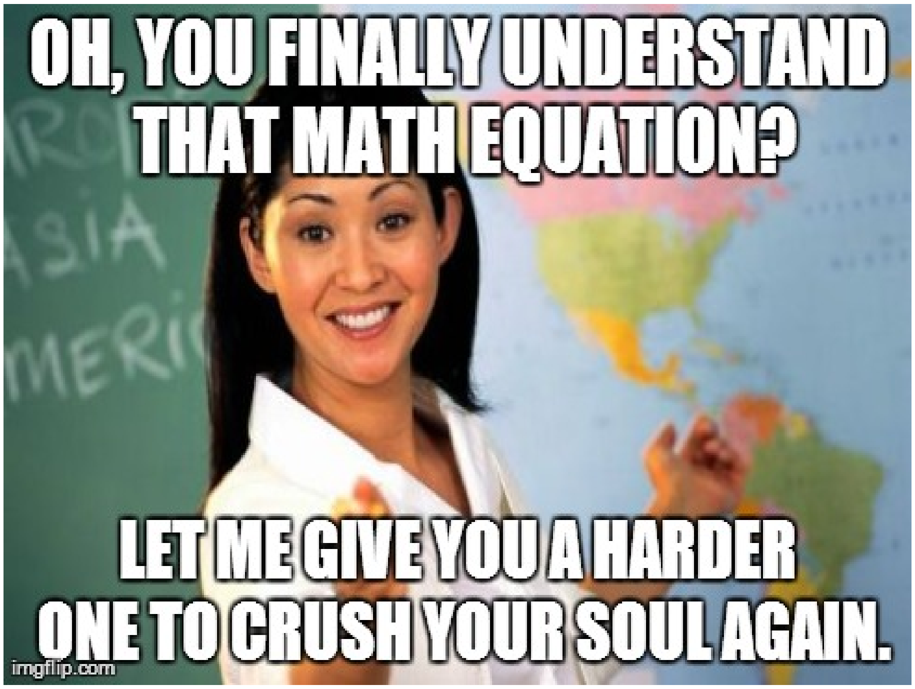

Figure 17: my master equation

3 Network Epidemics

3.1 random undirected social network

An undirected networking was made with 5000 students and a link density of 0.0016. we use matrix A to represent the network, as the network is undirected, we have A as a symmetric matrix, and with a link density of 0.0016, we generate A and plot the histogram of the degree distribution. The average degree of this network is around 8 and the degree follows a binomial distribution.

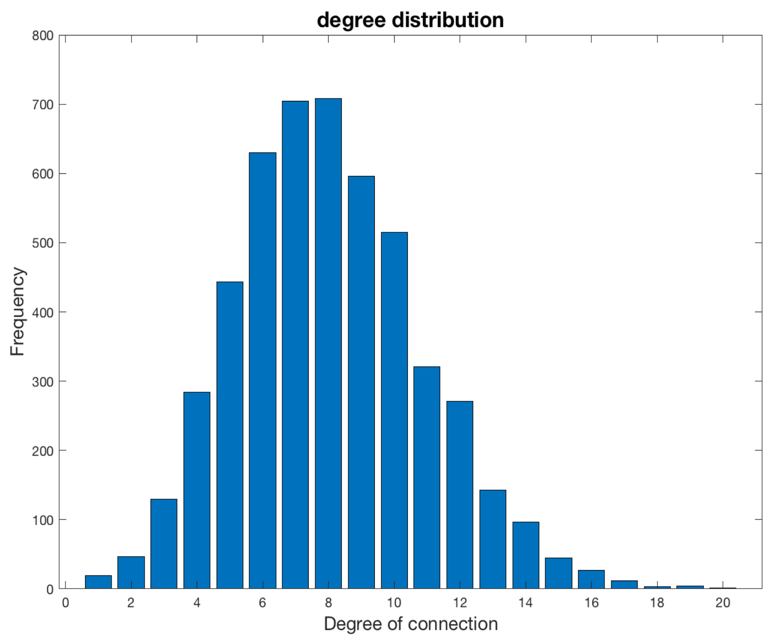

Figure 18: Frequency of the connection degree

3.2 Infection within the random network

A infection on above random network was modelled. In the beginning, there are 100 random infected students. The infection will spread according to:

1. An uninfected student will get infected depends on the infected student he is connected with. $P_{infected}(n) = 1 - e^{-pn}$.

2. An infected student will recover with the probability of r = 0.03.

3. A recovered student can get infected again.

To study the model, we set p = 0.01 and plot the number of infected students over time and it showed that the infection spreads very quick. At around 200 time, the total number of infected students are around 3000, and it stays stable around that number, which is more than half of the population. To further investigate, we run the model with p = 0.001:0.001:0.01, and the infection does not spread for $p \leq 0.003$, the infection will stay around the initial number when p = 0.004, and the infection will spread when $p \geq 0.005$, and the larger the p, the faster the spread of the infection and more infected students.

Finally, in Figure 21 we can see than the infection will die when $r/p \geq 10$ and spread very fast when $r/p \leq 5$.

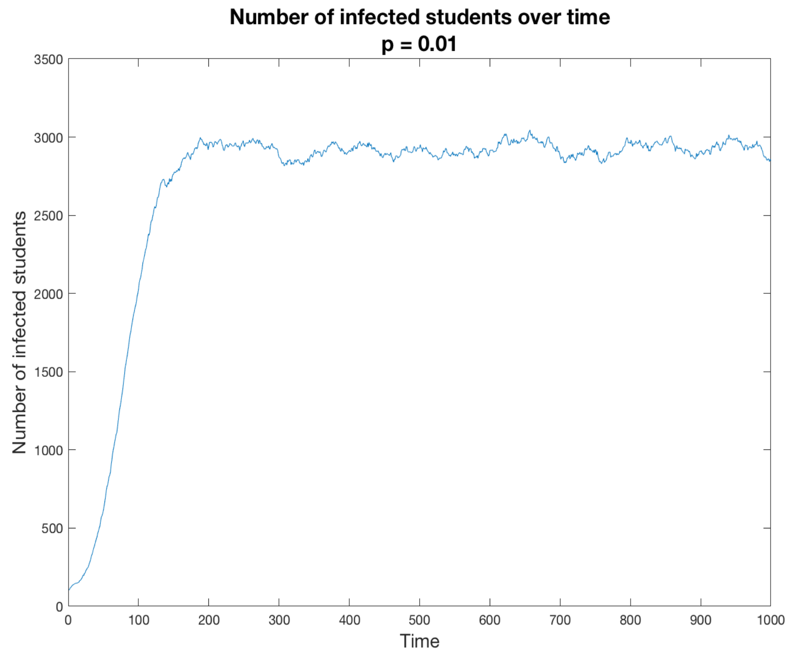

Figure 19: Number of infected individuals against time with p = 0.01

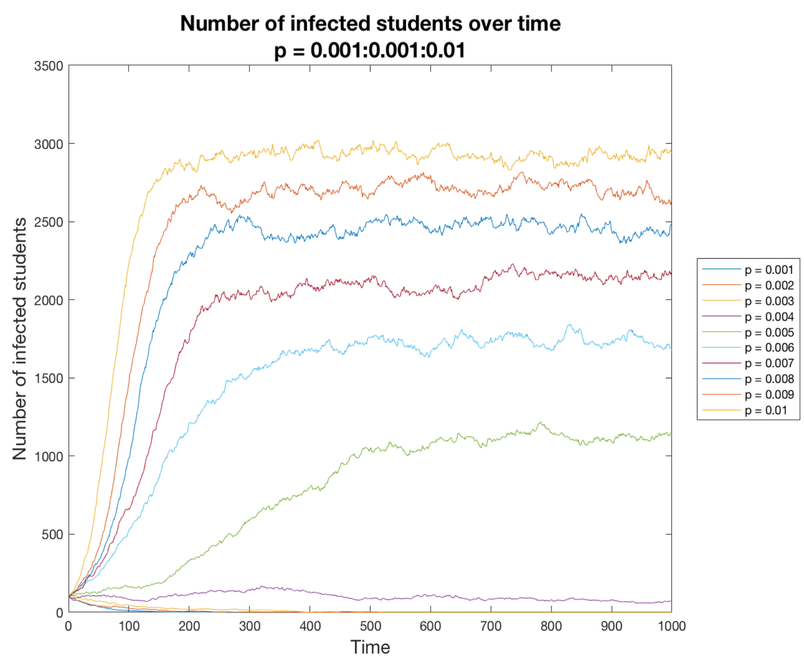

Figure 20: Number of infected individuals against time with different p

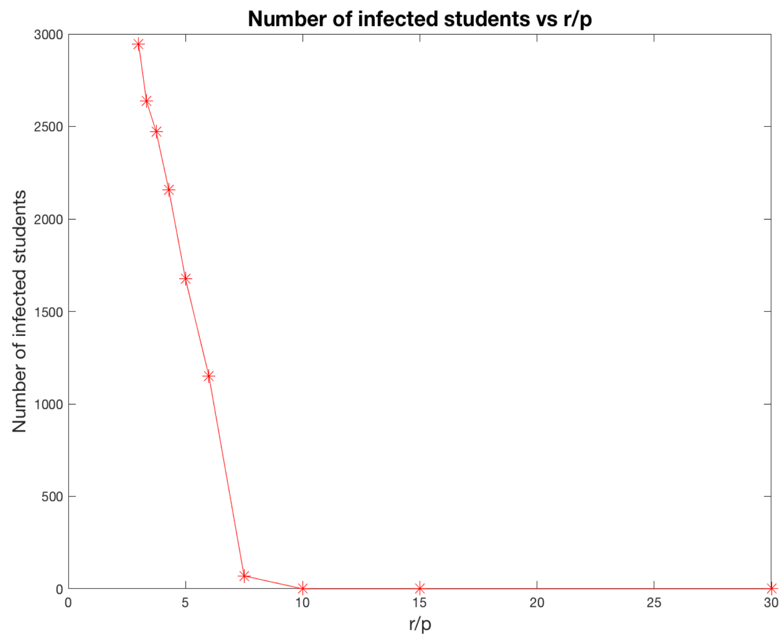

Figure 21: Number of infected individuals vs. r/p

3.3 Infection within the random network

In this part, we create a network with preferential attachment, starting at t_0, there are 2 students that are linked. Each time step, one new student is added to the network and he will chose to connect with a students at $p = \frac{k_i}{2(n-2)}$.

First, we can see that in Figure 22, the connection degree distribution follows the power law distribution. The average degree of the network is around 2. But to have a better idea, we take histogram at interval of 2^i. And in Figure 23, it eliminates the random noise, and shows a strong power law distribution for the preferential network.

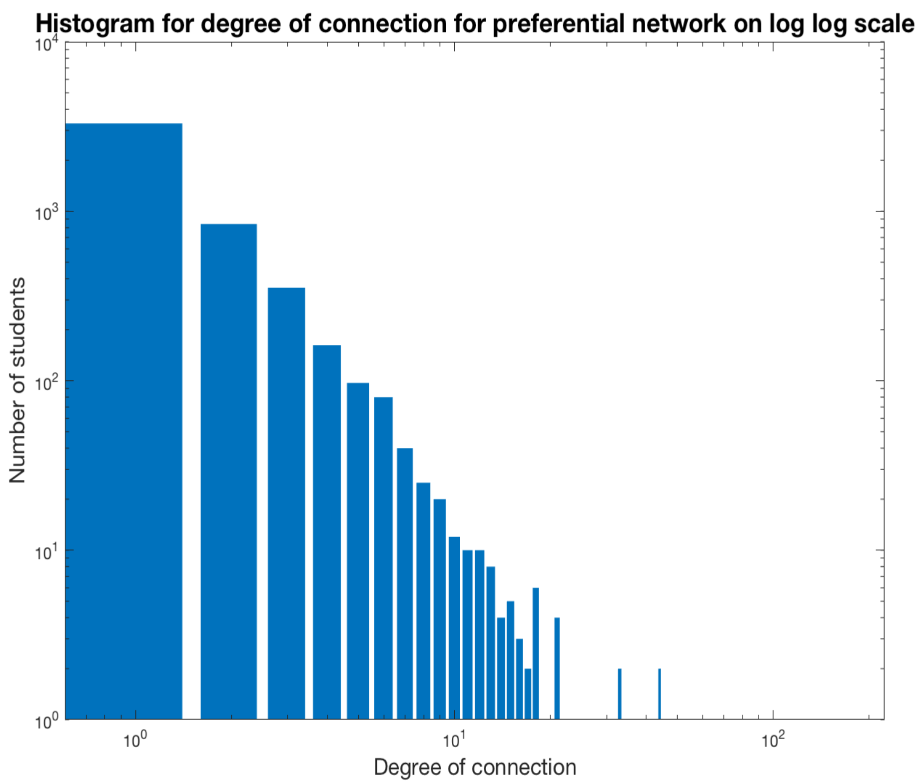

Figure 22: Histogram of the degree distribution

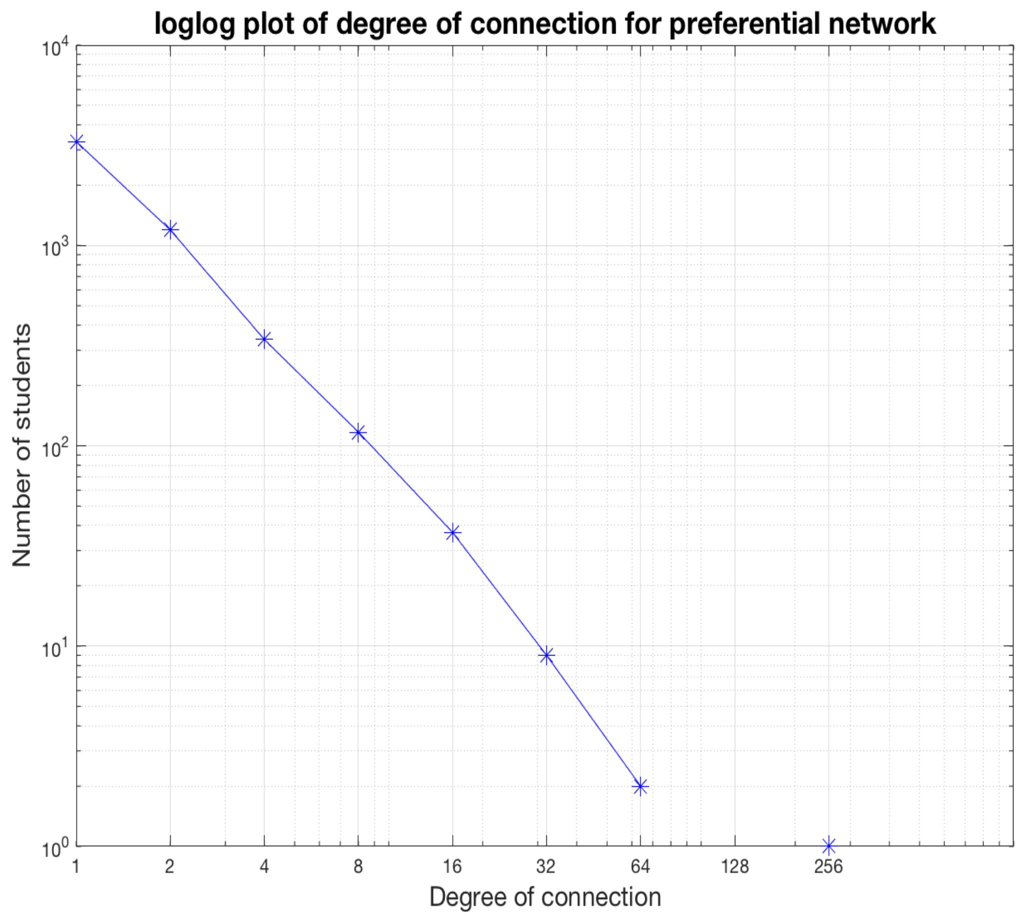

Figure 23: loglog plot of degree of connection

3.4 Master Equation

Below, I show my work for master equation.

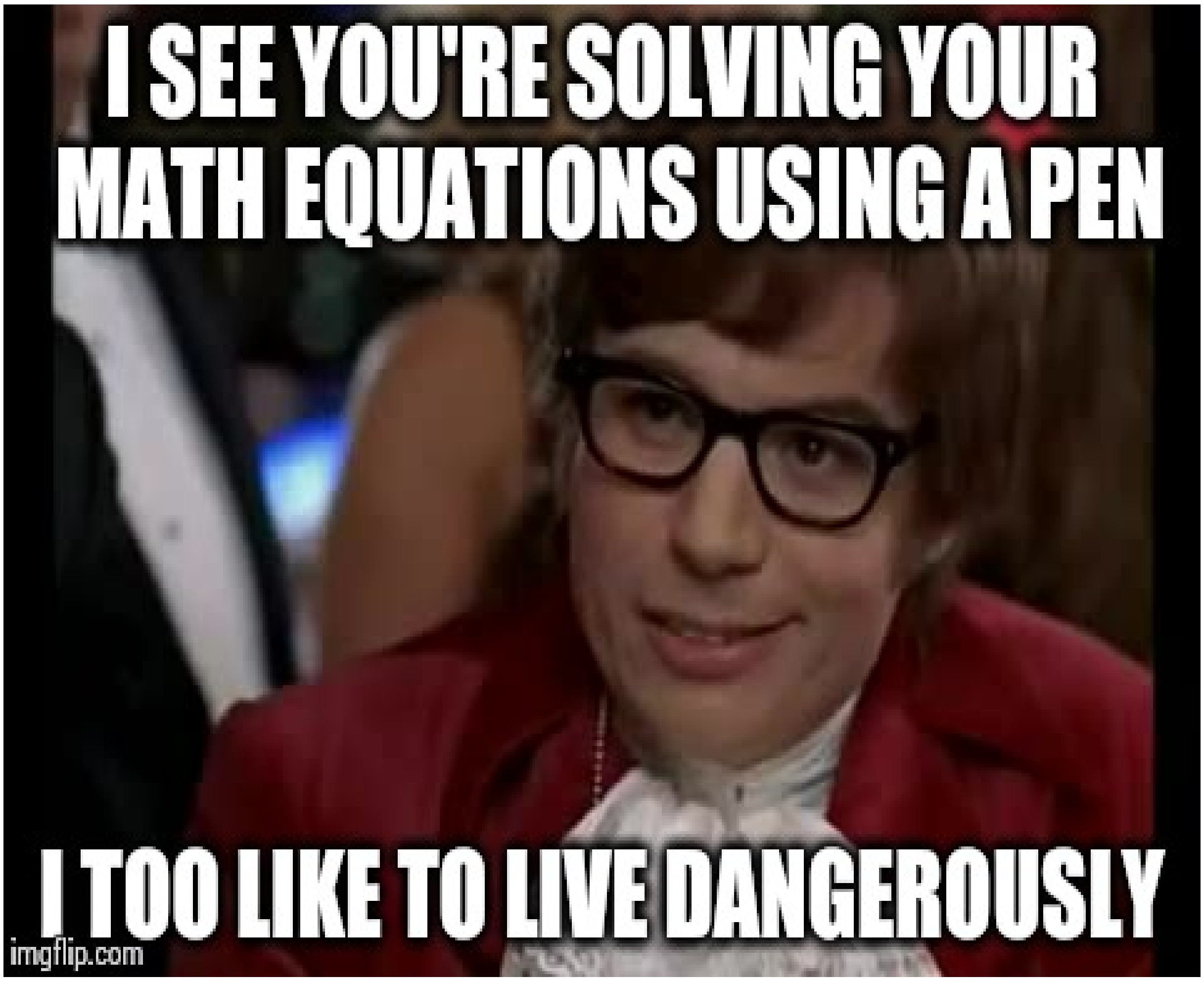

Figure 24: master equation for the preferential network

For the preferential network, we have the number of students n and time t, and at each time the probability of student with k links will get an extra link is $p_{k,n} = \frac{k}{2(n-2)}$.

Expected number of student with k links are:

$$E = \frac{k}{2(n-2)} \times p_{k,n} \times n \tag{15}$$

$$(n+1) \times p_{k,n+1} = n \times p_{k,n} + \frac{k-1}{2(n-2)} \times p_{k-1,n} \times n - \frac{k}{2(n-2)} \times p_{k,n} \times n \tag{16}$$

to find steady state, we set n and t to ∞. we get:

$$(n+1) \times p_k = n \times p_k + \frac{k-1}{2} \times p_{k-1} - \frac{k}{2} \times p_{k,n} \tag{17}$$

$$p_k = \frac{1}{2} \times ((k-1) \times p_{k-1} - k \times p_k) \tag{18}$$

$$p_k = \frac{(k-1)!}{(k+2)(k+1)(k)...4} \times p_1 \tag{19}$$

when k = 1:

$$p_1 = 1 - \frac{1}{2} \times p_1 \tag{20}$$

$$p_1 = \frac{2}{3} \tag{21}$$

$$p_k = \frac{(k-1)!}{(k+2)(k+1)(k)...4} \times \frac{2}{3} = \frac{4}{(k+2)(k+1)(k)} \tag{22}$$

Then we got our result c = 4.

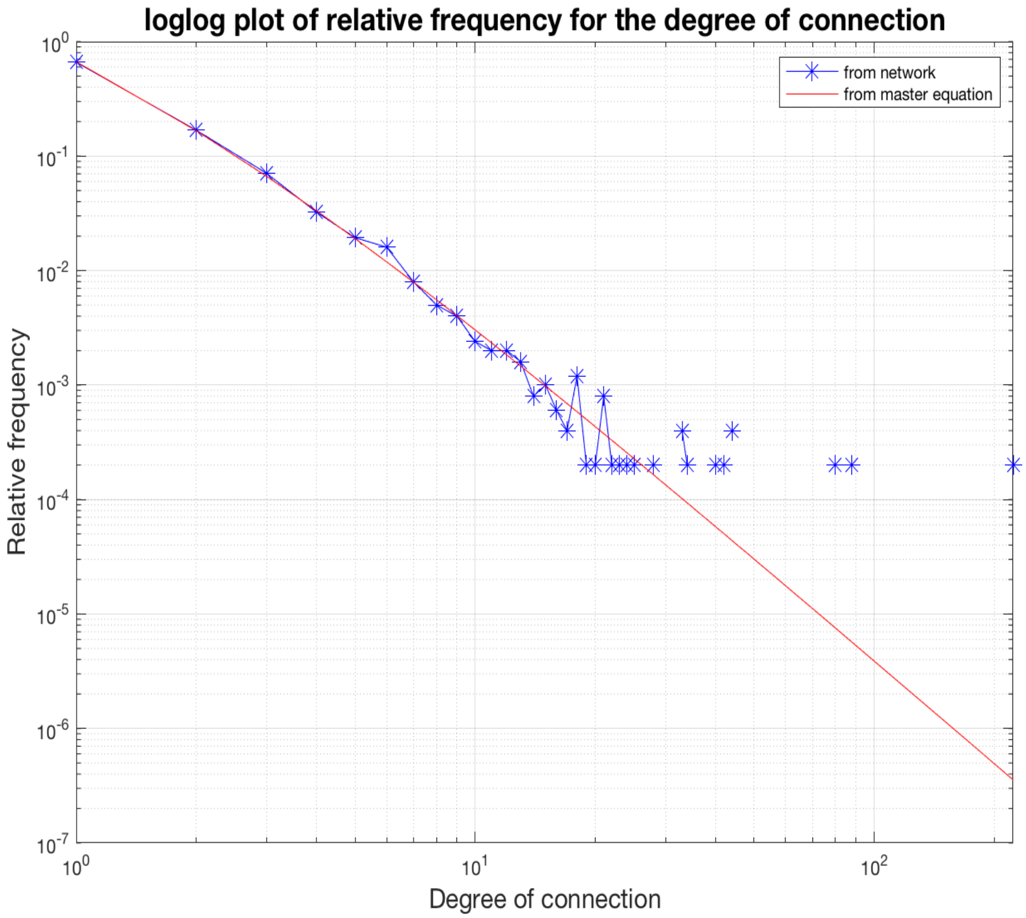

Figure 25: distribution of the connection from the network and master equation

The master equation aligns with the results from the network.

3.5 infection with the preferential attachment network

Here, the infection in 3.2 are simulated in the preferential network created in 3.3. We can see in Figure 26 that the number of infection will die when $p \leq 0.005$, the infected number will stay around the initial number or decrease a little when p is between 0.006 to 0.008, the infection will spread when $p \geq 0.009$.

The total number of infected students are less with same p in the preferential network compared to the random network as the average degree of network is lower at the preferential network. The infection are more difficult to spread in the preferential network than the random network. If a student with a lot of connections are infected then the infection will likely spread. But in a network with high r/p, the infected ones may recover sooner than spreading it to other students.

Using the spread of internet memes within the preferential network. the constant p here can be seen as how funny the memes is. If the memes is very funny, then the student will be very likely to hear from another students, thus become infected with the memes and spread it, if a key student with lots of connections become infected with the meme, the memes will take off and become viral. However we have a r = 0.03 recovery rate, can be seen as lost interest. While some students are sharing memes, some other students lost interests on the memes. When r/p ¿ 6, the memes are not likely to get spread out. When r/p are smaller than 4, the number of students that are interested in the memes are likely to remain the same as the initial number. If the r/p are decreased slightly under 4, the memes is likely to take off spreading very rapidly. However, this depends on the preferential network A. With the same rule in 3.3, sometimes the network can have key students with a large number of connections (100+), sometimes, the network does not have these big key student. It is suggested to run the model with different network A generated using the same methods in 3.3 to further investigate the preferential attachment network.

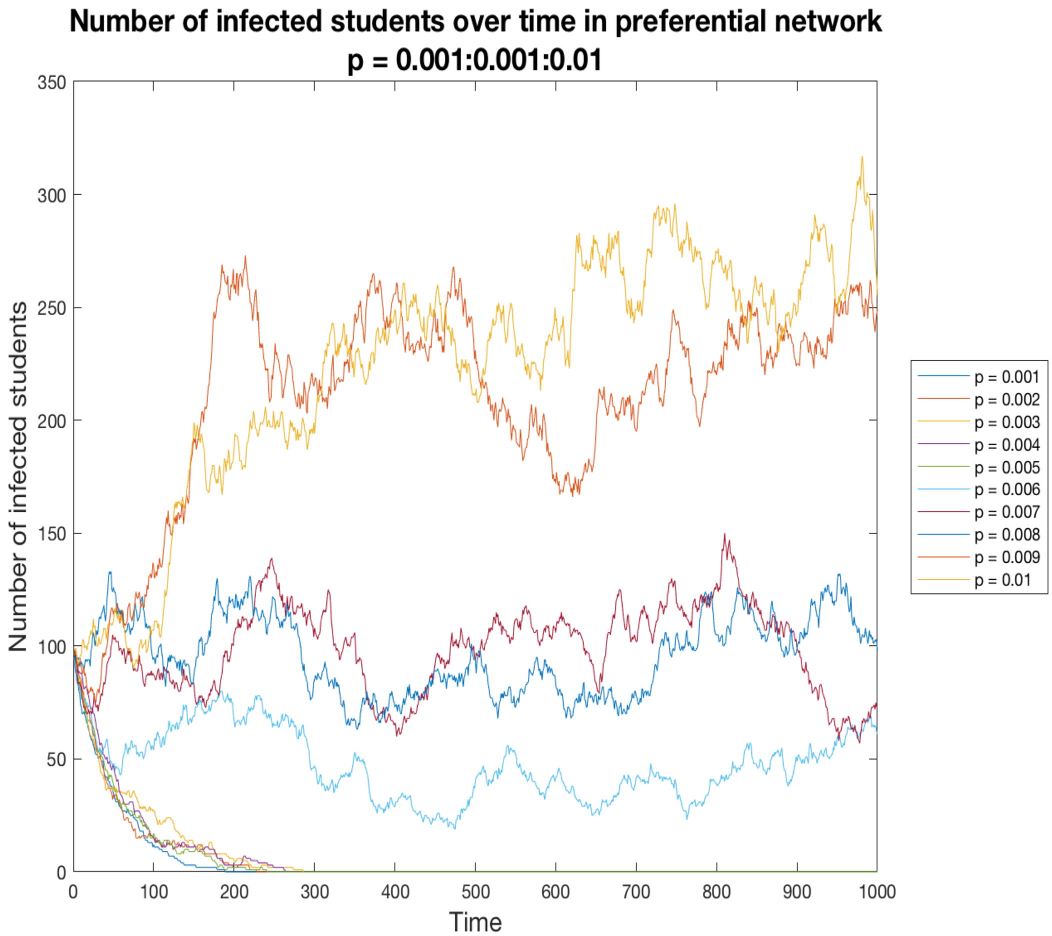

Figure 26: Number of infected individuals against time with different p

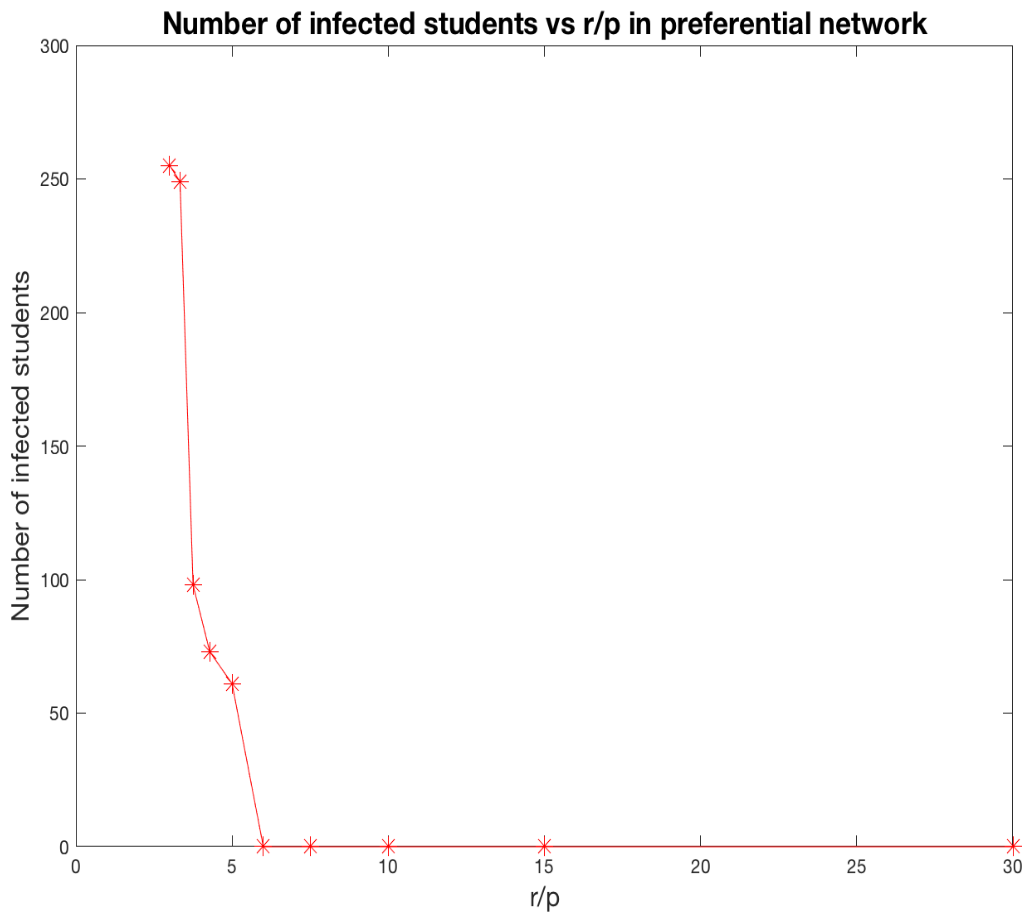

Figure 27: Number of infected individuals against time with different p

4 Flocks and Predators

4.1 Alignment

Run the Vicsek alignment model with 40 particles, domain size L = 10, angular noise e = 0.5, for 200 time steps, at each time, the direction of a particle is the average direction of its 4 nearest neighbours. The alignment varies from individual runs of the model, but generally, the particles all move towards the same direction very quick.

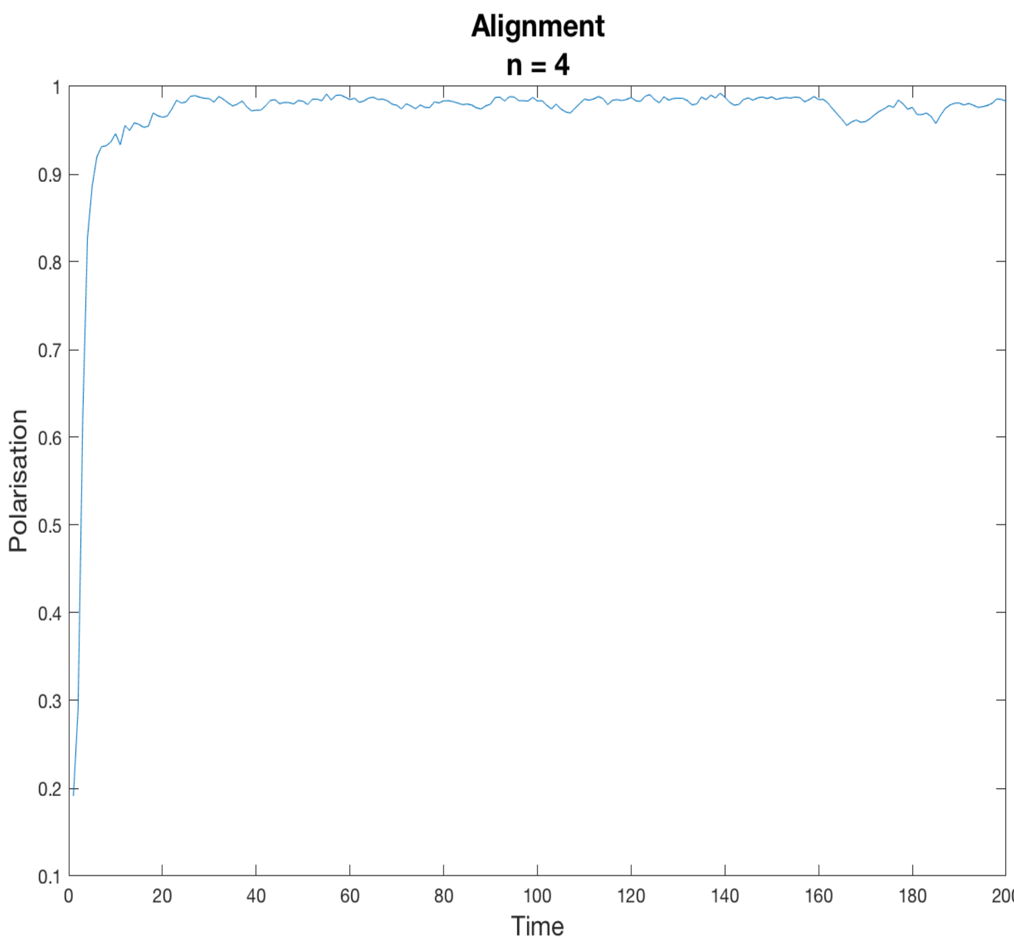

Figure 28: Alignment (4 nearest neighbour)

4.2 Add a small force

We add a small force that pulls the particles toward the center of mass. with coefficient 0.1. and we run the model with 40 particles, domain size L = 10, angular noise e = 0.1:0.1:6, nearest neighbour (n) = 1: 1:39 for 200 time steps with 10 replication. Then take average of the last half of the alignment. we get the heat map showing the steady-state alignment change with n and e.

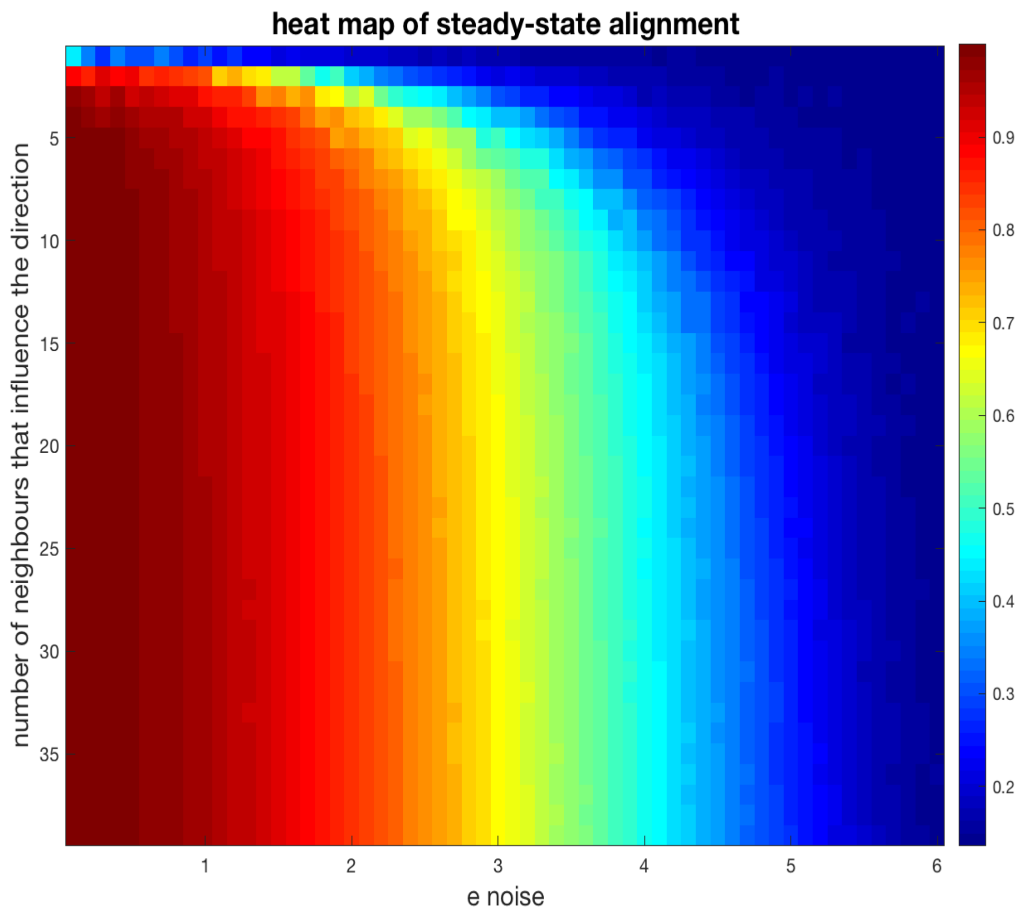

Figure 29: Heat map of steady state alignment

4.3 Predator

In this part, we will assign the particles to different settings.

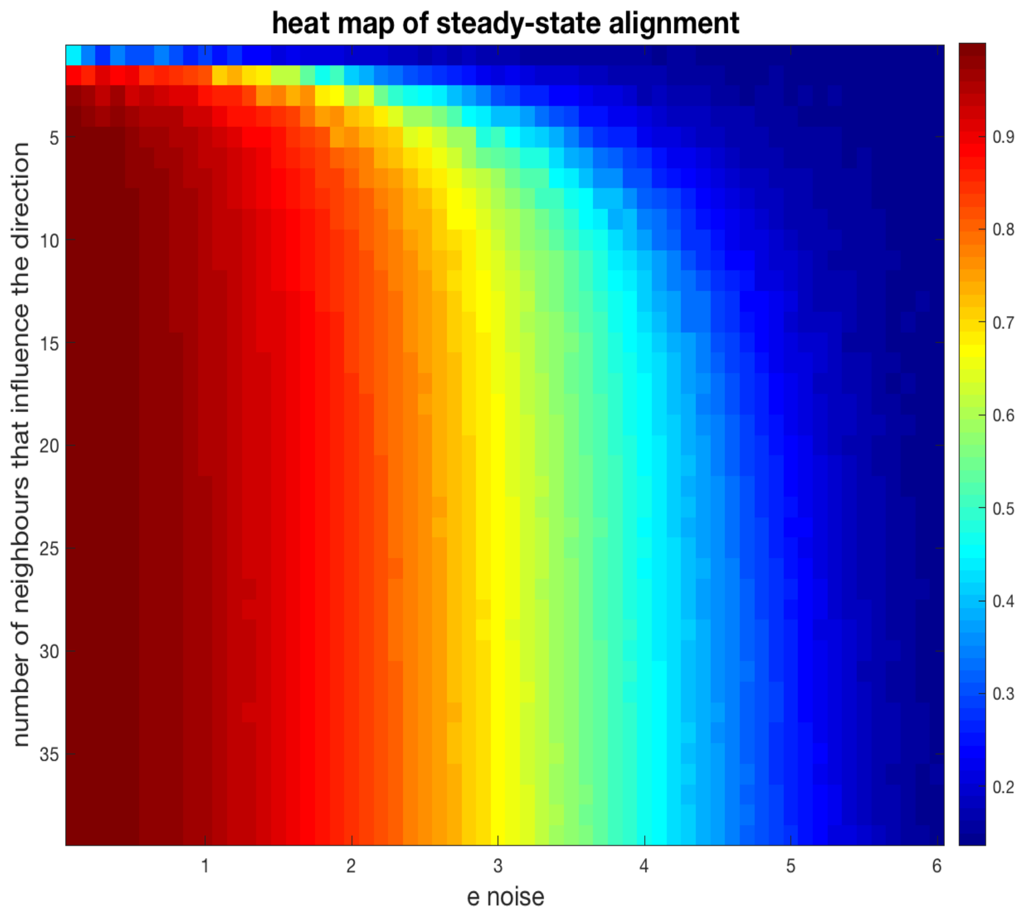

Figure 30: Heat map of steady state alignment

Here are the videos you can watch:

1. Hunting1 `<https://www.youtube.com/watch?v=WMfI2P52ros>`

2. Hunting2 `<https://www.youtube.com/watch?v=WMfI2P52ros>`

3. Hunting3 `<https://www.youtube.com/watch?v=WMfI2P52ros>`

4. Hunting4 <https://www.youtube.com/watch?v=WMfI2P52ros>

5. Hunting5 <https://www.youtube.com/watch?v=WMfI2P52ros>

5 Appendix

5.1 1firing brain code in matlab

- simulate single time of fire brain
- transition function
- initial state
- simulate 100 times of firing brain
- cell that move forward at one cell per time preserving the same shape
- cell that move forwad at one cell per time, launching other shapes behind them
- move forward at a rate of less than one cell per time step
- oscillate shape
- my cellular automata
- my cellular automata transit

5.2 Spread of memes

- simulation of spread of memes
- run single spread of memes
- mean field model
- phase transition
- probability for at least 25% are sharing
- simulation of spread of memes with new rules
- run single spread of memes
- mean field model
- phase transition for new rules
- lattice simulation for memes
- transition function for memes