UPPSALA
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Modelling complex systems

Project 2

Population Dynamics
Groups of Friends
Network Epidemics
Flocks and Predators

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1 Population Dynamics

In this part, we model the population with a stochastic model. There are n resource sites in the model world, and at time t =0, the population is A_0 and they are assigned randomly to one resource site. At each t step, the population rules are, if there are exactly two individuals on the same site. They reproduce b offsprings and these offsprings are assigned randomly to resources sites. If the number of individuals on a resources sites is any number other than 2, no offspring will be reproduced.

1.1 Matlab model

To begin, we can run this with different parameters and simulate in Matlab and observe the total number of population at different time step. The different parameters are:

1. b: number of offspring if reproduce
2. n: total of number of resource sites
3. A_0: initial population
4. t: the time steps we want to simulate the model

When set the initial population to a small number relative to the resource site, it would be difficult to have 2 individuals at the same site for reproduce, and even if they reproduce a large number of offspring the population dies out very quick. below are some plots showing the total number of population at different time steps with different parameters.

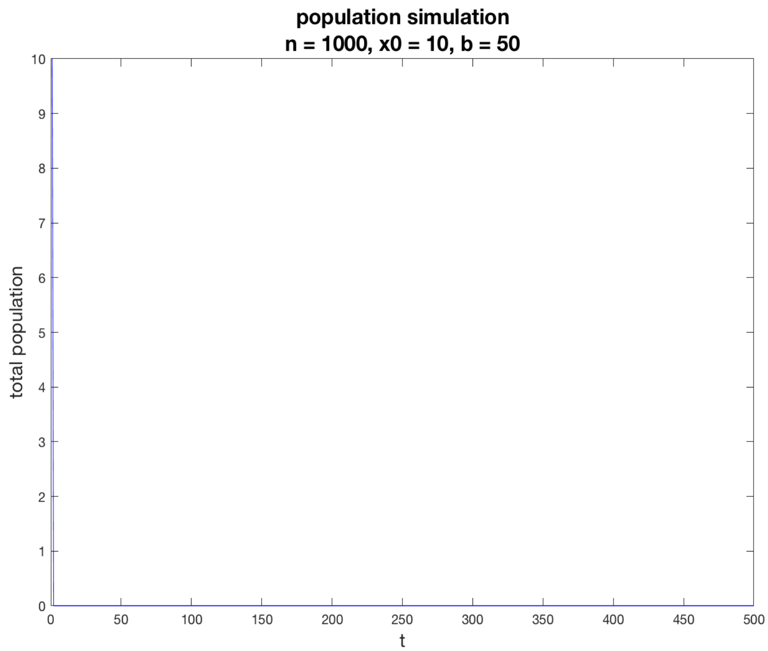

Figure 1: initial population of 10 b = 50 not reproducing

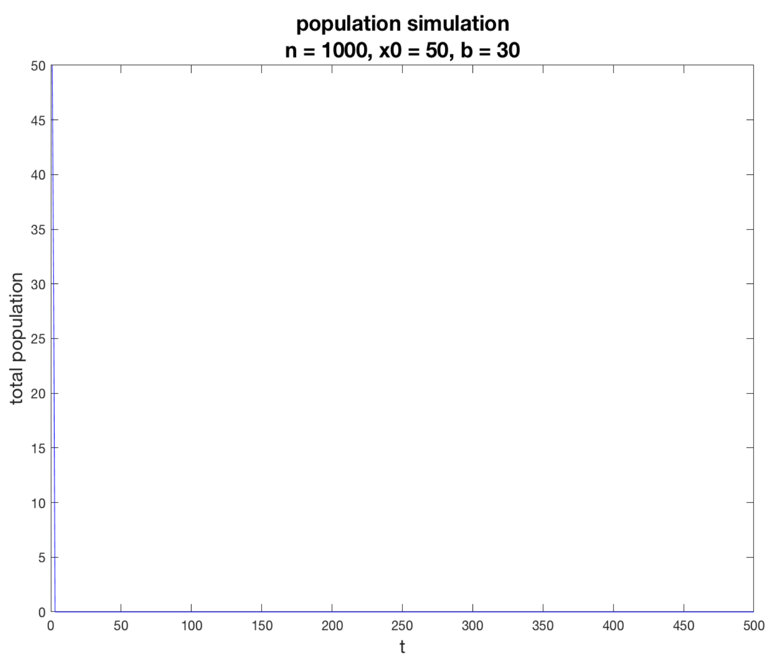

Figure 2: initial population of 50 and b =30 not reproducing

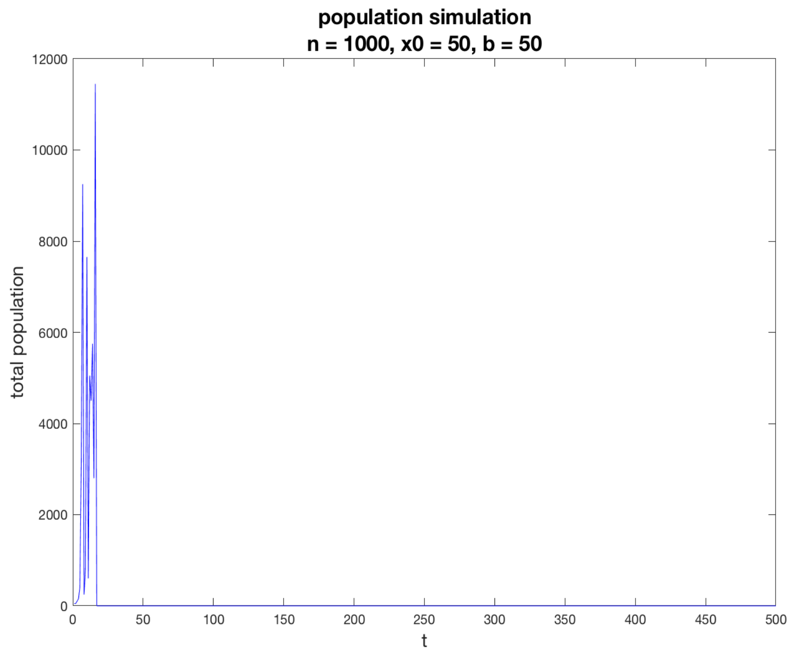

Figure 3: initial population of 50 and b = 50, in the beginning reproduce but quickly dies

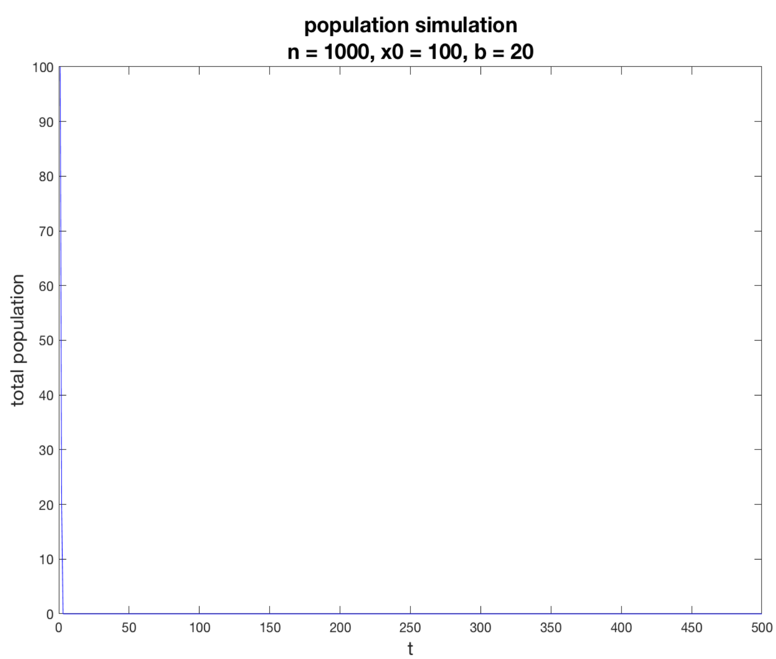

Figure 4: initial population of 100 and b = 20, it dies

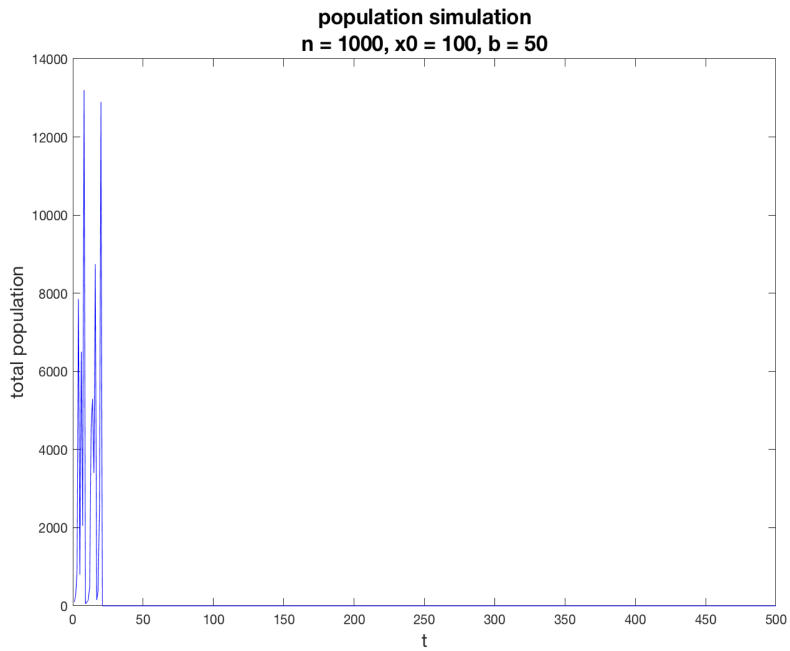

Figure 5: initial population of 100 and b = 50, it reproduce in the beginning but quickly dies

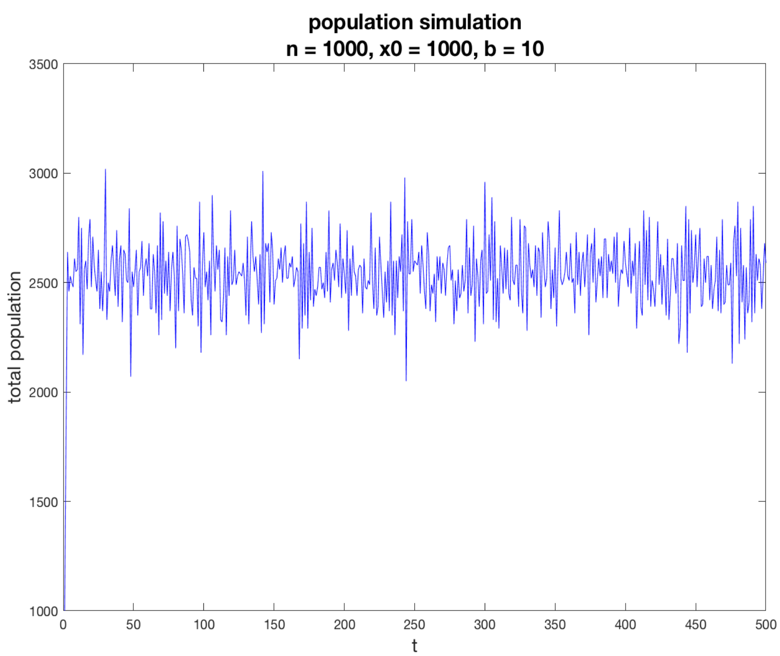

Figure 6: initial population of 1000 and b = 10, it show that the population oscillates around 2500

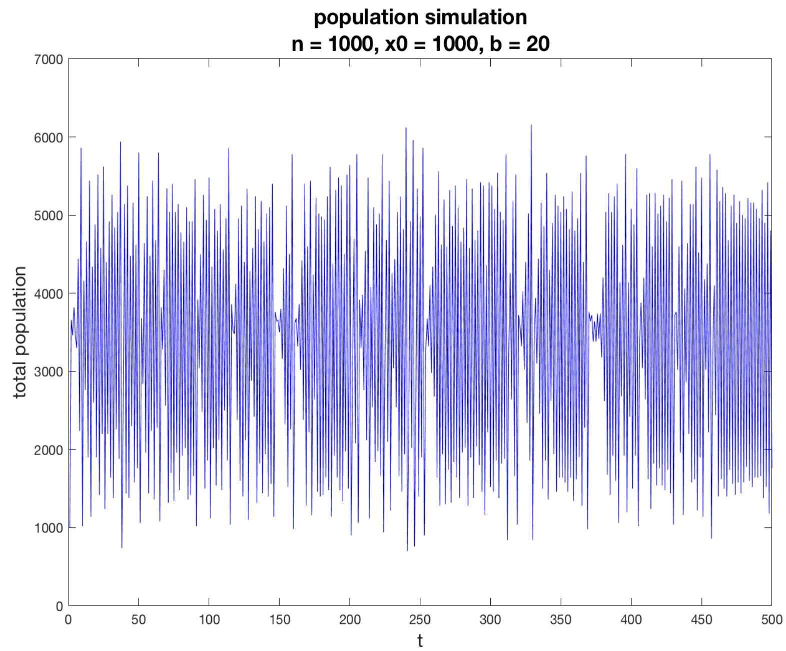

Figure 7: initial population of 1000 and b = 20, the populations starts to get chaotic

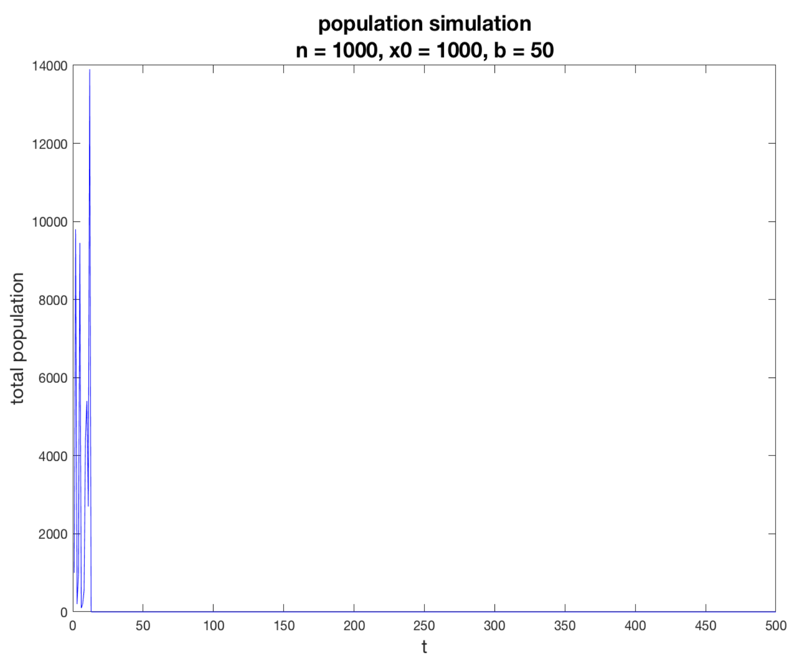

Figure 8: initial population of 1000 and b = 500, the populations dies very quick

To further study this model, we run it with $A_0 = 1000$, n= 1000, b = 1, 2, 3, 4, 5,...,48, 49, 50. and at each b value, run the simulation 100 times and plot a phase transition diagram. and we can see that when b = 5-15, the population increases steady and oscillates around a number. When $15 < b < 35$, the population will get more chaotic and at one time a lot of sites are reproducing, and then the next time steps, they dies because of overcrowding. When $b > 35$ the population will dies very quick due to overcrowding.

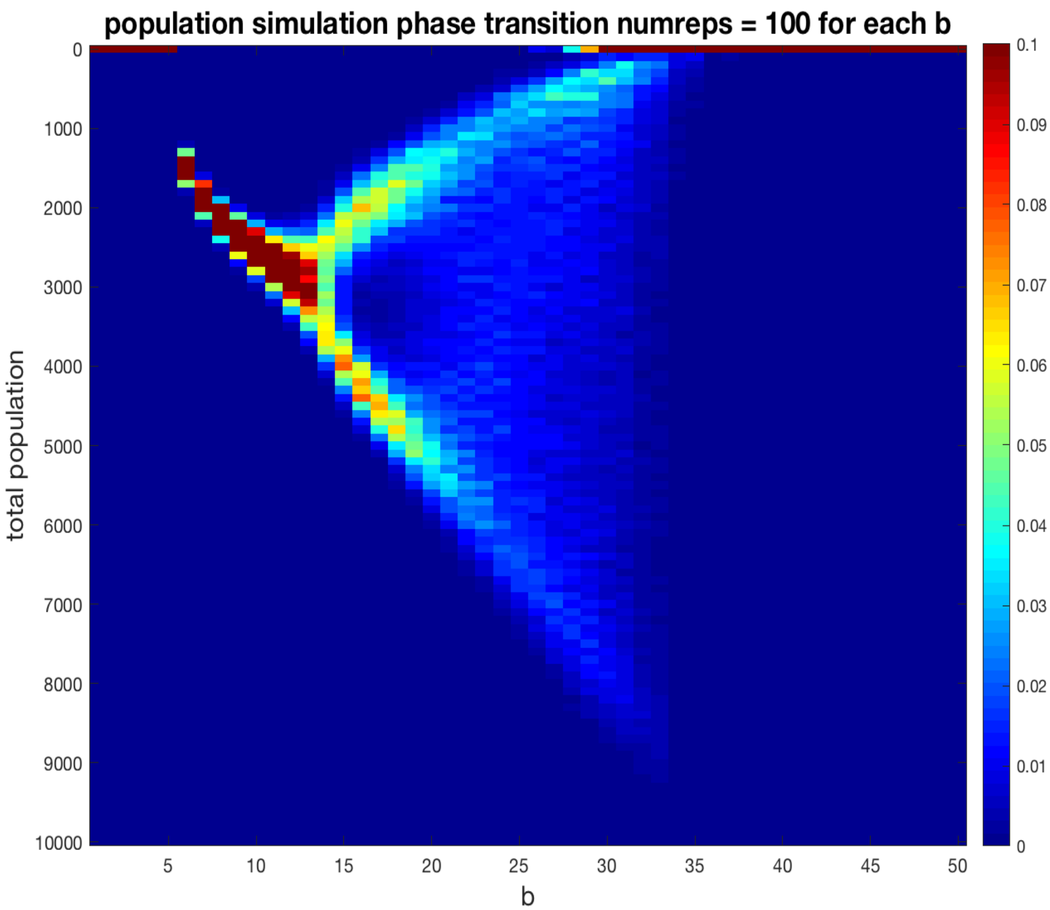

Figure 9: initial population of 1000 and b = 500, the populations dies very quick

1.2 Mean field model

We assume the sites are independent and number of individuals at sites is poisson distributed. I plot selected mean-field model here for b = 5, 10, 15, 20, 35, 50. The model are after the plot

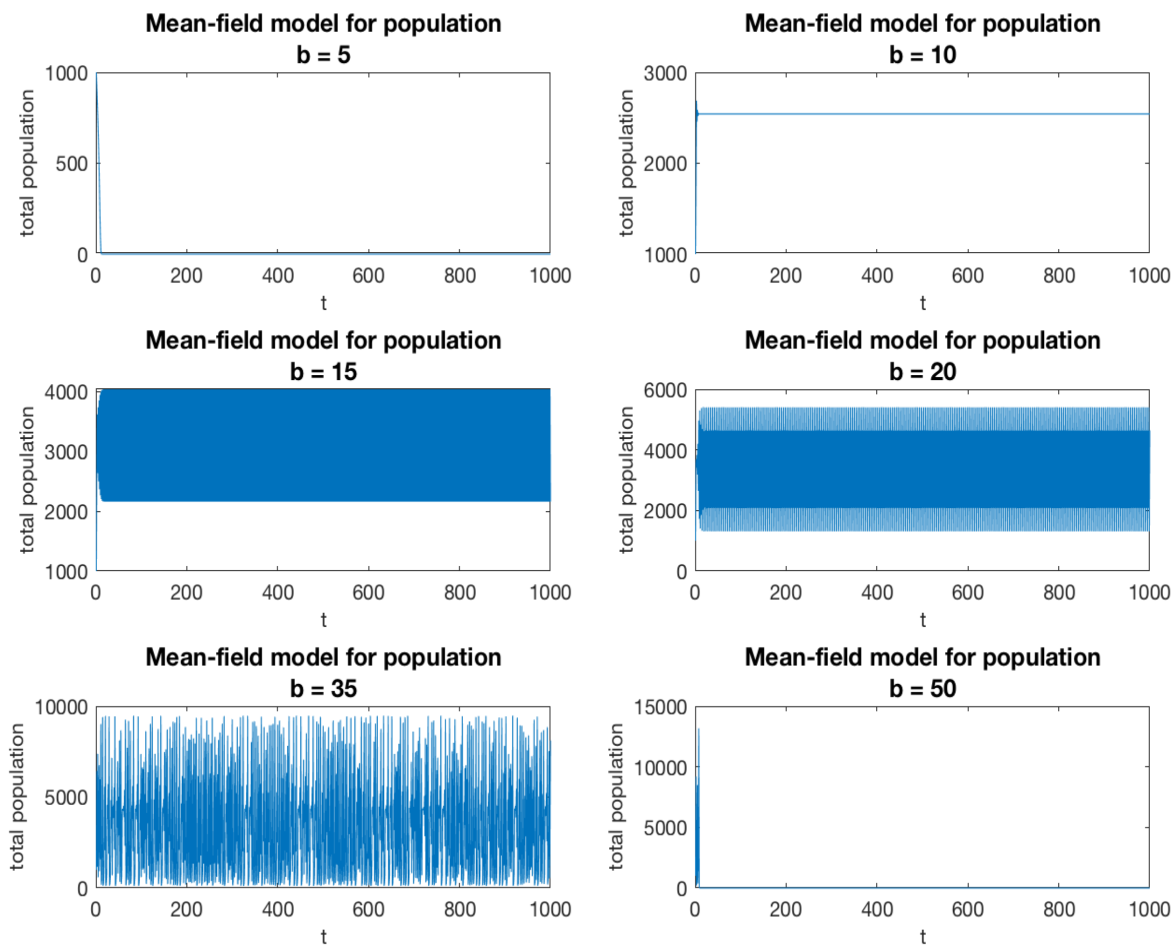

Figure 10: mean field model with selected b value

$$\begin{cases} E(A_{t+1}|A_t) = n \times \sum_{k=0}^{n} P_k \times \phi_k & (1) \\ \phi_k = \begin{cases} b, if k = 2 \\ 0, if k \neq 2 \end{cases} & (2) \end{cases}$$

following the mean field model, and the population only reproduce if there are exactly 2 individuals at same site, we can write the following:

$$A_{t+1} = n \times P(2 at site, A_t) \times b \tag{3}$$

$$A_{t+1} = n \times \frac{(\frac{A_t}{n})^2 \times e^{-\frac{A_t}{n}}}{2} \times b \tag{4}$$

For steady state, we have $A_{t+1} = A_t$.

$$A_t = n \times \frac{(\frac{A_t}{n})^2 \times e^{-\frac{A_t}{n}}}{2} \times b \tag{5}$$

When $A_t = 0$, left hand side always equal to right hand side, they are both 0. When $A_t \neq 0$, we have

$$1 = \frac{b \times A_t}{2n} \times e^{-\frac{A_t}{n}} \tag{6}$$

Let's rearrange equation (6) and let $\frac{A_t}{n} = x$, we have:

$$1 = \frac{b}{2} \times x \times e^{-x} \tag{7}$$

$$x \times e^{-x} = \frac{b}{2} \tag{8}$$

To find the conditions in terms of b for the existence of two further non-zero steady states, we need to find that $f1(x) = x \times e^{-x}$ has intersection with a horizontal line. b need to fulfill the condition that $\frac{2}{b} < 0.3679$, therefore we get: $b > 5.4366$, as b needs to be integer, we get $b > 5$.

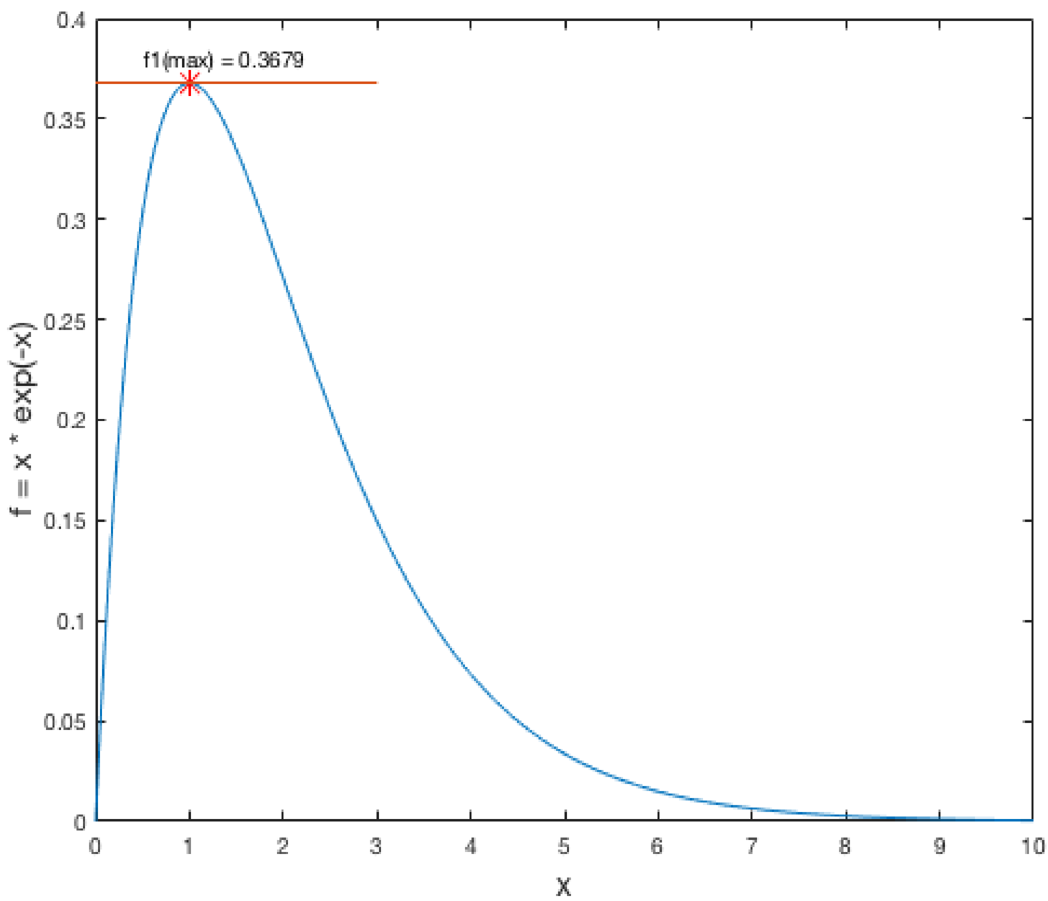

Figure 11: plot of f1(x) = x * exp(-x)

1.3 Lyapunov exponent

From the previous section, we have derived the equation for A_t. that we have

$$f(A_t) = n \times \frac{(\frac{A_t}{n})^2 \times e^{-\frac{A_t}{n}}}{2} \times b \tag{9}$$

$$f(a) = \frac{b}{2n} \times a^2 \times e^{-\frac{a}{n}} \tag{10}$$

$$f'(a) = \frac{b}{2n} \times (2a \times e^{-\frac{a}{n}} + a^2 \times e^{-\frac{a}{n}} \times (-\frac{1}{n})) \tag{11}$$

$$f'(a) = \frac{b}{2n} \times (2a \times e^{-\frac{a}{n}} - \frac{a^2}{n} \times e^{-\frac{a}{n}}) \tag{12}$$

$$|\Delta a_n| = |\Delta a_0| e^{\lambda n} \tag{13}$$

to compute Lyapunov exponent λ numerically, we use the following:

$$\lambda = \frac{1}{n} \sum_{t=0}^{n-1} ln|f'(a_t)| \tag{14}$$

The Lyapunov exponent are plotted below: we can see that when $\lambda > 0$ the diverges, that's when b is between 21-30, the population become chaotic. When $b \leq 5$ and $b \geq 31$ λ is -infinity that means the system converge very fast, and this explains why the population become extinct very fast for large b.

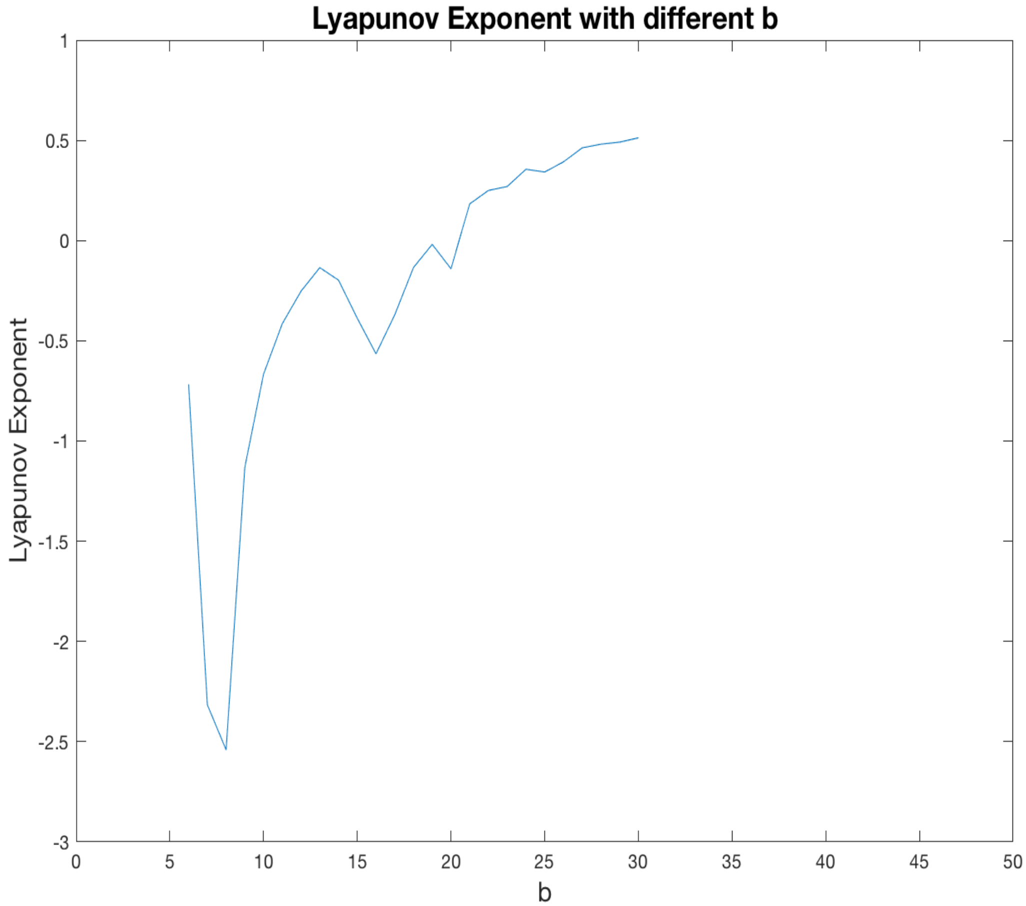

Figure 12: plot of Lyapunov exponent

Below are the results for a 40 x 40 grid where initially each cell has a probability of 0.3 being a firing(1) cell and all other neurons are ready. the figures below shows how the cells looks like after 10, 20, 100 and 1000 time steps.

The initial probability of being in a firing state is 0.3, which means at t = 0, there are around 480 cells that are being in a firing state. as we can see in figure 16, the total number of firing cell decreases over time. In the beginning, the number of cell decreases very fast, and the total number of cells gets stable around t = 300 400. When simulate 100 times to t = 1000, the average firing cell at t

= 400 is around 14 and at t = 1000, the average firing cell is around 12. At the equilibrium state, the shapes that remains are travelling forward at a constant rate preserving the same shape either in the same direction(up/down or left/right), or will never interact if there are shape that travel in the up/down direction and others in left/right direction. Over 100 simulation, the curve of average firing cell decreases in an exponential model, and a exponential model of y = a* exp(b*x) + c*exp(d*x) was fitted to the curve in figure 17.

fire10.png

Figure 13: 40x40 cell grid at t = 10

fire20.png

Figure 14: 40x40 cell grid at t = 20

fire100.png

Figure 15: 40x40 cell grid at t = 100

fire1000.png

Figure 16: 40x40 cell grid at t = 1000

firebrain100sim.png

Figure 17: average number of firing cells over time with 100 simulation of different initial conditions

sim_model.png

Figure 18: average number of firing cells over time with 100 simulation and exponential model fitting

and here are the links of video you can check out.

for simulation to t = 100

<https://youtu.be/CFUcpGHhj0O>

for simulation to t = 1000

<https://youtu.be/8EulLy_IRmw>

1.4 example of shapes

1.4.1 move forward at a rate of one cell per time step, while preserving the same shape

task2_1.png

Figure 19: shapes that move forward at a rate of one cell per time step preserving the same shape

1.4.2 move forward at a rate of one cell per time step, launching other shapes behind them

task2_2.png

Figure 20: shapes that move forward at a rate of one cell per time step, launching other shapes behind them

1.4.3 move forward at a rate of less than one cell per time step, while returning to the same shape after some period

task2_3.png

Figure 21: shapes that move less than one cell per time step, returning to same shape after some period

1.4.4 stay stationary but oscillate periodically

task2_4.png

Figure 22: shapes that stay stationary but oscillate periodically

1.5 create cellular automata

In this part, I create my own cellular automata. I used the two model in this project (a firing brain and spread of memes), Conway's game of life and several videos I watched online showing cellular automata as reference. I create the following to simulate a population the different life stage.

The states are:

- Waiting(0)
- Growing(1)
- Reproducing(2)
- Ageing (3)
- Dead (4)

The rules for the next time steps are:

- The waiting (0) cell needs at least 2 neighbour that are in the stage of reproducing(2) to be in grow. otherwise, it stays waiting.
- The growing (1) cell needs at least 1 neighbour that is in the stage of reproducing(2) and ageing(3) to become a reproducing (2) cell in the next time step. otherwise, it stays growing.
- The reproducing(2) celll has the probability of 0.5 to be ageing and 0.5 to remain reproducing
- The ageing cell (3)has the probability of 0.5 to be dead and 0.5 to remain ageing.
- The dead cell(4) has the the probability of 0.6 to be in waiting in the next time step, and 0.4 remain dead.

To simulate, I set up the initial condition:

- each cell has the probability of 0.05 to be a growing cell(1)
- each cell has the probability of 0.05 to be a reproducing cell(2)

- each cell has the probability of 0.15 to be in ageing (3)
- each cell has the probability of 0.10 to be dead(4)

I created this to simulate life from a society that is dominated by ageing population. The population will cluster together in the beginning and then different shapes will interact with each other.

mycell_automata.png

Figure 23: my cell automata at t = 10

and here are the links of video you can check out.

for simulation to t = 100

<https://youtu.be/gQ4c2FFDzDA>

for simulation to t = 1000

<https://youtu.be/OWazMF6is1c>

2 Spread of memes

In this part, a model is used to simulate the spread of internet memes. There are 3 different states, resting(0), sharing(1) and bored(2). The rules for the next time step are:

1. with probability p = 0.001, a person at rest will discover a new meme and become a sharer.(0 $\rightarrow$ 1 with p = 0.001)
2. with probability q = 0.01, a person sharing(1) will pick one person completely at random from the population to share the memes with. if the random person is at rest(0), that person will become a sharer(1), if that person is bored(2), then the sharing person will become bored(2).
3. bored(2) stays bored(2) forever. (2 is always 2).

2.1 some simulations in matlab

The simluation in matlab will run the model 1000 times with a population of 1000 to time at 2000 and show the change of number of resting, sharing and bored person over time. The initial condition is that there are one person sharing and one bored person. and below are the graphs showing the simulation.

memes_sim_1000times.png

Figure 24: simulation of spread of memes showing number of bored, sharing, resting person over time

The mean field difference equation model for the sharing of meme is:

Bored(B), Sharing(S), Resting(R), population(N).

$$
\begin{cases}
B(t+1) = B(t) + S(t) * q * B(t)/N & (15) \\
S(t+1) = S(t) + p * R(t) - S(t) * q * B(t)/N + S(t) * q * R(t)/N & (16) \\
R(t+1) = R(t) - R(t) * p - S(t) * q * R(t)/N & (17)
\end{cases}
$$

The figure below shows both the simulation and the mean field model memes_withmodel1000.png

Figure 25: simulation of spread of memes showing number of bored, sharing, resting person over time with mean field model

phasetransition.png

Figure 26: phase transition of total sharing person with simulation with t = 1000 and different B(0) condition

indi_phase.png

Figure 27: plot of sharing person in individual runs at t =1000 for 100 simulations of B0 = 1:1:999

To find the probability of at least 25% of the populations share a meme. I run the model with B(0) = 1:1:999 for 100 times to final time = 1000. and plot the probability in a heat map.

probability250_100sim.png

Figure 28: probability of more than 250 people are sharing with different B0 and time from 0 to 1000

2.2 changed the condition of a bored person

One condition was added to the bored person. A bored person will pick on person a random from the population with probability of q. If that person is resting then the bored person will become resting. otherwise she will continue to be bored. It was simulated in matlab for 1000 times with a population of 1000 to time = 2000 and at the same time plot together with the mean field model. The results were very different compared to the previous simulation. It seems that the model shows oscillation of bored person and sharing person over time but not in the simulation. In the model, we allow decimals for the number of persons. that's why the number of bored persons can slowly increase. In the simulation, it started with 1 bored person, and if that person meets someone at rest with probability of q. Then this bored person will be resting too. With this condition, a bored person soon finds a person at rest and then bored persons will be 0. The sharing person will increase gradually as a resting person will find a meme with $p = 0.001$. In the simulation one person can only be in one of the three states, that's why the simulations shows different results.

memes2_sim_1000times.png

Figure 29: 100 simulations of memes with different rules for bored person over t = 2000

memes2_withmodel1000.png

Figure 30: 100 simulations of memes with different rules for bored person over t = 2000 with mean field model plot

A phase transtion over q = 0.01:0.01:1 was made for the simulation. it is interesting to see that with probability q a person interact with another either to share or transtion from bored to rest. from q = 0.01 to q = 0.1, the total number of sharing persons at t = 1000 increase rapidly and almost everyone was sharing in the end when p > 0.1.

phasetransition2.png

Figure 31: phase transition of total sharing person with simulation with t = 1000 and different q condition

indi_phase2t1000withq.png

Figure 32: plot of sharing person in individual run at t = 1000

2.3 simulate on grids of 40x40 for spread of memes

In this part, we use the model above and in 2.2 to simulate that the person can only interact with the neighbours. the boundary conditions are set to be periodic. so that it goes from left to right, right to left, up to bottom, and bottom to up. After several runs. it can be observed that the number of sharer are increasing steadily. while the number of bored people tend to remain around at 1.the only way that bored person increase is that a sharing person meets a bored person. in theory the probability is p

= 1* 0.01*(1/1600). For the majority resting population, with a chance of 0.001 to discover a memes and become a sharer. in the initial state, there were 1588 resting people, that means that in theory 1.5 people will become a sharer. That's why it grows very fast, when t gets to 1000. almost everyone is sharing.

`memegrid.png`

Figure 33: initial condition for simulation

memegrid100.png

Figure 34: simulation of spread of meme to t = 100

The video of the simulation can be found at `<https://www.youtube.com/watch?v=WMfI2P52ros>`

3 Appendix

3.1 1firing brain code in matlab

- simulate single time of fire brain

- transition function

- initial state

- simulate 100 times of firing brain

- cell that move forward at one cell per time preserving the same shape

- cell that move forwad at one cell per time, launching other shapes behind them

- move forward at a rate of less than one cell per time step

- oscillate shape

- my cellular automata

- my cellular automata transit

3.2 Spread of memes

- simulation of spread of memes
- run single spread of memes
- mean field model
- phase transition
- probability for at least 25% are sharing
- simulation of spread of memes with new rules
- run single spread of memes
- mean field model
- phase transition for new rules
- lattice simulation for memes
- transition function for memes