1. Consider the monthly simple returns of Procter-Gamble (PG) stock and the Standard & Poor's 500 index from January 1961 to December 2010. The file has 3 columns namely date, PG, and SP5. It also has column names on top of the file.

   a) Compute the sample means, sample covariance matrix, and sample correlation matrix thee variables.

   b) Use different visualization plots to get a deeper understanding of the data, and summarize your findings.

   c) Split the data set into five decades, 1961-1970, 1971-1980, 1981-1990, 1991-2000, and 2001-2010 (similar to the "irris" data with three species), then repeat part (b).

2. Suppose that

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \sim N_4\left(\begin{pmatrix} 9 \\ 16 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 0.8 & -0.4 & 0 \\ 0.8 & 1 & -0.56 & 0 \\ -0.4 & -0.56 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right).$$

   a) Find the distribution of X_3.

   b) Find the distribution of $(X_2, X_4)^T$.

   c) Which pairs of random variables X_i and X_j are independent?

   d) Find the correlation $\rho(X_1, X_3)$.

3. Recall that if $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then the conditional distribution of $\boldsymbol{X}_1$ given that $\boldsymbol{X}_2 = \boldsymbol{x}_2$ is multivariate normal with mean $\boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\boldsymbol{x}_2 - \boldsymbol{\mu}_2)$ and covariance $\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$.

   Let $\sigma_{12} = \mathrm{Cov(Y,X)}$ and suppose Y and X follow a bivariate normal distribution

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N_2\left(\begin{pmatrix} 15 \\ 20 \end{pmatrix}, \begin{pmatrix} 64 & \sigma_{12} \\ \sigma_{12} & 81 \end{pmatrix} \right).$$

   a) If $\sigma_{12} = 10$ find $E(Y|X)$.

   b) If $\sigma_{12} = 10$, find $\mathrm{V}(Y|X)$.

   c) If $\sigma_{12} = 10$, find $\rho(Y, X)$, the correlation between Y and X.

   d) What is σ_{12} if Y and X are independent?

4. Let $\boldsymbol{X}$ be an $n \times p$ constant matrix and let $\boldsymbol{\beta}$ be a $p \times 1$ constant vector. Suppose $\boldsymbol{Y} \sim N_n(\boldsymbol{X}\boldsymbol{\beta}, \sigma^2 \boldsymbol{I})$. Find the distribution of $(\boldsymbol{I} - \boldsymbol{H})\boldsymbol{Y}$ if $(\boldsymbol{I} - \boldsymbol{H})^T = (\boldsymbol{I} - \boldsymbol{H}) = (\boldsymbol{I} - \boldsymbol{H})^2$ is an $n \times n$ matrix and if $\boldsymbol{H}\boldsymbol{X} = \boldsymbol{X}$. Simplify.

5. **Reading assignments:** Chapters 1 to 4 of the textbook.