2. Precision and Stability

Present your solutions to the following problems using latex, if you have figures make sure they are publication quality, include your code in the solutions.

1. Determine your machine precision to within a factor of two in double and single precision.

2. Within a factor of two, determine the minimum and maximum single and double precision real numbers that your computer supports.

3. Implement a double precision subroutine for the spherical Bessel functions, $j_n(x)$, using the recursion relation

$$j_{n+1}(x) + j_{n-1}(x) = \frac{2n+1}{x} j_n(x)$$

and the initial conditions

$$j_0(x) = \frac{\sin x}{x} \qquad\qquad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}.$$

Compare the results for $x \in (0, 1)$ for j_4, j_5, j_6 to the asymptotic expression given by

$$j_n(x) \to \frac{x^n}{(2n+1)!!}\left(1 - \frac{x^2}{2(2n+3)}\right).$$

Plot the relative error in each case (include the plots in your solution). Explain what you see.

4. The Lanczos algorithm is a method to tri-diagonalise a Hamiltonian by evaluating H in a basis that is generated from an initial vector. The basis is constructed with repeated applications of H, and therefore all basis vectors retain the symmetry of the initial vector. Once the ground state has been obtained, this property can be used to study excited states by ensuring that the initial vector is orthogonal to the ground eigenstate. Discuss the *stability* of this algorithm for (i) ground state computations, (ii) excited state computations. We will study the Lanczos algorithm later in the course.