Design of 3rd Butterworth Digital Filter
Using Convolution process and Python language

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Contents

1 Aim of the research work

Design of 3rd order Butterworth filter using Convolution process and Python language.

2 Problem statement

Noise is an unwanted signal which interferes and corrupts the parameters of the message signal. This alteration in the message signal, leads to the disturbance and degrades the quality of the signal.

In general, for removal of the noise, various filters are used. But, still there are chances of noise in the filtered signal.

In this research work, this problem has been overcome by designing 3rd order Butterworth filter using Convolution process and Python language.

3 Input parameters

The input parameters: A noisy audio signal (audio tone) in wav format and a Spectrum analyzer.
This noisy audio signal can be downloaded from the following link.

https://github.com/meertabresali-FWC-IITH/Module2/blob/master/ButterworthFilter/Input-noisy-tone.wav

Let the input noisy audio signal is denoted by $\mathbf{x(n)}$.

4 Spectrum of noisy audio signal

The noisy audio signal is taken on Spectrum analyzer. Figure 1 shows its spectrum. It can be seen that the signal content is available upto 4kHz (yellow marks) and noise is available upto 21.6kHz (red marks).

From the figure 1, we can consider the upper cut off frequency as 4kHz, because there is no signal content above 4kHz.

$$f_h = 4kHz$$

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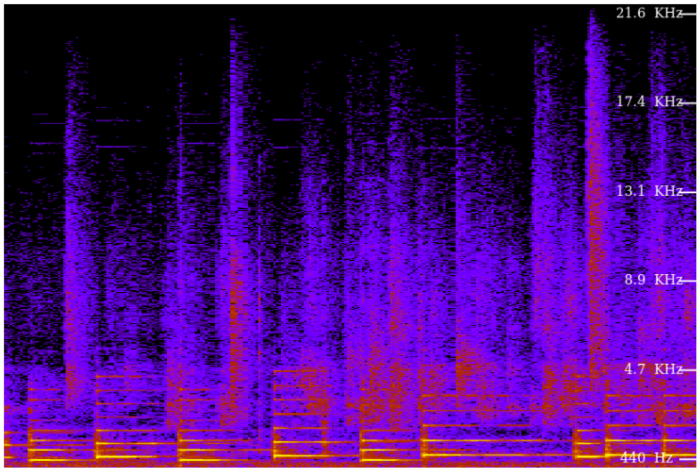


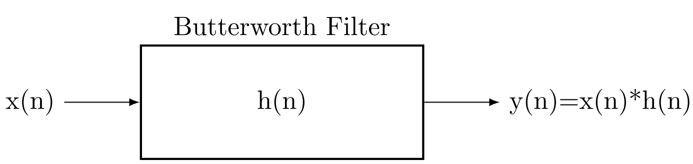
Figure 1: Spectrum of noisy audio signal $\mathbf{x(n)}$

5 Design of 3rd order Butterworth filter

Let the impulse response of the 3rd order butterworth filter is denoted by $\mathbf{h(n)}$.

Let the output audio signal (filtered audio signal) is denoted by $\mathbf{y(n)}$.

5.1 Block diagram of Butterworth filter



Block diagram of Butterworth filter

5.2 Coefficients of 3rd order Butterworth filter

General form of impulse response in Z-transform is

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}} \tag{5.2.1}$$

For 3rd order Butterworth filter, the co-efficients are as follows,

Denominator Coefficients:

$$a_0 = 1.00000000$$
$$a_1 = -1.87302725$$
$$a_2 = 1.30032695$$
$$a_3 = -0.31450204$$

Numerator Coefficients:

$$b_0 = 0.01409971$$
$$b_1 = 0.04229913$$
$$b_2 = 0.04229913$$
$$b_3 = 0.01409971$$

Impulse response given in equation 5.1.1 is in Z domain. It can be converted into time domain by applying inverse Z-transform as,

$\mathbf{h(n)} = Z^{-1}[H(Z)]$

$= 0.0140997$, for $n = 0$

$= (0.640832 - 3.68314 * 10^{(-17)} j) * 0.546882^n$
$-(0.29095 - 0.141431j) * (0.663072 - 0.36799j)^n$
$-(0.29095 + 0.141431j) * (0.663072 + 0.36799j)^n$, for $n > 0$

5.3 Plot of Impulse response

From the above equation of $\mathbf{h(n)}$, Impulse response is plotted using Python code. Plot of Impulse response is shown in the figure 2.

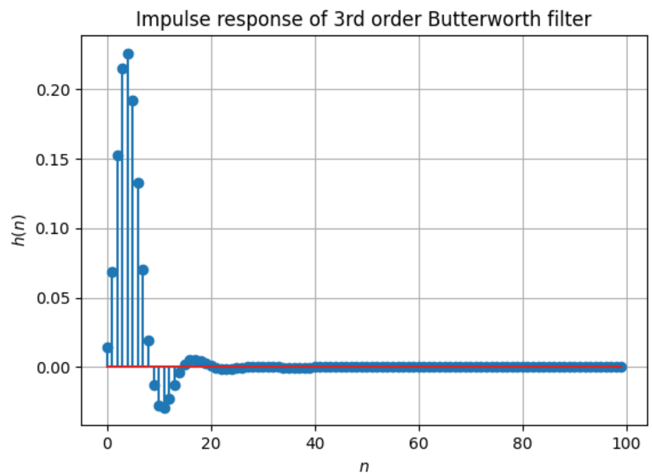


Figure 2: Impulse response of 3rd order Butterworth filter

5.4 Response of 3rd order Butterworth filter

The output response can be calculated in the following Two different ways.

5.4.1 Convolution process in Z-domain

Let the given Input noisy audio signal is represented in Z-domain as $\mathbf{X(Z)}$ and the Impulse response as $\mathbf{H(Z)}$. Then the output response of the filter can be obtained by applying Convolution process in Z-domain and taking inverse Z-transform as,

$$\mathbf{Y(Z)} = \mathbf{X(Z)} * \mathbf{H(Z)} \tag{5.4.1}$$

And,

$$\mathbf{y(n)} = Z^{-1}[\mathbf{Y(Z)}] \tag{5.4.2}$$

5.4.2 Convolution process in Time domain

On applying Convolution process directly in Time domain on Input noisy audio signal $\mathbf{x(n)}$ and Impulse response $\mathbf{h(n)}$, we get,

$$\mathbf{y(n)} = \mathbf{x(n)} * \mathbf{h(n)} \tag{5.4.3}$$

where y(n) is the filtered signal. It is the output response of 3rd order Butterworth filter.

6 Spectrum of output signal

Audio signal derived from the equation 5.4.2 (or 5.4.3) is in time domain.
It is the Output response of 3rd order Butterworth filter.
It is a reduced noise signal.

After applying this output signal on Spectrum analyzer, we will get the Spectrum of the output response of 3rd order Butterworth filter and it is shown in figure 3.

From the figure 3, it can be noticed that the noise is reduced upto 4kHz.

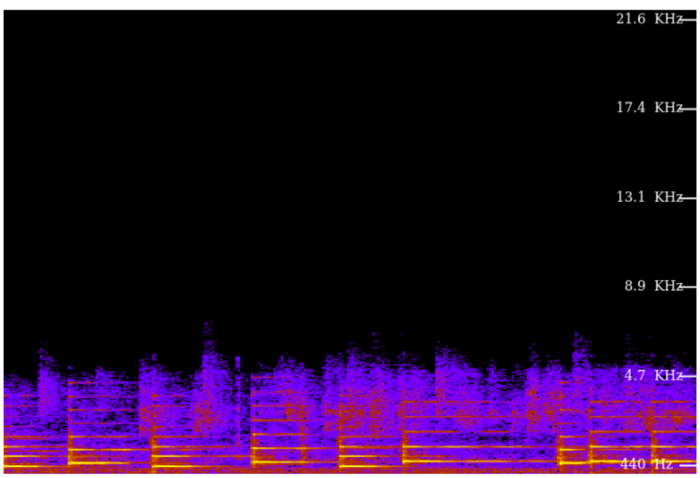


Figure 3: Spectrum of output audio signal $\mathbf{y(n)}$

7 Code in Python language

Python code can be downloaded from the following link.

https://github.com/meertabresali-FWC-IITH/
/Module2/blob/master/ButterworthFilter/codes/
ButterworthFilter.py

8 Conclusion

This research work, can be concluded with the following points.

1. Noise is an unwanted signal which interferes and corrupts the parameters of the message signal. This alteration in the message signal, leads to the disturbance and degrades the quality of the signal.

2. In general, for removal of the noise, various filters are used. But, still there are chances of noise in the filtered signal.

3. In the present research work, a noisy audio signal (tone) in wav format is taken as message signal.

4. It has been aimed to remove the noise upto 3rd order as this can reduce the noise as maximum as possible.

5. For achieving this, Butterworth filter has been designed by defining its impulse response upto 3rd order and then the noise is removed by using Convolution process.

6. The codes have been written in Python language and the output signal is generated by running the Python codes.

7. Input and output signals are taken on spectrum analyser. The spectrum of output signal can bee seen noise free.

9 References

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