Design of 3rd order Butterworth Digital Filter
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1 Aim of the research work

Design of 3rd order Butterworth filter using convolution process and Python language.

2 Problem statement

Audio signals may contain noise. For removal of the noise, various filters are used. But there are chances of noise with filtered audio signal.
In the present research work, noise is removed upto 3rd order. This problem can be overcome by designing 3rd order Butterworth filter using convolution process and Python language.

3 Input parameters

The input parameters: a noisy audio signal in wav format and a Spectrum analyzer
It can be downloaded from the following link.

https://github.com/meertabresali-FWC-IITH/Module2/blob/master/ButterworthFilter/Input-noisy-tone.wav

Let the input noisy audio is denoted by $\mathbf{x(n)}$

4 Spectrum of noisy audio signal

The given audio signal is taken on spectrum analyzer. Figure 1 shows its spectrum. It can be seen that the signal content is available upto 4kHz (yellow marks) and noise is available upto the top 21.6kHz (red marks).

From the figure 1, we can consider the upper cut off frequency as 4kHz, because there is no signal content above 4kHz.

$$f_h = 4kHz$$

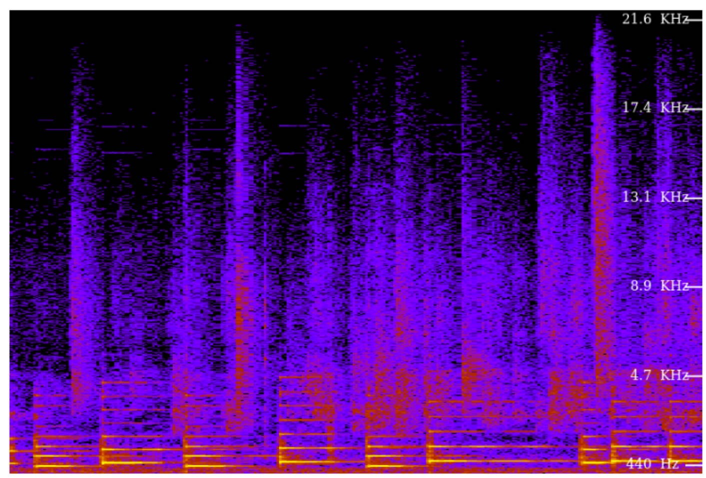


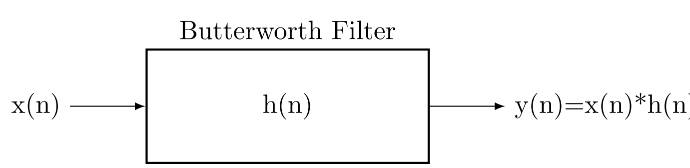
Figure 1: Spectrum of noisy audio signal

5 Design of 3rd order Butterworth filter

Let the input audio signal (in wav file) is denoted by $\mathbf{x(n)}$.

Let the impulse response of the 3rd order butterworth filter is denoted by $\mathbf{h(n)}$.

Let the output audio signal (filtered audio signal) is denoted by $\mathbf{y(n)}$.



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Block diagram of Butterworth filter

5.1 Coefficients of 3rd order Butterworth filter

General form of impulse response in Z-transform is

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}} \tag{5.1.1}$$

For 3rd order Butterworth filter, the co-efficients are as follows

Denominator Coefficients:

a_0= 1.00000000
a_1=-1.87302725
a_2= 1.30032695
a_3=-0.31450204

Numerator Coefficients:

b_0= 0.01409971
b_1= 0.04229913
b_2= 0.04229913
b_3= 0.01409971

Impulse response given in equation 5.1.1 is in Z domain. It can be converted into time domain by applying inverse Z-transform as,

$\mathbf{h(n)} = Z^{-1}[H(Z)]$

$= 0.0140997$, for $n = 0$

$= (0.640832 - 3.68314 * 10^{(-17)}j) * 0.546882^n$
$-(0.29095 - 0.141431j) * (0.663072 - 0.36799j)^n$
$-(0.29095 + 0.141431j) * (0.663072 + 0.36799j)^n$, for $n > 0$

5.2 Plot of Impulse response

From the above equation of $\mathbf{h(n)}$, Impulse response is plotted using python code.

Plot of Impulse response is shown in the figure 2.

5.3 Response of 3rd order Butterworth filter

The output response can be found in the following Two different ways.

1. By applying convolution on input $\mathbf{X(Z)}$ and impulse response $\mathbf{H(z)}$ in Z-domain and taking inverse Z-transform :

Let the given audio signal is represented in Z transform as $\mathbf{X(Z)}$. Then the output response of the filter can be written as,

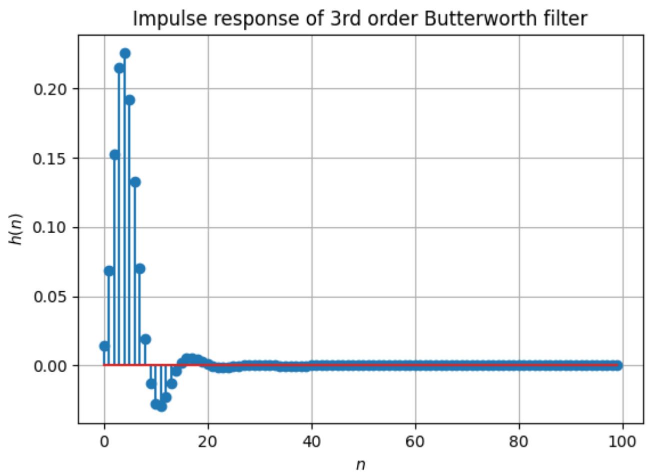

Figure 2: Spectrum of noisy audio signal

$$\mathbf{Y(Z) = X(Z) * H(Z)} \tag{5.3.1}$$

from the above equation, the output response of the filter can be found by taking inverse Z-transform,

$$\mathbf{y(n)} = Z^{-1}[\mathbf{Y(Z)}] \tag{5.3.2}$$

2. By applying convolution directly on input signal $\mathbf{x(n)}$ and impulse response $\mathbf{h(n)}$:

$$\mathbf{y(n) = x(n) * h(n)} \tag{5.3.3}$$

where y(n) is the filtered signal. It is the output response of 3rd order Butterworth filter.

6 Spectrum of output signal

Audio signal derived from the equation 5.2.2 (or 5.2.3) is in time domain.
It is a reduced noise signal.

After applying this output signal on Spectrum analyzer, we will get the spectrum of the output response of 3rd oreder Butterworth filter and it is shown in figure 2.

From the figure 3, it can be noticed that the noise is reduced upto 4kHz.

7 Code in Python language

Python code can be downloaded from the following link.

https://github.com/meertabresali-FWC-IITH/
/Module2/blob/master/ButterworthFilter/codes/
ButterworthFilter.py

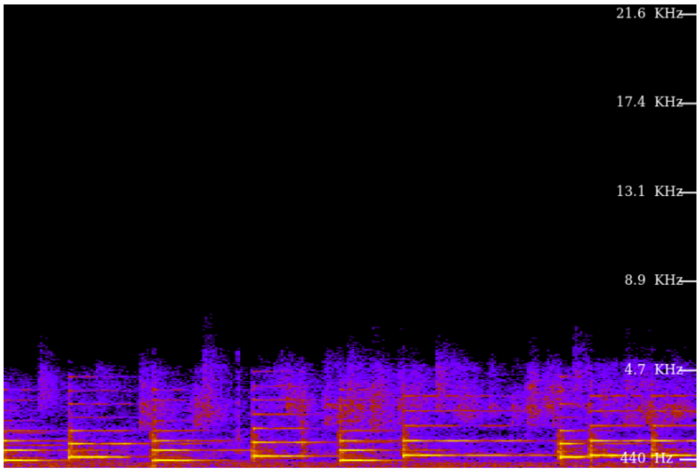

Figure 3: Spectrum of output audio signal

8 Conclusion

This research work, can be concluded with the following points.

1. Noise is an unwanted signal which interferes and corrupts the parameters of the message signal. This alteration in the message signal, leads to the disturbance and degrades the quality of the signal.

2. In general, for removal of the noise, various filters are used. But, still there are chances of noise in the filtered signal.

3. In the present research work, a noisy audio signal (tone) in wav format is taken as message signal.

4. It has been aimed to remove the noise upto 3rd order and this can reduce the noise as maximum as possible.

5. For achieving this, Butterworth filter has been designed by defining its impulse response upto 3rd order and then the noise is removed by using convolution process.

6. The codes have been written in Python language and the output signal is generated by running the Python codes.

7. Input and output signals are taken on spectrum analyser. The spectrum of output signal can bee seen noise free.

9 References

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