Finding the maxima and minima of the function Using Advanced Optimization
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Meer Tabres Ali and G V V Sharma

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1 Problem statement

Determine the points of maxima and minima of the function $f(x)$=$\frac{1}{8}lnx - bx + x^2, x > 0$, where $b \geq 0$ is a constant.

2 Considerations

As per given data, the following table has been prepared.

Symbol	Value	Description
f(x)	$\frac{1}{8}lnx - bx + x^2$	function
b	7	b$\geq$ 0
a	$\frac{1}{8}$	Coeff. of lnx
c	1	Coeff. of x^2

Table 1: Considerations

*Meer Tabres Ali as an intern with FWC IIT Hyderabad. *The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

3 Plot to find maxima and minima of the function

Plot of the function $\frac{1}{8}lnx - bx + x^2$ is shown in the figure 1.

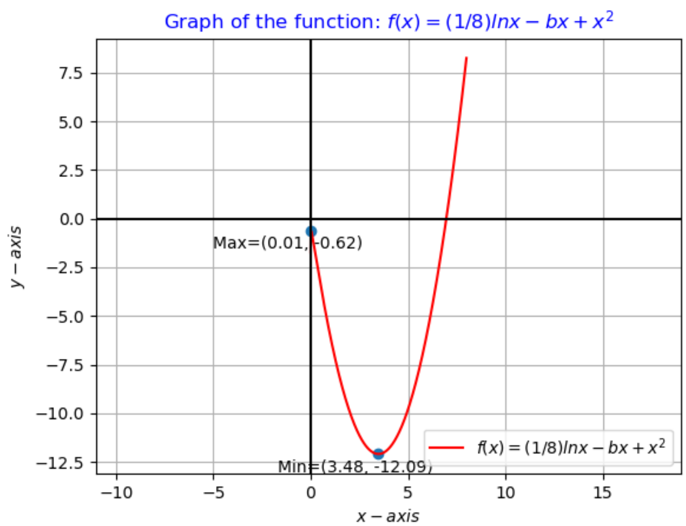

Figure 1: Plot of f(x) to find Maxima and Minima

4 Solution

Given function is,

$$f(x) = \frac{1}{8}lnx - bx + x^2 \tag{4.0.1}$$

where $x > 0$ and $b \geq 0$

4.1 Calculation of Maxima and Minima using normal differentiation

Differentiating above Eq. (4.0.1), we get,

$$\frac{df(x)}{dx} = \frac{d(\frac{1}{8}lnx - bx + x^2)}{dx}$$

$$\implies \frac{df(x)}{dx} = \frac{1}{8x} - b + 2x$$

$$\implies 0 = \frac{1}{8x} - b + 2x$$

$$\implies 16x^2 - 8bx + 1 = 0$$

On simplifying,

$$x_{max} = \frac{-b - \sqrt{b^2 - 1}}{4}, \quad x_{min} = \frac{-b + \sqrt{b^2 - 1}}{4} \tag{4.1.1}$$

Assuming b=7,

$$x_{max} = 0.01$$

$$x_{min} = 3.48$$

By submitting x_{max} and x_{min} in Eq. 4.0.1, we get,

$$f(x)_{max} = -0.62$$

$$f(x)_{min} = -12.09$$

Therefore,

$$Maximum\ value\ of\ x\ is\ \textbf{-0.62}\ \ at\ \ x = \textbf{0.01} \tag{4.1.2}$$

$$Minimum\ value\ of\ x\ is\ \textbf{-12.09}\ \ at\ \ x = \textbf{3.48} \tag{4.1.3}$$

4.2 Calculation of Maxima using gradient ascent algorithm

Maxima of the above equation (4.0.1), can be calculated from the following expression,

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{4.2.1}$$

$$\implies x_{n+1} = x_n - \alpha \left(\frac{1}{8}(1/x_n) - 7 + 2x_n \right) \tag{4.2.2}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Maxima = -0.62}}$$

$$\boxed{\text{Maxima Point = 0.01}}$$

4.3 Calculation of Minima using gradient descent algorithm

Maxima of the above equation (4.0.1), can be calculated from the following expression,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{4.3.1}$$

$$\implies x_{n+1} = x_n + \alpha \left(\frac{1}{8}(1/x_n) - 7 + 2x_n \right) \tag{4.3.2}$$

Taking $x_0 = 1.5, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Minima = -12.09}}$$

$$\boxed{\text{Minima Point = 3.48}}$$

5 Software

Download the codes given in the link below and execute them.

https://github.com/meertabresali-FWC-IITH/project/blob /main/Asgn8.opt.advance/codes/optadv.py

6 Conclusion

1. At first, the given function has been differentiated and it is solved by setting f'(x) equal to zero. By using x values, f(x) values are calculated.

2. Later, the given function f(x) is solved by gradient ascent algorithm to find maxima and the point at which f(x) is maximum.

3. Then, the given function f(x) is solved by gradient descent algorithm to find minima and the point at which f(x) is is minimum.

4. Maxima and Minima and related points are,

Maxima point, Max=(0.01, -0.62) and

Minima point, Min=(3.48, -12.09)