

Map Reduce and PageRank

Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer i and produces the list of pairs (p, i) such that p is a prime divisor of i . For example, $\text{map}(12) = [(2, 12), (3, 12)]$.

The reduce function is addition. That is, $\text{reduce}(p, [i_1, i_2, \dots, i_k])$ is $(p, i_1 + i_2 + \dots + i_k)$.

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

Map Tasks:

$\text{Map}(15) = [(3, 15), (5, 15)]$

$\text{Map}(21) = [(3, 21), (7, 21)]$

$\text{Map}(24) = [(2, 24), (3, 24)]$

$\text{Map}(30) = [(2, 30), (3, 30), (5, 30)]$

$\text{Map}(49) = [(7, 49)]$

Above outputs will be the inputs to the reduce tasks.

Reduce Tasks:

$\text{Reduce}(3, [15, 21, 24, 30]) = \text{reduce}(3, [15+21+24+30]) = (3, 90)$

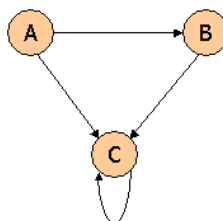
$\text{Reduce}(2, [24, 30]) = (2, [24+30]) = (2, 54)$

$\text{Reduce}(5, [15, 30]) = (5, 15+30) = (5, 45)$

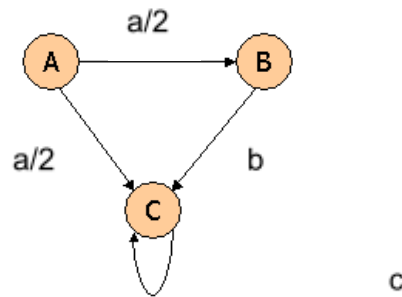
$\text{Reduce}(7, [21, 49]) = (7, 21+49) = (7, 70)$

Question 2:

Consider three Web pages with the following links:



Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRanks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRanks a , b , and c of the three pages A, B, and C, respectively.



Here $n = 3$, $\beta = 0.7$
Page rank formula;

$$r_j = \sum_i \frac{r_i}{d_i} \beta + (1 - \beta)(1/n)$$

So, here for node A,

$$\begin{aligned} a &= 0 + (1 - 0.7) \frac{1}{3} \\ &= 0 + (0.3) \frac{1}{3} \\ &= 0 + 0.1 \\ \underline{a} &= \underline{0.1} \end{aligned}$$

For node B,

$$\begin{aligned} b &= \beta \cdot \frac{a}{2} + (1 - \beta) \frac{1}{3} \\ &= ((0.7)(0.1)(0.5)) + \frac{(1-0.7)}{3} \\ &= 0.035 + \frac{0.3}{3} \\ \underline{b} &= \underline{0.135} \end{aligned}$$

For node C,

$$\begin{aligned} c &= \beta c + \beta b + \beta \cdot \frac{a}{2} + (1 - \beta) \frac{1}{3} \\ &= (0.7)c + (0.7)(0.135) + \frac{(0.7)(0.1)}{2} \\ \underline{c} &= \underline{0.765} \end{aligned}$$

Now, $a + b + c = 0.1 + 0.135 + 0.765 = 1$

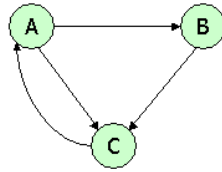
But the constraint is $a + b + c = 3$, so multiplying all the values with 3.

$$\begin{aligned} a &= 3 * 0.1 = 0.3 \\ b &= 3 * 0.135 = 0.405 \\ c &= 3 * 0.765 = 2.295 \end{aligned}$$

Now the value of, $a + b + c = 0.3 + 0.405 + 2.295 = 3$

Question 3:

Suppose we compute PageRank with $\beta=0.85$. Write the equations for the PageRanks a , b , and c of the three pages A, B, and C, respectively



3)

$n=3, \beta=0.85$

Page rank equation: $r_j = \left(\sum_{i \rightarrow j} \frac{x_i}{d_i} \beta \right) + \frac{(1-\beta)}{n}$

for node A: $a = \beta \cdot \frac{c}{1} + \frac{(1-\beta)}{3}$

$$= 0.85c + \frac{(1-0.85)}{3}$$

$$a = 0.85c + 0.05 \quad \text{--- ①}$$

for node B:

$$b = \beta \cdot \frac{a}{2} + \frac{(1-\beta)}{3}$$

$$= \frac{(0.85)(a)}{2} + \frac{(1-0.85)}{3}$$

$$b = 0.425a + 0.05 \quad \text{--- ②}$$

for node C:

$$c = \beta \cdot \frac{a}{2} + \beta \cdot b + \frac{(1-\beta)}{n}$$

$$= (0.85)(0.5)a + (0.85)b + \frac{0.15}{3}$$

$$c = 0.425a + 0.85b + 0.05 \quad \text{--- ③}$$

③ - ② $\Rightarrow c - b = 0.85b$

$$\boxed{c = 1.85b} \quad \text{--- ④}$$

④ in ① $\Rightarrow a = (0.85)(1.85)b + 0.05$

$$a = 1.5725(0.425a + 0.05) + 0.05$$

$$\Rightarrow a = 0.39$$

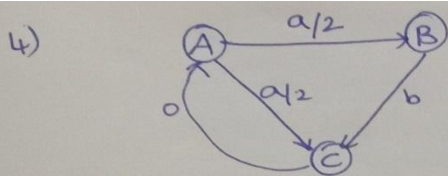
$$\Rightarrow b = 0.425(0.39) + 0.05 = 0.22$$

$$\Rightarrow c = 1.85(0.22) = 0.407$$

Question 4:



Assuming no "taxation," compute the PageRanks a , b , and c of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank $a = b = c = 1$. Compute as far as the 5th iteration, and also determine what the PageRanks are in the limit.



Initially $a=b=c=1$

page rank formula without taxation : $r_j = \sum_{i \rightarrow j} \frac{r_i}{d_i}$

0th iteration : $a=b=c=1$

1st iteration :

| | | |
|-------|-----------|---------------|
| $a=c$ | $b = a/2$ | $c = a/2 + b$ |
| $a=1$ | $b = 1/2$ | $c = 3/2$ |

2nd iteration :

| | | |
|-----------------------|-----------|---------------------|
| $a=c$ | $b = a/2$ | $c = a/2 + b$ |
| $\Rightarrow a = 3/2$ | $b = 1/2$ | $c = 1/2 + 1/2 = 1$ |

3rd iteration :

| | | |
|-------|-------------------|-------------------|
| $a=1$ | $b = \frac{3}{4}$ | $c = \frac{5}{4}$ |
|-------|-------------------|-------------------|

4th iteration :

| | | |
|-------------------|-------------------|-------------------|
| $a = \frac{5}{4}$ | $b = \frac{3}{2}$ | $c = \frac{5}{4}$ |
|-------------------|-------------------|-------------------|

5th iteration :

| | | |
|-------------------|-------------------|-------------------|
| $a = \frac{5}{4}$ | $b = \frac{5}{8}$ | $c = \frac{9}{8}$ |
|-------------------|-------------------|-------------------|

$a \text{ \& } c > 1$ and $b < 1$