UNIVERSITY OF LONDON

CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathematics

Release date: Monday 8 March 2021 at 12 midday Greenwich Mean Time

Submission date: Tuesday 9 March 2021 by 12 midday Greenwich Mean Time

Time allowed: 24 hours to submit

INSTRUCTIONS TO CANDIDATES:

Section A of this assessment paper consists of a set of **TEN** Multiple Choice Questions (MCQs) which you will take separately from this paper. You should attempt to answer **ALL** the questions in Section A. The maximum mark for Section A is 40.

Section A will be completed online on the VLE. You may choose to access the MCQs at any time following the release of the paper, but once you have accessed the MCQs you must submit your answers before the deadline or within **4 hours** of starting, whichever occurs first.

Section B of this assessment paper is an online assessment to be completed within the same 24-hour window as Section A. We anticipate that approximately **1 hour** is sufficient for you to answer Section B. Candidates must answer **TWO** out of the THREE questions in Section B. The maximum mark for Section B is **60**.

Calculators are not permitted in this examination. Credit will only be given if all workings are shown.

You should complete **Section B** of this paper and submit your answers as **one document**, if possible, in Microsoft Word or a PDF to the appropriate area on the VLE. You are permitted to upload 30 documents. However, we advise you to upload as few documents as possible. Each file uploaded must be accompanied by a coversheet containing your **candidate number**. In addition, your answers must have your candidate number written clearly at the top of the page before you upload your work. Do not write your name anywhere in your answers.

SECTION B

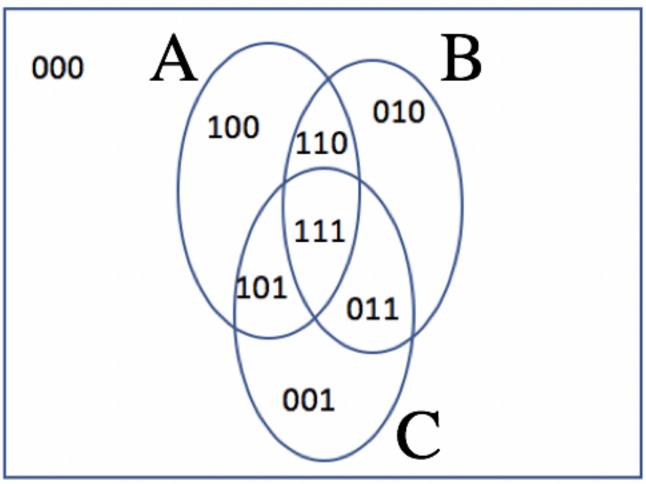
Candidates should answer any **TWO** questions from Section B.

Question 1

(a) Let A and B be two finite sets such that $|A| = 30$, $|B| = 35$ and $|A \cup B| = 45$, find $|A \cap B|$.

[3]

(b) Consider the following Venn diagram representing three sets A, B and C intersecting in the most general way. Three binary digits are used to refer to each one of the eight regions in this diagram. In terms of A, B and C, write the set representing the area comprising the regions 001, 010, 011, 100 and 101.



[3]

(c) Let p q and r be three propositions for which p and q are true, and r is false. Determine the truth value of for each of the following: [4]

i. $p \leftrightarrow (q \rightarrow r)$

ii. $p \rightarrow (r \rightarrow q)$

iii. $(p \oplus r) \rightarrow \neg q$

iv. $p \wedge (r \rightarrow q)$

(d) Let $P(x)$ be the statement " $x^2 > 1$ " and $Q(x)$ be the statement "$x+1 < 4$". The universe of discourse consists of all real numbers. What are the truth values for the following:

i. $\forall x(P(x) \rightarrow Q(x))$

ii. $\exists x(P(x) \rightarrow Q(x))$

iii. $\forall x(P(x) \wedge Q(x))$

iv. $\exists x(P(x) \wedge \neg Q(x))$

[8]

(e) Decide if the following arguments are valid or invalid. State the Rule of Inference or fallacy used.

i. If it snows, then school is closed
School is open
$\therefore$ it is not snowing.

ii. If the movie is long, I will fall asleep
I do fall asleep.
$\therefore$ the movie was long.

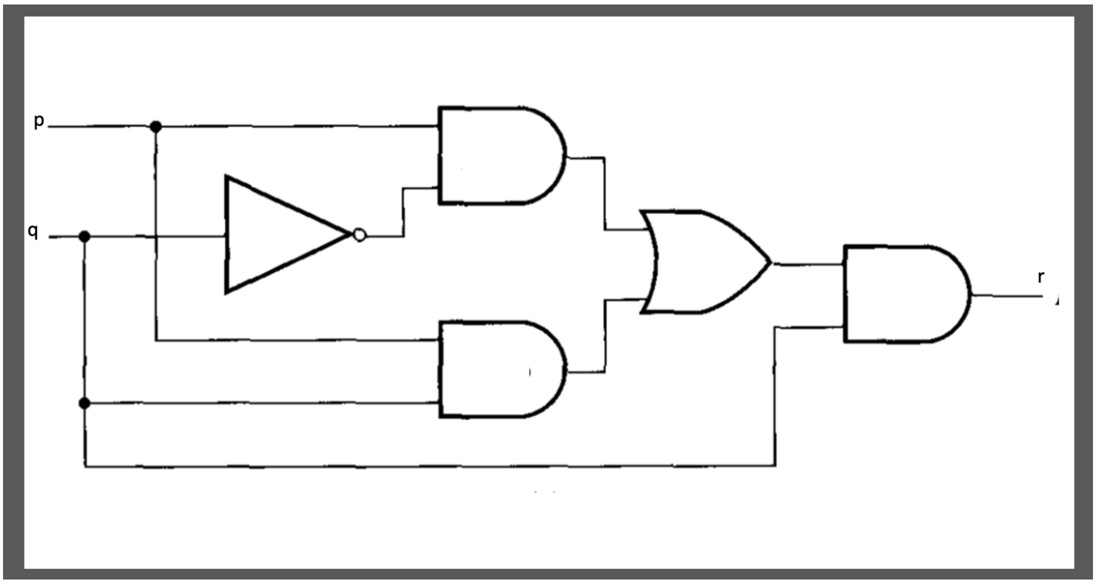
[4]

(f) Given a universal set U and three sets A, B and C, subsets of U. Prove that the expression $\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C})$ is equivalent to $\overline{A} \cup \overline{B} \cup \overline{C}$ by re-writing the expression using algebraic laws, state the name of each law used. [4]

(g) Suppose there are 6 questions in an exam paper , find the number of ways in which a student can attempt one or more questions. [4]

Question 2

(a) Given the following logical circuit with two inputs p and q:



i. Identify the logical gates used in this circuit. [2]

ii. Use the boolean algebra notation and write down the boolean expression of the output, r, of this circuit. [4]

iii. Simplify the logical expression in (ii). Explain your answer. [4]

iv. Draw the resulting simplified circuit. [2]

(b) i. List two properties a function has to satisfy to be a bijective function. [2]

ii. Consider the following function $f : \mathbb{R} \to \mathbb{R}^+$ with $f(x) = 2^{x+3}$.

(1) Show f is a bijective function. [4]

(2) Find the inverse function f^{-1}. [4]

(c) Let $S_n = \sum_{k=1}^{k=n} k * k!$ for all $n \in \mathbb{Z}^+$.

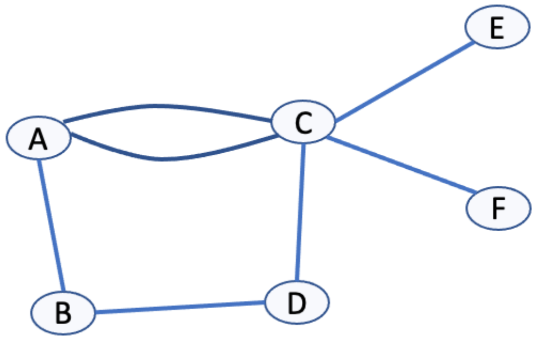
i. Find S_2 and S_3. [2]

ii. Prove by induction that $S_n = (n+1)! - 1$ for all $n \in \mathbb{Z}^+$. [6]

Question 3

(a) i. Give a definition of a Hamiltonian graph. [2]

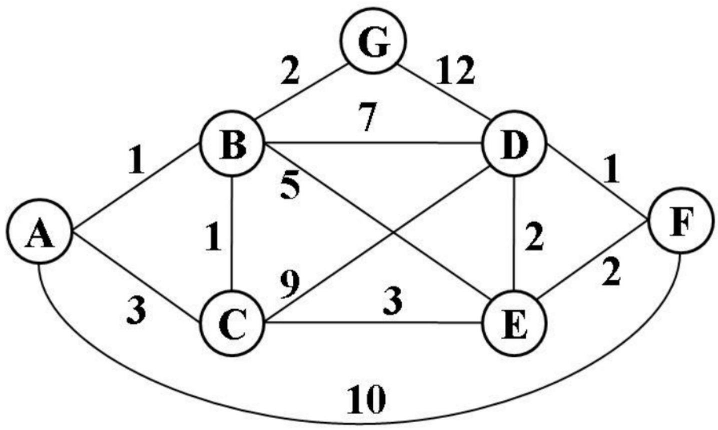
ii. Does the following graph have a Hamiltonian Circuit? Explain your answer.



[2]

(b) Draw the first 3 levels of a binary search tree which holds 4000 records and find the height of this tree. [5]

(c) Given the following undirected, weighted graph:



i. Use Dijkstra's algorithm to calculate the single-source shortest paths from A to every other vertex. [4]

ii. Find the lowest-cost path from the vertex A to F.

[2]

(d) Given S is the set of integers $\{2, 3, 4, 6, 7, 9\}$. Let $\mathcal{R}$ be a relation defined on S by the following condition such that,
for all $x, y \in S$, xRy if $3|(x - y)$ which means 3 divides $(x - y)$.

i. Draw the digraph of $\mathcal{R}$. [2]

ii. Say with reason whether or not $\mathcal{R}$ is

- reflexive;
- symmetric;
- anti-symmetric;
- transitive.

In the cases where the given property does not hold, provide a counter example to justify this. [4]

iii. is $\mathcal{R}$ a partial order? Explain your answer. [1]

iv. is $\mathcal{R}$ an equivalence relation? If the answer is yes, write down the equivalence classes for this relation and if the answer is no, explain why. [3]

(e) Use the proof by contradiction to show that for any integer n, $7n + 4$ is not divisible by 7. [5]

END OF PAPER

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