Discrete Mathematics
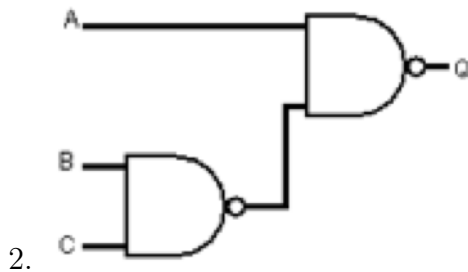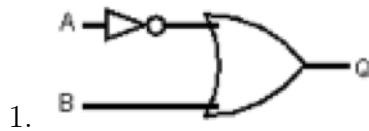
Tutorial sheet

Boolean Algebra
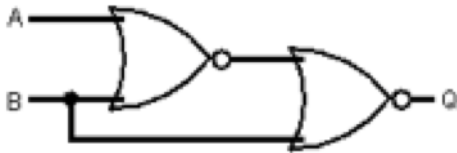
Question 1.

What is the output for each of the following logic circuits:

1.

2.

Question 2.

Write down the truth table for the output Q of the following circuit.

Question 3.

Simplify each Boolean expression to one of the following expressions: $0, 1, A, B, AB, A+B, \overline{AB}, \overline{A} + \overline{B}, \overline{A}B$ and $A\overline{B}$

1. $\overline{\overline{A} + \overline{B}}$

2. $A(A + \overline{A}) + B$

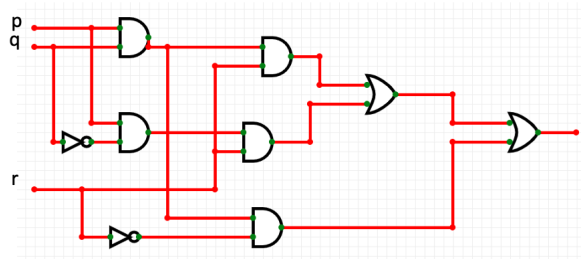3. $(A + B)(\overline{A} + B)\overline{B}$

Question 4.

1. Use the laws of boolean algebra to simplify the boolean expression:
   $a + \overline{a}b = a + b$.

2. Use the truth table prove that $a + \overline{a}b = a + b$.

3. Use he results from 1 and 2 to find a simplified circuit for the following logic circuit:
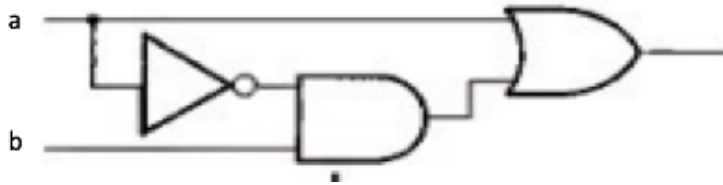
Question 5.

1. What is the output of the following logical circuit:

2. Simplify the output form the circuit above and find a simpler circuit which has the same effect.

Question 6.
Use the truth table prove De Morgan's laws: $\overline{ab} = \bar{a} + \bar{b}$ and $\overline{a + b} = \bar{a}.\bar{b}$

Question 7.
Use the laws of boolean algebra to simplify the boolean expression:

$$\overline{ab}(\bar{a} + b)(\bar{b} + b)$$

Question 8.
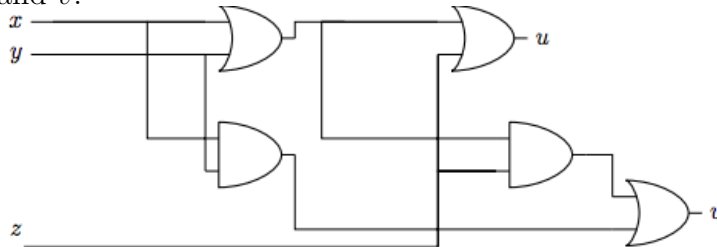Use the laws of boolean algebra to simplify the boolean expression:

$$\bar{a}(a + b) + (b + aa)(a + \bar{b})$$

Question 9.
Prove that in a boolean algebra $a^2 = a$. You are required to explain your answer by making a reference to a boolean algebra axioms (laws).
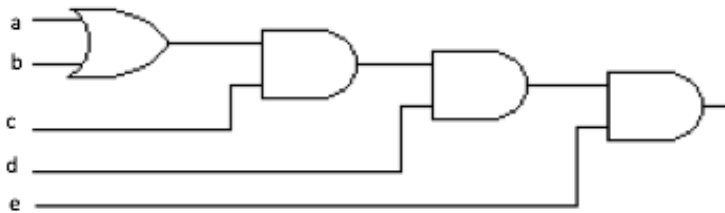
Question 10.

The following diagram shows a circuit with three inputs and two outputs, u and v.

1. List the logic gates used in this circuit.

2. Describe each output u and v as a Boolean expression in terms of x, y and z.

Question 11.

Derive the Boolean expression for the following logic circuit shown below

Question 12.

1. Write down a boolean expression for the following input/output behaviour.

x	y	z	u
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

2. Construct the corresponding circuit of the above expression using not-gates, and-gates and or-gates only.

Question 13.

Given the following boolean expression $\overline{\overline{(x+y)} + \bar{z}}$.

1. Construct its corresponding circuit.

2. Use DeMorgan's laws to find a simpler form for this expression

3. Construct the circuit the simplified expression.

Question 14.

Simplifying the following boolean expression using Karnaugh Map

$$\bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + ab\bar{x}$$

Question 15.

Given the following boolean function

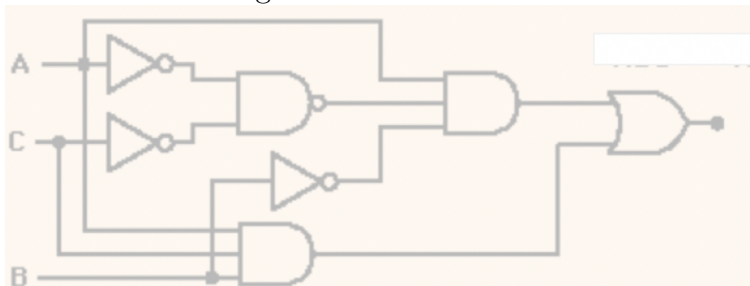$$f(a, b, c, d) = \bar{a}\bar{b}cd + \bar{a}bcd + abcd + a\bar{b}cd + ab\bar{c}\bar{d} + ab\bar{c}d + abc\bar{c}$$

1. Fill in the missing value in the following Karnaugh map of $f(a, b, c)$:

ab \ cd	00	01	11	10
00				
01				
11				
10				

2. Use K-map in (1) to find the minimum sum of products of $f(a, b, c)$.

Question 16.

Given the following circuit:

1. Find the output of this circuit.

2. Use the laws of algebra to give a simpler expression for this output.

3. Use the result in 2 to draw a simpler circuit equivalent circuit.

Question 17.
A set of three sensors in a factory detects whether the pollution level it is outputting from an incinerator exceeds the safety limit. In which case the incinerator is shut down. An alarm A goes off if at least two the three sensors s_1, s_2 and s_3 detect a pollution level above the limit. Draw a logic circuit for the system showing the inputs s_1, s_2 and s_3 and the output A.

End of questions