1) Given three sets A, B and C, prove that: $|A \cup B \cup C|=|A|+|B|+|C|-|A \cap B|-|A \cap C|-|B \cap C|+|A \cap B \cap C|$

When we find cardinality, we count values only once, and do not count duplicates, therefore we need to count each part only once. We can use principle of the inclusion-exclusion, but let us verify this using Venn diagram:

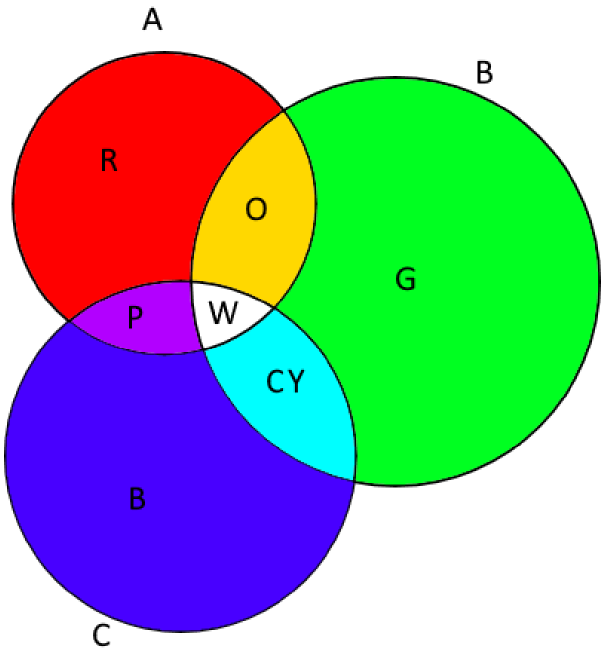

Figure 1 – Venn diagram

$|A \cup B \cup C|=|Red|+|Green|+|Blue|+|Orange|+|Purple|+|Cyan|+|White|$. Let us establish each part (to shorten the writing we will use first capital letters):

$\|R\|=\|A\|-\|O\|-\|P\|-\|W\|$	$\|O\|=\|A \cap B\|-\|W\|$
$\|G\|=\|B\|-\|O\|-\|CY\|-\|W\|$	$\|P\|=\|A \cap C\|-\|W\|$
$\|B\|=\|C\|-\|P\|-\|CY\|-\|W\|$	$\|CY\|=\|B \cap C\|-\|W\|$
	$\|W\|=\|A \cap B \cap C\|$

Then, $|A \cup B \cup C|=|A| \underline{- |O| - |P| - |W|} + |B| - |O| \underline{- |CY|} - |W| + |C| - |P| - |CY| -|W| + + \underline{|O| + |P| + |CY| + |W|} =$

$= |A| + |B| + |C| - |O| - |P| - |CY| - |W| - |W| =$

$= |A| + |B| + |C| - |A \cap B| \underline{+ |W|} - |A \cap C| \underline{+ |W|} - |B \cap C| + |W| \underline{- |W| - |W|} =$

$= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |W| =$

$= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

2) Let A and B two subsets of the universal set U= {x: x ϵ Z and 0 $\leq$ x < 20}. A is the set of even numbers in U, where B is the set of odd numbers in U.

Use the listing method to list the elements of the following sets: $A \cap \overline{B}$, $\overline{A \cap B}$, $\overline{A \cup B}$, $\overline{A \oplus B}$

First, let us define subsets A and B using listing method:

A = {0, 2, 4, 6, 8, 10, 12, 14, 16, 18}, and B = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}.

We can also state that subsets A and B are a partition of the universal set U ($A \cup B = U$ and $A \cap B = \emptyset$), and they are complementing sets (i.e. $\overline{A} = B$ and vice versa), therefore:

2.1) $A \cap \overline{B} = A \cap A = A = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$;

2.2) $\overline{A \cap B} = \overline{A} \cup \overline{B} = B \cup A = U = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19\}$;

Alternatively, we can show that: $\overline{A \cap B} = \overline{\emptyset} = U = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19\}$;

2.3) $\overline{A \cup B} = \overline{A} \cap \overline{B} = B \cap A = \emptyset$;

Alternatively, we can show that: $\overline{A \cup B} = \overline{U} = \emptyset$;

2.4) $\overline{A \oplus B} = \overline{(A \cup B)\text{-}(A \cap B)} = \overline{U\text{-}\emptyset} = \overline{U} = \emptyset$;

1) Given three sets A, B and C, prove that:
$|A \cup B \cup C| = |A|+|B|+|C|-|A \cap B|-|A \cap C|-|B \cap C|+|A \cap B \cap C|$

When we find cardinality, we count values only once, and do not count duplicates, therefore we need to count each part only once. We can use principle of the inclusion-exclusion, let substitute ($A \cup B$) for D:

1.1) We use the inclusion-exclusion principle for the $D \cup C$:

$$|A \cup B \cup C| = |D \cup C| = |D| + |C| - |D \cap C| \quad (1)$$

1.2) We use the inclusion-exclusion principle for the $A \cup B$:

$$|D| = |A \cup B| = |A| + |B| - |A \cap B| \quad (2)$$

1.3) We use the inclusion-exclusion principle for the $(A \cap C) \cup (B \cap C)$:

$$\begin{aligned} |D \cap C| &= |(A \cup B) \cap C| = |A \cap C| \cup |B \cap C| \\ &= |A \cap C| + |B \cap C| - |A \cap C \cap B \cap C| \\ &= |A \cap C| + |B \cap C| - |A \cap B \cap C| \quad (3) \end{aligned}$$

1.4) Now we use (2) and (3) and put them into (1):

$$\begin{aligned} &|A \cup B \cup C| = |D \cup C| = |D| + |C| - |D \cap C| = \\ &= |A| + |B| - |A \cap B| + |C| - |A \cap C| - |B \cap C| + |A \cap B \cap C| = \\ &= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|. \end{aligned}$$