1. Assume that we built a linear regression model with $n = 22$ observations and $p = 5$ predictors. Determine the minimum value of R^2 for which at least one of the predictors is statistically significant when $\alpha = 0.01$.

F-test

$$F = \frac{(TSS - RSS)/P}{RSS/(n-P-1)} = \frac{(TSS - RSS)/5}{RSS/(22-5-1)} \geqslant 4.4374$$

$$F_{P, n-P-1, \alpha} = F_{5, 22-5-1, 0.01} = 4.4374$$

using table

$$\frac{(TSS - RSS)/5}{RSS/16} \geqslant 4.4374$$

$$\frac{16(TSS - RSS)}{5RSS} \geqslant 4.4374 \longrightarrow \frac{TSS - RSS}{RSS} \geqslant 1.3866$$

$$\boxed{R^2 = 1 - \frac{RSS}{TSS}}$$

$$\frac{TSS}{RSS} - 1 \geqslant 1.3866 \longrightarrow$$

$$\frac{1}{1-R^2} - 1 \geqslant 1.3866 \longrightarrow \frac{1}{1-R^2} \geqslant 2.3866 \longrightarrow$$

$$\longrightarrow -R^2 + 1 \leqslant \frac{1}{2.3866} \longrightarrow R^2 \geqslant 1 - \frac{1}{2.3866}$$

$$\boxed{R^2 \geqslant 0.581}$$

2